Complex static potential in lattice QCD

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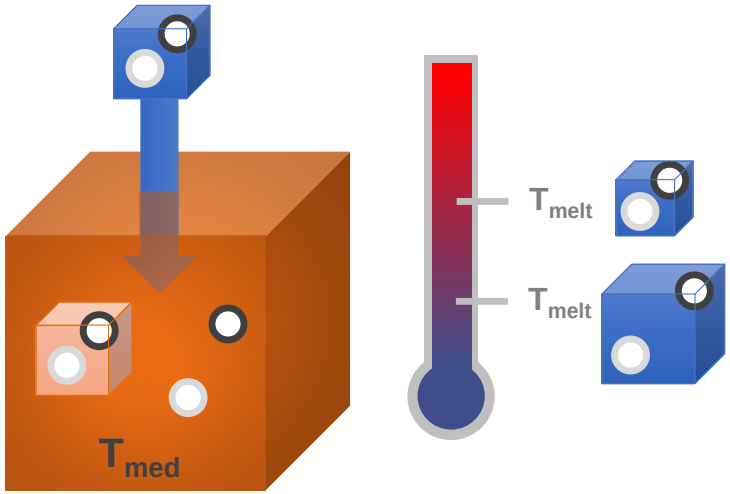
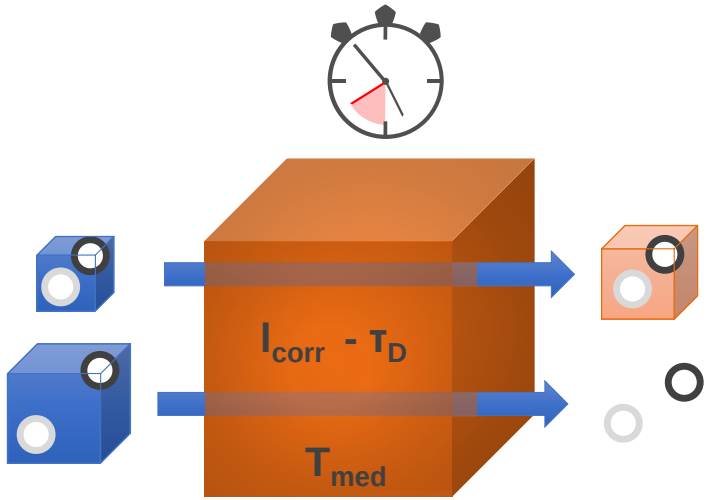
Outline

Quarkonium suppression

legacy static picture



modern dynamical picture

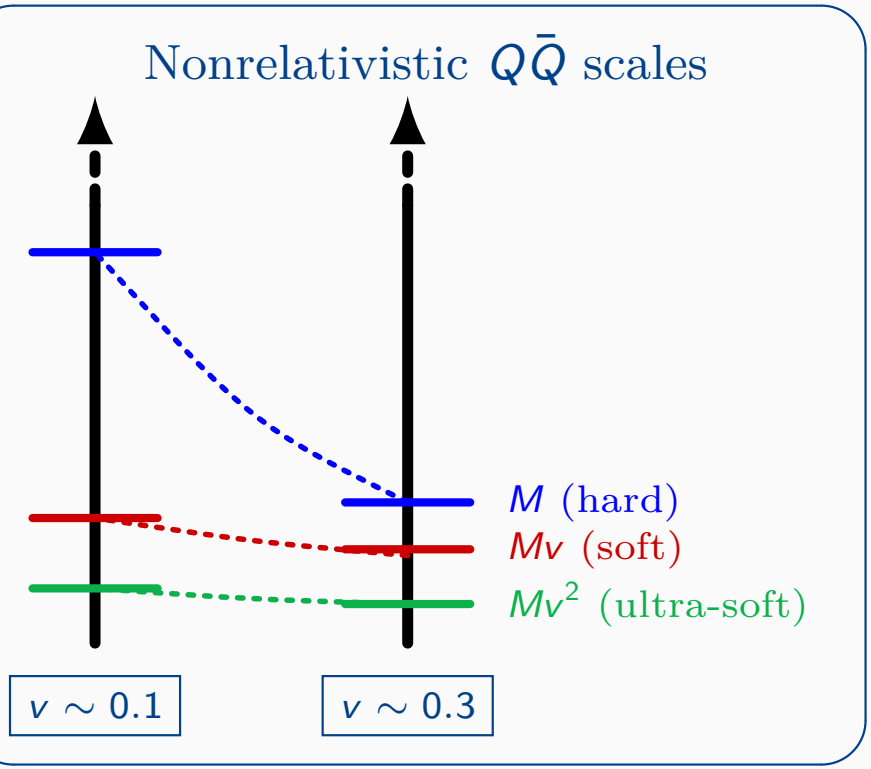


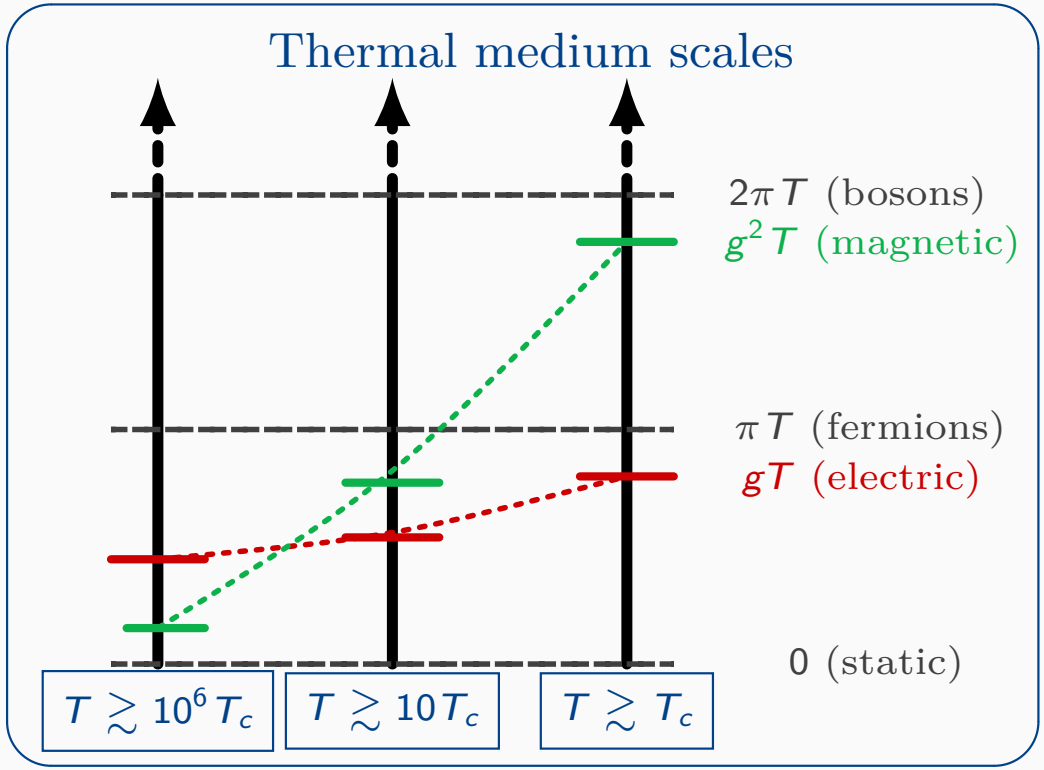
source: Rothkopf, Phys.Rept. 858 (2020) 1-117

- Quarkonium suppression as a characteristic feature of QGP
- Sequential suppression[1] suggested as indicator of temperature
- Static picture applies only for very long time scales $t \gtrsim 1/T$
- Quarkonium is governed by time scale $t \sim 1/Mv^2 \sim r/\alpha_s$

[1] Matsui, Satz, Phys.Lett.B 178 (1986) 416-422

Scales governing in-medium quarkonium





- Separation of nonrelativistic scales permits EFT (NRQCD, pNRQCD)
- Static $Q\bar{Q}$ energy E and potential V_s agree only for $Mv^2 \sim \alpha_s/r \ll \Lambda_{\mathrm{QCD}}$
- Separation of thermal scales: weak-coupling picture holds for $T \simeq \# T_c$?
- Intertwined thermal and nonrelativistic scales permit various hierarchies, with or without substantial thermal modification of the quarkonium

Static energy at zero temperature

- Real-time Wilson loop or Wilson line correlator obeys the EOM

$$W(r,T,t) = \int_{-\infty}^{+\infty} d\omega\ \rho(r,T,\omega) e^{-i\omega t},$$

$$i\partial_t W(r,T,t) = \Phi(r,T,t) W(r,T,t),$$

  where $\lim_{t\to\infty} \Phi(r,T,t) = E(r,T)$ if potential picture is applicable

- Infer ground state $E(r)$ (*static energy*) from large time limit or in terms of the Fourier transform of the spectral function

$$E(r) = \lim_{t\to\infty} i\partial_t \ln W(r,T,t) = \lim_{t\to\infty} \frac{\int d\omega\ \omega\ e^{-i\omega t}\rho(r,\omega)}{\int d\omega\ e^{-i\omega t}\rho(r,\omega)}$$

- $\rho(T=0,\omega)$: sum of separated delta peaks + high ω continuum
- Peak positions in $\rho(T=0,\omega)$ are operator-independent
- Note: operators do not mix with pair of static D mesons

Static energy at finite temperature

- $\rho(r, T > 0, \omega)$: sum of smeared peaks (tails!) + high ω continuum
- Assuming existence of a time independent, complex static potential infer that in the vicinity of the lowest peak $\omega \simeq \mathrm{Re}\,[E]$

$$\rho(T > 0, \omega) = \frac{1}{\pi} e^{\mathrm{Im}[\sigma_\infty]} \frac{|\mathrm{Im}[E]| \cos[\mathrm{Re}[\sigma_\infty]] - (\mathrm{Re}[E] - \omega) \sin[\mathrm{Re}[\sigma_\infty]]}{\mathrm{Im}[E]^2 + (\mathrm{Re}[E] - \omega)^2}$$
$$+ c_0 + c_1 t_{Q\bar{Q}} (\mathrm{Re}[E] - \omega) + c_2 t_{Q\bar{Q}}^2 (\mathrm{Re}[E] - \omega)^2 + \cdots$$

where E, σ_∞, $t_{Q\bar{Q}}$, c_0, c_1, c_2 are functions[2] of r and T

- Peak shape in some narrow interval is a skewed Breit-Wigner
- Peak sitting on top of a nontrivial, underlying pedestal

- At small times dynamics of bound state formation dominant
- At large times the low ω tail dominates the correlator
- $E(T > 0)$ determines the correlator only at times $\sim 1/E(T > 0)$

$\Rightarrow$ $E(T > 0)$ can be defined only via spectral function $\rho(T > 0, \omega)$

[2] precise definition in Burnier, Rothkopf, Phys.Rev.D 87 (2013) 114019

Complex static energy in perturbation theory

- pNRQCD describes how quarkonium is modified by the medium
- For $\alpha_s/r \gg T \gg m_D$ non-potential, thermal corrections to E
- For $1/r \gg T \gg m_D$ thermal corrections to E scale with powers of rT and modify the potential, i.e.[3]

$$E(T,r) = -C_F \alpha_s(1/r) + \left[\left\{ \#_1 \frac{\Delta V}{T} + \#_2 \frac{m_D^2}{T^2} + \#_3 \frac{m_D^3}{T^3} \right\} + \mathrm{i} \left\{ \#_4 \frac{\Delta V^2}{T^2} + \#_5 \frac{m_D^2}{T^2} \right\} \right] g^2 r^2 T^3$$

- For $T \gg 1/r \sim m_D$: $\mathrm{Re}\,[E] = F_S$ @$\mathcal{O}(g^3)$ and $\mathrm{Im}\,[E] \sim g^2 T$

$$E(T,r) = -C_F \alpha_s(\nu) \left\{ \frac{e^{-m_D(\nu) r}}{r} + m_D(\nu) + \mathrm{i} T \phi(r m_D(\nu)) \right\}, \quad \phi(x) = 2 \int_0^\infty \frac{dz\ z}{(z^2+1)^2} \left\{ 1 - \frac{\sin(zx)}{zx} \right\}$$

  implies HTL-resummed propagators, $Q\bar{Q}$ at time scale $\sim 1/g^2 T$
- Scale separation $\mathrm{Im}\,[E] \sim g^2 T \ll m_D \sim gT$ needed to have a screened potential, otherwise vacuum-like $\mathrm{Re}\,[E]$ with dissipation

[3] coefficients $\#_i$, $i = 1, \ldots, 5$ given in Brambilla et al., Phys. Rev. D78, 014017 (2008)

Lattice setup

- We use tree-level Symanzik-improved gauge action with (2+1) quark flavors via highly improved staggered quark (HISQ) action
- Physical strange; light quarks at either $m_l = m_s/20$ or $m_s/5$ corresponding to $m_\pi = 160\,\text{MeV}$ or $320\,\text{MeV}$ in the continuum
- We use ${N_\sigma}^3 \times N_\tau$ lattices, temperature defined as $T = 1/aN_\tau$
- Observables are Wilson line correlators in Coulomb gauge or Wilson loops with HYP smeared spatial links

- High statistics ensembles with aspect ratio $N_\sigma/N_\tau = 4$ and lattice cutoff with $N_\tau = 16,\ 12,\ 10$; bare lattice coupling $\beta = 10/g_0^2$
- Temperature varied via $a(\beta)$: $120\,\text{MeV} \lesssim T \lesssim 2.2\,\text{GeV}$;
- Generated by HotQCD and TUMQCD for studies of equation of state[4][5], Polyakov loop correlators[6], and the extraction of $\alpha_s(M_Z)$[7]

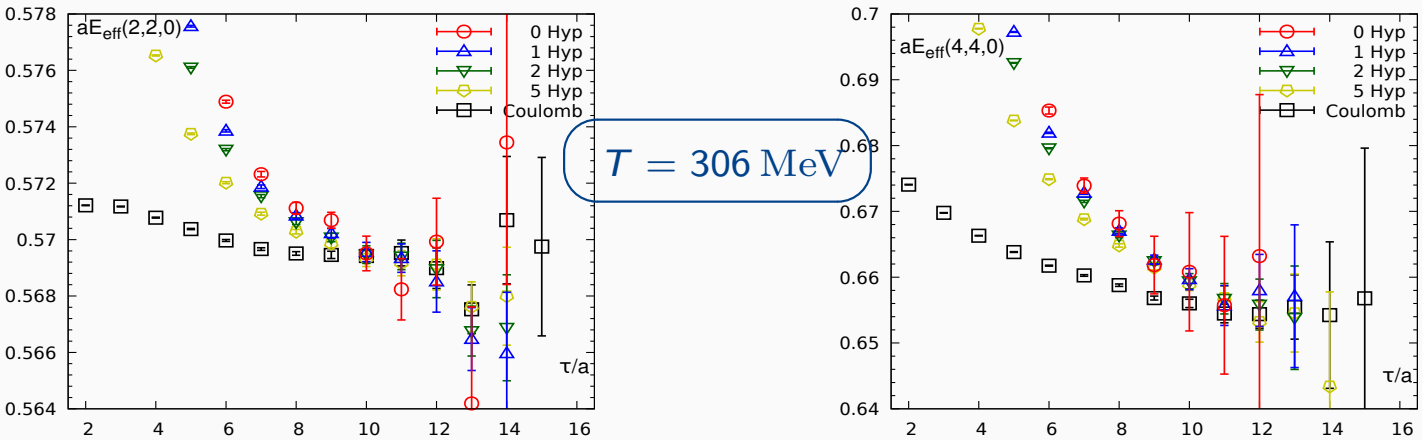
[4] HotQCD: Phys.Rev.D 90 (2014) 094503;
[5] Bazavov, Petreczky, JHW: Phys.Rev.D 97 (2018) 1, 014510
[6] TUMQCD: Phys.Rev.D 98 (2018) 5, 054511;
[7] TUMQCD: Phys.Rev.D 100 (2019) 11, 114511

Moments of correlators

- We define moments of the correlators to compare and understand key features of the lattice data and weak-coupling results

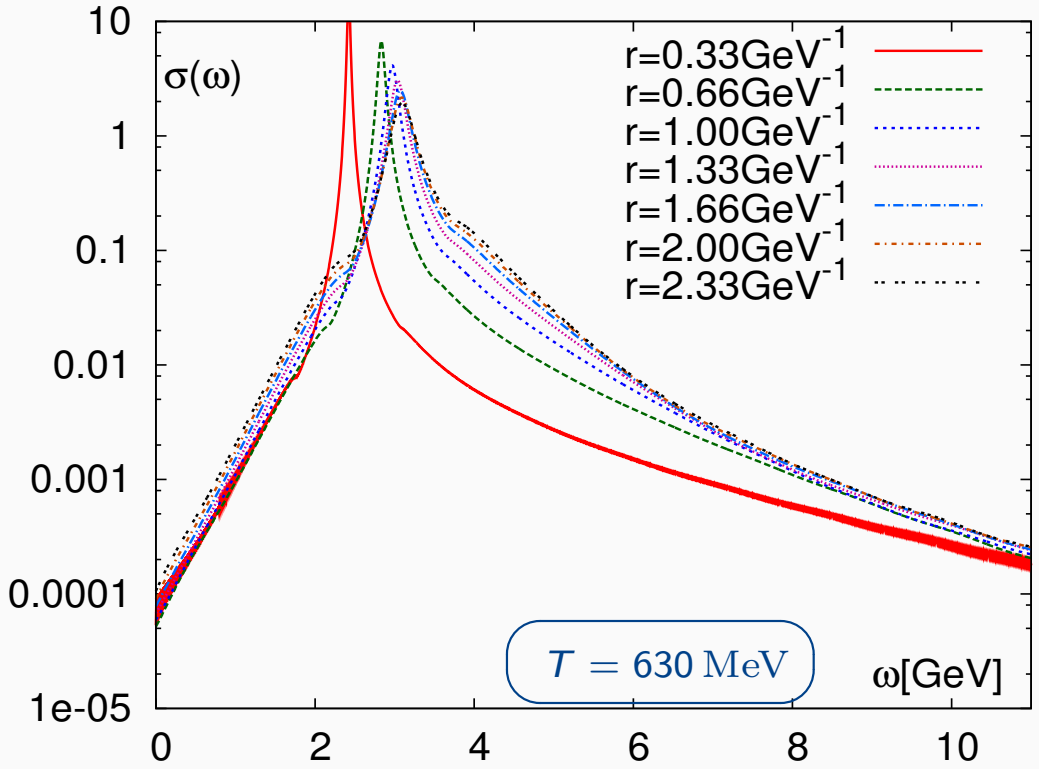
$$m_1(r, T, \tau) = -\partial_\tau \ln W(r, T, \tau) = \ln\left[W(r, T, \tau)/W(r, T, \tau + a)\right] + \ldots,$$
$$m_n(r, T, \tau) = \partial_\tau m_{n-1}(r, T, \tau), \quad n > 1$$

- For $1/\tau \sim$ peak: $m_1(r, T, \tau)$ is independent of the operator
- Elsewhere: $m_1(r, T, \tau)$ has T independent structure (excited states, high ω continuum, operator dependence, lattice cutoff, ...)



HTL spectral function as a baseline

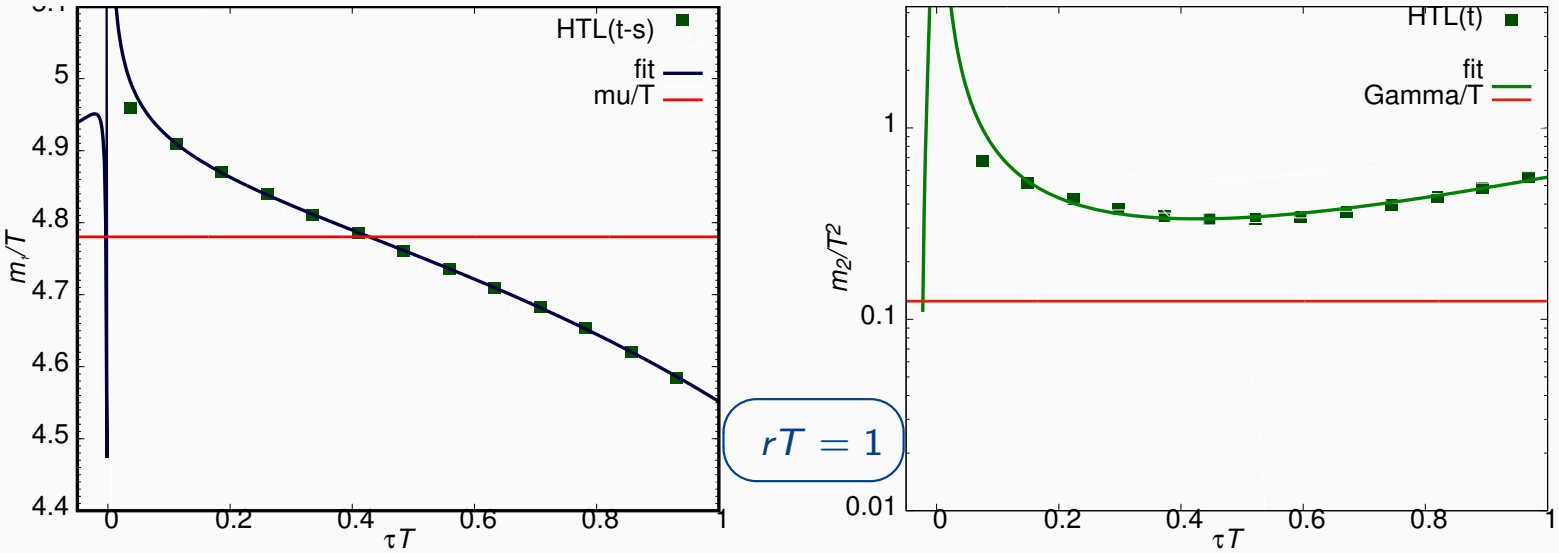
- In HTL the correlator, spectral function and potential are known



- At $T = 630\,\mathrm{MeV}$ ($2.33\,T_c$ in pure gauge theory):
  - Breit-Wigner type peak
  - Shoulder-like structure at high ω side gets more pronounced for larger r
  - Decays exponentially for both very low or very high ω

HTL baseline for the moments

- For HTL we construct a correlator with $N_\tau - 1$ points from ρ, obtain the moments, and compare to the known input



source:Petreczky, JHW, Nucl.Phys.A 967 (2017)

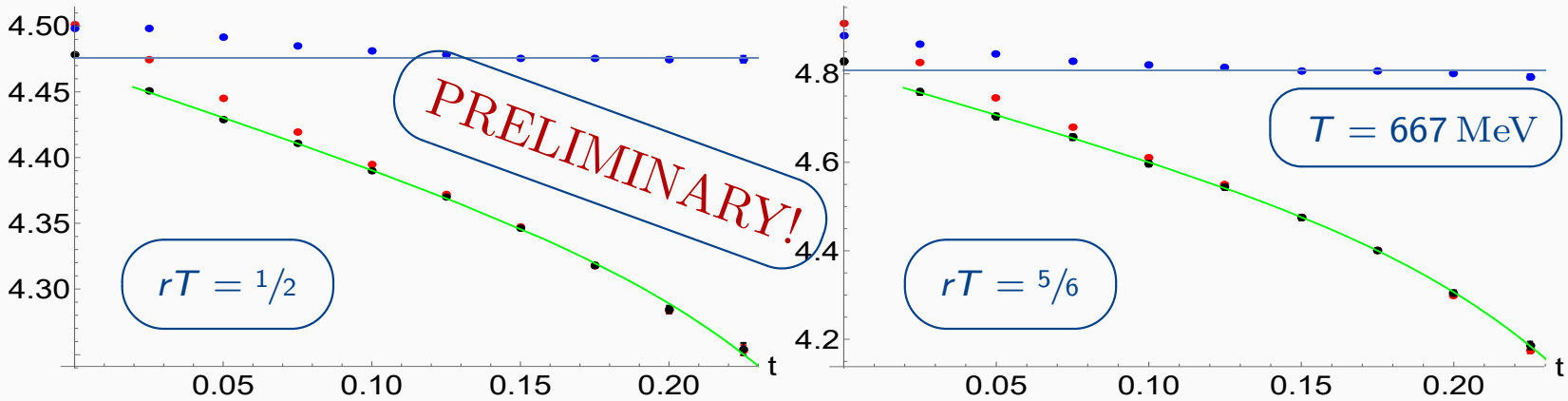
- Slope of the 1st moment indicates the width, curvature indicates higher moments (skewness, etc.)
- 2nd moment is much larger than the peak width, indicating the large contribution from the tails/shoulders/pedestal

IR/UV decompostion and subtracted correlators

- Approximate T independence of m_1 at small τ suggests an IR/UV decomposition as $\rho(T,\omega) \approx \rho_{\text{low}}(T,\omega) + \rho_{\text{high}}(\omega)$

$\Rightarrow$ determine UV part at $T = 0$ and subtract from $T > 0$ correlator

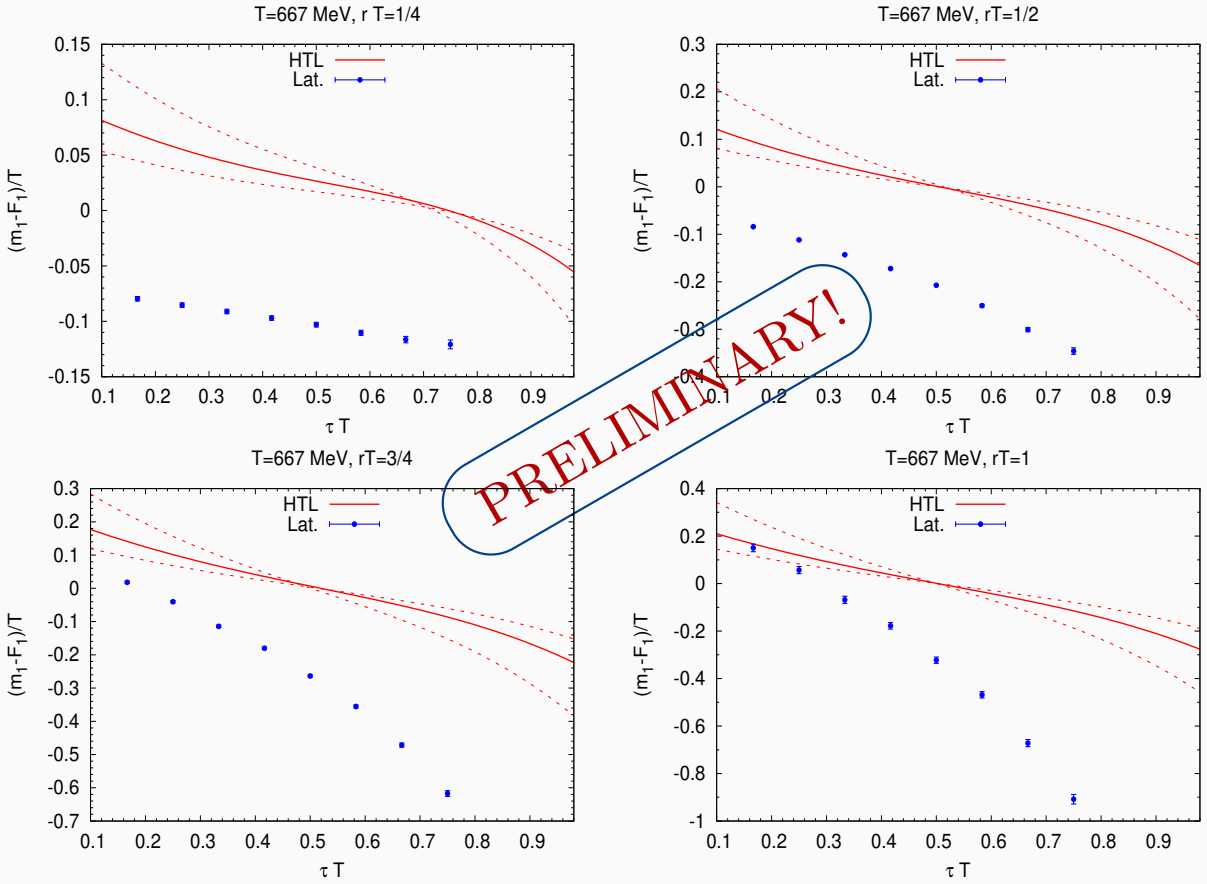
- Assume $\rho_{\text{low}}(T = 0,\omega) \propto \delta(\omega - E)$, the rest is taken as UV part
- Subtraction only possible if $T = 0$ lattices with same β available



- In-medium modification of m_1 is clearly visible
- m_1 of subtracted correlator can always be fitted with a second order polynomial — access to only first three moments on lattice
- At small τ the first two moments are dominant (after subtraction)

HTL versus lattice at $T = 667\,\mathrm{MeV}$

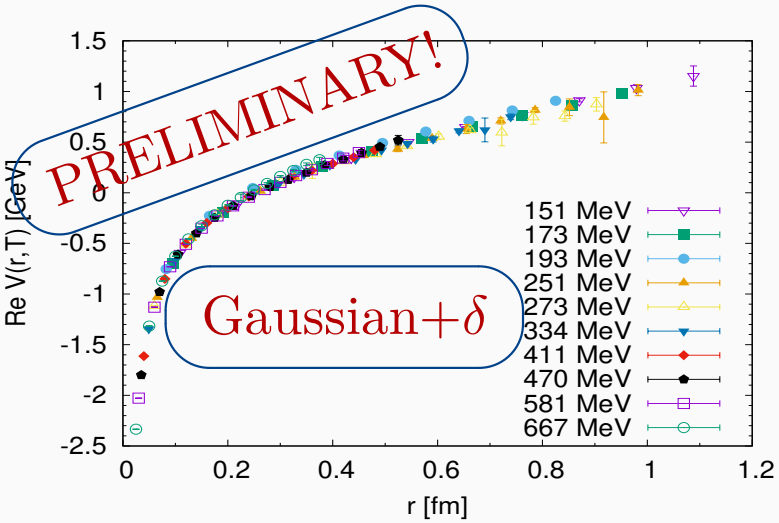
- Static energy is scheme dependent (lattice vs dim. reg.)
- Consider scheme-independent difference $m_1(r, T, \tau) - F_S(r, T)$
- Estimate HTL truncation error via scale variation $\nu = \{1, 2, 4\}\pi T$

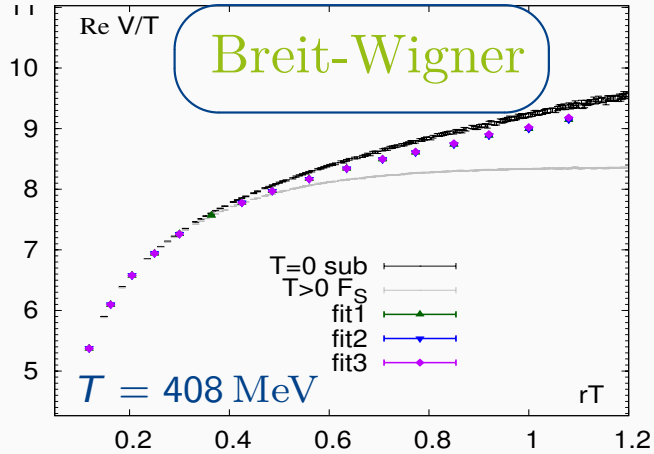


- Situation does not get much better even at $T \approx 2\,\mathrm{GeV}\ldots$

Real part of the potential

- Fit $m_1(r, T, \tau)$ of subtracted correlator for small enough τ with Gaussian peak + delta function (mimicking the low ω tail)





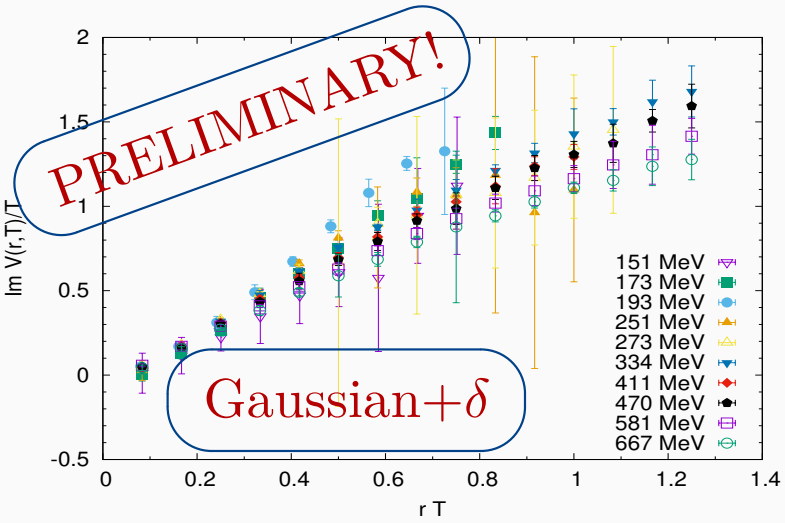
source:Petreczky, JHW, Nucl.Phys.A 967 (2017)

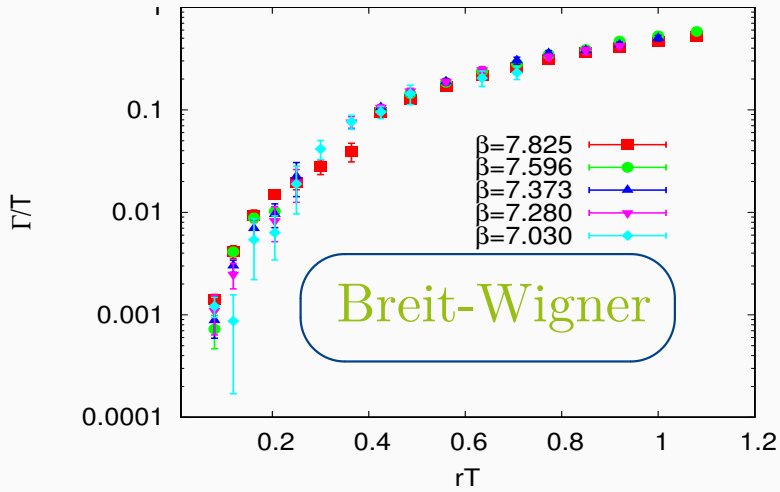
- Hardly any in-medium modification is visible in $\mathrm{Re}\,[E](r, T)$
- In particular: no evidence for screening could be identified
- Previous analysis[8] with (skewed) Breit-Wigner peak and low ω cutoff (no extra delta function) did not find screening either

[8] Petreczky, JHW, Nucl.Phys.A 967 (2017)

Imaginary part of the potential

- Fit $m_1(r, T, \tau)$ of subtracted correlator for small enough τ with Gaussian peak + delta function (mimicking the low ω tail)





source: Petreczky, JHW, Nucl.Phys.A 967 (2017)

- Temperature independent scaling of $\mathrm{Im}\,[E](r, T)/\tau$ as function of rT, reminiscent of HTL perturbation theory
- Previous analysis[9] with (skewed) Breit-Wigner peak and low ω cutoff (no extra delta function): smaller value of $\mathrm{Im}\,[E](r, T)$

[9] Petreczky, JHW, Nucl.Phys.A 967 (2017)

Lattice results viewed through the lens of HTL

- Recently, a new idea inspired by HTL was suggested by Bala & Datta[10] to extract the complex static energy from lattice
- Assuming approximately linear behavior at $\tau \sim 1/(2T)$ rewrite lattice correlator

$$W(r,T,\tau) \equiv e^{w(r,T,\tau)} \times W(r, 1/(2T)),$$
$$w(r,T,\tau) \simeq -(\tau - 1/(2T))\,\mathrm{Re}\,[E](r,T) + \ldots$$

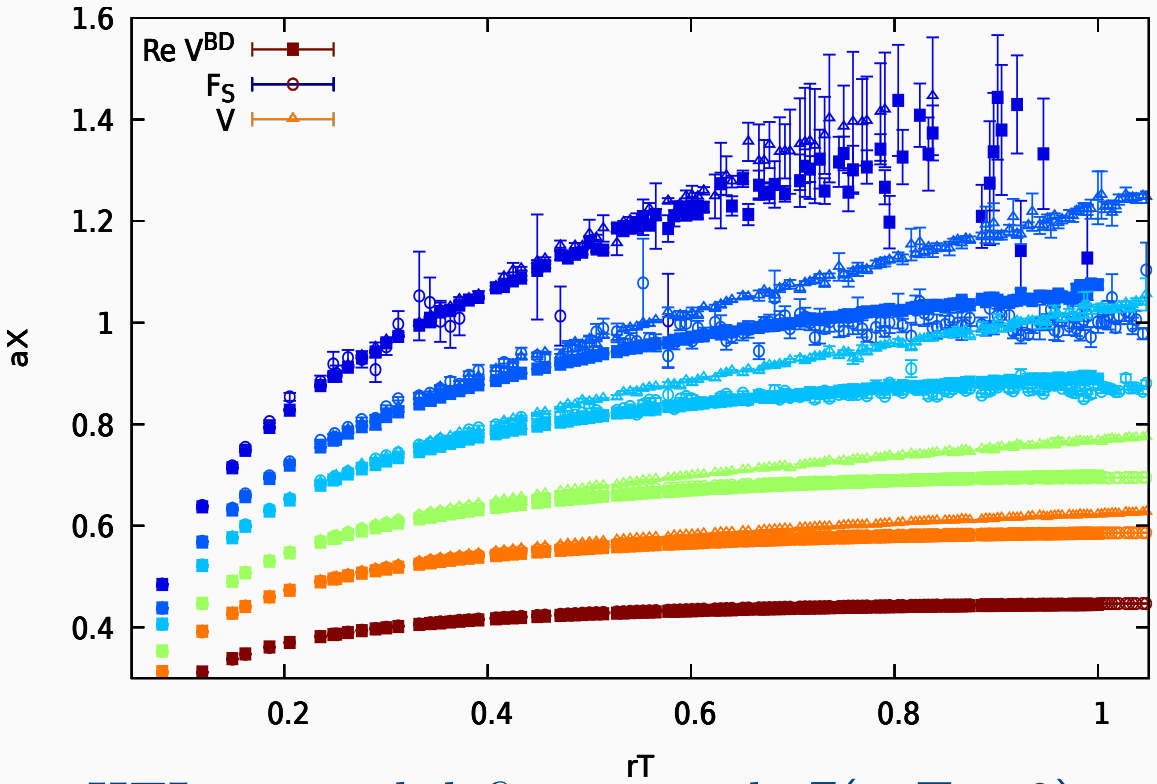
- Study symmetric and anti-symmetric combinations

$$W(r,T,\tau) \equiv W^a(r,T,\tau) \times W^p(r,T,\tau),$$
$$W^a(r,T,\tau) \equiv \sqrt{\frac{W(r,T,\tau)}{W(r,T,(1/T)-\tau)}},$$
$$W^p(r,T,\tau) \equiv \sqrt{W(r,T,\tau) \times W(r,T,(1/T)-\tau)}$$

- Then interpret $-\ln W^a(r,T,\tau)/(1/(2T)-\tau)$ as $\mathrm{Re}\,[E](r,T)$

[10] Bala, Datta, Phys. Rev. D 101, (2020) 034507

Real part through the lens of HTL



- Compare HTL inspired definition with $E(r, T = 0)$ and $F_S(r, T)$
- $-\ln W^a(r, T, \tau)/(1/(2T) - \tau)$ is very close to $F_S(r, T)$ over the full range from $T \approx 150\,\mathrm{MeV}$ to $2\,\mathrm{GeV}$ ($N_\tau = 12$), but differences are resolved with high enough accuracy in $SU(3)$ pure gauge theory[11]

[11] Bala, Datta, Phys. Rev. D 101, (2020) 034507

Imaginary part through the lens of HTL

- In HTL perturbation theory spectral decomposition[12] for

$$A(T,\tau) \equiv \ln W^p(T,\tau) = \int_{-\infty}^{+\infty} d\omega\, \sigma(T,\omega) \frac{e^{-\omega\tau} + e^{-\omega((1/T)-\tau)}}{2} + \text{const}$$

- $\sigma(T,\omega)$ has no peak structure, but is instead related to spectral function of longitudinal and transverse gluons in HTL approx.
- Interpret leading singularity of

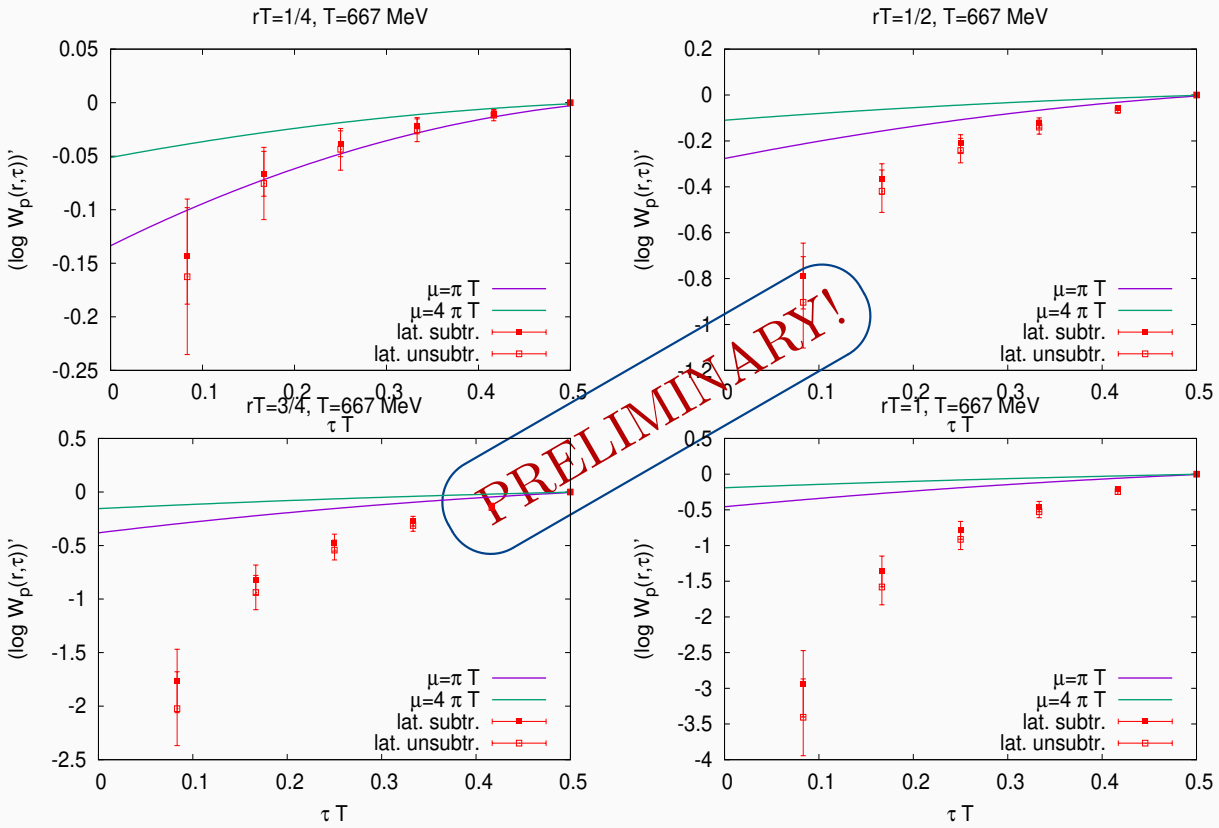
$$\sigma(T,\omega) = (1 + n_B(\omega)) \sum_{i=0}^{\infty} c_i \omega^{2i-1}$$

as imaginary part $\mathrm{Im}\,[E] \equiv \pi T c_0$

[12] Note: $\sigma(T,\omega)$ and $\rho(T,\omega)$ are distinct objects, not related in a simple fashion

Confronting the HTL perturbation theory with the HTL fit

- HTL perturbation theory is only consistent with this HTL inspired analysis of lattice data for very small r



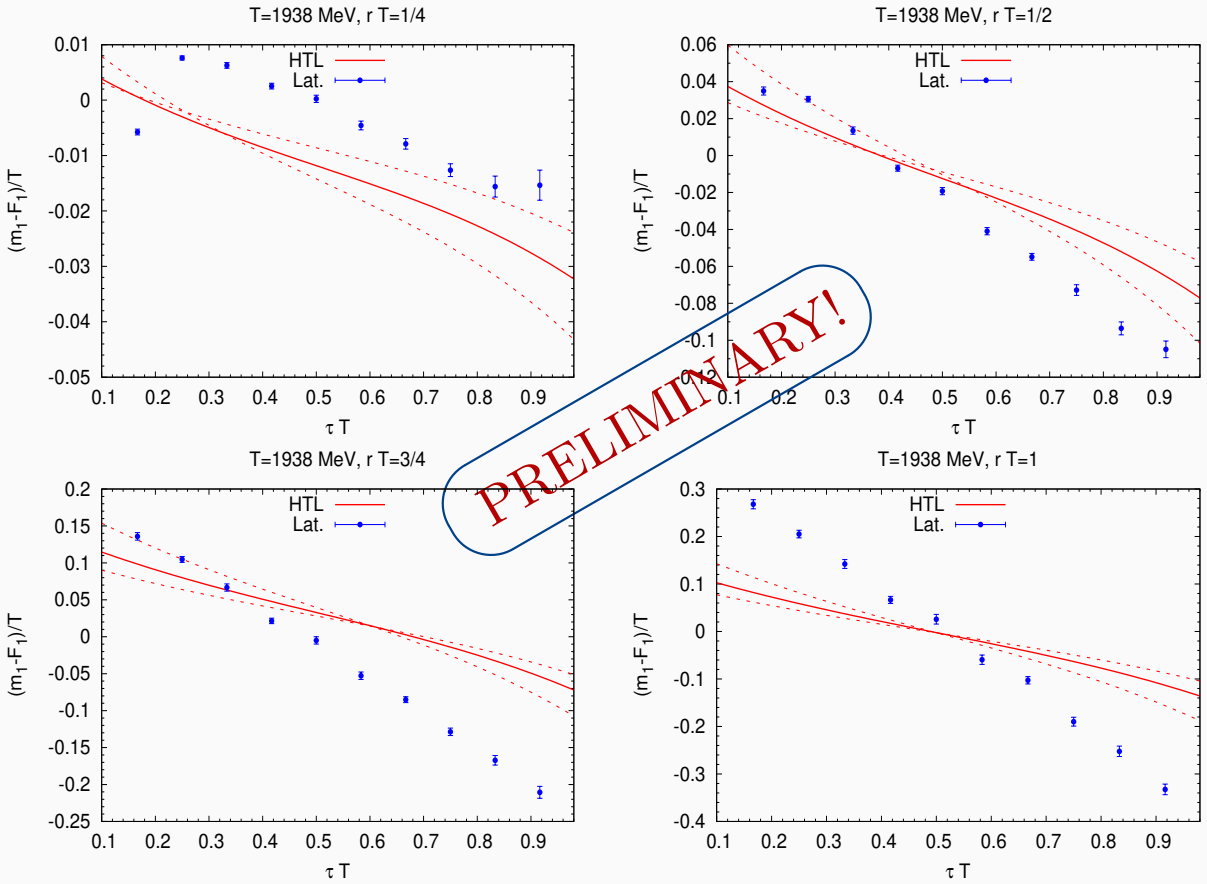
Open issues with the complex static energy on the lattice

- Robust subtraction of UV contribution feasible via fits of $T = 0$ data
- Reliable extraction of the first three moments of the static $Q\bar{Q}$ correlator (or spectral function) is feasible on the lattice with $N_\tau \geq 12$
- On the level of moments, HTL perturbation theory at LO is inconsistent with the lattice result for phenomenologically relevant temperatures, cannot be used naively to interpret the lattice results
- Lattice definition of potential only possible via spectral function; diverse toolbox for extracting its lowest feature from lattice data; similar answers when assuming Gaussian or Breit-Wigner peaks
- Current lattice data do not allow yet to reliably discriminate between a genuine width of a potential peak or contributions from the tails
- So far, no reliable prediction of the complex static energy from lattice, but understanding of issues and systematics is constantly improving

Current best guess: quarkonium dissociates at phenomenologically interesting temperatures before it can even begin to rearrange due medium effects.

HTL versus lattice at $T = 1938\,\mathrm{MeV}$

- Static energy is scheme dependent (lattice vs dim. reg.)
- Consider scheme-independent difference $m_1(r, T, \tau) - F_S(r, T)$
- Cannot subtract excited states at such high T (no vacuum result)
- Estimate HTL truncation error via scale variation $\nu = \{1, 2, 4\}\pi T$

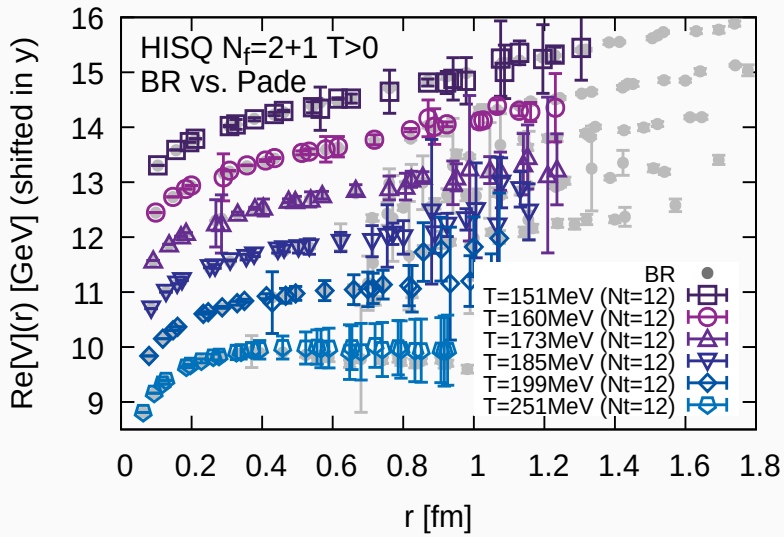


Extracting the lattice results for $\mathrm{Re}\,[E](r,T)$

- Reconstruction of spectral function on $\mathcal{O}(10^3)$ frequency bins from lattice correlator with $(O)(10^1)$ data is a classic ill-posed problem
- Proceed with two complementary approaches
  1. **Bayesian reconstruction (BR) method**

     $$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] = \exp[-L+\alpha S_{\mathrm{BR}}]$$

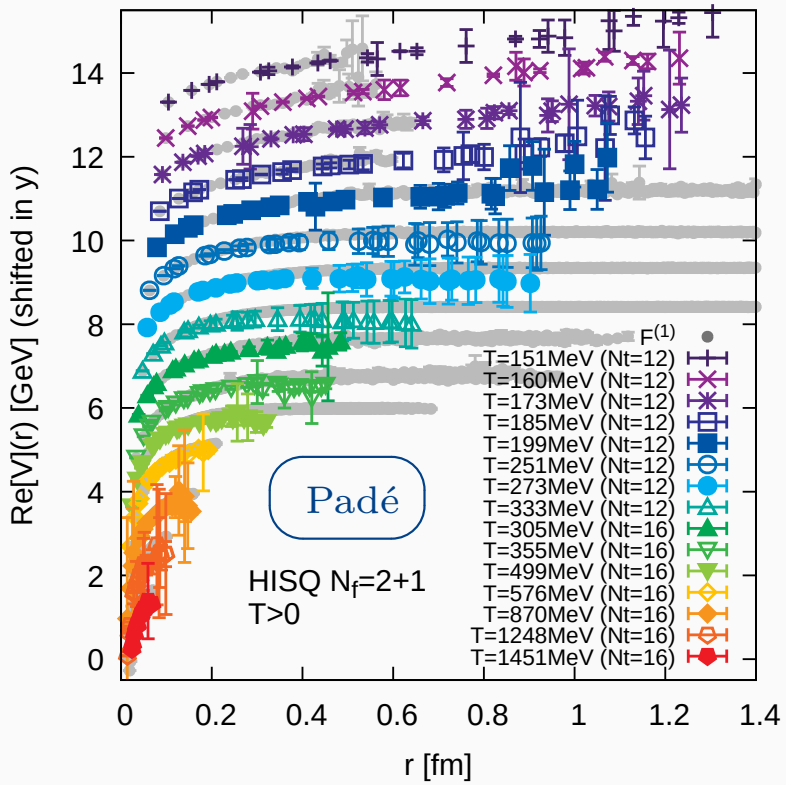
     not reliable at high temperatures due to ringing artifacts
  2. **Padé approximation with analytic continuation**
     can only obtain reliables estimate of the peak position,
     but fails to reproduce width or spectral function away from the peak

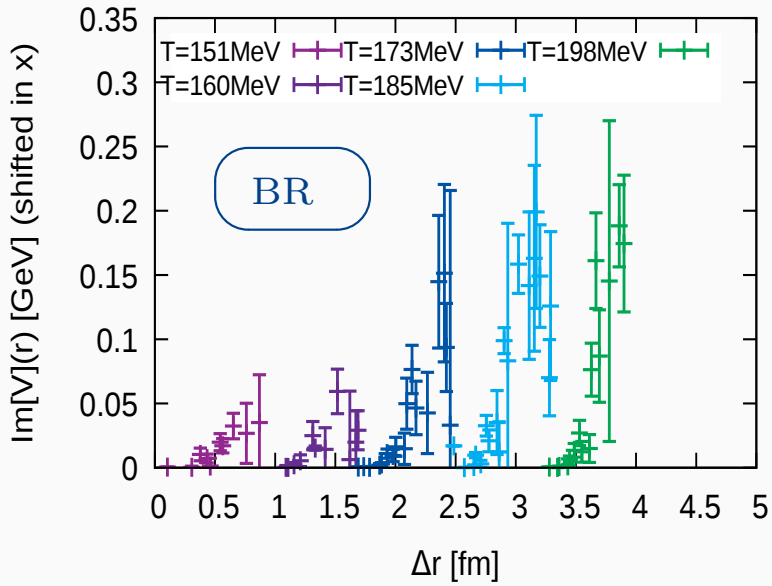


- BR method: vacuum-like, linear rise at $T \lesssim 200\,\mathrm{MeV}$, with a not so small width
- Padé approx.: free energy-like, screened at $T \gtrsim 180\,\mathrm{MeV}$, with a smaller width at most

Preliminary lattice results for $\mathrm{Re}\,[E](r,T)$ and $\mathrm{Im}\,[E](r,T)$

- Use Padé approx. for $\mathrm{Re}\,[E]$, BR method for $\mathrm{Im}\,[E]$ (low T)





source:
Petreczky, Rothkopf, JHW, Nucl.Phys.A 982 (2019)

- Results are inconclusive, compatible with HTL inspired analysis

Bayesian reconstruction method

- Posterior probability $P[\rho|D,I]$ defined in terms of data D and prior probability $P[\rho|I]$; BR method still subject to refinements

$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] = \exp[-L+\alpha S_{\rm BR}],$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N_d}(D_i - D_i^\rho)C_{ij}^{-1}(D_j - D_j^\rho),$$

$$S_{\rm BR} = \int d\omega\Big(1 - \frac{\rho(\omega)}{m(\omega)} + \log\Big[\frac{\rho(\omega)}{m(\omega)}\Big]\Big)$$

- S_{BR} regulates flat directions in parameter space
- Default model $m(\omega)$ usually taken as flat ($m(\omega) = 1$)
- Hyperparameter α may be marginalized over (or not ...)

$$P[\rho|D,I,m] \propto P[D|\rho,I]\int_0^\infty d\alpha P[\rho|m,\alpha]P[\alpha]$$

- BR method may have ringing artifacts $\Rightarrow$ avoid via *low-gain* BR

Padé approximation

- Fourier transform correlator

$$W_{||}(r_k, \tilde{\omega}_n) = \sum_{j=0}^{N_\tau - 1} e^{2\pi i \frac{jn}{N_\tau}} W_{||}(r_k, \tau_j)$$

- Interpret via eigenvalues ω_n of discretized momentum operator instead of naive frequencies $\tilde{\omega}_n$ (reduces discretization errors)

$$\omega_n = \frac{2}{a} \sin\left(\frac{\pi n}{2N_\tau}\right)$$

- Fit with Padé approximant in terms of continued fractions

$$C_{N_\tau}(i\omega) = \frac{W(r_k, \omega_0)}{1+} \frac{a_0(\omega - \omega_0)}{1+} \frac{a_1(\omega - \omega_1)}{1+} \cdots \frac{a_{N_\tau - 1}(\omega - \omega_{N_\tau - 1})}{1+}$$

- Continue analytically $C_{N_\tau}(i\omega) \to C_{N_\tau}(\omega)$, obtain spectral function

$$\rho(\omega) \approx -\frac{1}{\pi} \mathrm{Im}[C_{N_\tau}(\omega)]$$