Bottomonium suppression in open quantum systems

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N. Brambilla, M.-A. Escobedo, M.S., A. Vairo,
P. Vander Griend, and J.H. Weber, arXiv:2012.01240 and forthcoming

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KENT STATE UNIVERSITY
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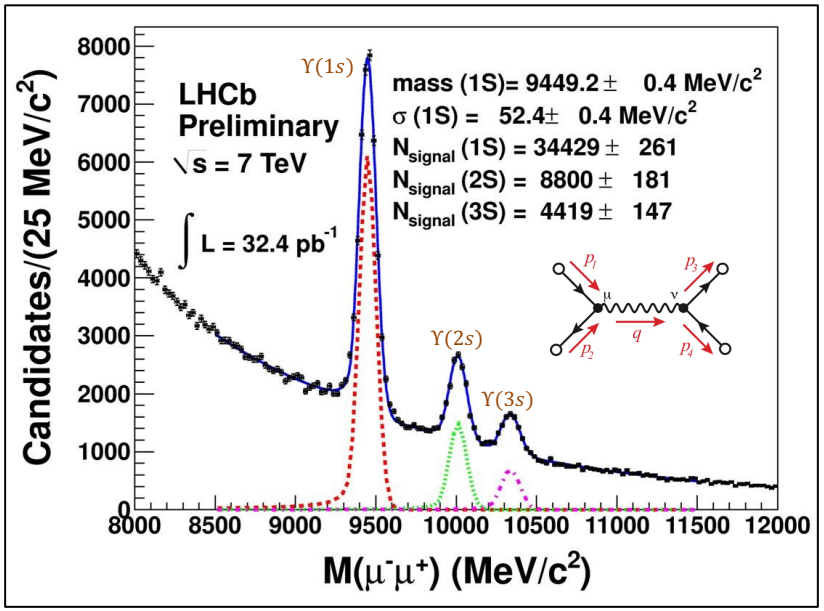
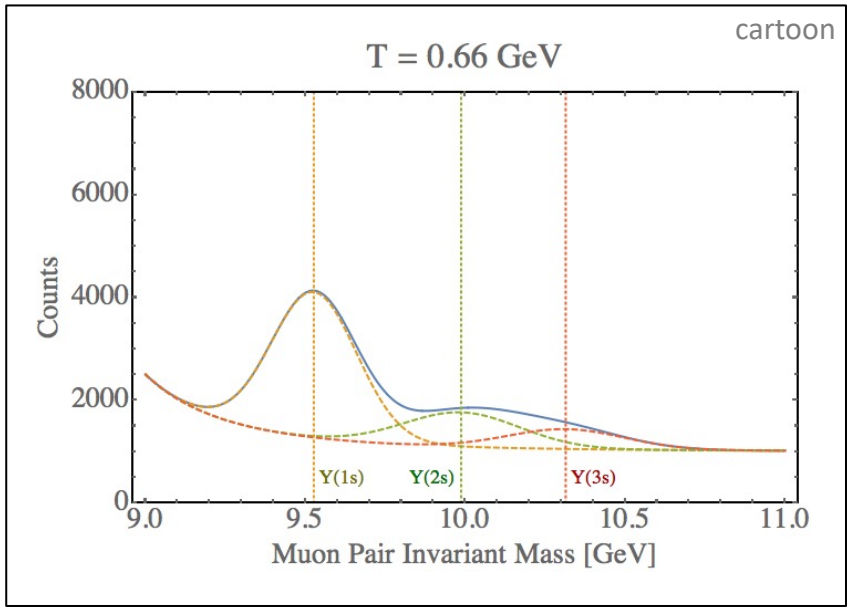
Heavy Quarkonium Suppression

- In a high temperature quark-gluon plasma we expect **weaker color binding** (Debye screening)

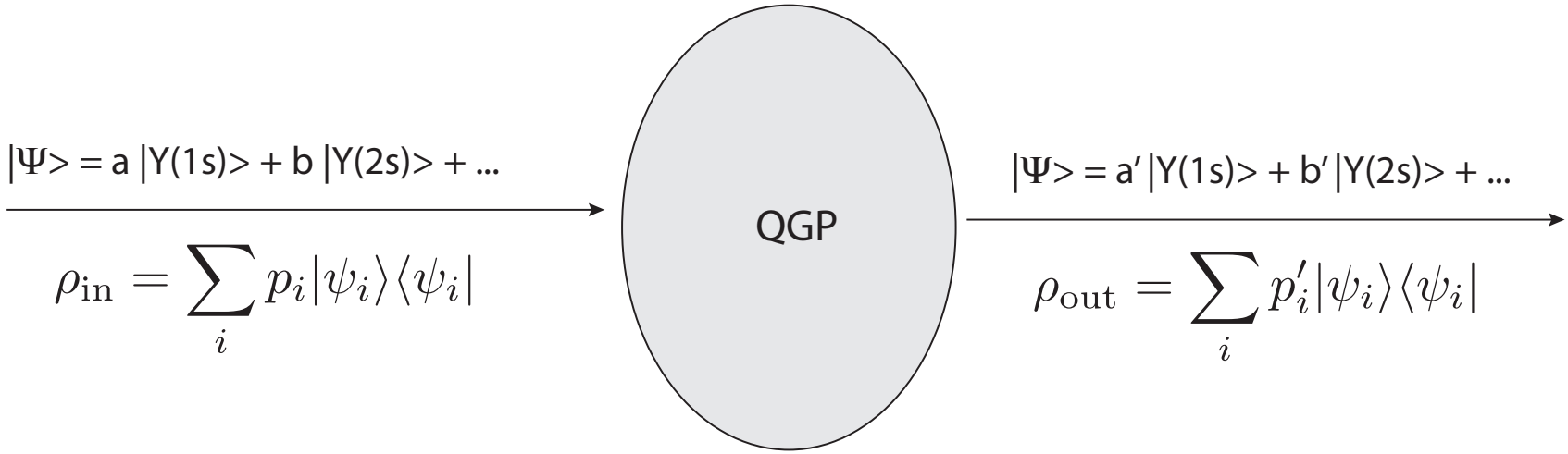
  E. V. Shuryak, Phys. Rept. 61, 71–158 (1980)
  T. Matsui, and H. Satz, Phys. Lett. B178, 416 (1986)
  F. Karsch, M. T. Mehr, and H. Satz, Z. Phys. C37, 617 (1988)

- Also, high energy plasma particles which slam into the bound states cause them to have shorter lifetimes → **larger spectral widths** (singlet/octet and Landau damping)



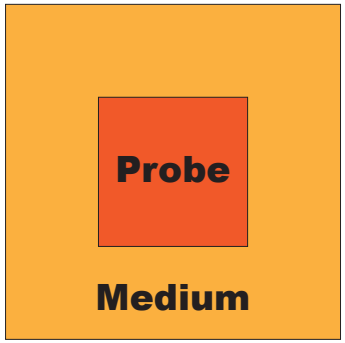


Conceptual problem



- Bottomonium states have a large binding energy and are produced locally (hard processes) at early times in hard collisions (t < 1 fm/c).
- They then propagate through the plasma and interact with the medium.
- Bound states can break up and potentially re-form due to in-medium transitions induced by screening and, more importantly, gluon absorption and emission.

Open quantum system approach I



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm medium} + I_{\rm probe} \otimes H_{\rm medium} + H_{\rm int}$$

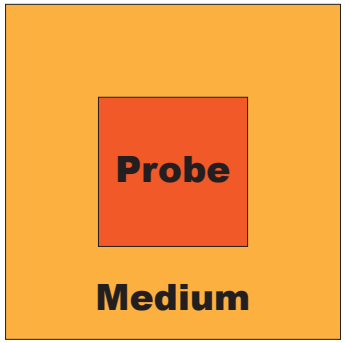
- Total density matrix

$$\rho_{\rm tot} = \sum_k \frac{1}{Z_{\rm tot}} e^{-E_k/T} |E_k\rangle\langle E_k| \longrightarrow \frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

- Reduced density matrix

$$\rho_{\rm probe} = {\rm Tr}_{\rm medium}[\rho_{\rm tot}] \longrightarrow$$ Evolution equation?

Open quantum system approach II



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Separation of time scales
  - Medium relaxation time scale $\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/t_{\rm M}}$
  - Intrinsic probe time scale $t_P \sim \dfrac{1}{\omega_i - \omega_j}$
  - Probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}}$

Lindblad equation

$$\xrightarrow[t_{\rm rel}\,,\ t_{\rm P} \gg t_{\rm M}]{} \boxed{\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\}\right)}$$

G. Lindblad Commun. Math. Phys. 48 (1976) 119
V. Gorini, et.al. J. Math. Phys. 17 (1976) 821

Open quantum system approach III

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n \, \rho_{\rm probe} \, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

- H_{probe} is a Hermitian operator (includes singlet and octet states)
- C_n are the **collapse (or jump) operators** (connect different internal states)
- Partial and **total decay widths** are

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

- Can reorganize Lindblad equation by defining

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$

← **Non-Hermitian effective Hamiltonian**

$$\longrightarrow \quad \frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n \, \rho_{\rm probe} \, C_n^\dagger$$

Hamiltonian and collapse operators from pNRQCD

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

Six collapse operators cover

- singlet → octet,
- octet → singlet
- octet → octet

$$\gamma \equiv \frac{g^2}{6\,N_c}\,{\rm Im}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\kappa \equiv \frac{g^2}{6\,N_c}\,{\rm Re}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

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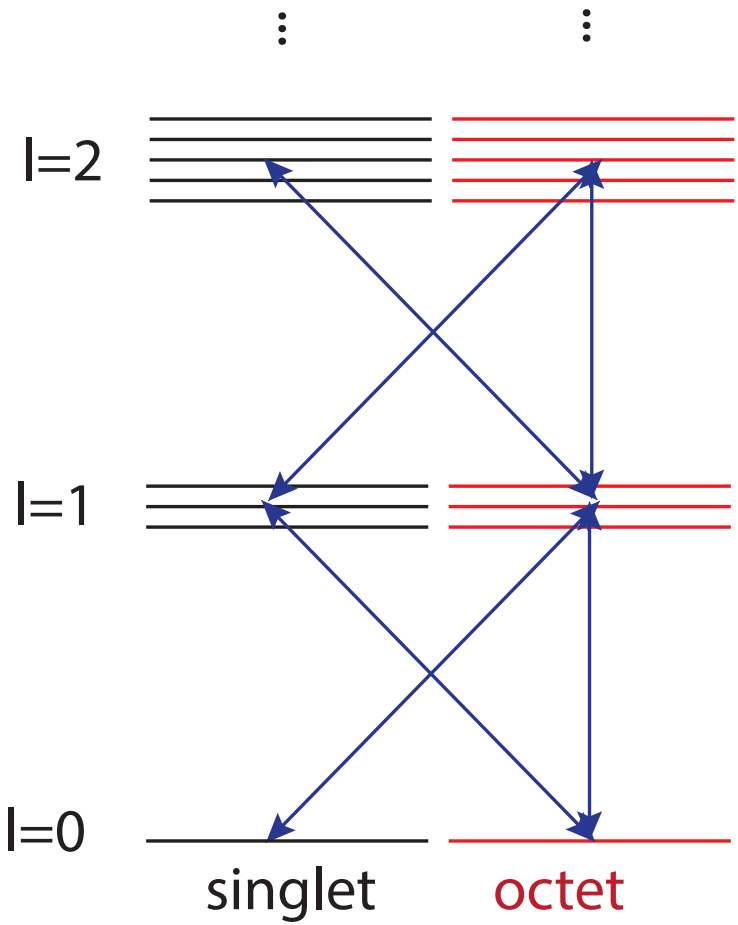

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Values of $\hat{\kappa}$ and $\hat{\gamma}$ used

- We used NLO fits to recent lattice measurements of the heavy quark transport coefficient $\hat{\kappa} \equiv \kappa/T^3$.
  - N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078

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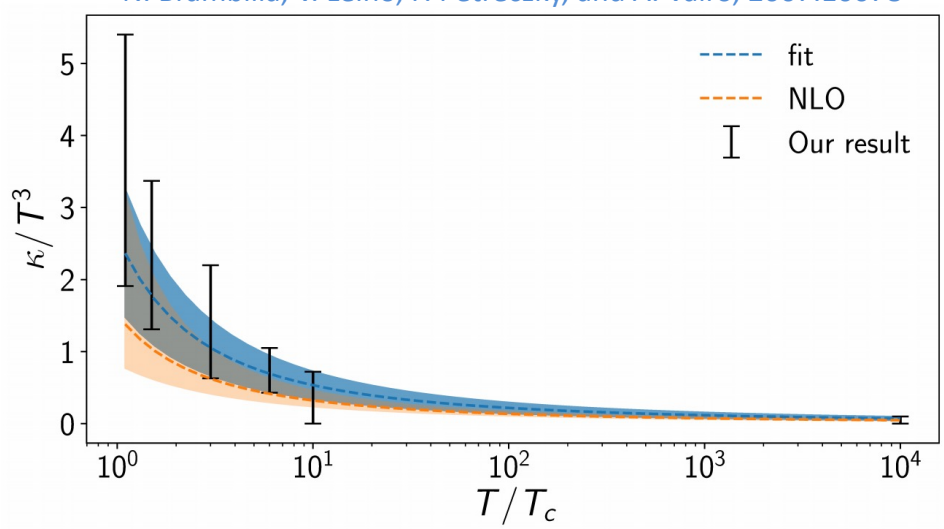
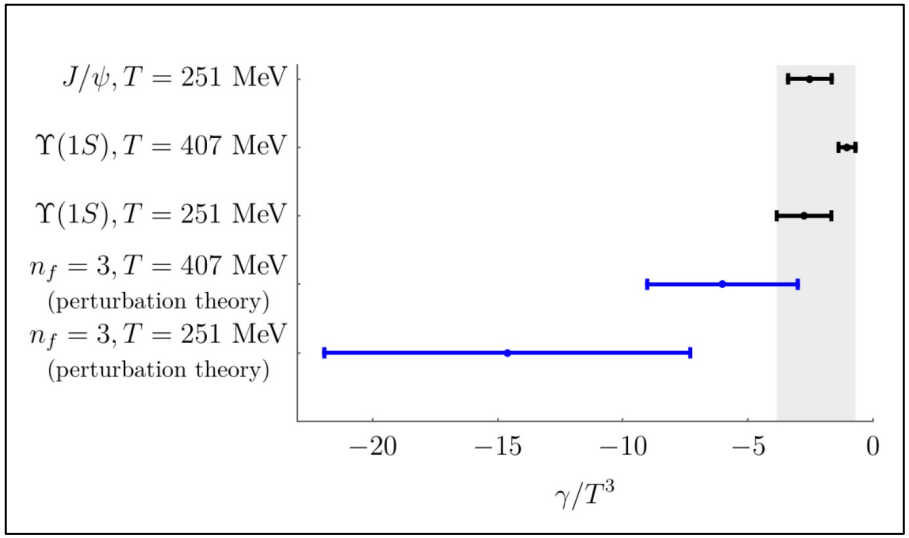


- The value of $\hat{\gamma} \equiv \gamma/T^3$ is less constrained, we vary it in the range -3.5 < $\hat{\gamma}$ < 0.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1711.04515.
  - N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.



N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.

How can one numerically solve these equations?

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\,\rho_{\rm probe}\,C_n^{\dagger}$$

- Each block of the density matrix in color space can be decomposed into orbital angular momentum.
- Upon truncating in angular momentum ($l \leq l_{max}$) one can reduce both the singlet and octet blocks of the reduced density matrix to size $(l_{max}+1)*(l_{max}+1)$. N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515
- One can then discretize the radial wavefunction (N = # of lattice points) and evolve the reduced density matrix using standard differential equation and matrix solvers gives $\sim N*N*(l_{max}+1)*(l_{max}+1)$ matrix size.
- Need to describe bound and unbound states with highly localized initial wave function, so the box must be large and have small lattice spacing → large N.
- As N and l_{max} become large, **<u>the computation becomes very challenging</u>**.
- **Need a better/faster method which we can easily parallelize.**

A parallelizable approach: Quantum trajectories

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2012.01240

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

Non-unitary "no jump" evolution

Can treat this "quantum jump" term stochastically

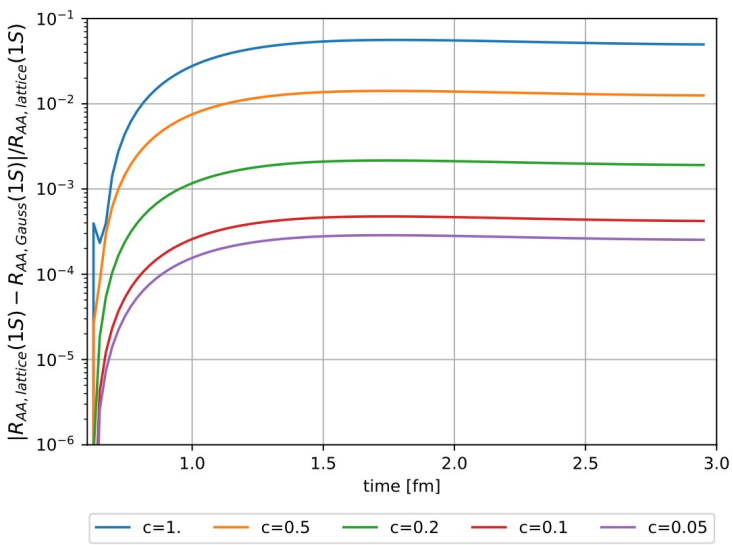
- Can be reduced to the solution of a large set of "quantum trajectories" in which we solve a 1D Schrödinger equation with a **non-Hermitian Hamiltonian $H_{\rm eff}$**, subject to **stochastic quantum jumps**.
- The evolution with the non-Hermitian $H_{\rm eff}$ preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode all transitions between different color/angular momentum states (subject to selection rules).
- For each **physical trajectory** (path through the QGP) we average over a large set of **<u>independent</u> quantum trajectories** → **Embarrassingly parallel**
- **Added benefit: Can describe all angular momentum states (no cutoff) .**

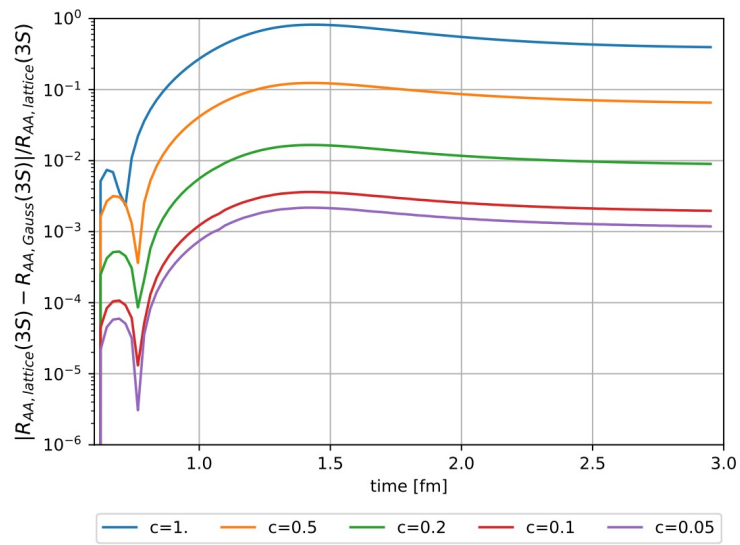
Initial bottomonium wavefunction

- We took the initial wavefunction to be given by a smeared delta function (local production due to large mass, Δ ~ 1/M) of the form

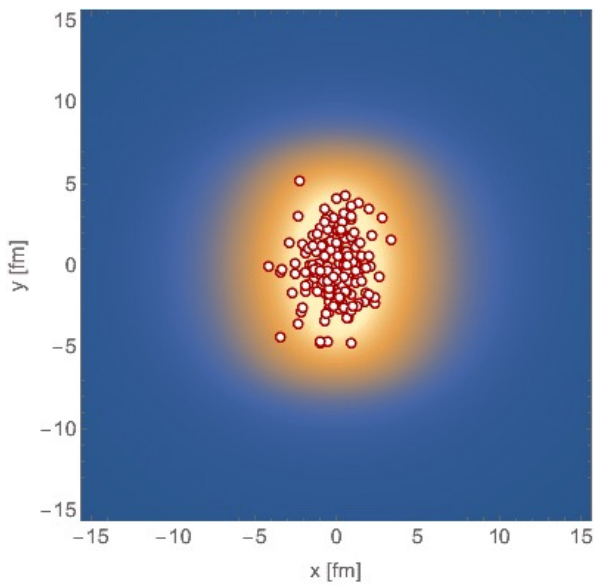
$$u_\ell(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2)$$

- For a given l, the **initial state is a quantum linear superposition** of the eigenstates of H.
- **Includes both bound and unbound states.**
- We took $\Delta = 0.2\ a_0$ which reproduces results obtained with a true delta to within 1%.





Quantum trajectories (QTraj) implementation



Survival probability

$$SP(n,l) = \frac{|\langle n,l|\psi(t_f)\rangle|^2}{|\langle n,l|\psi(t_0)\rangle|^2}$$

- We solved the real-time Schrodinger equation (SE) with a complex potential and stochastic jumps.
- Used ~ **100k quantum trajectories** per impact parameter.
- We sampled bottomonium production points and momentum using Monte Carlo sampling

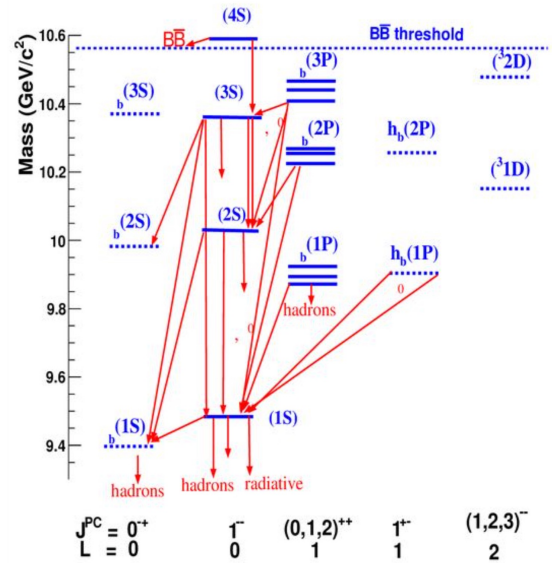
$$N_{\rm bottom}(x,y,p_T) \propto \frac{N_{\rm bin}(x,y)}{(p_T^2+M^2)^2}$$

- We averaged over the ensemble to obtain a trajectory-averaged temperature as a function of impact parameter.
- 4D temperature profiles provided by aHydroQP; good description of soft hadron spectra and flow
- We then solved for the survival probability for S- and P-wave states.

Feed-down implementation

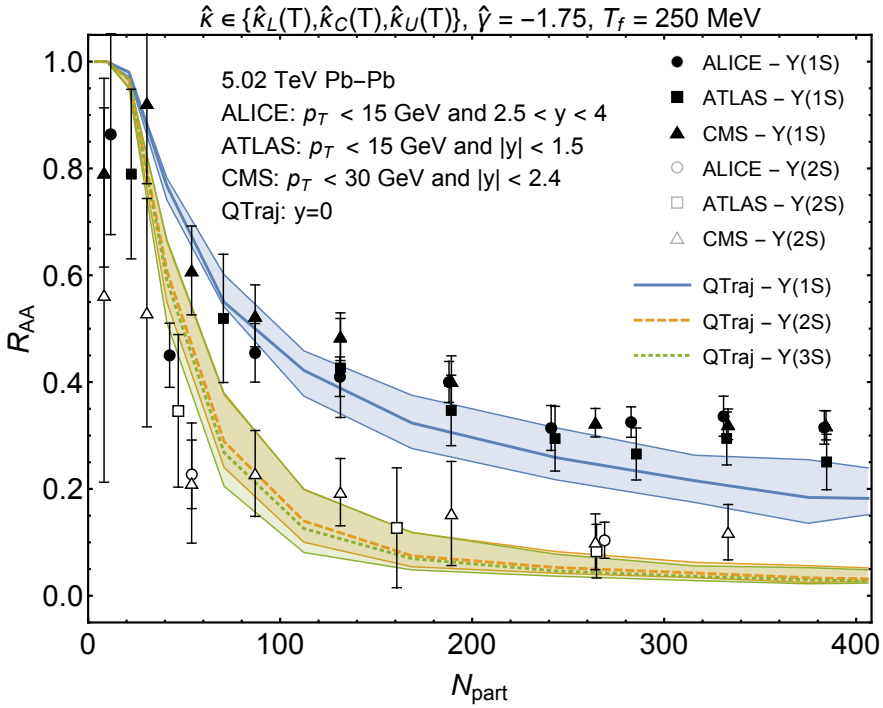
$$\vec{N}_{\text{observed}} = F\vec{N}_{\text{direct}}$$

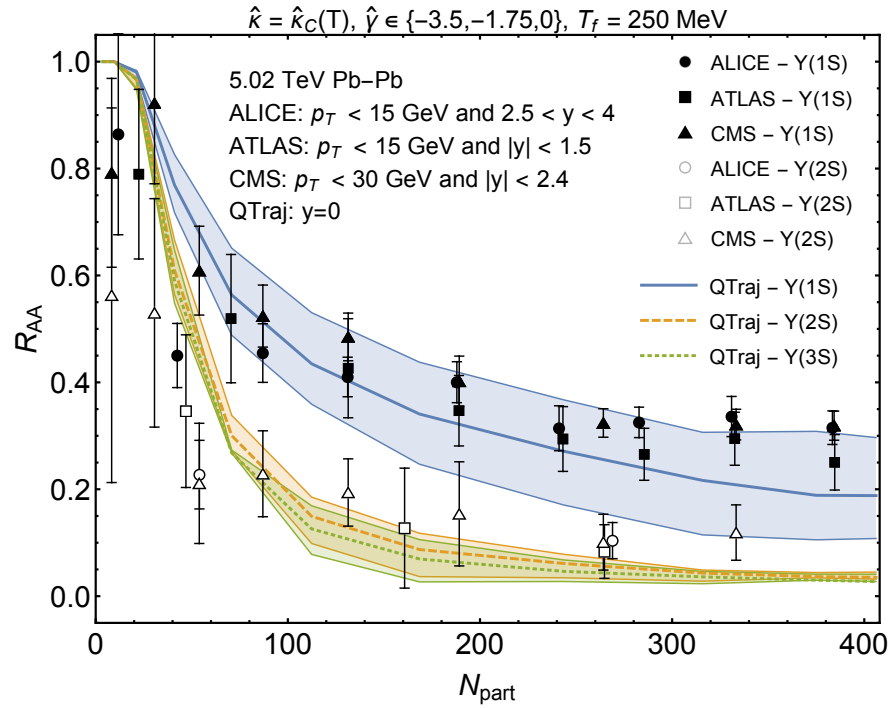
$$F = \begin{pmatrix} 1 & 0.2645 & 0.0194 & 0.352 & 0.18 & 0.0657 & 0.0038 & 0.1153 & 0.077 \\ 0 & 1 & 0 & 0 & 0 & 0.106 & 0.0138 & 0.181 & 0.089 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0091 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0051 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



- N_{QGP} corresponds to $(N_{1S}, N_{2S}, N_{1P} \times 3, N_{3S}, N_{2P} \times 3, N_{2D})^T$ where, e.g., N_{1S} is the final number of Y(1S) states that can decay in the dilepton channel.
- N_{QGP} can be obtained using $\langle N_{bin}(b)\rangle * \sigma_{direct}$ * (Survival probability)
- After feed down, we then normalized to the pp collision result scaled to AA → R_{AA}.
- We averaged over trajectories in each centrality/p_T bin to obtain R_{AA} vs centrality and p_T. (Can also obtain v_2, will present today)

QTraj predictions for R_{AA} vs N_{part}

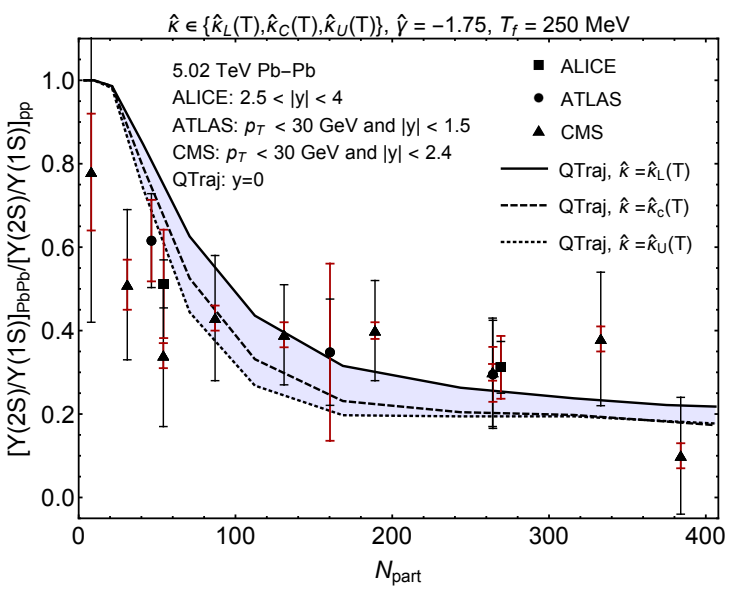


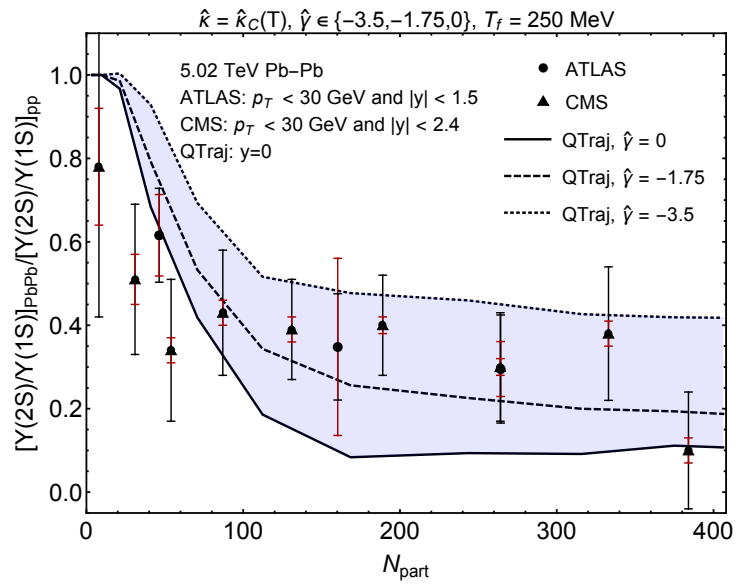


- **Left panel**: Result including feed down, when varying $\hat{\kappa}$ over the theoretical uncertainty.
- **Right panel**: Result including feed down, when varying $\hat{\gamma}$ over the theoretical uncertainty
- Bands also include statistical uncertainty associated with average over quantum trajectories.

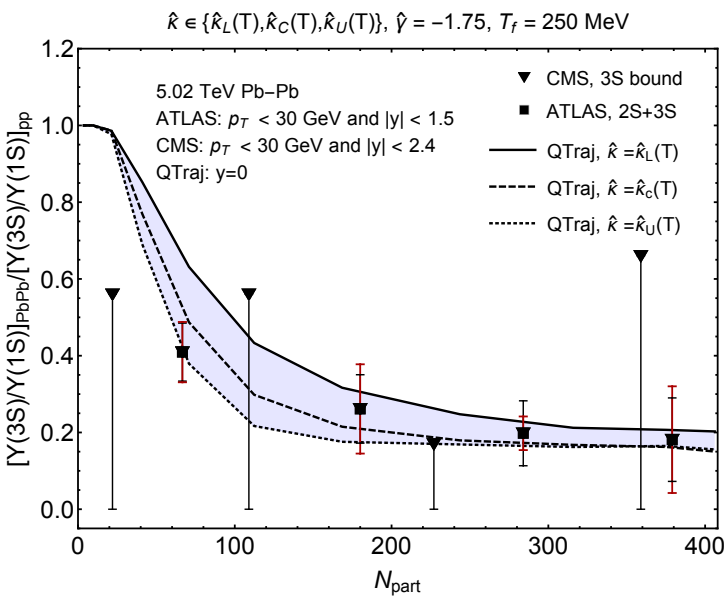
2S/1S and 3S/1S double ratios

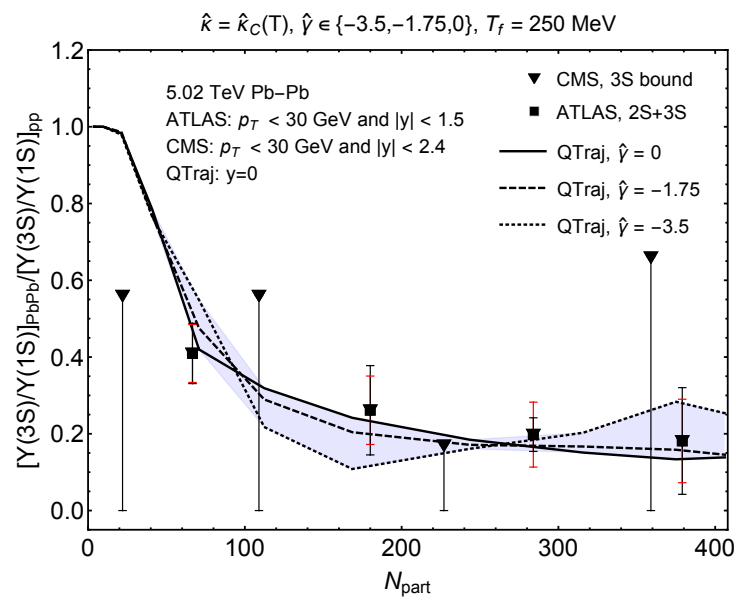
2S/1S





3S/1S

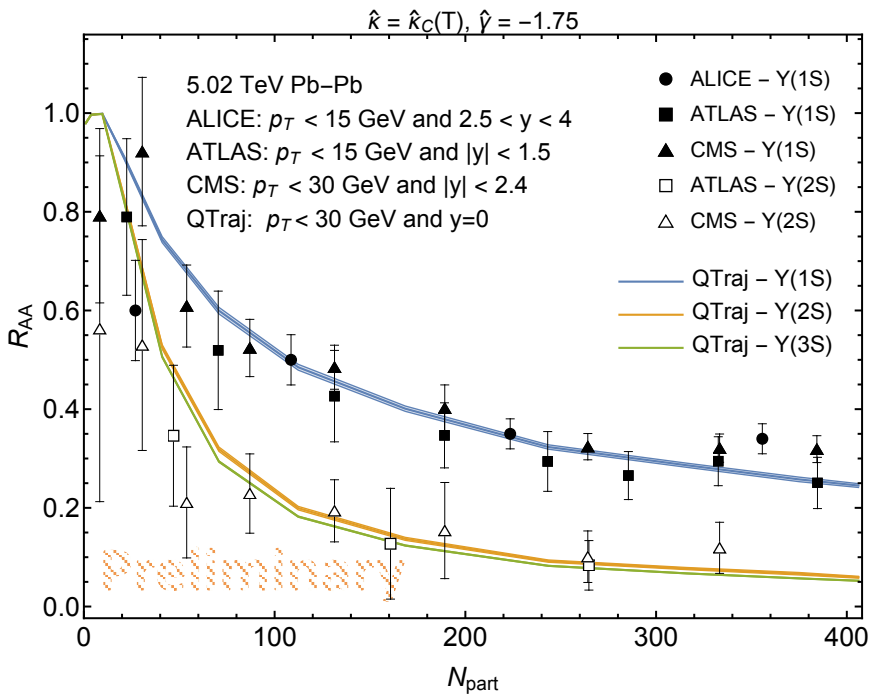


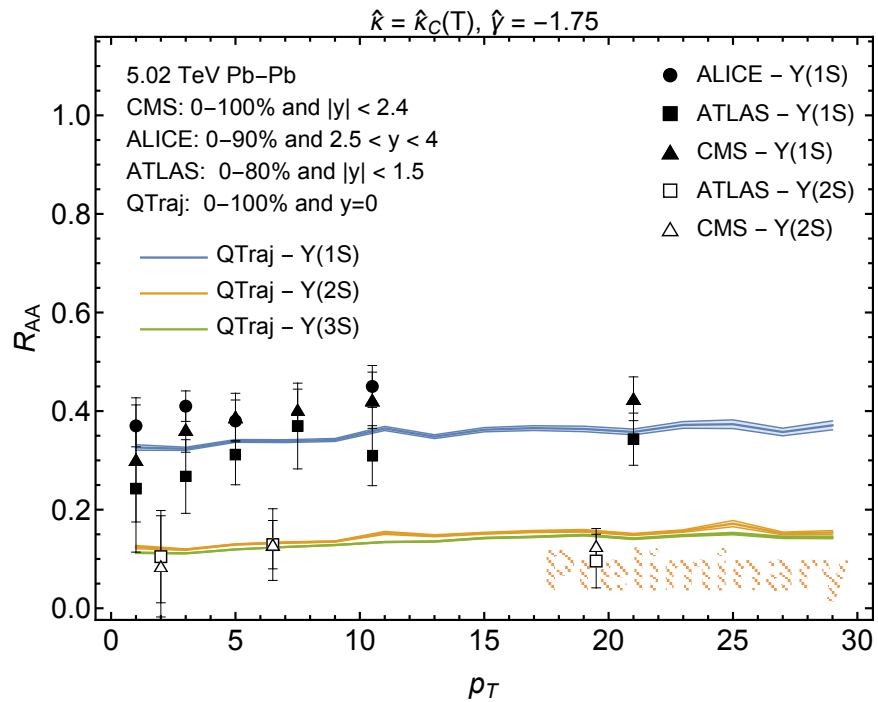


Forthcoming expanded results

In forthcoming a paper, we expand our study to include

- Use the temperature along each Monte-Carlo sampled physical trajectory
- Better statistics due to various computational optimizations
- Our goal is to make quantitative predictions for both R_{AA} and v_2 for all S-wave states





N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

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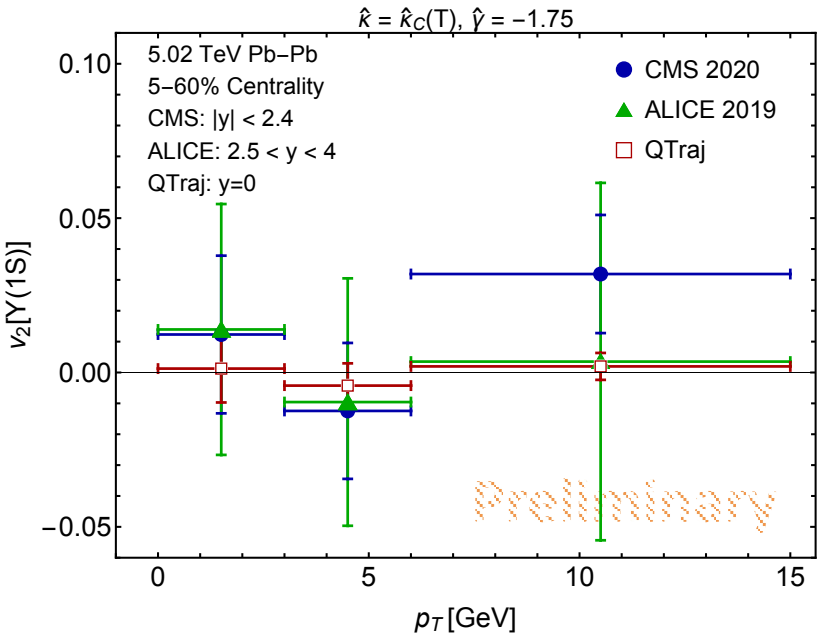

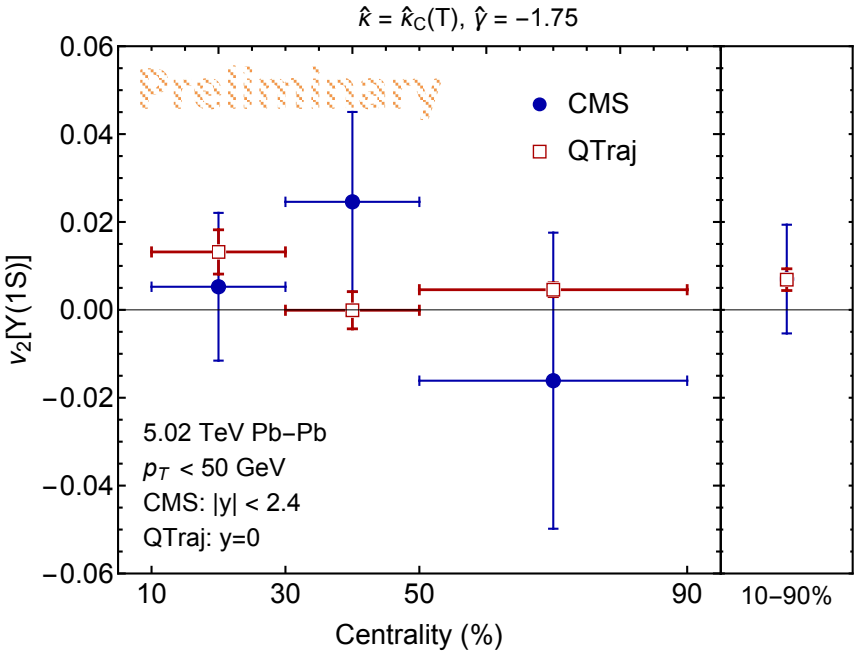

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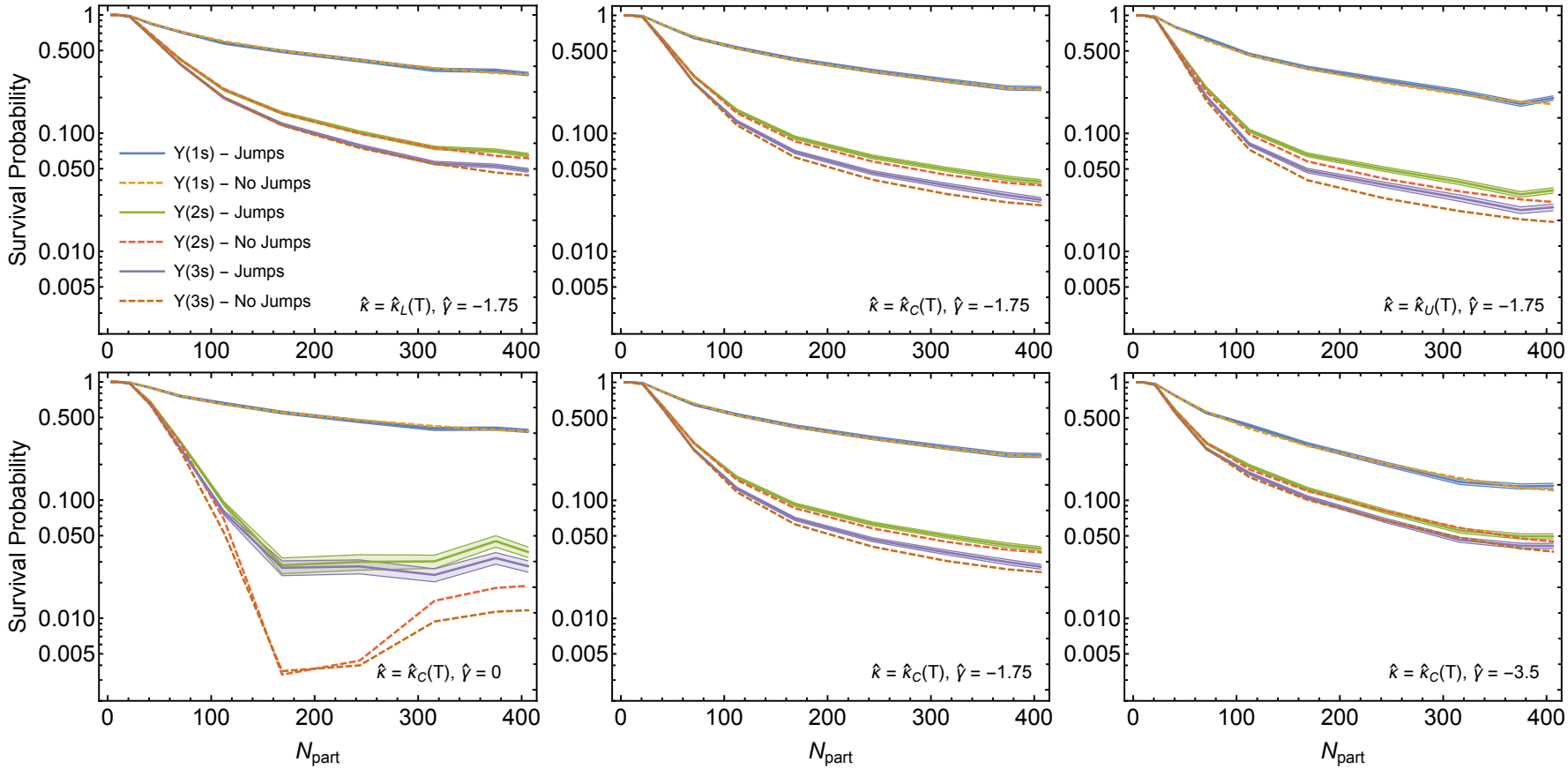
Conclusions and Outlook

- pNRQCD-based approach works reasonably well to describe the suppression and “flow” seen at LHC
- Transport coefficients used were constrained by independent lattice measurements.
- Demonstrated that Upsilon R_{AA} can be used to provide experimental constraints on these transport coefficients.
- The **quantum trajectory algorithm** (implemented in qTraj) allowed us to include effect of quantum jumps between color and angular momentum states in a computationally scalable manner.
- One outstanding issue is how to describe the low-temperature bottomonium dynamics (T <~ 200-250 MeV). For low-temperatures, a different formal approach is required.

Additional slides

Effects of quantum jumps



- **Solid lines show result with jumps**
- **Dashed lines show result without jumps (H_{eff} evolution)**

Predictions for 2S and 3S flow

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- Better statistics due to computational optimizations
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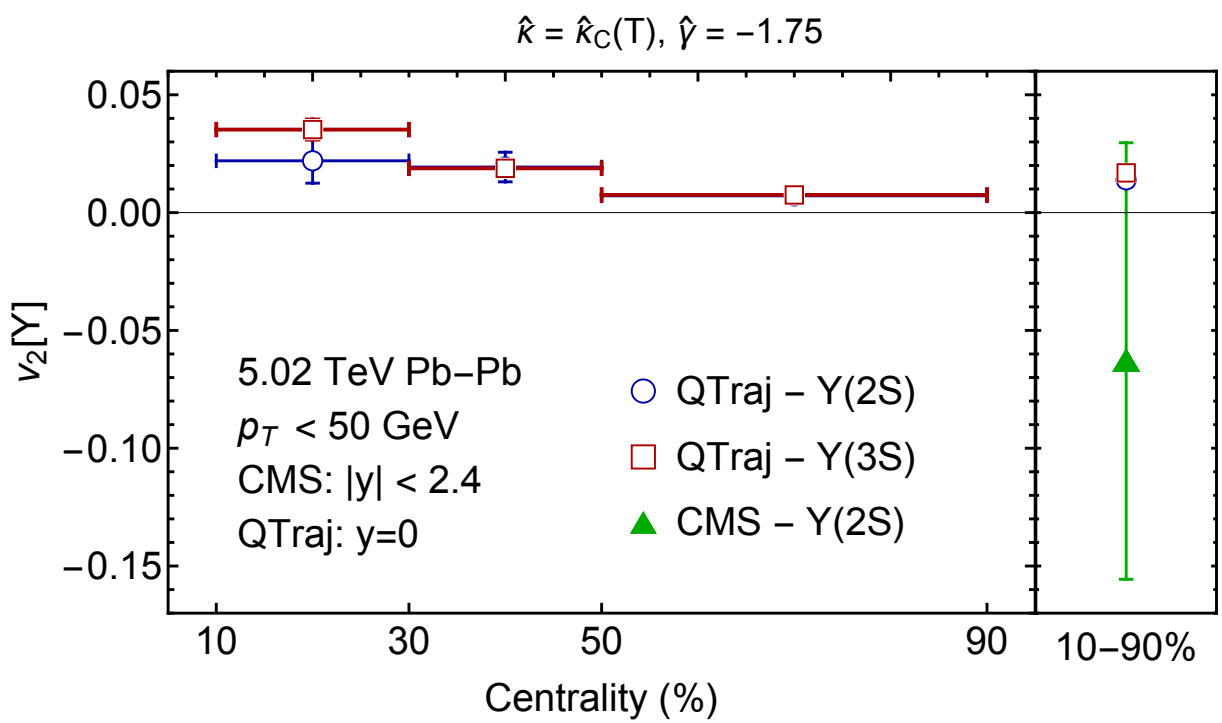

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

Dependence on T_f