INCLUSIVE HADROPRODUCTION OF P-WAVE HEAVY QUARKONIA IN POTENTIAL NRQCD

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NRQCD FACTORIZATION

- NRQCD provides a factorization formalism for inclusive production cross sections of quarkonia.

NRQCD matrix elements

$$\sigma = \sum_n \sigma_{Q\bar{Q}(n)} \langle \Omega | \mathcal{O}_n | \Omega \rangle$$

Perturbatively calculable
$Q\bar{Q}$ cross sections

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- In general it is *not known how to compute matrix elements* from first principles, so they are usually determined from *fits to cross section measurements*. So far this approach has not lead to a comprehensive description of measurements.

- We aim to ***compute*** the matrix elements in potential NRQCD, which is obtained by integrating out scales above mv^2.

QUARKONIUM IN PNRQCD

- We work in the strong coupling regime where $mv^2 \ll \Lambda_{\rm QCD}$, which applies to non-Coulombic quarkonia, such as P-wave quarkonia. The degree of freedom is the singlet field $S(x_1,x_2)$, which describe $Q\bar{Q}$ in a color-singlet state.

$$\mathcal{L}_{\rm pNRQCD} = {\rm Tr}\{S^\dagger (i\partial_0 - h) S\}$$

Pineda, Soto, NPB Proc. Suppl. 64, 428 (1998)
Brambilla, Pineda, Soto, Vairo, NPB566, 275 (2000)
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77, 1423 (2005)

- Matching to NRQCD is done nonperturbatively.
- pNRQCD provides expressions for decay matrix elements in terms of wavefunctions and universal gluonic correlators.
- We ***extend the formalism for production matrix elements***.

MATCHING IN PNRQCD

- Matching to NRQCD in strongly coupled pNRQCD is done nonperturbatively in expansion in powers of $1/m$:

NRQCD Hamiltonian
$$H_{\rm NRQCD} = H^{(0)}_{\rm NRQCD} + H^{(1)}_{\rm NRQCD}/m + \ldots$$
$$H^{(0)}_{\rm NRQCD} = \frac{1}{2}\int d^3x\,(\boldsymbol{E}^a\cdot\boldsymbol{E}^a + \boldsymbol{B}^a\cdot\boldsymbol{B}^a) - \sum_{k=1}^{n_f}\int d^3x\,\bar{q}_k i\boldsymbol{D}\cdot\boldsymbol{\gamma}q_k$$
$$H^{(1)}_{\rm NRQCD} = -\frac{1}{2}\int d^3x\,\psi^\dagger\boldsymbol{D}^2\psi - \frac{c_F}{2}\int d^3x\,\psi^\dagger\boldsymbol{\sigma}\cdot g\boldsymbol{B}\psi + \text{c.c.}$$

Eigenstates
$$|\underline{\rm n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle = |\underline{\rm n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle^{(0)} + \frac{1}{m}|\underline{\rm n};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle^{(1)} + \ldots$$

- $|\underline{0};\,x_1,\,x_2\rangle$ is the ground state, $x_1,\,x_2$ are positions of $Q,\,\bar{Q}$.
- A quarkonium state in *vacuum* is described by

$$\int d^3x_1 d^3x_2\,\phi(\boldsymbol{x}_1,\boldsymbol{x}_2)|\underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle$$

quarkonium wavefunction

PRODUCTION MATRIX ELEMENTS

- Production matrix elements for production of quarkonium $\mathcal{Q}$

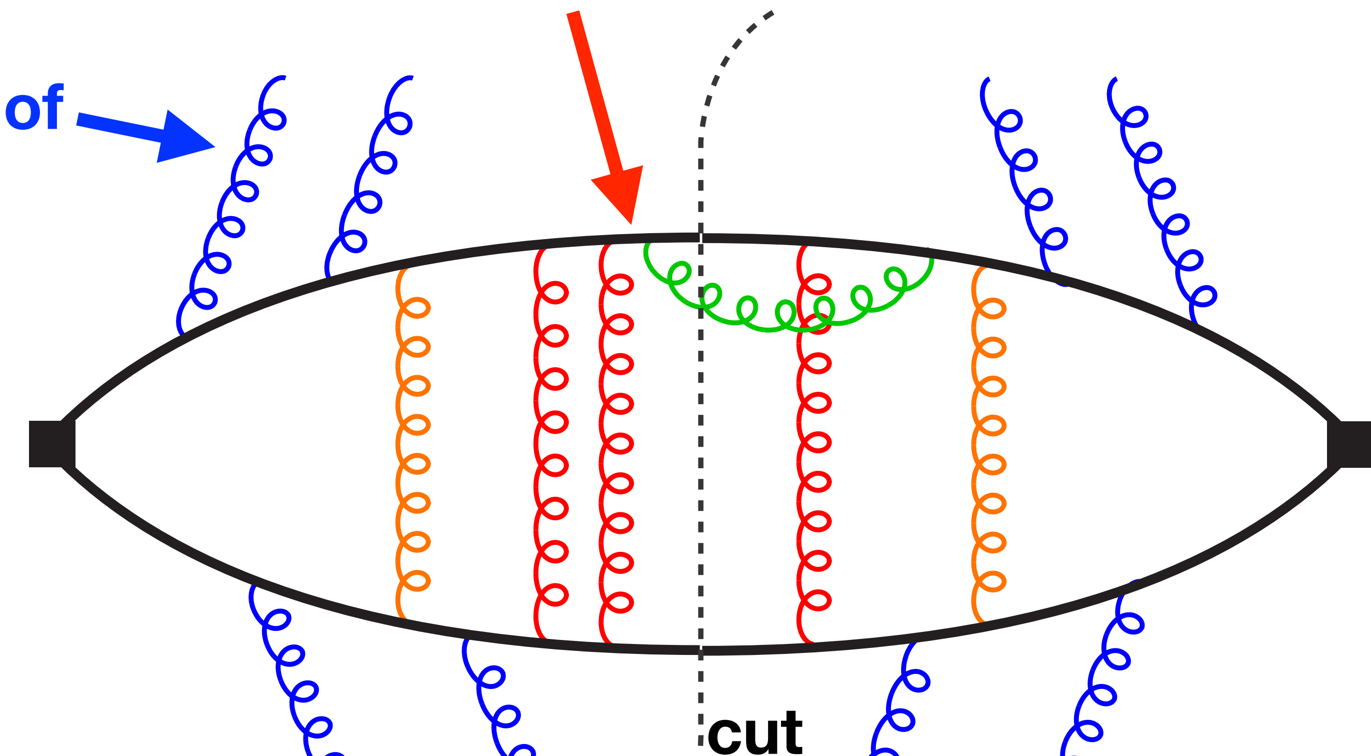
$$\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$$

color/spin matrices and covariant derivatives, gauge-completion Wilson lines (color octet)

- $\mathcal{P}_{\mathcal{Q}} = \sum_X |\mathcal{Q} + X\rangle\langle \mathcal{Q} + X| = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$

  : projection onto quarkonium $\mathcal{Q}$ + *anything*

- A schematic view of production matrix element



PRODUCTION MATRIX ELEMENTS

- We want to compute in pNRQCD

$$\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$$

color/spin matrices and covariant derivatives, gauge-completion Wilson lines (color octet)

- To compute production matrix elements we need to

1. Express $\mathcal{P}_{\mathcal{Q}} = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$ in terms of $|\underline{\mathrm{n}};\, x_1,\, x_2\rangle$ states.

2. Describe a quarkonium in background of light particles in terms of *wavefunctions*. While quarkonium is always color singlet, background can be color octet.

QUARKONIUM PROJECTION OPERATOR

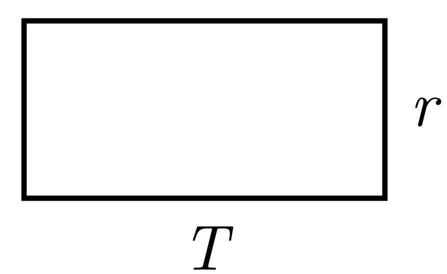
- $\mathcal{P}_{\mathcal{Q}} = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$ is essentially a number operator : $\mathcal{P}_{\mathcal{Q}}$ and $H_{\rm NRQCD}$ are *simultaneously diagonalizable*. *Simultaneous eigenstates* are given by
$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}(n)}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$
- We obtain the expression $\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$
- For $n{=}0$, $|\mathcal{Q}(0)\rangle$ is just the quarkonium in vacuum and $\phi_{\mathcal{Q}(0)}$ is the usual quarkonium wavefunction.
- For $n{>}0$, $|\mathcal{Q}(n)\rangle$ describe quarkonium + light particles. The "wavefunctions" $\phi_{\mathcal{Q}(n)}$ are in general unknown for $n{>}0$.

QUARKONIUM WAVEFUNCTIONS

- We need to identify "wavefunctions" ϕ of quarkonium in the background of gluons and light particles.

- Potential for quarkonium in vacuum is given in terms of the *vacuum expectation value (VEV) of a Wilson loop*:

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log \langle \Omega |$$

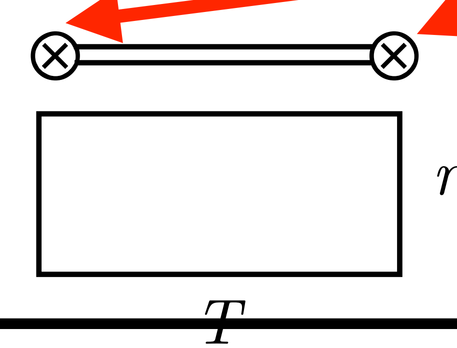


$$| \Omega \rangle$$

- For the potential for the $n>0$ states, the *light excitations* in the $|\underline{\mathrm{n}};\ x_1,\ x_2\rangle$ states should be included.

gluonic operators

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log \langle \Omega |$$



$$| \Omega \rangle$$

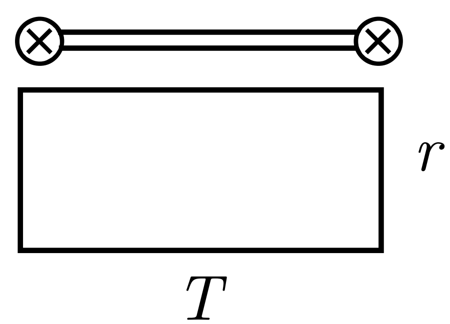
QUARKONIUM WAVEFUNCTIONS

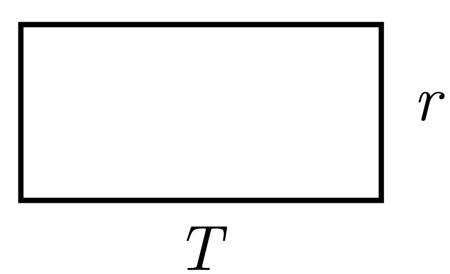
- In general, VEVs of products of color-singlet operators factorize into products of VEVs of individual operators.

$$\langle\Omega|AB|\Omega\rangle = \langle\Omega|A|\Omega\rangle\langle\Omega|B|\Omega\rangle[1+O(1/N_c^2)]$$

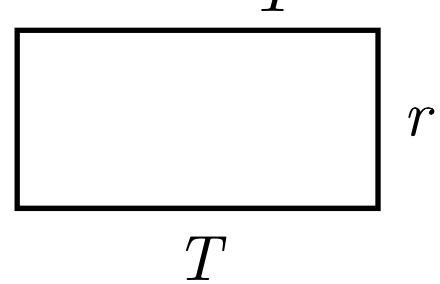
Makeenko, Migdal, PLB88, 135 (1979)
Witten, NATO Sci. Ser. B 59, 403 (1980)

- So the $n>0$ potentials reduce to

$$V(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\Omega|$$

$$|\Omega\rangle$$

$$= \lim_{T\to\infty}\frac{i}{T}\log\Big(\langle\Omega|$$

$$|\Omega\rangle \times \langle\Omega|$$

$$|\Omega\rangle\Big)$$

$$= \lim_{T\to\infty}\frac{i}{T}\log\langle\Omega|$$

$$|\Omega\rangle + \text{constant}$$

QUARKONIUM WAVEFUNCTIONS

- Hence, the $n{>}0$ potentials are just the $n{=}0$ potential plus constants.
- Because constant shifts in the potential have no effect to the wavefunctions, $\phi_{\mathcal{Q}(n)}$ are independent of n, and the projection operator is just

$$\mathcal{P}_{\mathcal{Q}} = \int d^3x_1 d^3x_2 d^3x_1' d^3x_2' \phi(\boldsymbol{x}_1, \boldsymbol{x}_2)\phi^*(\boldsymbol{x}_1', \boldsymbol{x}_2') \times \sum_n |\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle\langle\underline{\mathrm{n}}; \boldsymbol{x}_1', \boldsymbol{x}_2'|$$

- This is valid up to corrections of relative order $1/N_c^2$.

PRODUCTION MATRIX ELEMENTS IN PNRQCD

- Now we can compute the production matrix elements as

$$\langle\Omega|\chi^\dagger\mathcal{K}\psi\mathcal{P}_{\mathcal{Q}}\psi^\dagger\mathcal{K}'\chi|\Omega\rangle = \int d^3x_1 d^3x_2 \int d^3x_1' d^3x_2' \phi(\boldsymbol{x}_1,\boldsymbol{x}_2)\phi^*(\boldsymbol{x}_1',\boldsymbol{x}_2')$$
$$\sum_n \langle\Omega|\chi^\dagger\mathcal{K}\psi|\underline{\mathrm{n}};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle\langle\underline{\mathrm{n}};\boldsymbol{x}_1',\boldsymbol{x}_2'|\psi^\dagger\mathcal{K}'\chi|\Omega\rangle$$

Contact term,
computed order by order in $1/m$

- ***Contact terms*** can be computed in terms of *universal gluonic correlators* and *differential operators* that act on wavefunctions.

- This allows ***calculation of production matrix elements in strongly coupled pNRQCD***.

PRODUCTION OF P-WAVE QUARKONIA

- We apply this formalism to $\chi_{QJ}\,(Q{=}c$ or $b,\ J{=}1,\,2)$

- At leading order in v, the cross section is given by

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$$

Bodwin, Braaten, Yuan, Lepage, PRD46, R3703 (1992)
Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

color singlet : $$\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) = \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\psi\mathcal{P}_{\chi_{Q0}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\chi$$

color octet : $$\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) = \chi^\dagger\sigma^i T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\chi_{Q0}}\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c\chi$$

Nayak, Qiu, Sterman, PLB613, 45 (2005)

- We compute both color singlet and color octet matrix elements in strongly coupled pNRQCD.

P-WAVE PRODUCTION MATRIX ELEMENTS

- Color-singlet matrix element: $\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]}) \rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$ we *reproduce* the known result in the vacuum-saturation approximation. $R(r)$ is the radial wavefunction of χ_Q.

- Color-octet matrix element: result is given in terms of a *universal gluonic correlator*.

$$\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]}) \rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$

- $\mathcal{E}$ is a universal quantity that *does not depend on quark flavor or radial excitation*. Determination of $\mathcal{E}$ directly leads to determination of all χ_{cJ} and $\chi_{bJ}(nP)$ cross sections, as well as h_c and h_b production rates.

P-WAVE PRODUCTION MATRIX ELEMENTS

- The correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE at time t and t', with Wilson lines extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty t\, dt \int_0^\infty t'\, dt' \langle \Omega | \Phi_\ell^{\dagger ab} \Phi_0^{\dagger da}(0,t) gE^{d,i}(t) gE^{e,i}(t') \Phi_0^{ec}(t',0) \Phi_\ell^{bc} | \Omega \rangle.$$

- $\mathcal{E}$ has a one-loop scale dependence that is consistent with the evolution equation for NRQCD matrix elements

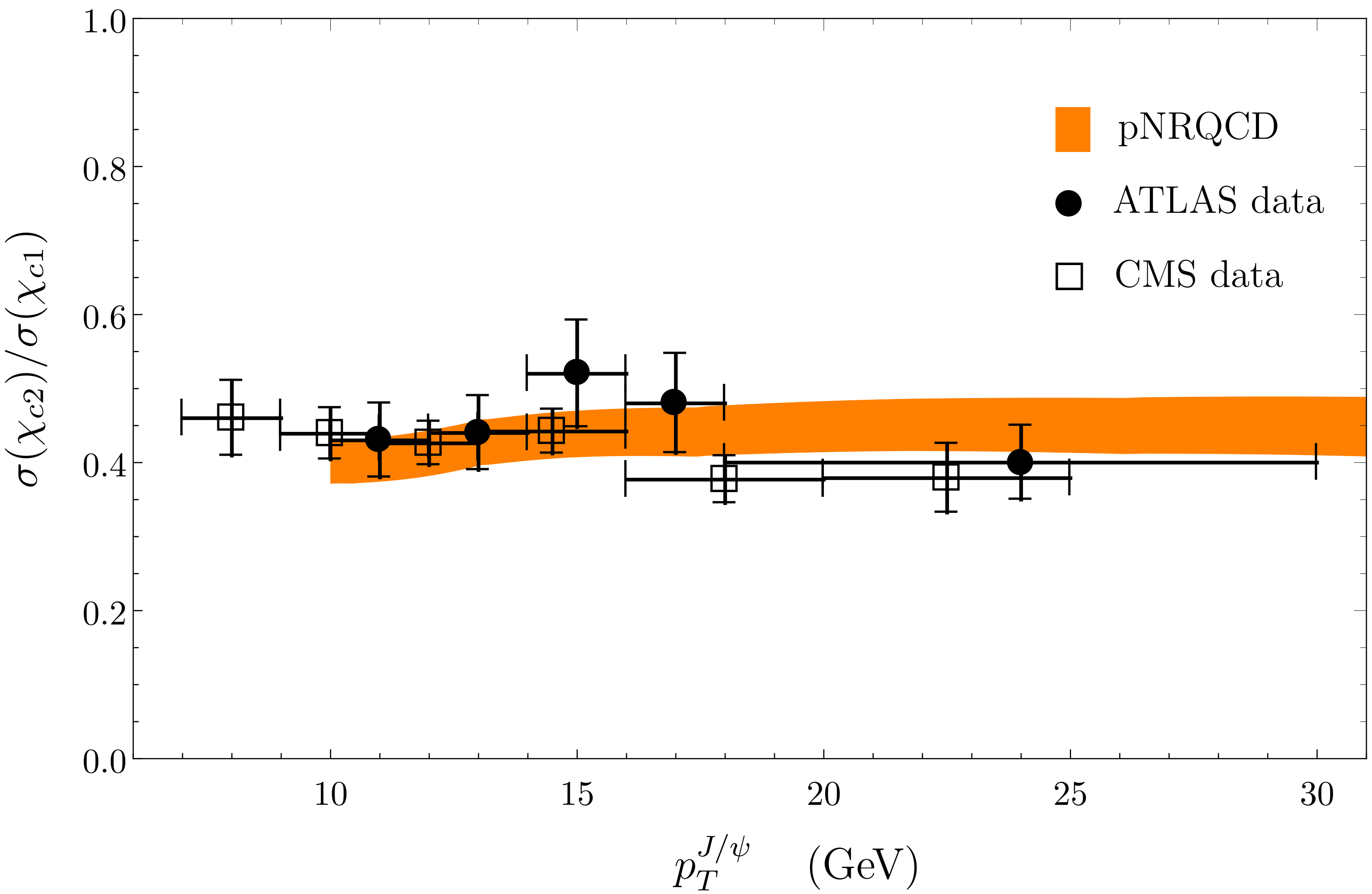
$$\frac{d}{d\log\Lambda}\mathcal{E}(\Lambda) = 12 C_F \frac{\alpha_s}{\pi} \qquad \frac{d}{d\log\Lambda}\langle \mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})\rangle = \frac{4C_F\alpha_s}{3N_c\pi m^2}\langle \mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})\rangle$$

- In principle, $\mathcal{E}$ can be determined from lattice QCD. Since lattice calculation is unavailable, we determine $\mathcal{E}$ from measured χ_{cJ} cross section ratios to obtain $\mathcal{E}(\Lambda = 1.5\ \mathrm{GeV}) = 1.97 \pm 0.06$

P-WAVE CHARMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ at the LHC compared to ATLAS and CMS data.

CMS, EPJC72, 2251 (2012)
ATLAS, JHEP07, 154 (2014)



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

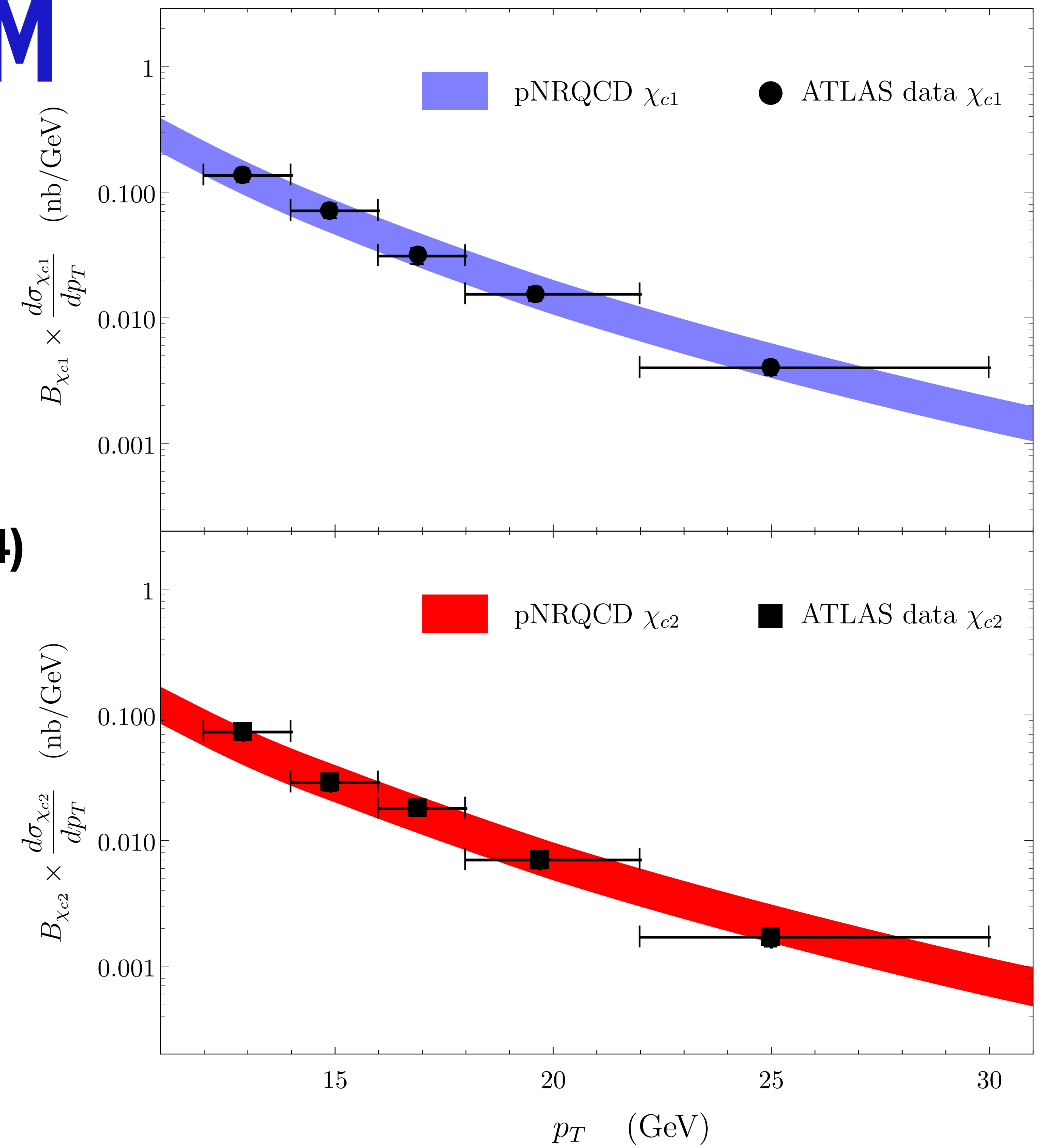
P-WAVE CHARMONIUM PRODUCTION

- χ_{c1} and χ_{c2} cross sections at the LHC, compared to ATLAS data.

ATLAS, JHEP07, 154 (2014)

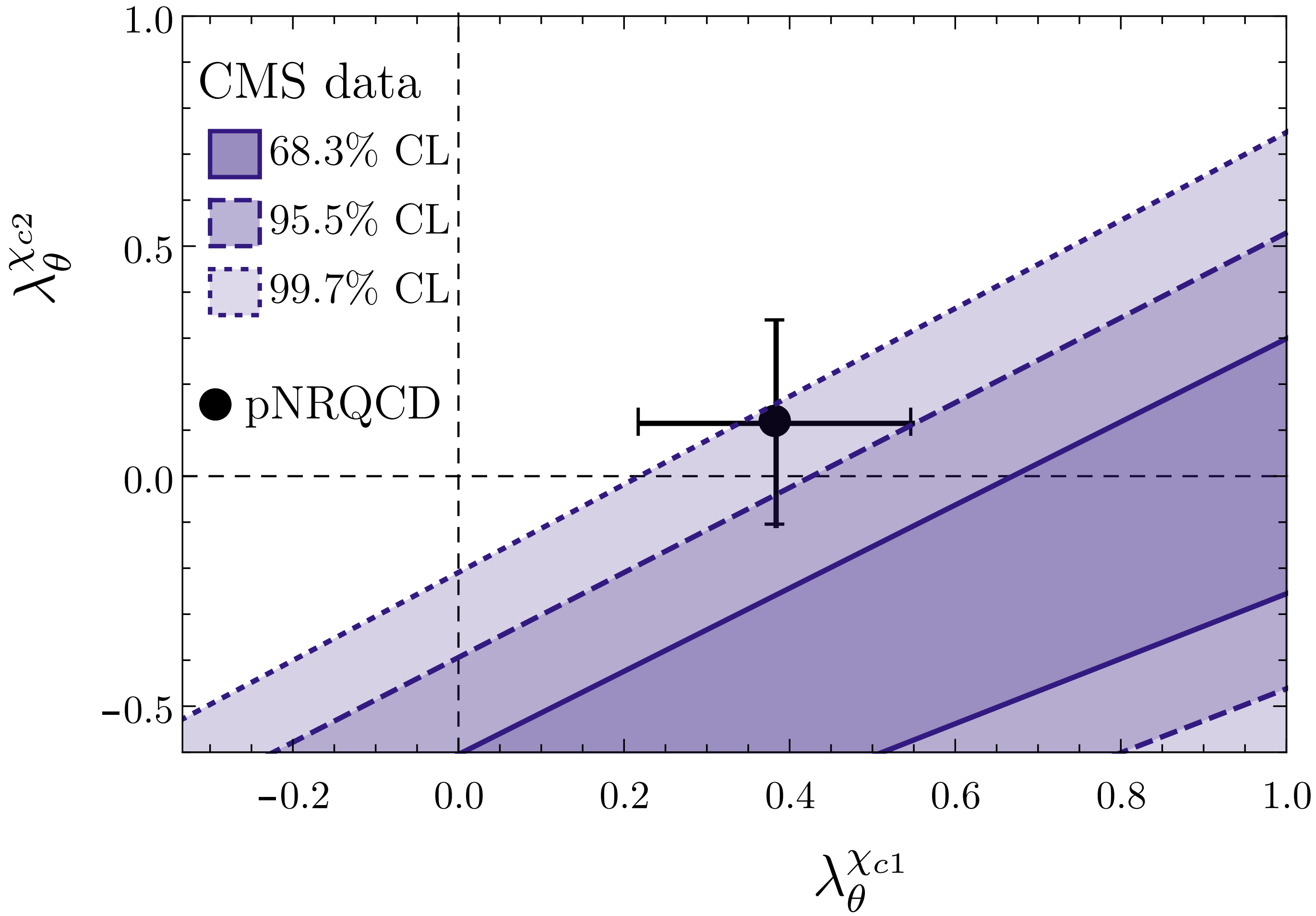
- Wavefunctions at the origin obtained from two-photon decay rates of χ_{c2} and χ_{c0}.

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)



P-WAVE CHARMONIUM POLARIZATION

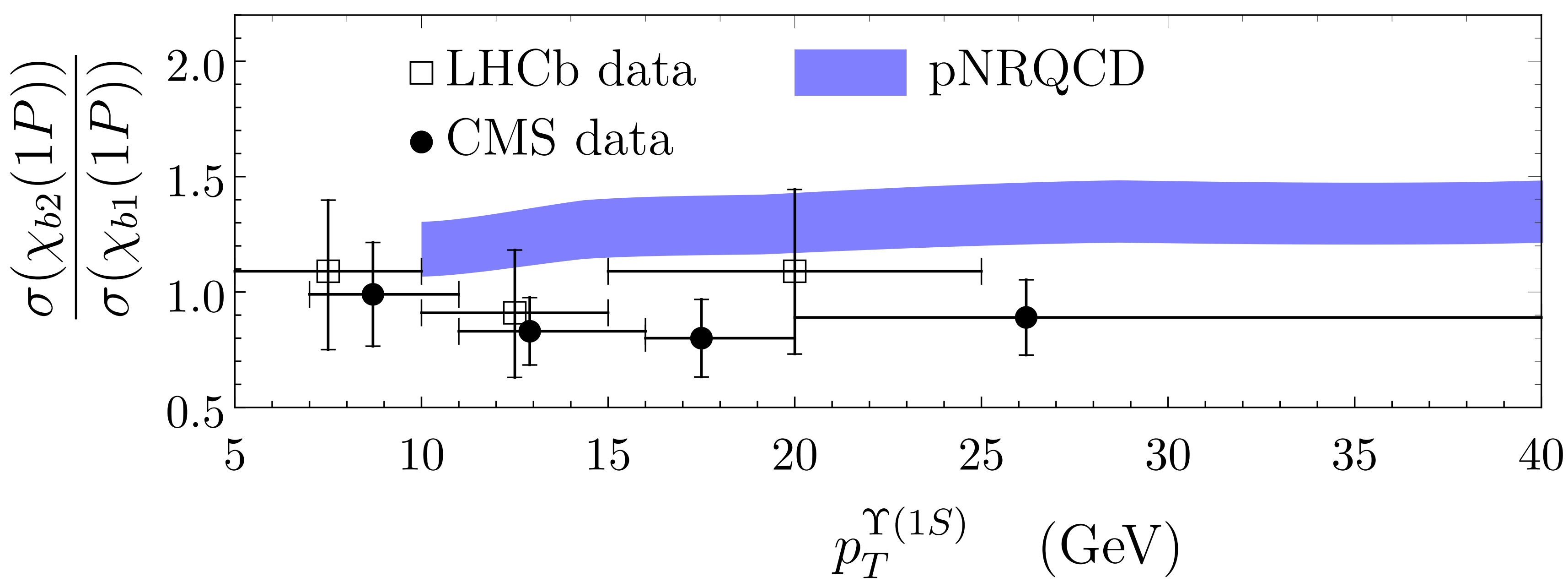
- χ_{c1} and χ_{c2} polarization at the LHC compared to experimental constraints from CMS. **CMS, PRL124, 162002 (2020)**



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

P-WAVE BOTTOMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{b2})/\sigma(\chi_{b1})$ for $1P$ states at the LHC compared to LHCb and CMS measurements.



LHCb, JHEP10, 088 (2014)
CMS, PLB743, 383 (2015)

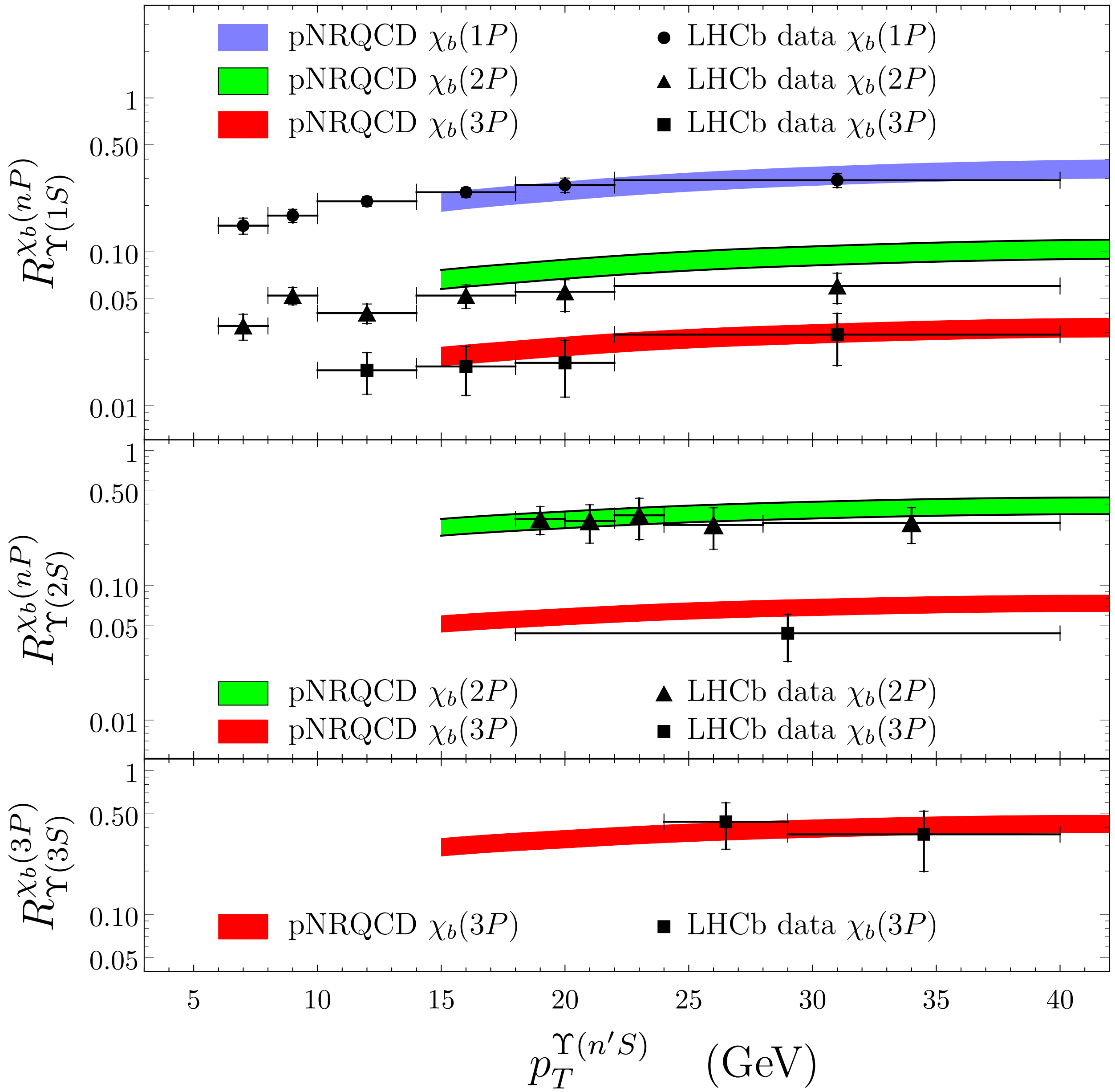
Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

P-WAVE BOTTOMONIUM PRODUCTION

- $\chi_{bJ}(nP)$ production rates relative to $\Upsilon(n'S)$ cross sections at the LHC compared to LHCb measurement of feeddown fractions.

LHCb, EPJC74, 3092 (2014)

$$R^{\chi_b(nP)}_{\Upsilon(n'S)} = \sum_{J=1,2} \frac{\sigma_{\chi_{bJ}(nP)} \times \mathrm{Br}_{\chi_{bJ}\to\Upsilon(nS)+\gamma}}{\sigma_{\Upsilon(n'S)}}$$



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

χ_{bJ} **wavefunctions computed from potential models**

$\Upsilon(nS)$ **matrix elements taken from fits to data in Han, Ma, Meng, Shao, Zhang, Chao, PRD94, 014028 (2016)**

SUMMARY AND OUTLOOK

- We developed a formalism for inclusive production of heavy quarkonium in strongly coupled potential NRQCD.
- For the first time, this allows first-principles determination of color-octet production matrix elements.
  A single gluonic correlator leads to determination of all P-wave charmonium and bottomonium cross sections.
- We computed production rates of χ_{cJ} and χ_{bJ} at the LHC, which are in agreements with measurements.
- This formalism can be applied to other quarkonium states, and may be extended to exotics such as hybrids.
- Lattice determination of the gluonic correlator is desirable.