

An **EFT** approach to **Quarkonium** at small transverse momentum

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Quarkonium and TMDs

Quarkonium production is considered as one of the most important processes to access unpolarized and polarized gluon TMDs. Some example of relevant processes are:

$$ep/pp \rightarrow \psi + X/\psi + \psi + X/\psi + \gamma + X/\dots$$

arXiv:1202.6585 (S. J. Brodsky, F. Fleuret, C. Hadjidakis, J. P. Lansberg)

arXiv:1208.3642 (D. Boer, C. Pisano)

arXiv:1401.7611 (W. J. den Dunnen, J.-P. Lansberg, C. Pisano, M. Schlegel)

arXiv:1406.5476 (G-P. Zhang)

arXiv:1809.02056 (A. Bacchetta, D. Boer, C. Pisano, P. Taelis)

and many others

Quarkonium and TMDs

Many attempts that approach the problem in CEM and CSM: Cannot be improved the same way EFTs can. CEM and CSM fail in other regions/aspects of quarkonium production

arXiv:hep-ph/0404158 (E. L. Berger, J. Qiu, Y. Wang)

arXiv:hep-ph/0411026 (E. L. Berger, J. Qiu, Y. Wang)

NRQCD attempt (including evolution)*: Assumption: NRQCD factorization holds at low p_T ?

arXiv:1210.3432 (P. Sun, C-P. Yuan, and F. Yuan)

arXiv:1405.3373 (J.P. Ma, J.X. Wang, S. Zhao)

arXiv:1509.04421 (J.P. Ma, C. Wang)

*Plethora of other studies that use NRQCD factorization also make the same assumption.

Usually the CSS kernel is assumed to be sufficient for resummation and evolution.

CGC + NRQCD methods: small- x resummation/ NO p_T/M resummation, sufficient for charmonium at LHC.

arXiv:1408.4075 (Y-Q. Ma, R. Venugopalan)

arXiv:1503.07772 (Y-Q. Ma, R. Venugopalan, and H-F. Zhang)

In this talk

NRQCD: a brief review

- Scales - soft modes
- Factorization conjuncture
- Factorization breakdown

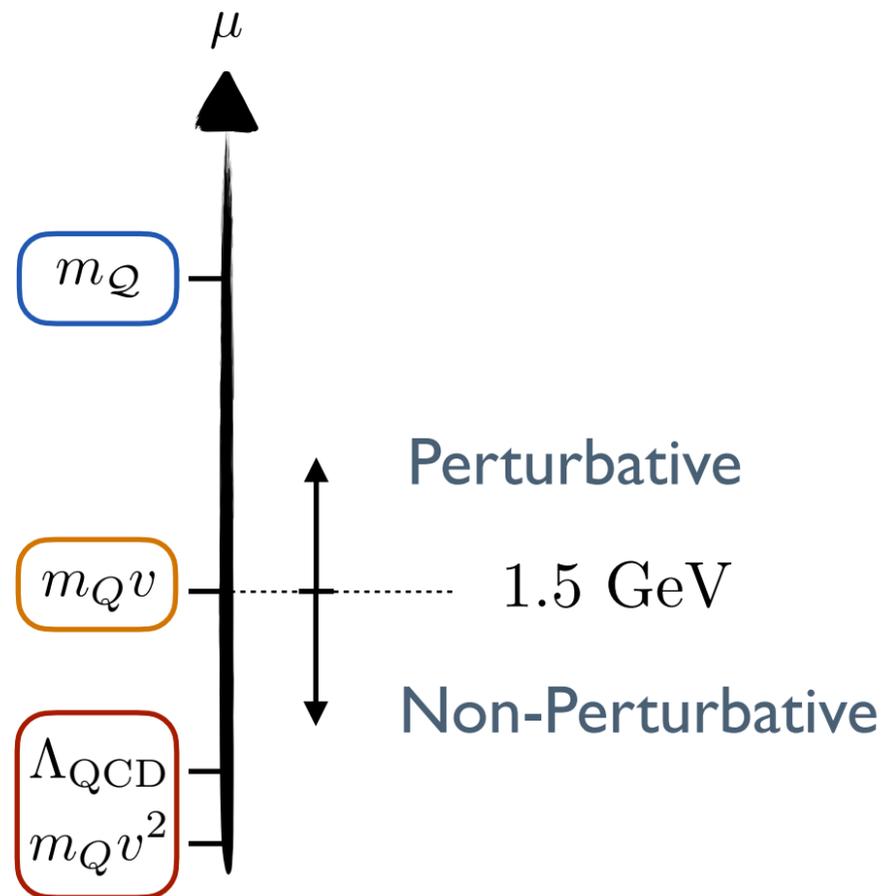
SCET-Q: a fusion of SCET and NRQCD

- New approach to deal with TMDs
- The operators
- Re-parameterization transformation

Case-Study: P wave

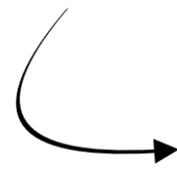
- Operators
- Factorization and Shape functions
- IR safety and channel mixing
- 2D-RG and resummation

NRQCD in brief (scales)



$b\bar{b}$: $v^2 \sim 0.1$ bottomonium

$c\bar{c}$: $v^2 \sim 0.3$ charmonium



Relative velocity of the heavy quark and antiquark in the quarkonium

typical momentum of heavy quark: $|\mathbf{p}_Q| \sim m_Q v$ (soft)

typical kinetic energy of heavy quark: $K_Q \sim m_Q v^2$ (ultra-soft)

NRQCD in brief (Lagrangian)

arXiv:hep-ph/9910209 (M. E. Luke, A. V. Manohar, I. Z. Rothstein)

$$\mathcal{L} = \sum_{\mathbf{p}} \psi_{\mathbf{p}}^{\dagger} \left\{ \overset{\text{ultra-soft}}{\downarrow} iD^0 - \frac{(\mathbf{p} - \overset{\text{subleading}}{\downarrow} i\mathbf{D})^2}{2m} \right\} \psi_{\mathbf{p}}$$

soft:	$p_s^\mu \sim m_Q v(1, 1, 1, 1)$
ultra-soft:	$p_{us}^\mu \sim m_Q v^2(1, 1, 1, 1)$

$$-4\pi\alpha_s \sum_{q, q', \mathbf{p}, \mathbf{p}'} \left\{ \frac{1}{q^0} \psi_{\mathbf{p}'}^{\dagger} [A_{q'}^0, A_q^0] \psi_{\mathbf{p}} + \frac{g^{\nu 0} (q' - p + p')^\mu - g^{\mu 0} (q - p + p')^\nu + g^{\mu\nu} (q - q')^0}{(\mathbf{p}' - \mathbf{p})^2} \psi_{\mathbf{p}'}^{\dagger} [A_{q'}^\nu, A_q^\mu] \psi_{\mathbf{p}} \right\}$$

soft

$$+ \sum_{\mathbf{p}, \mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p} - \mathbf{q})^2} \psi_{\mathbf{q}}^{\dagger} T^A \psi_{\mathbf{p}} \chi_{-\mathbf{q}}^{\dagger} \bar{T}^A \chi_{-\mathbf{p}} + \dots$$

NRQCD in brief (factorization)

arXiv:hep-ph/9407339 (G.T. Bodwin, E. Braaten, G. P. Lepage)

LDME: Long Distance Matrix Elements

$$d\sigma(a + b \rightarrow Q + X) = \sum_n \left(d\sigma(a + b \rightarrow Q\bar{Q}(n) + X) \right) \langle \mathcal{O}_n^Q \rangle$$

↓ Perturbative expansion in the strong coupling.
↓ NRQCD Scaling Rules

$$d\sigma_0(1 + \alpha_s C_1 + \alpha_s^2 C_2 + \dots) \quad \langle \mathcal{O}(^{2S+1}L_J^{[1,8]}) \rangle \sim v^{3+2L+2E+4M}$$

$$Q\bar{Q}(n) \xrightarrow{\langle \mathcal{O}_n^Q \rangle} Q$$

NRQCD is an expansion of QCD in the relative velocity of the heavy quark pair

$$\mathcal{O}_n^Q = \mathcal{O}_2^{n\dagger} \left(\sum_X |X + Q\rangle \langle X + Q| \right) \mathcal{O}_2^n$$

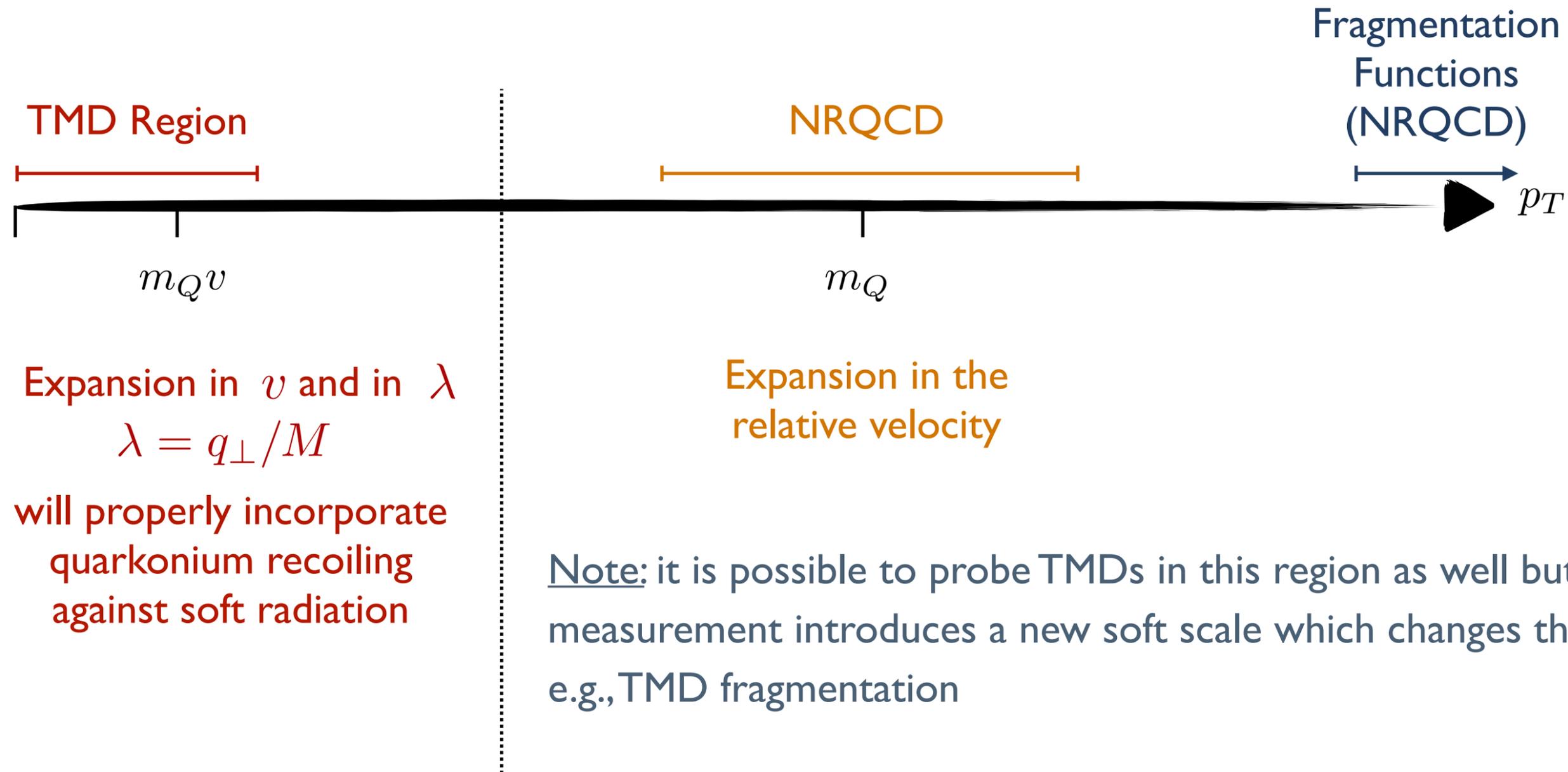
$$\mathcal{O}_2^n = \psi^\dagger \mathcal{K}^n \chi$$

$$n = ^{2S+1}L_J^{[c]}$$

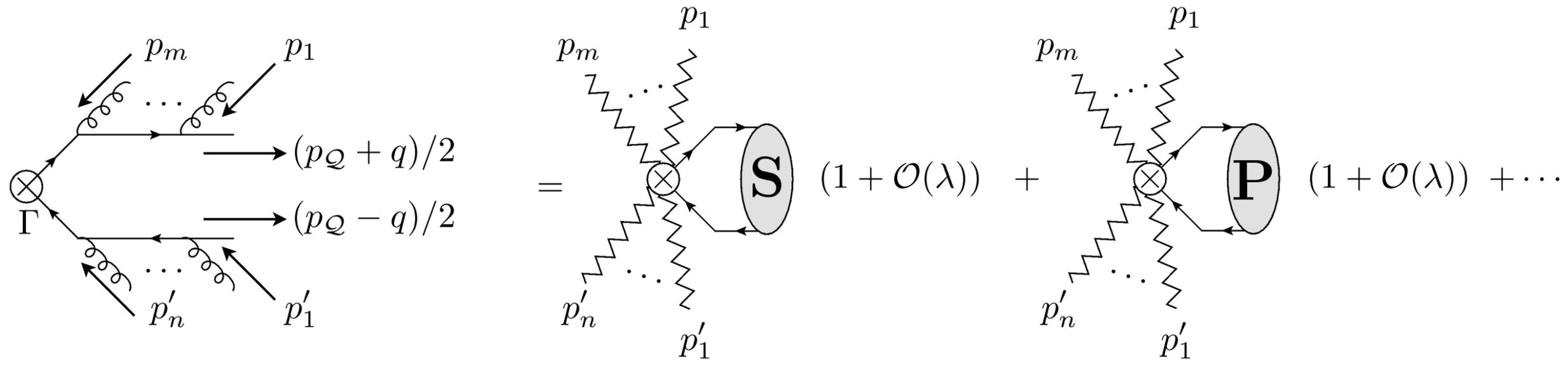
↑ ultra-soft + soft

NRQCD in brief (regimes)

Quarkonium spectrum vs EFT regions



Diagrammatic analysis at tree level



$$d_{\Gamma}(m, n) = d_{\Gamma}^{(0)}(m, n) (1 + \mathcal{O}(\lambda)) + d_{\Gamma}^{(1)}(m, n) (1 + \mathcal{O}(\lambda)) + \dots$$

S-wave:
simple result

$$S_v = \sum_n \sum_{\text{perms}} \frac{g^n}{n!} \prod_{s=1}^n \left[\frac{A_{n+1-s}^0}{p_t^0(s)} \right]$$

$$d_{\Gamma}^{(0)} = \left(u^{(0)} \right)^\dagger S_v^\dagger \Gamma^{(0)} S_v v^{(0)}$$

$$S_v(x, -\infty) = \text{P} \left[\exp \left(-ig \int_{-\infty}^0 d\tau v \cdot A_{\text{soft}}(x^\mu + v^\mu \tau) \right) \right]$$

Diagrammatic analysis at tree level

$$d_{\Gamma}(m, n) = d_{\Gamma}^{(0)}(m, n) (1 + \mathcal{O}(\lambda)) + d_{\Gamma}^{(1)}(m, n) (1 + \mathcal{O}(\lambda)) + \dots$$

S-wave:
simple result

$$d_{\Gamma}^{(0)} = \left(u^{(0)}\right)^{\dagger} S_v^{\dagger} \Gamma^{(0)} S_v v^{(0)}$$

For S-wave, color-singlet heavy quark pair the soft gluon contribution, at leading order, cancels.

For color-octet contributions the net effect is similar to jet and top-pair productions: new soft wilson-lines enter the soft matrix element.

Diagrammatic analysis at tree level

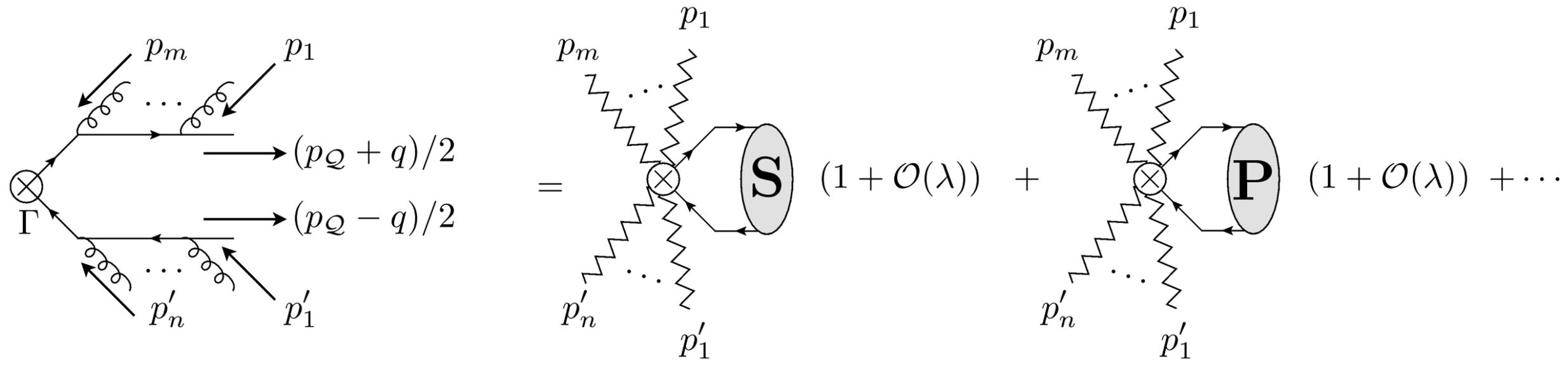
$$d_{\Gamma}(m, n) = d_{\Gamma}^{(0)}(m, n) (1 + \mathcal{O}(\lambda)) + d_{\Gamma}^{(1)}(m, n) (1 + \mathcal{O}(\lambda)) + \dots$$

P-wave: Not so simple result

$$B_s^{\mu} = -\frac{1}{g} S_v^{\dagger} [(\mathcal{P}^{\mu} - gA^{\mu}) S_v]$$

$$d_{\Gamma}^{(1)} = \frac{g}{2m} (u^{(0)})^{\dagger} \left\{ S_v^{\dagger} \Gamma^{(0)} S_v, \left[\frac{1}{v \cdot \mathcal{P}} \mathbf{q} \cdot \mathbf{B}_s \right] \right\} v^{(0)} + (u^{(0)})^{\dagger} S_v^{\dagger} \mathbf{q} \cdot \left(\Gamma^{(1)} - \frac{1}{4m} \{ \Gamma^{(0)}, \gamma \} \right) S_v v^{(0)}$$

Diagrammatic analysis at tree level



$$d_{\Gamma}(m, n) = d_{\Gamma}^{(0)}(m, n) (1 + \mathcal{O}(\lambda)) + d_{\Gamma}^{(1)}(m, n) (1 + \mathcal{O}(\lambda)) + \dots$$

P-wave: Not so simple result

Also through RPI transformations

$$d_{\Gamma}^{(1)} = \frac{g}{2m} (u^{(0)})^{\dagger} \left\{ S_v^{\dagger} \Gamma^{(0)} S_v, \left[\frac{1}{v \cdot \mathcal{P}} \mathbf{q} \cdot \mathbf{B}_s \right] \right\} v^{(0)} + (u^{(0)})^{\dagger} S_v^{\dagger} \mathbf{q} \cdot \left(\Gamma^{(1)} - \frac{1}{4m} \{ \Gamma^{(0)}, \gamma \} \right) S_v v^{(0)}$$

Re-parameterization transformations

$$\psi_{\mathbf{p}}^\dagger S_v^\dagger \Gamma^{(0)} S_v \chi_{\mathbf{p}} \longrightarrow \psi_{\mathbf{p},+}^\dagger S_{v_+}^\dagger \Gamma^{(0)} S_{v_-} \chi_{\mathbf{p},-}$$

$$v_{\pm}^\mu = \left(\sqrt{1 + \frac{\mathbf{q}^2}{4m^2}}, \pm \frac{\mathbf{q}}{2m} \right)$$

$$= v^\mu + \frac{q^\mu}{2m} + O\left(\frac{q^2}{m^2}\right)$$

$$\psi_{\pm} \psi_{\mathbf{p},\pm} = \psi_{\mathbf{p},\pm}$$

$$\psi_{\pm} \chi_{\mathbf{p},\pm} = -\chi_{\mathbf{p},\pm}$$

$$\left(v + \frac{q}{2m} \right) \cdot (\mathcal{P} - gA)(S_v + \delta S_v) = 0$$

$$\psi_{\mathbf{p},\pm} = \psi_{\mathbf{p}} \pm \frac{\not{q}}{4m} \psi_{\mathbf{p}} \quad \chi_{\mathbf{p},\pm} = \chi_{\mathbf{p}} \mp \frac{\not{q}}{4m} \chi_{\mathbf{p}}$$

$$S_{v,+} = S_v + \frac{1}{2m} \frac{1}{v \cdot (\mathcal{P} - gA)} q \cdot (\mathcal{P} - gA) S_v$$

$$= S_v + \frac{1}{2m} S_v \frac{1}{v \cdot \mathcal{P}} S_v^\dagger q \cdot (\mathcal{P} - gA) S_v$$

$$= S_v - \frac{g}{2m} S_v \frac{1}{v \cdot \mathcal{P}} q \cdot B.$$

HQET:

arXiv: hep-ph/9205228 (M. E. Luke and A.V. Manohar)

SCET:

arXiv: hep-ph/0204229 (A.V. Manohar, T. Mehen, D. Pirjol and I.W. Stewart)

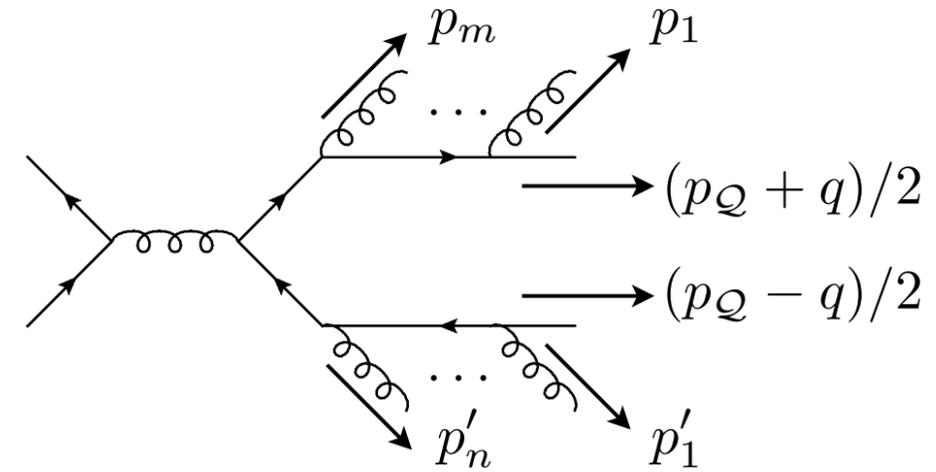
Case studies

Next in this talk

	Case-1	Case-2	Case-3
Process:	P-wave decay to light quarks	Photo-production of S-wave	TMD-Fragmentation
Partonic process:	$Q\bar{Q}[n] \rightarrow i + j + X$	$\gamma g \rightarrow Q\bar{Q}[n] + X$	$q \rightarrow Q\bar{Q}[n] + X$
Mechanisms involved:	$n \in \{^3P_J^{[1]}, ^3S_1^{[8]}\}$	$n \in \{^1S_0^{[8]}, ^3P_J^{[8]}, ^3S_1^{[1]}\}$	$n \in \{^3S_1^{[8]}, ^3S_1^{[1]}\}$
Challenge:	channel mixing	sub-leading factorization	competing the LO result
	arXiv:1910.03586 (S. Fleming, Y. Makris, and T. Mehen)	In progress: (S. Fleming, Y. Makris, T. Mehen, and J. Lieffers)	arXiv:2007.05547 (M. G. Echevarria, Y. Makris, and I. Scimemi)

Case study: P-wave Decay

Decay to light quarks nice example that exhibits operator mixing as well as color-octet modification of the CSS kernel:



S-wave octet: ${}^3S_1^{(8)}$

$$\mathcal{S}_v^{cd} \left(\psi^\dagger \sigma^i T^d \chi \right) \times \left(\bar{\chi}_{\bar{n}} \gamma^i S_{\bar{n}}^\dagger T^c S_n \chi_n \right)$$

LO matching

$$C_{q\bar{q}}({}^3S_1^{[8]}) = \alpha_s \frac{\pi}{m^2}$$

NRQCD

SCET

P-wave singlet: ${}^3P_J^{(1)}$

$$\sqrt{\frac{2}{N_c}} \times \mathcal{S}_v^{cd} \left[\frac{g B_s^{d,j}}{m v \cdot \mathcal{P}} \right] \left[\psi^\dagger \frac{\sigma^i \vec{\mathcal{P}}^j}{2\sqrt{2}N_c} \chi \right] \times \left(\bar{\chi}_{\bar{n}} \gamma^i S_{\bar{n}}^\dagger T^c S_n \chi_n \right) \longrightarrow C_{q\bar{q}}({}^3P_J^{[1]}) = C_{q\bar{q}}({}^3P_J^{[8]}) = C_{q\bar{q}}({}^3S_1^{[8]})$$

Confirmed at LO:

Factorization

$$\frac{d\Gamma}{dz_1 dz_2 d\mathbf{q}_\perp} = \Gamma_0 \sum_{ij} H_{3S_1^8}^{ij} D_{i/H_1}^\perp(b, z_1) D_{j/H_2}^\perp(b, z_2) S_{ij}^\perp(b)$$

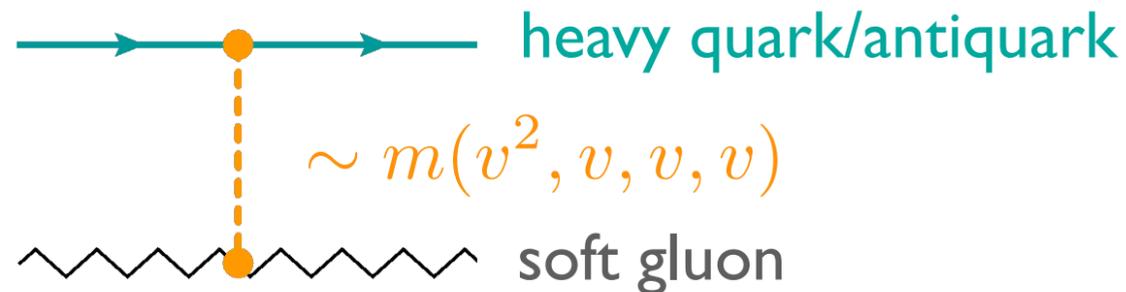
Hard function: The same for S and P wave mechanism for this process. Importance consequence for IR finiteness of the observable

Standard unsubtracted TMD fragmentation functions

Quarkonium shape functions

$$S_{ij}^\perp(b) = \sum_{n \in \{^3S_1^8, ^3P_J^1\}} S_{ij}^{[n]\perp}(b)$$

Quarkonium TMD-shape functions* encode both soft and non-perturbative quarkonium related effects. Further factorization is not possible due to Coulomb-like interactions:



*See also: arXiv:1907.06494 (M. Echevarria)

TMD-shape functions

$$\frac{d\Gamma}{dz_1 dz_2 d\mathbf{q}_\perp} = \Gamma_0 \sum_{ij} H_{3S_1^8}^{ij} D_{i/H_1}^\perp(b, z_1) D_{j/H_2}^\perp(b, z_2) S_{ij}^\perp(b)$$

Quarkonium shape functions

$$S_{ij}^\perp(b) = \sum_{n \in \{^3S_1^8, ^3P_J^1\}} S_{ij}^{[n]\perp}(b)$$

S-wave octet:

$$S_{\chi_J \rightarrow ^3S_1^{[8]}}^\perp(\mathbf{k}_\perp) = \frac{d-2}{(d-1)t_F} \text{tr} \left\langle \chi_J \left| \psi^\dagger \sigma^i T^a \chi \mathcal{S}_v^{ba} (S_{\bar{n}}^\dagger T^b S_n) \delta^{(2)}(\mathbf{k}_\perp - \mathcal{P}_\perp) \times (S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \chi^\dagger \sigma^i T^d \psi \right| \chi_J \right\rangle$$

Ultra-soft gluons (Wilson-lines)
are not shown to keep the
expressions simple

P-wave singlet:

$$S_{\chi_J \rightarrow ^3P_J}^\perp(\mathbf{k}_\perp) = (2J+1) \frac{g^2}{N_c^2 t_F} \mathcal{A}_J^{ij} \text{tr} \left\langle \chi_J \left| \psi^\dagger \sigma \cdot \overleftrightarrow{\mathcal{P}} \chi \left[\frac{B_s^{a,i}}{m v \cdot \mathcal{P}} \right] \mathcal{S}_v^{ba} (S_{\bar{n}}^\dagger T^b S_n) \right. \right. \\ \left. \left. \times \delta^{(2)}(\mathbf{k}_\perp - \mathcal{P}_\perp) (S_n^\dagger T^c S_{\bar{n}}) \mathcal{S}_v^{dc} \left[\frac{B_s^{d,j}}{m v \cdot \mathcal{P}} \right] \chi^\dagger \sigma \cdot \overleftrightarrow{\mathcal{P}} \psi \right| \chi_J \right\rangle$$

Factorization@NLO

The NLO S-wave shape function:

$$S_{\chi \rightarrow {}^3S_1^{[8]}}^{\perp, \text{NLO}}(\mathbf{k}_{\perp}; \mu, \nu) = \frac{d-2}{d-1} \left\{ \left[S_{\text{DY}}^{\perp}(\mathbf{k}_{\perp}) + \frac{\alpha_s C_A}{2\pi} \left(\frac{1}{\epsilon} \delta^{(2)}(\mathbf{k}_{\perp}) - 2\mathcal{L}_0(\mathbf{k}_{\perp}^2, \mu^2) \right) \right] \langle {}^3S_1^{[8]} \rangle_{\text{LO}} \right. \\ \left. + \delta^{(2)}(\mathbf{k}_{\perp}) \left[\frac{4\alpha_s}{3\pi m^2} \left(C_F \sum_J \langle {}^3P_J^{[1]} \rangle_{\text{LO}} + B_F \sum_J \langle {}^3P_J^{[8]} \rangle_{\text{LO}} \right) \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) \right] \right\}$$

Divergence and logarithm associated with the color octet final state. An additional term in the anomalous dimension will thus change the evolution kernel

The hard function:

$$H(\mu) = 1 - \frac{\alpha_s C_F}{2\pi} \left\{ \ln^2 \left(\frac{\mu^2}{M^2} \right) + 3 \ln \left(\frac{\mu^2}{M^2} \right) + \frac{\pi^2}{6} - 2B({}^3S_1^{[8]}) \right\} - \frac{\alpha_s C_A}{2\pi} \ln \left(\frac{\mu^2}{M^2} \right)$$

arXiv:hep-ph/9707223 (A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, M.L. Mangano)

Factorization@NLO

The NLO S-wave shape function:

$$S_{\chi \rightarrow {}^3S_1^{[8]}}^{\perp, \text{NLO}}(\mathbf{k}_{\perp}; \mu, \nu) = \frac{d-2}{d-1} \left\{ \left[S_{\text{DY}}^{\perp}(\mathbf{k}_{\perp}) + \frac{\alpha_s C_A}{2\pi} \left(\frac{1}{\epsilon} \delta^{(2)}(\mathbf{k}_{\perp}) - 2\mathcal{L}_0(\mathbf{k}_{\perp}^2, \mu^2) \right) \right] \langle {}^3S_1^{[8]} \rangle_{\text{LO}} \right. \\ \left. + \delta^{(2)}(\mathbf{k}_{\perp}) \left[\frac{4\alpha_s}{3\pi m^2} \left(C_F \sum_J \langle {}^3P_J^{[1]} \rangle_{\text{LO}} + B_F \sum_J \langle {}^3P_J^{[8]} \rangle_{\text{LO}} \right) \left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right) \right] \right\}$$

Mixing terms in the renormalization.
The same cancelation as in NRQCD.

The NLO P-wave shape function:

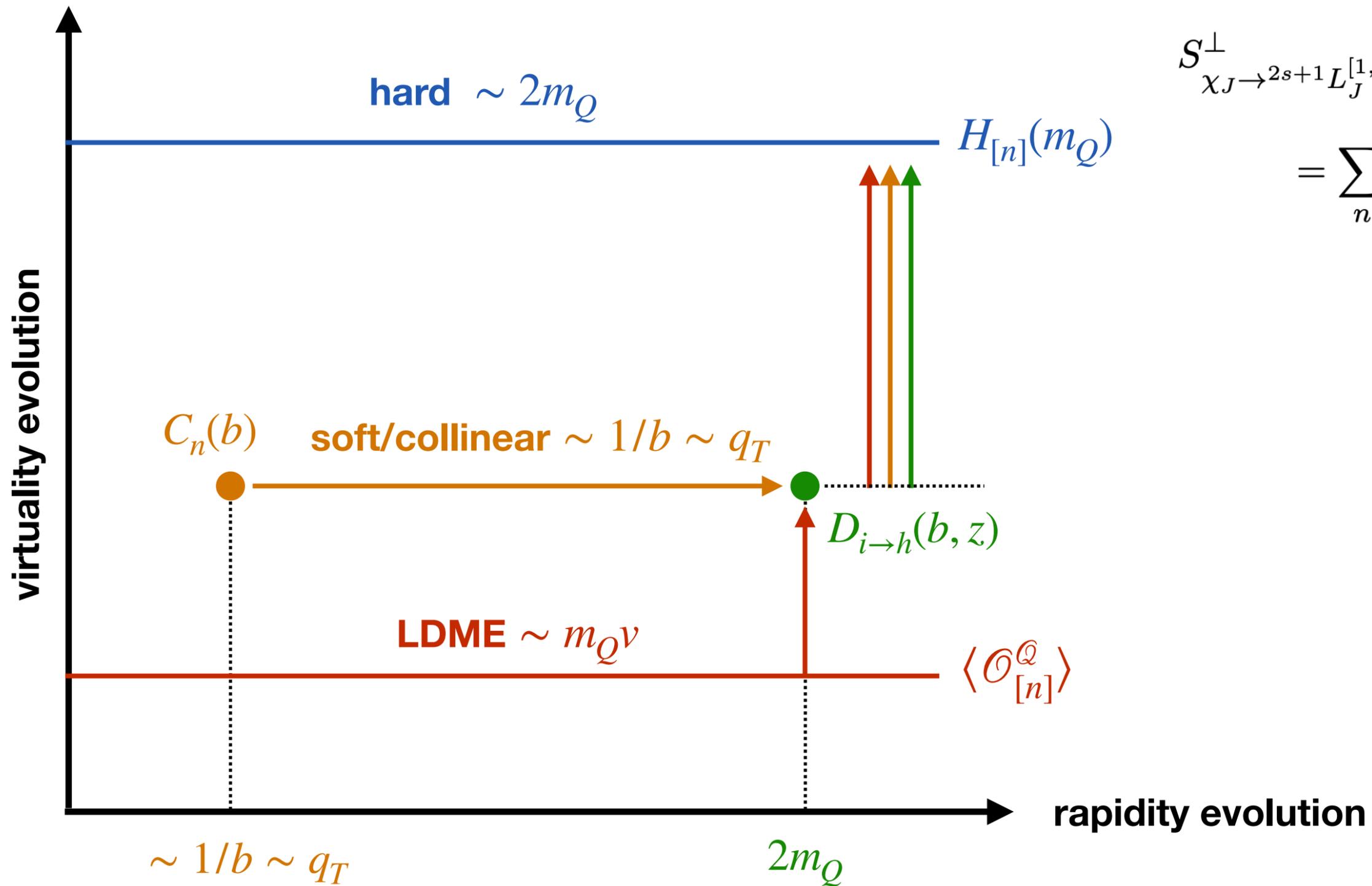
$$S_{\chi \rightarrow {}^3P_J^{[1]}}^{\perp, \text{NLO}}(\mathbf{k}_{\perp}) = \frac{8\alpha_s C_F}{9\pi m^2} \sum_J \langle {}^3P_J^{[1]} \rangle_{\text{LO}} \left(\frac{1}{\epsilon} \delta^{(2)}(\mathbf{k}_{\perp}) - 2\mathcal{L}_0(\mathbf{k}_{\perp}^2, \mu^2) + c_J \right)$$

The hard function:

$$H(\mu) = 1 - \frac{\alpha_s C_F}{2\pi} \left\{ \ln^2 \left(\frac{\mu^2}{M^2} \right) + 3 \ln \left(\frac{\mu^2}{M^2} \right) + \frac{\pi^2}{6} - 2B({}^3S_1^{[8]}) \right\} - \frac{\alpha_s C_A}{2\pi} \ln \left(\frac{\mu^2}{M^2} \right)$$

arXiv:hep-ph/9707223 (A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, M.L. Mangano)

2-dimensional RG evolution



$$S_{\chi_J \rightarrow 2s+1 L_J^{[1,8]}}^\perp(\mathbf{k}_\perp; \mu_0, \nu_0)$$

$$= \sum_n C_n(\mathbf{k}_\perp; \mu_0, \nu_0, \mu) \times \langle \mathcal{O}^{[n]} \rangle_{\chi_J}^\mu$$

Summary

- Presence of TMD-shape functions reveals new NP-effects unique to quarkonium production
- New TMD evolution associated with the color-octet channels
- Operator mixing is necessary for the IR-safe definition of quarkonium production/decay of P-waves

Factorization in NR-SCET

Quarkonium at the kinematic end-point:

arXiv:hep-ph/9705286 (M. Beneke, I. Z. Rothstein, and M. B. Wise)
arXiv:hep-ph/0211303 (S. Fleming and A. K. Leibovich)
arXiv:hep-ph/010631 (C. W. Bauer, C-W Chiang, S. Fleming, A. K. Leibovich, and I. Low)
arXiv:hep-ph/0306139 (S. Fleming, A. K. Leibovich, and T. Mehen)
arXiv:hep-ph/0607121 (S. Fleming, A. K. Leibovich, and T. Mehen)
arXiv:0705.3230 (A. K. Leibovich and X. Liu)

Dark matter bound-state decay spectrum (NRDM-SCET):

arXiv:1409.4415 (M. Baumgart, I. Z. Rothstein, and V. Vaidya)
arXiv:1409.7392 (M. Bauer, T. Cohen, R. J. Hill, and M. P. Solon)
arXiv:1409.8294 (G. Ovanessian, T. R. Slatyer, and I. W. Stewart)
arXiv:1412.8698 (M. Baumgart, I. Z. Rothstein, and V. Vaidya)
arXiv:1510.02470 (M. Baumgart and V. Vaidya)
arXiv:1612.04814 (G. Ovanessian, N. L. Rodd, T. R. Slatyer, and I. W. Stewart)
arXiv:1712.07656 (M. Baumgart, T. Cohen, I. Moulton, N. L. Rodd, T. R. Slatyer, P. Solon, I. W. Stewart, and V. Vaidya)
arXiv:1808.04388 (L. Rinchuso, N. L. Rodd, I. Moulton, E. Moulin, M. Baumgart, T. Cohen, T. R. Slatyer, I. W. Stewart, and V. Vaidya)
arXiv:1808.08956 (M. Baumgart, T. Cohen, E. Moulin, I. Moulton, L. Rinchuso, N. L. Rodd, T. R. Slatyer, I. W. Stewart, and V. Vaidya)