NNLO Radiative Corrections to Charmonium Exclusive Production at B factories

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based on arXiv:1901.08447 & arXiv:2008.04898

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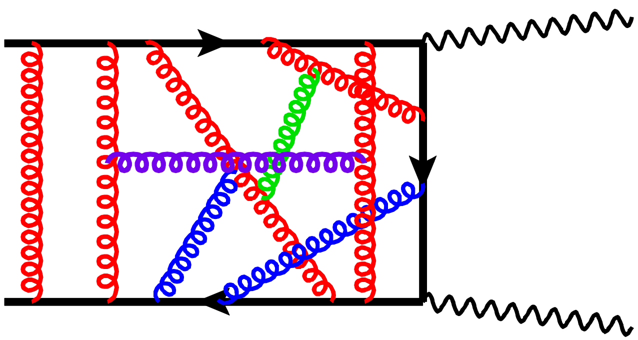
Outline

- Brief Introduction
- study on $e^+e^- \to \chi_{cJ} + \gamma$
- study on $e^+e^- \to J/\psi + \eta_c$
- Summary

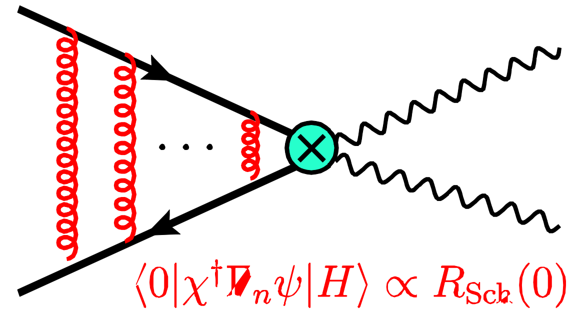
NRQCD factorization formalism

Bodwin, Braaten, Lepage, PRD (1995)

Quarkonium is a QCD bound state involving several distinct scales



$\longrightarrow \quad \sum_n C_n(m) \times$



$\langle 0|\chi^\dagger \mathcal{K}_n \psi|H\rangle \propto R_{\rm Sch}(0)$

Short–distance coefficient	*wave function at the origin*
perturbatively calculable	*nonperturbative yet universal*

Separate the short-distance effect and long-distance dynamics

Asymptotic freedom: $\alpha_s(m) \ll 1$, one can invoke perturbation theory

NRQCD is the mainstream tool in studying quarkonium (see Brambilla et al. EPJC 2011 for a review)

Nowadays, NRQCD becomes standard approach to tackle various quarkonium production and decay processes:

charmonium: $v^2/c^2 \sim 0.3$ not truly non-relativistic to some extent

bottomonium: $v^2/c^2 \sim 0.1$ a better "non-relativistic" system

Exemplified by

$e^+e^- \to J/\psi + \eta_c$ at B factories (**exclusive charmonium production**)

Unpolarized/polarized J/ψ production at hadron colliders (**inclusive**)

Very active field in recent years (Chao's group, Kniehl's group, Bodwin's group, Qiu's group, Wang's group, …)

The ubiquitous symptom of NRQCD factorization: often plagued with huge QCD radiative correction

Table 1: Quarkonium energy scales

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
$\alpha_s(M)$	0.35	0.22	0.11
$\alpha_s(Mv)$	0.52	0.35	0.16
$\alpha_s(Mv^2)$	$\gg 1$	$\gg 1$	0.35

Most of the NRQCD successes based on the NLO QCD predictions.

However, the NLO QCD corrections are often large:

$e^+e^- \to J/\psi + \eta_c$ K factor: $1.8 \sim 2.1$ Zhang *et.al.*

$e^+e^- \to J/\psi + J/\psi$ K factor: $-0.31 \sim 0.25$ Gong *et.al.*

$p + p \to J/\psi + X$ K factor: ~ 2 Campbell *et.al.*

$J/\psi \to \gamma\gamma\gamma$ K factor: ≤ 0 Mackenzie *et.al.*

… …

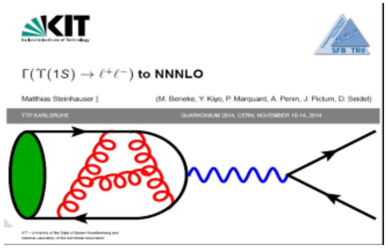
The existing NNLO corrections to quarkonium decay and production

1. $J/\psi \to \gamma^* \to \ell\ell$

NNLO corrections were first computed by two groups in 1997:

Czarnecki and Melkinov; Beneke, Smirnov, and Signer;

N^3LO corrections available recently: Steinhausser et al. (2013)



2. $\eta_c \to 2\gamma$

NNLO correction was computed by Czarnecki and Melkinov (2001) : (neglecting light-by-light diagrams)；

Feng, Jia, Sang (2017): (including the light-by-light diagrams)

3. $B_c \to \ell\nu$

NNLO correction computed by Onishchenko, Veretin (2003);

Chen and Qiao (2015)

The existing NNLO corrections to quarkonium decay and production

4. $\chi_{cJ} \to 2\gamma$

NNLO corrections were available by our group (arXiv: 1511.06288)

5. $\eta_{c2}(^1D_2) \to 2\gamma$

NNLO corrections were available very recently (arXiv: 2010.14364)

6. $\eta_c \to \mathrm{LH}$

NNLO correction was computed by our group (arXiv: 1707.05758)

The existing NNLO corrections to quarkonium decay and production

7. the electromagnetic form factor of $\gamma\gamma^* \to \eta_c$

NNLO corrections were available (arXiv: 1505.02665)

8. $e^+e^- \to \eta_c + \gamma$

NNLO corrections were obtained very recently by Chen, Liang and Qiao (arXiv: 1710.07865)

Perturbative convergence of some processes appears to be rather poor for some process

$$\text{with } \mu_R = \mu_\Lambda = m_c$$

$$\Gamma(J/\psi \to \ell\ell) = \Gamma^{(0)}\left[1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41\, n_f)\left(\frac{\alpha_s}{\pi}\right)^2\right]^2$$

$$+(-2091 + 120.66\, n_f - 0.82\, n_f^2)\left(\frac{\alpha_s}{\pi}\right)^3$$

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma^{(0)}\left[1 - 1.69\frac{\alpha_s}{\pi} - (55.39 + 0.38 n_f)\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]^2$$

$e^+e^- \to \eta_c + \gamma$ at B factories

Chen, Liang, Qiao, JHEP(2018)

NLO=LO+$\mathcal{O}(\alpha_s)$

NNLO=LO+$\mathcal{O}(\alpha_s)$+$\mathcal{O}(\alpha_s^2)$

σ(fb)	LO	NLO	NNLO
$\eta_c(1.4)$	89.7	75.2	44.6
$\eta_c(1.5)$	82.8	68.5	45.2
$\eta_b(4.7)$	2.50	1.77	1.75
$\eta_b(4.8)$	2.07	1.47	1.46

The NLO & NNLO corrections are considerable, however not so huge! So the convergence may be not so worse.

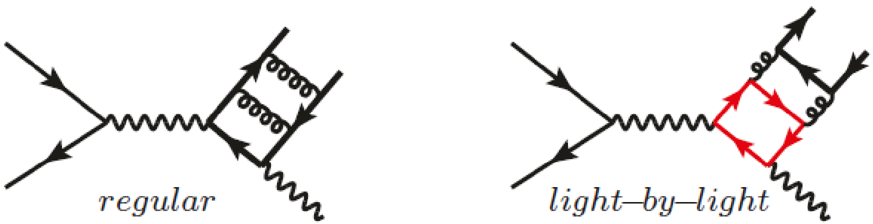
More quarkonium involved processes may need to be studied at higher order to testify the convergence of the perturbative expansion and check the predictive power of NRQCD.

The main steps used in our numerical computation

- Feynman Diagrams && Amplitudes (Packages: **FeynArts / QGraf**)
- Trace && Contraction (Packages: **FeynCalc / FormLink / self-writing functions**)
- Partial Fraction && IBP Reduction (Packages: **Apart / FIRE C++**)
- Master Integrals by Sector Decomposition (Packages: **FIESTA / self-writing functions**)
- Numerical Integration (Packages: **Cubpack / HCUBATURE**)
- Other Processing – Plots etc. (**Mathematica**)

Feynman Diagrams

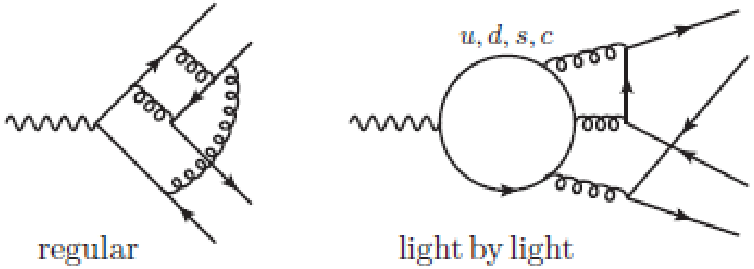
- Feynman Diagrams && Amplitudes (Package: **FeynArts / QGraf**)



NNLO

There are about **120** two loop Feynman diagrams for

$$e^+e^- \to \chi_{cJ} + \gamma$$



c) NNLO

and around **2000** two loop Feynman diagrams for

$$e^+e^- \to J/\psi + \eta_c$$

study on $e^+e^- \to \chi_{cJ} + \gamma$

Experimental data by BELLE collaboration (Phys. Rev. D98, 092015 (2018))

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) \quad = \quad (17.3^{+4.2}_{-3.9}(\text{stat.}) \pm 1.7(\text{syst.}))fb \;\; @\sqrt{s} = 10.58\text{GeV}$$

However, **no** significant excesses for χ_{c0} and χ_{c2}.

OBSERVATION OF $e^+e^- \to \gamma\chi_{c1}$ ...

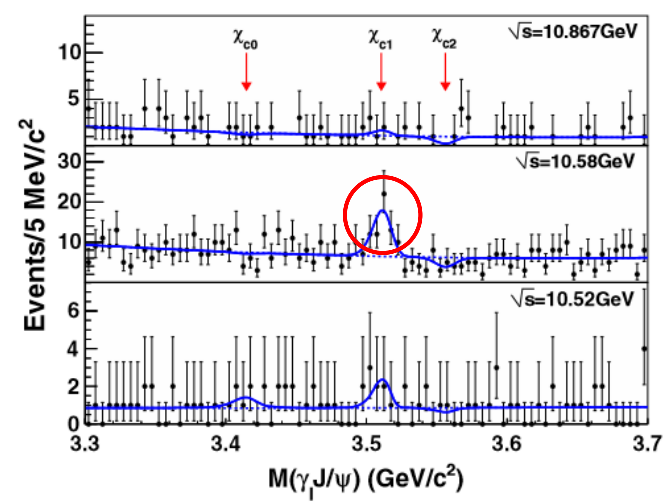


FIG. 2. The $\gamma_l J/\psi$ invariant mass spectra at $\sqrt{s} = 10.52$ (bottom), 10.58 (middle), and 10.867 GeV (top) together with fit results. The points with error bars show the data and the solid curves are the fit functions; the dashed curves show the fitted backgrounds contributions. The arrows show the expected peak positions for the χ_{c0}, χ_{c1}, and χ_{c2} states.

Theoretical prediction based on NRQCD factorization formalism arXiv:2008.04898

NRQCD factorization formula

$$\sigma(\chi_{cJ}+\gamma) \quad = \quad F_1({}^3P_J)\langle\mathcal{O}({}^3P_J)\rangle + \mathcal{O}(\sigma v^2)$$

where

$$\langle\mathcal{O}({}^3P_J)\rangle \equiv \left|\langle\chi_{cJ}|\psi^\dagger\mathcal{K}_{{}^3P_J}\chi|0\rangle\right|^2,$$

$$\mathcal{K}_{{}^3P_0} = \frac{1}{\sqrt{3}}\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma}\right), \quad (1)$$

$$\mathcal{K}_{{}^3P_1} = \frac{1}{\sqrt{2}}\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma}\right), \quad (2)$$

$$\mathcal{K}_{{}^3P_2} = -\frac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}. \quad (3)$$

$$F_1({}^3P_J) = F_1^{(0)}({}^3P_J)\left(1 + c_1\frac{\alpha_s}{\pi} + c_2\frac{\alpha_s^2}{\pi^2} + \cdots\right)$$

Hoang, Ruiz-Femenia (vNRQCD/RGE)
Phys. Rev. D74,114016 (2006)

Theoretical prediction based on NRQCD factorization formalism

$$\gamma_{\chi_{c0}} = -\pi^2\left(\frac{C_A C_F}{6} + \frac{2C_F^2}{3}\right)$$
$$\gamma_{\chi_{c1}} = -\pi^2\left(\frac{C_A C_F}{6} + \frac{5C_F^2}{12}\right)$$
$$\gamma_{\chi_{c2}} = -\pi^2\left(\frac{C_A C_F}{6} + \frac{13C_F^2}{60}\right)$$

By taking charm pole mass: m=1.4 GeV

$$F(\chi_{c0}) = F^{(0)}(\chi_{c0})\left\{1 + \frac{\alpha_s}{\pi}(1.9332) + \frac{\alpha_s^2}{\pi^2}\left[\frac{1}{4}\beta_0 \ln\frac{\mu_R^2}{4m^2}(1.9332) + \gamma_{\chi_{c0}} \ln\frac{\mu_\Lambda^2}{m^2} + \left(0.867143(3)n_H - 1.6338020(7)n_L + 5.17(4)\mathrm{lbl} - 9.020(3)\right)\right]\right\},$$

$$F(\chi_{c1}) = F^{(0)}(\chi_{c1})\left\{1 + \frac{\alpha_s}{\pi}(-3.1597) + \frac{\alpha_s^2}{\pi^2}\left[\frac{1}{4}\beta_0 \ln\frac{\mu_R^2}{4m^2}(-3.1597) + \gamma_{\chi_{c1}} \ln\frac{\mu_\Lambda^2}{m^2} + \left(3.7950(1)\times 10^{-2}n_H - 0.5954237(4)n_L - 4.191(3)\mathrm{lbl} - 17.337(2)\right)\right]\right\},$$

$$F(\chi_{c2}) = F^{(0)}(\chi_{c2})\left\{1 + \frac{\alpha_s}{\pi}(-9.0312) + \frac{\alpha_s^2}{\pi^2}\left[\frac{1}{4}\beta_0 \ln\frac{\mu_R^2}{4m^2}(-9.0312) + \gamma_{\chi_{c2}} \ln\frac{\mu_\Lambda^2}{m^2} + \left(2.205168(2)n_H + 4.1844189(5)n_L + 3.456(3)\mathrm{lbl} - 60.504(2)\right)\right]\right\}$$

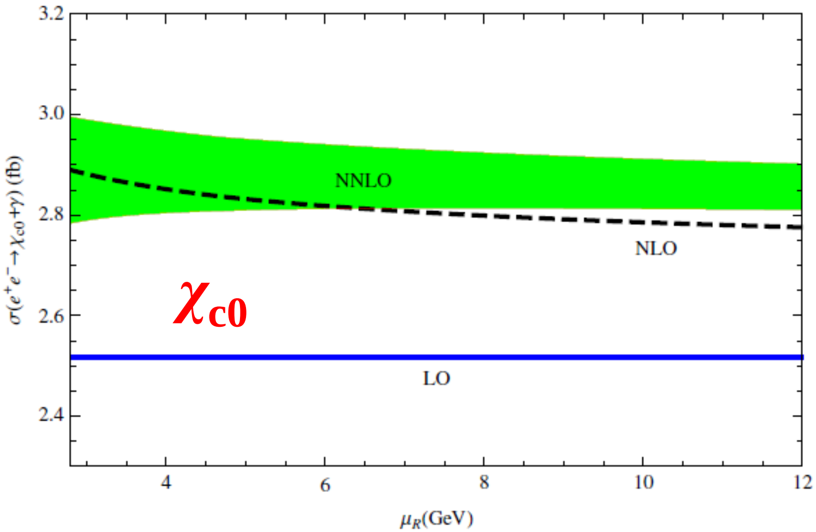
Theoretical prediction based on NRQCD factorization formalism

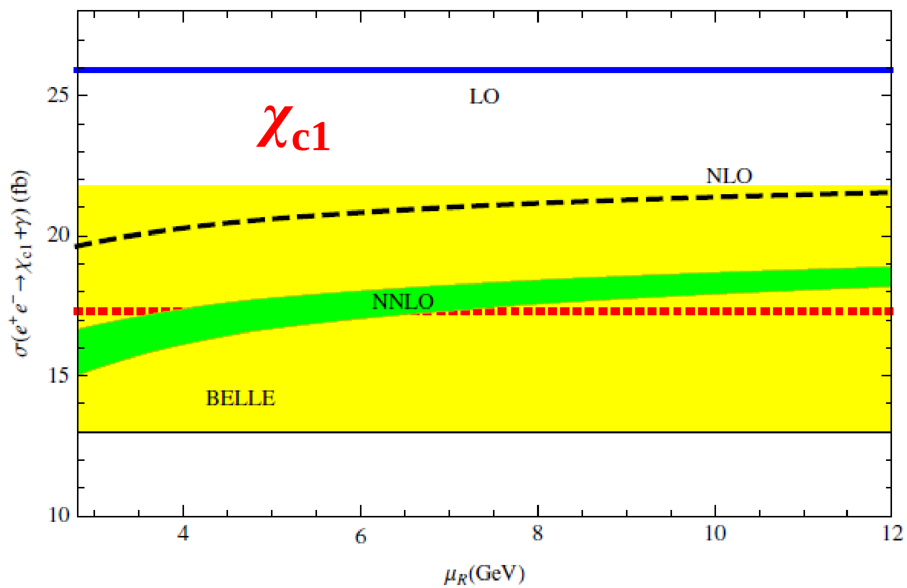
Table 1: NRQCD predictions to $\sigma(\chi_{cJ}+\gamma)$ at various levels of accuracy in α_s at B factory. The LDME $\langle\mathcal{O}(^3P_J)\rangle = 0.107\ \text{GeV}^5$ is taken from Buchmüller-Tye (BT) potential model. The errors are estimated by sliding the renormalization scale μ_R from $2m$ to $\sqrt{s}$. NLO=LO+$\mathcal{O}(\alpha_s)$
NNLO=LO+$\mathcal{O}(\alpha_s)$+$\mathcal{O}(\alpha_s^2)$

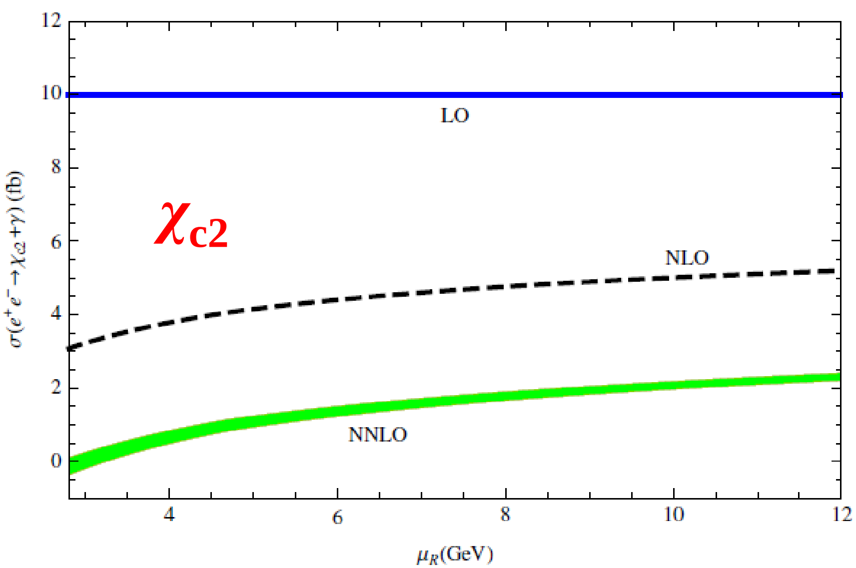
$\sigma(e^+e^- \to \chi_{c1}+\gamma) = (17.3^{+4.2}_{-3.9} \pm 1.7)\text{fb}$ @$BELLE$

$m = 1.40$ GeV			$\mu_\Lambda = 1$ GeV	$\mu_\Lambda = m$
σ (fb) Order / χ_{cJ}	LO	NLO	NNLO	NNLO
$\chi_{c0}+\gamma$	2.52	$2.83^{+0.06}_{-0.04}$	$2.96^{+0.05}_{-0.04}$	$2.82^{+0.01}_{-0.03}$
$\chi_{c1}+\gamma$	25.96	$20.72^{+0.75}_{-1.05}$	$17.91^{+0.89}_{-1.21}$	$16.83^{+1.20}_{-1.79}$
$\chi_{c2}+\gamma$	10.02	$4.24^{+0.83}_{-1.16}$	$1.34^{+0.92}_{-1.23}$	$1.03^{+1.01}_{-1.40}$
$m = 1.68$ GeV				
$\chi_{c0}+\gamma$	1.18	$1.39^{+0.03}_{-0.03}$	$1.48^{+0.03}_{-0.03}$	$1.38^{+0.01}_{-0.01}$
$\chi_{c1}+\gamma$	15.98	$12.25^{+0.54}_{-0.50}$	$10.87^{+0.45}_{-0.37}$	$9.84^{+0.75}_{-0.72}$
$\chi_{c2}+\gamma$	6.60	$2.84^{+0.54}_{-0.50}$	$1.03^{+0.58}_{-0.52}$	$0.71^{+0.67}_{-0.63}$

The results explain why the other two states are not observed !







NRQCD predictions for the cross sections of $\chi_{cJ}+\gamma$ as a function of μ_R at various levels of accuracy in α_s with ***m*=1.4** GeV

The uncertainty in the theoretical prediction corresponds to the change of μ_Λ from 1 GeV to ***m***. We did not consider the uncertainties from the input parameters.

Some drawbacks in our study

1. We did not include the relativistic corrections, where the LDMEs are of relatively large uncertainties.

Brambilla, Chen, Jia, Shtabovenko, Vairo.
Phys. Rev. D 97 (2018) 096001

2. The results strongly depend on the charm quark mass and NRQCD matrix element (obtained from potential models)!

Study on $e^+e^- \to J/\psi + \eta_c$

Experimental data by BELLE & BABAR (very important measurment)

(PRL89,142001(2002), PRD70, 071102 (2004), PRD72, 031101 (2005))

$$\begin{aligned}
\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>4} &= 33^{+7}_{-6} \pm 9 \text{ fb} \quad \text{@BELLE}, \\
\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} &= 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad \text{@BELLE}, \\
\sigma(e^+e^- \to J/\psi + \eta_c) \times \mathcal{B}_{>2} &= 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad \text{@BABAR},
\end{aligned}$$

where $\mathcal{B}_{>2}$ signifies that branching fraction of η_c decay into the final states with more than 2 charged tracks

Study on $e^+e^- \to \gamma^* \to J/\psi + \eta_c$

arXiv:1901.08447

Time-like electromagnetic (EM) form factor (Lorentz invariant & P-conservation)

$$\langle J/\psi(P_1,\lambda) + \eta_c(P_2)|J^{\mu}_{\rm EM}|0\rangle = i\,F(s)\,\epsilon^{\mu\nu\rho\sigma}P_{1\nu}P_{2\rho}\varepsilon^*_{\sigma}(\lambda),$$

The cross section reads

$$\sigma[e^+e^- \to J/\psi + \eta_c] = \frac{4\pi\alpha^2}{3}\left(\frac{|\mathbf{P}|}{\sqrt{s}}\right)^3 |F(s)|^2$$

The form factor can be factorized as

$$\begin{aligned} F(s) &= \sqrt{4M_{J/\psi}M_{\eta_c}}\langle J/\psi|\psi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}\chi|0\rangle\langle\eta_c|\psi^\dagger\chi|0\rangle \\ &\times \left[f + g_{J/\psi}\langle v^2\rangle_{J/\psi} + g_{\eta_c}\langle v^2\rangle_{\eta_c} + \cdots\right], \end{aligned}$$

Braaten, Lee, PRD(2003) , Liu, He, Chao, PLB(2003) Zhang, Gao, Chao, PRL(2006) , He, Fan, Chao, PRD(2007), Bodwin, Lee, Yu, PRD(2008), Gong, Wang, PRD(2008), Dong, Feng, Jia, PRD(2012) … …

$$\begin{aligned} f &= f^{(0)} + \frac{\alpha_s}{\pi} f^{(1)} + \frac{\alpha_s^2}{\pi^2} f^{(2)} + \cdots \\ g_H &= g_H^{(0)} + \frac{\alpha_s}{\pi} g_H^{(1)} + \cdots \end{aligned}$$

Study on $e^+e^- \to J/\psi + \eta_c$

$$\gamma_{J/\psi} = -\pi^2\left(\frac{C_A C_F}{4} + \frac{C_F^2}{6}\right)$$

$$\gamma_{\eta_c} = -\pi^2\left(\frac{C_A C_F}{4} + \frac{C_F^2}{2}\right)$$

Our main result

$$f^{(2)} = f^{(0)}\left\{\frac{\beta_0^2}{16}\ln^2\frac{s}{4\mu_R^2} - \left(\frac{\beta_1}{16} + \frac{1}{2}\beta_0 \hat{f}^{(1)}\right)\ln\frac{s}{4\mu_R^2} + (\gamma_{J/\psi} + \gamma_{\eta_c})\ln\frac{\mu_\Lambda^2}{m^2} + \mathsf{F}(r)\right\},$$

$$\mathrm{Re}\,\mathsf{F}(r = \frac{4m^2}{s}) = -25 \pm 4 \quad @\, m = 1.4\ \mathrm{GeV},$$

$$\mathrm{Re}\,\mathsf{F}(r = \frac{4m^2}{s}) = -21 \pm 5 \quad @\, m = 1.68\ \mathrm{GeV},$$

F is insensitive to the charm mass!

Study on $e^+e^- \to J/\psi + \eta_c$

Selection of LDMEs

Bodwin,Chung, Kang, Lee,Yu, Phys. Rev. D77, 094017 (2008)

$$\begin{aligned}
\langle \mathcal{O} \rangle_{J/\psi} &= \left| \langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \right|^2 = 0.440 \, \text{GeV}^3, \\
\langle \mathcal{O} \rangle_{\eta_c} &= \left| \langle \eta_c | \psi^\dagger \chi | 0 \rangle \right|^2 = 0.437 \, \text{GeV}^3, \\
\langle v^2 \rangle_{J/\psi} &= 0.441 \, \text{GeV}^2/m^2, \\
\langle v^2 \rangle_{\eta_c} &= 0.442 \, \text{GeV}^2/m^2
\end{aligned}$$

Study on $e^+e^- \to J/\psi + \eta_c$

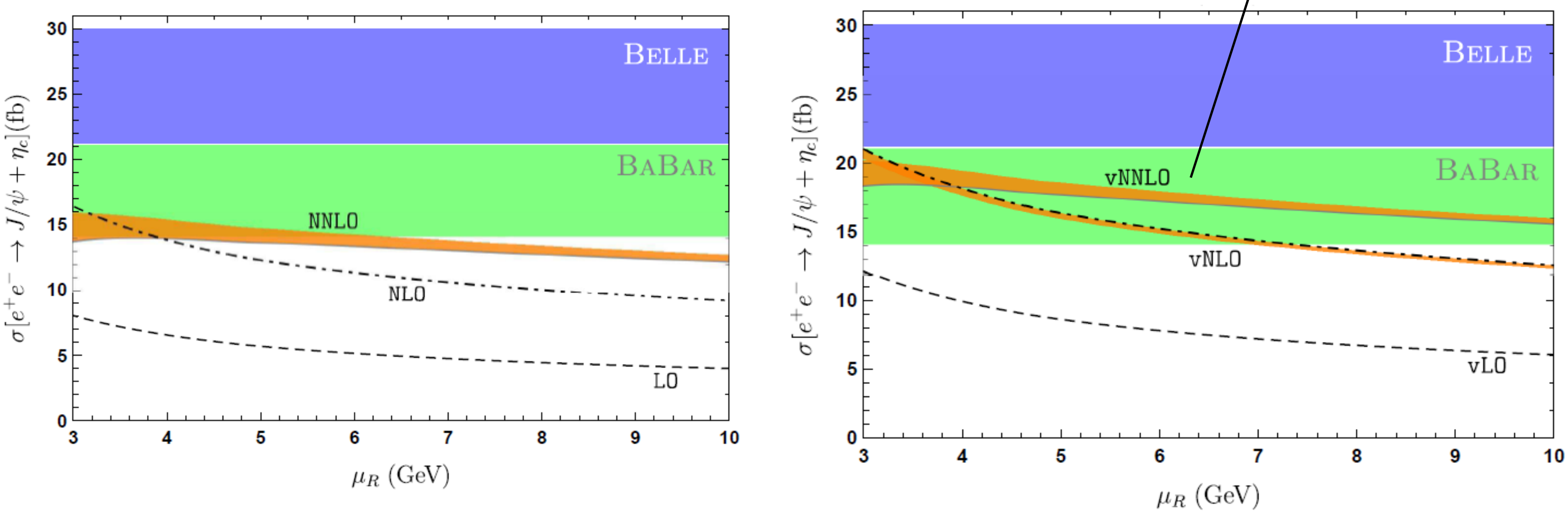
Table 1: We fix $\mu_\Lambda = m$. The two upper rows and the two lower rows correspond to $m = 1.4$ GeV and $m = 1.68$ GeV, respectively.

μ_R	LO	$\mathcal{O}(v^2)$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s v^2)$	$\mathcal{O}(\alpha_s^2)$	Total	$17.6 \pm 2.8^{+1.5}_{-2.1}$ fb @BABAR
$2m$	8.48	4.36	8.64	0.34	$-3.7(5)$	18.1(5)	m=1.4 GeV
$\frac{\sqrt{s}}{2}$	5.52	2.84	6.48	1.18	1.6(2)	17.6(2)	
$2m$	5.59	1.44	4.71	-0.33	$-1.4(4)$	10.0(4)	m=1.68 GeV
$\frac{\sqrt{s}}{2}$	4.16	1.07	4.08	0.06	0.7(2)	10.1(2)	

1. The 2-loop corrections are smaller than the 1-loop corrections;
2. By including all the corrections, theoretical predictions with *m*=1.4 GeV agree with BABAR measurement;
3. The prediction is insensitive to renormalization scale.

$vLO=LO+\mathcal{O}(v^2)$
$vNLO=NLO+\mathcal{O}(v^2)+\mathcal{O}(\alpha_s v^2)$
$vNNLO=NNLO+\mathcal{O}(v^2)+\mathcal{O}(\alpha_s v^2)$

Study on $e^+e^- \to J/\psi + \eta_c$



We take $m = 1.4$ GeV. The brown bands represents the uncertainty due to varying μ_Λ from 1 to m. The left panel only includes the perturbative correction, while the right panel also includes the $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s v^2)$ corrections

Summary

- ✓ We computed the NNLO radiative corrections to $\chi_{cJ}+\gamma$ and $J/\psi+\eta_c$ productions at B factories. We found the $O(\alpha_s^2)$ corrections are smaller than the $O(\alpha_s)$ corrections. So the perturbative expansion may seem to be better.

- ✓ When taking the LDME from BT potential model and m_c=1.4 GeV, the NNLO predictions to $\chi_{c1}+\gamma$ agree with Belle measurement.

- ✓ Up to NNLO, the cross section of $J/\psi+\eta_c$ productions exhibit a much flatter μ_R dependence. By using the LDMEs fitted from experiment and m_c=1.4 GeV, the theoretical prediction (including radiative and relativistic corrections) is consistent with BABAR experiment.

- ✓ The cross sections are sensitive to the charm mass.

Thank you!

Backup Slides

The techniques used in our numerical computation

- Some subtlety in Sector Decomposition

$$\mathcal{F}(a_1, \cdots, a_n) = \int \cdots \int \frac{d^d k_1 \cdots d^d k_l}{E_1^{a_1} \cdots E_n^{a_n}}$$

where k_i are the loop momenta and the denominators E_i are either quadratic or linear with respect to the loop momenta k_i of the graph.

Perform Feynman parametrization and integrate over the loop momenta

$$\mathcal{F} = (i\pi^{d/2})^l \frac{\Gamma(A - ld/2)}{\Pi_{j=1}^n \Gamma(a_j)} \int_{x_j \geq 0} dx_1 \cdots dx_n \delta(1 - \sum x_i)(\Pi x_j^{a_j - 1}) \frac{U^{A-(l+1)d/2}}{(F - i0^+)^{A - ld/2}}$$

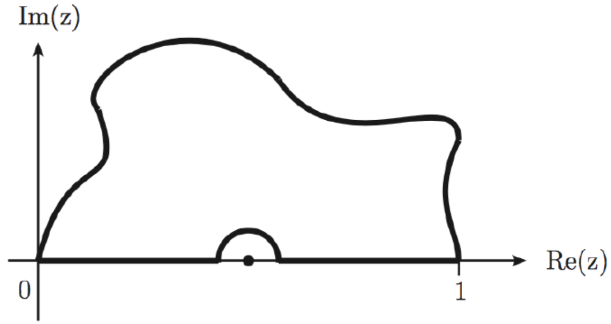
The singularity in the endpoints can be treated by sector decomposition. However *F* **may dispear at some intermediate *x* points** !

Backup Slides

The techniques used in our numerical computation

- **Contour Deformation:**

Deformation of the integration contour



making use of Cauchy's theorem to avoid the poles on the real axis by a **deformation of the integration contour** into the complex plane

suggested by **Borowka and Heinrich** in **arXiv: 1209.6345**

$$\vec{z} = \vec{x} - i\vec{\tau}(\vec{x}), \qquad \tau_k = \lambda_k x_k (1 - x_k) \frac{\partial F}{\partial x_k}$$

$$F(\vec{z}(\vec{x})) = F(\vec{x}) - i0^+ - i\lambda_k \sum_j x_j (1 - x_j) (\frac{\partial F}{\partial x_j})^2 + \mathcal{O}(\lambda^2)$$

Backup Slides

The MIs hard to compute!