QWG 2021 - The 14th International Workshop on Heavy Quarkonium

Quarkonium fragmentation function and test of NRQCD factorization

Peng Zhang
Peking University

March 18, 2021

PEKING UNIVERSITY

Outline

- Introduction
- Test of NRQCD factorization
- Discussion
- Summary

NRQCD factorization

- Factorization formula

Bodwin, Braaten, Lepage, *Phys. Rev.* **D51** (1995) 1125-1171

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}+X}\langle \mathcal{O}^H(n)\rangle$$

  - Long-distance matrix elements (LDMEs)
    - lattice, potential model, experiment
  - Short-distance coefficients (SDCs)
    - matching from the Full QCD calculation
- Phenomenological success
  - IR divergence in NLO calculation of P wave
  - Excess production of $\psi(2S)$
- Proof of NRQCD factorization
  - proved all orders for decays
  - proved to two-loop for production under eikonal approximation

Nayak, Qiu, *Sterman*. *Phys.Rev. D* **72** (2005) 114012
Bodwin, Chung, Ee, Kim, Lee, *Phys.Rev. D* **101** (2020) 096011

Fragmentation Function (FF)

- QCD Collinear Factorization

Collins, Soper, Sterman, *Adv. Ser. Direct. High Energy Phys.* **5** (1989) 1-91

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}\left(\frac{p_T}{z},\mu\right) \otimes D_{i\to H}(z,\mu) + \mathcal{O}\left(\frac{1}{p_T^2}\right)$$

fragmentation function (FF)

- Definition

$$D_{g\to H}(z,\mu_0) = \frac{-g_{\mu\nu} z^{D-3}}{2\pi P_c^+ (N_c^2-1)(D-2)} \int_{-\infty}^{+\infty} \mathrm{d}x^- e^{-iP_c^+ x^-} \times \langle 0|G_c^{+\mu}(0)\mathcal{E}^\dagger(0,0,\mathbf{0}_\perp)_{cb}\mathcal{P}_{H(P)}\mathcal{E}(0,x^-,\mathbf{0}_\perp)_{ba}G_a^{+\nu}(0,x^-,\mathbf{0}_\perp)|0\rangle$$

  - $\mathcal{E}(0,x^-,\mathbf{0}_\perp)_{ba}$ denotes eikonal operator

- Apply NRQCD to FF

$$D_{i\to H}(z,\mu_0) = \sum_n d_n(z,\mu_0,\mu_f)\langle \mathcal{O}^H(n)\rangle$$

  - SDCs: IR-safe
  - LDMEs:
    - absorb all IR divergences
    - independent of eikonal-line direction
  - IR divergence in full-QCD calculation of FF is independent of eikonal-line direction

FF of $g \to Q\bar{Q}(^3P_J^{[1/8]}) + X$

- LO: IR divergence in FF can be absorbed by LDME
- NLO: IR structure?
- Squared amplitude for FF at $\mathcal{O}(q^2)$

$$\mathcal{A}(q,q') = (q\cdot q')\mathcal{A}_1 + \frac{(q\cdot n)(q'\cdot n)}{(P\cdot n)^2}\mathcal{A}_2$$

  - All IR divergences must be absorbed by LDMEs
  - LDMEs are independent of eikonal direction n
  - If NRQCD factorization is to hold, $\mathcal{A}_2$ is free of IR divergence
- FF :

$$\int d\Phi\, I^{\beta\beta'} \frac{\partial}{\partial q^\beta \partial q'^{\beta'}} \mathcal{A}(q,q')$$

- Projection operator:

$$I^{\beta\beta'} = -g^{\beta\beta'} + \frac{P^\beta P^{\beta'}}{P^2}$$

- Mix of $\mathcal{A}_1$ and $\mathcal{A}_2$

FF of $g \to Q\bar{Q}(^3P_J^{[1/8]}) + X$

- Extract $\mathcal{A}_2$ only
- use projection operator:

$$P^{\beta\beta'} = (D-2)I^{\beta\beta'} - (D-1)d^{\beta\beta'}$$

where

$$d^{\beta\beta'} = -g^{\beta\beta'} + \frac{P^\beta n^{\beta'} + n^\beta P^{\beta'}}{P\cdot n} - \frac{P^2 n^\beta n^{\beta'}}{(P\cdot n)^2}$$

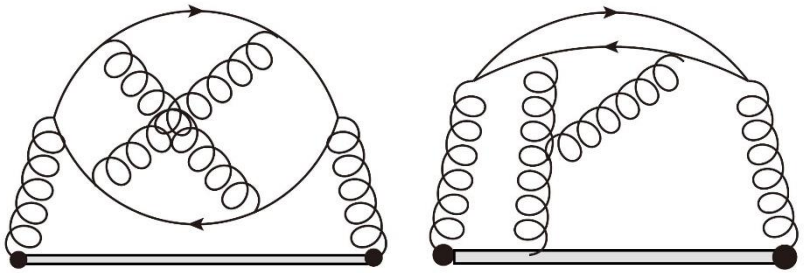
then

$$P^{\beta\beta'}\frac{\partial}{\partial q^\beta \partial q'^{\beta'}}\mathcal{A}(q,q') = (D-2)\frac{1}{P^2}\mathcal{A}_2$$

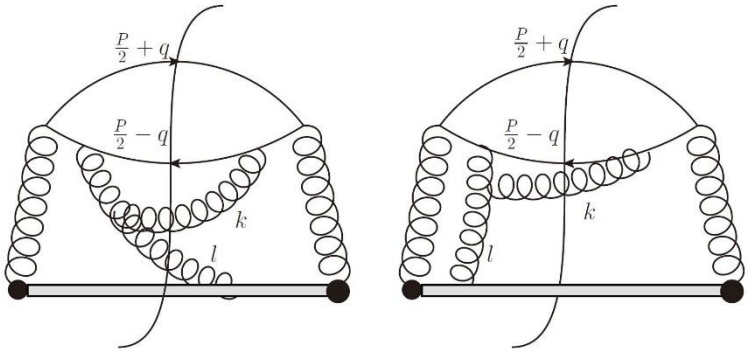
- same computation method for FFs
  - use $I^{\beta\beta'}$ to get the full-QCD FFs
  - use $P^{\beta\beta'}$ to get the part proportional to $\mathcal{A}_2$

Typical Feynman diagrams

- Two typical cut diagrams:



- Cut method (take the last diagram as an example)



- Real SDCs:

$$\int \mathrm{d}\Phi_{\text{real}} \prod_i \frac{1}{E_i^{a_i}} = \frac{P\cdot n}{2z^2} \int \frac{\mathrm{d}^D k}{(2\pi)^{D-1}} \frac{\mathrm{d}^D l}{(2\pi)^{D-1}} \delta_+(k^2)\delta_+(l^2)\delta\left(k\cdot n + l\cdot n - \frac{1-z}{z} P\cdot n\right) \prod_i \frac{1}{E_i^{a_i}}$$

propagator denominators

- Virtual SDCs:

$$\int \mathrm{d}\Phi_{\text{loop}} \int \frac{\mathrm{d}^D l}{(2\pi)^D} \prod_i \frac{1}{F_i^{a_i}} = \frac{P\cdot n}{z^2} \int \frac{\mathrm{d}^D k}{(2\pi)^{D-1}} \frac{\mathrm{d}^D l}{(2\pi)^D} \delta_+(k^2)\delta\left(k\cdot n - \frac{1-z}{z} P\cdot n\right) \prod_i \frac{1}{F_i^{a_i}}$$

- Multi-loop techniques (see backup)

Test of NRQCD factorization

- When use $P^{\beta\beta'}$ as projection operator
  - UV divergences are canceled between real, virtual and counter terms
  - No IR divergences are left
  - IR divergences in $\mathcal{A}(q,q')$ are free of eikonal line vector n
  - NRQCD factorization is verified to hold at two-loop order
- When use $I^{\beta\beta'}$ as projection operator
  - obtain the full-QCD FF $D_{\rm NLO}[g \to Q\bar{Q}(^3P_J^{[1/8]})]$
- NRQCD renormalization

$$
\begin{aligned}
D_{\rm NLO}[g \to Q\bar{Q}(^3P_J^{[1/8]})] = \; & d_{\rm NLO}[g \to Q\bar{Q}(^3P_J^{[1/8]})]\langle \mathcal{O}^{^3P_J^{[1/8]}}(^3P_J^{[1/8]})\rangle_{\rm LO} && ② \\
① \quad & +d_{\rm LO}[g \to Q\bar{Q}(^3P_J^{[1/8]}))]\langle \mathcal{O}^{^3P_J^{[1/8]}}(^3P_J^{[1/8]})\rangle_{\rm NLO} && \boxed{\text{no IR divergence}} \\
& +d_{\rm NLO}[g \to Q\bar{Q}(^3S_1^{[8]})]\langle \mathcal{O}^{^3P_J^{[1/8]}}(^3S_1^{[8]})\rangle_{\rm LO} && ③ \\
\boxed{\delta(1-z)} \quad & +d_{\rm LO}[g \to Q\bar{Q}(^3S_1^{[8]})]\langle \mathcal{O}^{^3P_J^{[1/8]}}(^3S_1^{[8]})\rangle_{\rm NLO}\,, && ④
\end{aligned}
$$

  - The divergence structure of (①-③) is proportional to $\delta(1-z)$. Other structures such as $\left[\frac{\ln^i(1-z)}{1-z}\right]_+$ are canceled.
  - Give another verification of NRQCD factorization

NRQCD LDMEs matching

- NRQCD LDMEs in part ④ can be matched from the divergence part of (①-③)

$$\langle \mathcal{O}^{^3P_J^{[1]}}(^3S_1^{[8]})\rangle_{\rm NLO} = -C_F \frac{\alpha_s^2}{27\pi^2 m_Q^2}\left(\frac{4\pi\mu_r^2}{\mu_\Lambda^2}\right)^{2\epsilon}\Gamma(1+\epsilon)^2\left(\frac{9b_0}{\epsilon^2}+\frac{5n_f+(12\pi^2-47)N_c}{\epsilon}\right)$$

$$\langle \mathcal{O}^{^3P_J^{[8]}}(^3S_1^{[8]})\rangle_{\rm NLO} = -B_F \frac{\alpha_s^2}{27\pi^2 m_Q^2}\left(\frac{4\pi\mu_r^2}{\mu_\Lambda^2}\right)^{2\epsilon}\Gamma(1+\epsilon)^2\left(\frac{9b_0}{\epsilon^2}+\frac{5n_f+(3\pi^2-47)N_c}{\epsilon}\right)$$

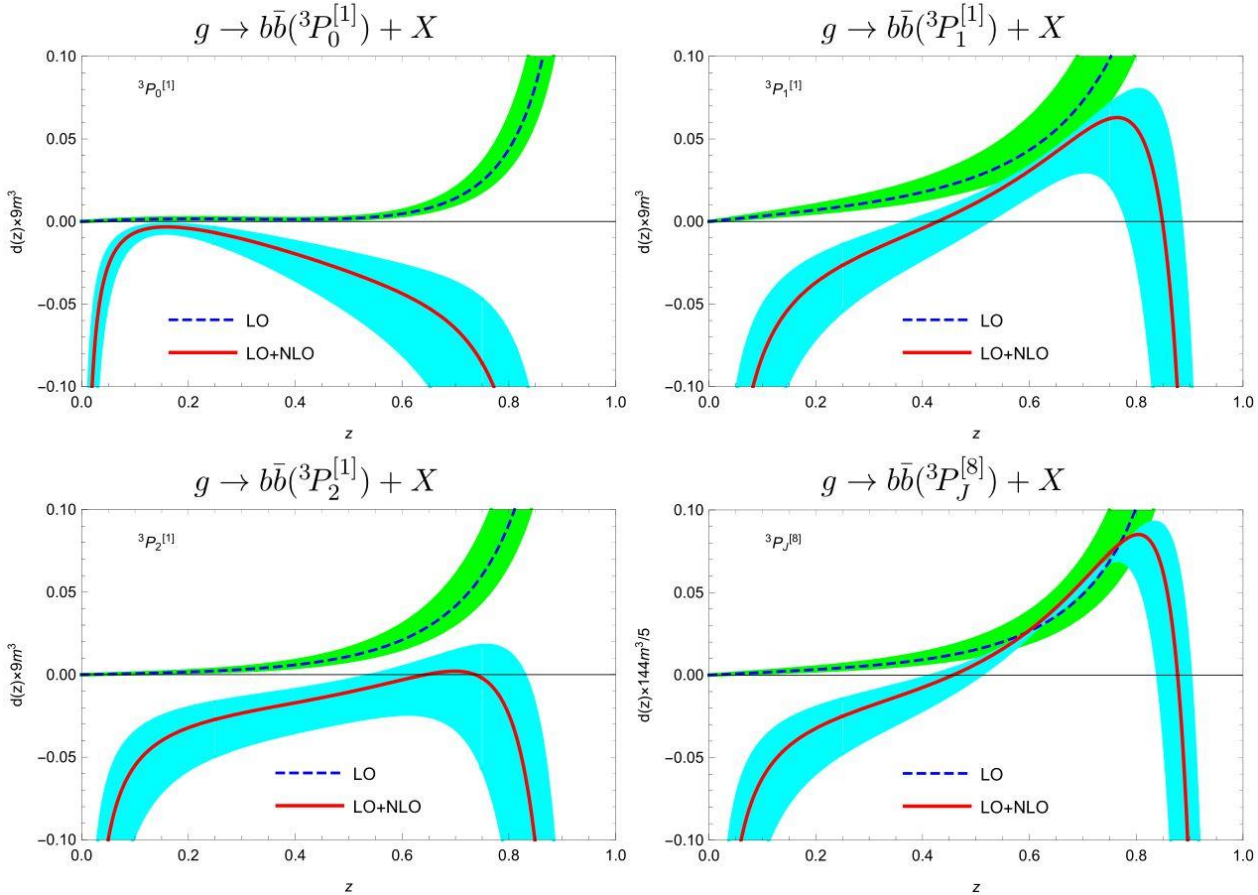
- Finite part of ① gives SDCs: $d_{\rm NLO}[g \to Q\bar{Q}(^3P_J^{[1/8]})]$

$$d_{\rm NLO}[g \to Q\bar{Q}(^3P_0^{[1]})] = \frac{\alpha_s^3}{3\pi N_c m_Q^5}\left(p_\delta\delta(1-z)+\sum_{i=0}^{2} p_i\left[\frac{\ln^i(1-z)}{1-z}\right]_+ + p(z) + \ln\left(\frac{\mu_r^2}{4m_Q^2}\right) b_0\, d_{\rm LO}[g \to Q\bar{Q}(^3P_0^{[1]})]\right)$$

- $p(z)$: piecewise function of the expansions at singularities
- divergence at $z \to 1$: $\delta(1-z)$ and $\left[\frac{\ln^2(1-z)}{1-z}\right]_+$

Numerical FF

- Take $\mu_r = 2m$ as the center line
- The bands are obtained by varying the renormalization scale μ_r by a factor of 2



- singularities at $z \to 0$: $1/z$
- singularities at $z \to 1$: $\ln^2(1-z)/(1-z)$

Moments of SDCs

- Moments integral (take ${}^3P_J^{[8]}$ as an example)

$$\int_0^1 \mathrm{d}z\, z^n (d_{\mathrm{LO}}(z) + d_{\mathrm{NLO}}(z)) \times \frac{144m^5}{5}$$

- Partonic hard parts behave as z^n with n larger than 4

n	4	6	8
K-factor	1.14	0.83	0.58

- K-factors range from 1.14 to 0.58 for physical cross sections

Summary

- We compute the partonic FF of $g \to Q\bar{Q}({}^3P_J^{[1/8]}) + X$ at NLO.
- We find that IR divergences in partonic FF is independent of eikonal line direction, which verifies the validity of NRQCD factorization at two-loop order.
- We obtain the perturbative expansion of NRQCD LDMEs at two-loop order.

Thank you!

Backup

Multi-loop technique

- Reverse unitarity relation

Anastasiou, Melnikov, *Nucl. Phys. B* **646** (2002) 220-256

$$(2\pi)\delta(\mathcal{D}_i^c) = \frac{\mathrm{i}}{\mathcal{D}_i^c + \mathrm{i}0^+} + \frac{-\mathrm{i}}{\mathcal{D}_i^c - \mathrm{i}0^+}$$

  - cut condition ⟶ propagator
  - map phase-space integrals onto pure loop integrals
- IBP reduction

Chetyrkin, Tkachov, *Nucl. Phys.* **A15** (1981) 159-204
Smirnov, *Comput. Phys. Commun.* **189** (2015) 182-191

  - reduce to master integrals (MIs)
- Differential equations

Lee, Smirnov, Smirnov, *JHEP* **03** (2018) 008
Liu, Ma, Wang, *Phys. Lett.* **B779** (2018) 353-357

$$\frac{\mathrm{d}\boldsymbol{I}(\epsilon, z)}{\mathrm{d}z} = A(\epsilon, z)\boldsymbol{I}(\epsilon, z)$$

- Asymptotic expansions

$s = a + b\,\epsilon$

Henn, *J. Phys. A* **48** (2015) 153001

$$I_k(z,\epsilon)|_{z_0} = \sum_s \sum_{i=0}^{n_s} (z - z_0)^s \ln^i(z - z_0) \sum_{j=0}^{\infty} I_k^{s\,i\,j}(\epsilon)(z - z_0)^j$$

- Boundary conditions
  - region analysis: real correction
  - auxiliary mass flow (AMF): virtual correction

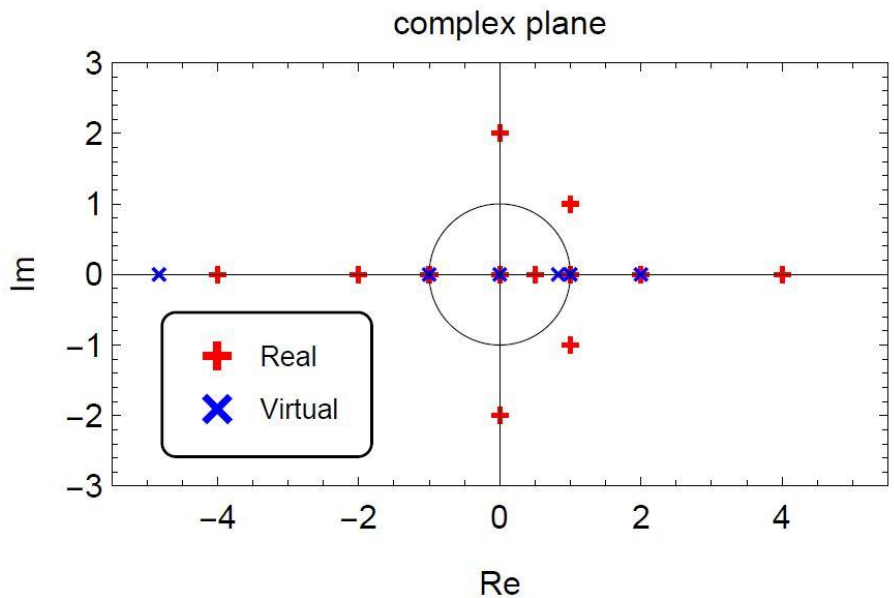
Auxiliary mass flow (AMF)

Liu, Ma, Wang, *Phys. Lett. B* **779** (2018) 353-357
Liu, Ma, Tao, Zhang, *Chin. Phys. C* **45**, (2021) 013115

- use AMF to calculate the boundary conditions
  - add auxiliary mass η on inverse propagators with loop momentum
  - set up DEs w.r.t. η
  - at $\eta \to \infty$ with $z = z_0$, regions of the boundary are clear
  - flow $\eta \to \infty$ to $\eta \to 0$ with DEs
  - get boundary of initial MIs at $z = z_0$

Expansion at singularities

- Singularities in DEs: $0, 1/2, 1$



- Estimate values of MIs in regions $0 \sim \frac{1}{4}, \frac{1}{4} \sim \frac{3}{4}, \frac{3}{4} \sim 1$ respectively by the asymptotic expansions of MIs at $z = 0,1/2,1$

Renormalization

- Counter term
- Operator renormalization

$$d_{\rm O}(z) = -\frac{\alpha_s}{2\pi}\frac{\Gamma(1+\epsilon)}{\epsilon}\left(\frac{4\pi\mu_r^2}{\mu_f^2}\right)^{\epsilon}\int_z^1 \frac{{\rm d}y}{y} P_{gg}(y) d_{\rm LO}\left(\frac{z}{y}\right)$$

where $P_{gg}(z) = \frac{11N_c - 2n_f}{6}\delta(1-z) + 2N_c\left(\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right)$

- Plus function exist in $d_{\rm LO}(z)$
- Convolution between two plus functions is difficult
- Change plus function in $d_{\rm LO}(z)$ back to $(1-z)^{-1+n\epsilon} f(z)$

$$\int_z^1 \frac{{\rm d}y}{y}\frac{1}{[1-y]_+}(1-z/y)^{-1+n\epsilon} f(z/y)$$

$\eta = 1 - z$

$$=\eta^{-1+n\epsilon}\int_0^1 \frac{{\rm d}t}{1-t}\left(t^{-1+n\epsilon}\left(f(1-\eta t) - f(1)\right) - \frac{1-\eta}{1-\eta t}\left(f(1-\eta) - f(1)\right)\right)$$

$$+\eta^{-1+n\epsilon} f(1)\left(-H(n\epsilon) + \frac{1}{n\epsilon} - \ln(1-\eta)\right)$$

$$+\eta^{-1+n\epsilon}\ln\eta\, f(1-\eta)$$