Quarkonium TMD fragmentation functions in NRQCD

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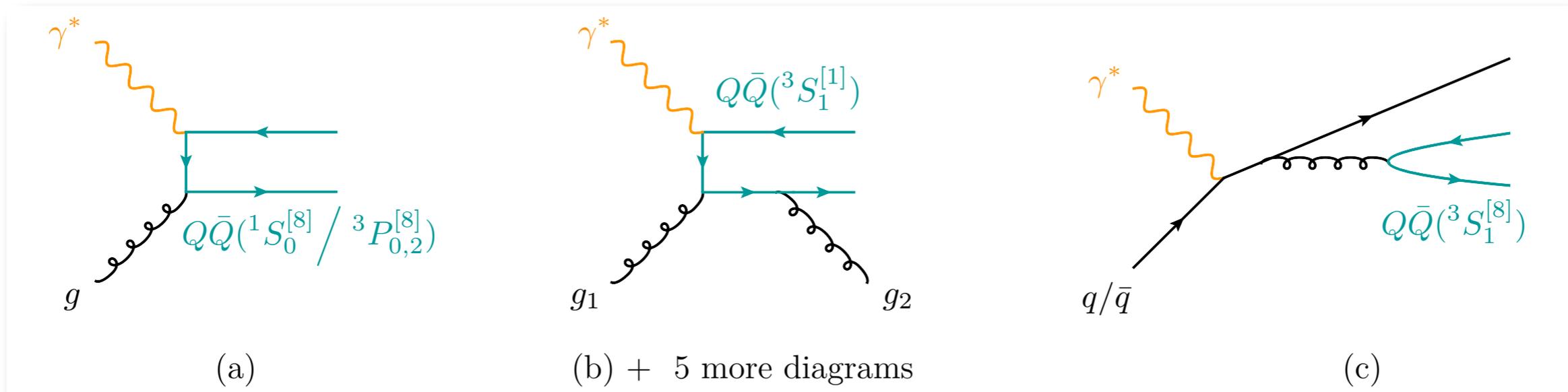
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In collaboration with Y. Makris and I. Scimemi

INTRODUCTION

- Goal: asses how relevant it is the fragmentation process at low pₜ in SIDIS (EIC).
- Quarkonium production at the EIC is considered a useful way to probe gluon TMDs
- If we expand in alpha_s we have:

Photon-gluon fusion described in terms of gluon TMDPDFs and TMD-shape functions

[MGE 1907.06494]
[Fleming-Makris-Mehen 1910.03586]

Single-parton fragmentation

●Follows the same steps as for standard SIDIS…

$$\ell(l) + h(p) \rightarrow \ell(l') + H(P) + X$$

$$d\sigma = \frac{2}{s - m^2} \frac{\alpha_{\text{em}}^2}{(q^2)^2} L_{\mu\nu} W^{\mu\nu} \frac{d^3 l'}{2E'} \frac{d^3 P}{2E_H}$$

$$L_{\mu\nu} = e^{-2} \langle l'|J_\mu(0)|l\rangle \langle l|J_\nu^\dagger(0)|l'\rangle,$$

$$W_{\mu\nu} = e^{-2} \int \frac{d^4 x}{(2\pi)^4} e^{-i(xq)} \sum_X \langle p|J_\mu^\dagger(x)|P, X\rangle \langle P, X|J_\nu(0)|p\rangle$$

$$Q^2 = -q^2 = -(l - l')^2$$

$$x = \frac{Q^2}{2(pq)}$$

$$y = \frac{(pq)}{(pl)}$$

$$z = \frac{(pP)}{(pq)}$$

- We use SCET to factorize the hadronic tensor
- The relevant collinear, anti-collinear and soft matrix elements are:

$$\Phi_{1,q\leftarrow h}(x,b) =$$

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda p^+} \sum_X \langle h(p)|\bar{q}(n\lambda + b)W_n^\dagger(n\lambda + b)\frac{\gamma^+}{2}|X\rangle\langle X|W_n(0)q(0)|h(p))\rangle,$$

$$\Delta_{q\to H}(z,b) =$$

$$\frac{1}{2zN_c} \int \frac{d\lambda}{2\pi} e^{i\lambda P^-/z} \sum_X \langle 0|\frac{\gamma^-}{2}W_{\bar{n}}(\bar{n}\lambda + b)q(\bar{n}\lambda + b)|H(P),X\rangle\langle H(P),X|\bar{q}(0)W_{\bar{n}}^\dagger(0)|0\rangle$$

$$S(b) = \frac{\text{Tr}_c}{N_c} \langle 0| \, T\left[S_n^{T\dagger}\tilde{S}_{\bar{n}}^T\right](0^+,0^-,\boldsymbol{b}_T)\bar{T}\left[\tilde{S}_{\bar{n}}^{T\dagger}S_n^T\right](0)\,|0\rangle$$

These have spurious rapidity-divergences...

FACTORIZATION (3/5): DEFINITION OF TMDS

$$k_n \sim Q(1, \lambda^2, \lambda)$$
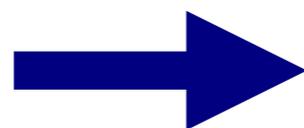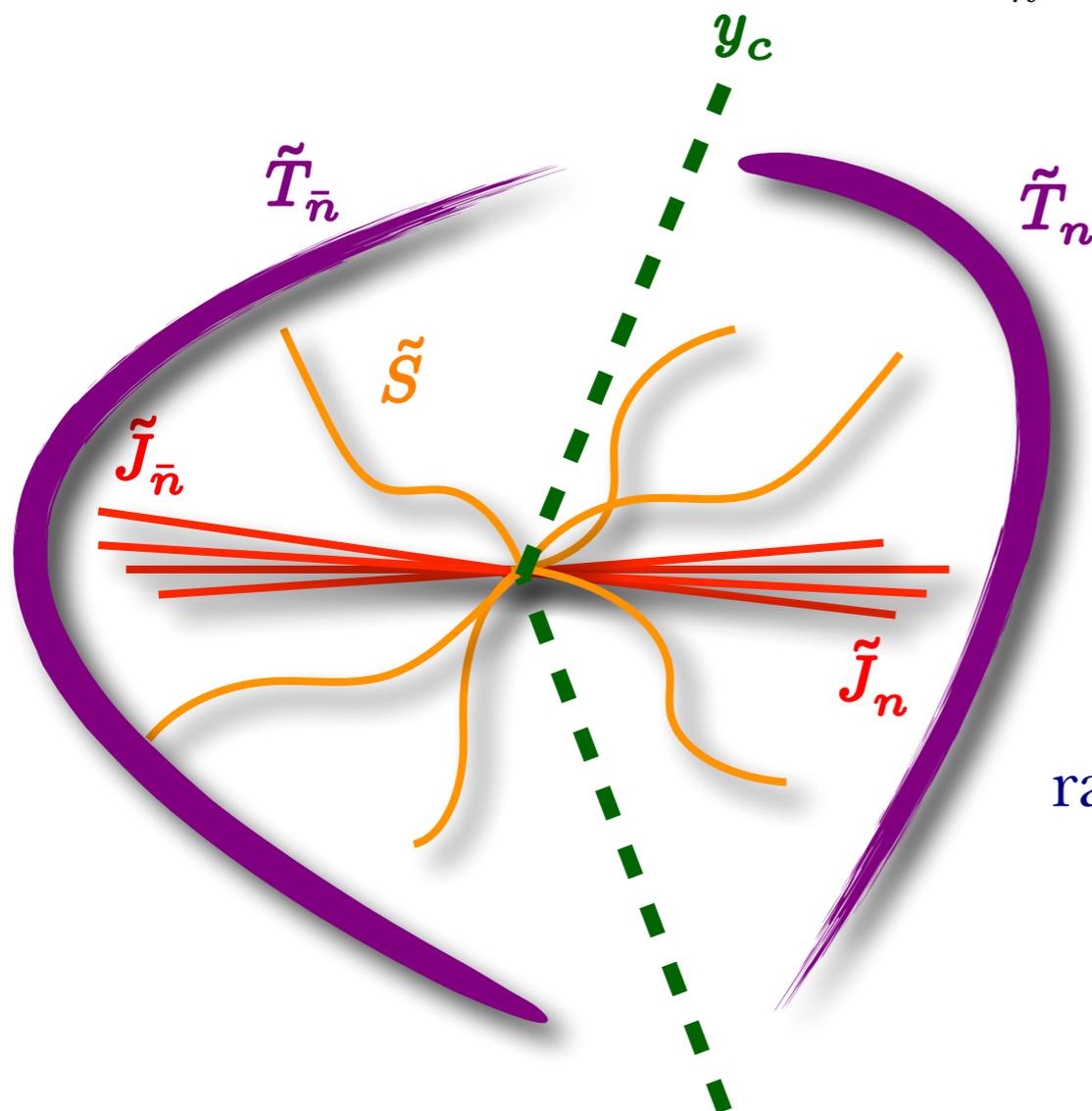$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda)$$
$$k_s \sim Q(\lambda, \lambda, \lambda)$$

$$y = \frac{1}{2}\ln\left|\frac{k^+}{k^-}\right|$$

Different rapidities
(mixed under boosts)

$$k_n^2 \sim k_{\bar{n}}^2 \sim k_s^2 \sim Q^2\lambda^2$$

Same invariant mass!

y_c

$\tilde{T}_{\bar{n}}$ $\tilde{T}_n$

$\tilde{S}$

$\tilde{J}_{\bar{n}}$

$\tilde{J}_n$

$$\zeta_A = (p^+)^2 e^{-2y_c}$$

$$\tilde{T}_n(x_A, \vec{b}_\perp, S_A; \zeta_A, \mu) = \tilde{J}_n \sqrt{\tilde{S}}$$
$$\tilde{T}_{\bar{n}}(x_B, \vec{b}_\perp, S_B; \zeta_B, \mu) = \tilde{J}_{\bar{n}} \sqrt{\tilde{S}}$$

Cancel spurious
rapidity divergences

$$\zeta_B = (\bar{p}^-)^2 e^{+2y_c}$$

[Collins' book '11]
[MGE-Idilbi-Scimemi 1111.4996, 1211.1947, 1402.0869]
[MGE-Kasemets-Mulders-Pisano 1502.05354]

These are the quantities that one can probe

FACTORIZATION (4/5): TMD EVOLUTION

TMDs depend on **two scales**: renormalization and rapidity scales

●The dependence on the **renormalization** scale is:

$$\frac{d}{d\ln\mu}\ln\tilde{T}_{j\leftrightarrow A}^{[pol]}(x,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = \gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right)$$

Known at 3-loops.
Numerical at 4-loops

$$\gamma_j\left(\alpha_s(\mu),\ln\frac{\zeta_A}{\mu^2}\right) = -\Gamma_{\text{cusp}}^j(\alpha_s(\mu))\ln\frac{\zeta_A}{\mu^2} - \gamma_{nc}^j(\alpha_s(\mu))$$

[Moch-Vermaseren-Vogt hep-ph/0507039, hep-ph/0403192]
[Moch-Ruijl-Ueda-Vermaseren-Vogt 1707.08315]

●The dependence on the **rapidity** scale is:

$$\frac{d}{d\ln\zeta_A}\ln\tilde{T}_{j\leftrightarrow A}^{[pol]}(x,\boldsymbol{b}_\perp,S_A;\zeta_A,\mu) = -D_j(b_T;\mu)$$

Known at 3-loops (almost 4-loops)

$$\frac{dD_j}{d\ln\mu} = \Gamma_{\text{cusp}}^j(\alpha_s(\mu))$$

Cusp does not
completely determine Dj

Indirect NLO: [Becher, Neubert EPJC71(2011)]
Direct NLO: [MGE, Scimemi, Vladimirov PRD93(2016)]

Direct NNLO:
[Li, Zhu PRL118(2017)]
[Vladimirov PRL118(2017)]

QCDxQED corrections for both
anomalous dimensions in *[Bacchetta-MGE 1810.02297]*

FACTORIZATION (5/5)

● Putting everything together:

TMDPDF   **TMDFF**

$$\frac{d\sigma}{dxdzdQ^2 d\vec{P}_\perp^2} = \sigma_0 \, |C_V(Q^2,\mu^2)|^2 \int_0^\infty bdb \, J_0(b|\vec{q}_T|) f_{1,f\leftarrow h}(x_S, b; \mu, \zeta_1) \, D_{f\to H}(z_S, b; \mu, \zeta_2)$$

$$\vec{q}_T^2 \simeq \frac{\vec{P}_\perp^2}{z^2}$$

$$x_S \simeq x$$

$$z_S \simeq z$$

We apply known results & fits

We calculate it within NRQCD

$$\tilde{T}_{i\leftrightarrow A}(x, b_T; \zeta, \mu) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{i\leftrightarrow j}(x, \hat{b}_T; \mu_b^2, \mu_b) \otimes t_{j\leftrightarrow A}(x; \mu_b)$$

$$\times \exp\left[\int_{\mu_b}^\mu \frac{d\hat{\mu}}{\hat{\mu}} \, \gamma_j\left(\alpha_s(\hat{\mu}), \ln\frac{\zeta}{\hat{\mu}^2} \right) \right] \left(\frac{\zeta}{\mu_b^2} \right)^{-D_j(\hat{b}_T; \mu_b)}$$

$$\times \tilde{T}^{NP}_{i\leftrightarrow A}(x, b_T; \zeta)$$

QUARKONIUM TMD FRAGMENTATION AT LO (1/3)

●Let us calculate the quarkonium TMDFF using NRQCD factorization:

$$D_{q \to H}(z, b; \mu, \zeta) = \sum_n d_{q \to Q\bar{Q}(n)}(z, b; \mu, \zeta) \frac{\langle \mathcal{O}^H(n) \rangle}{N_{\text{col.}} N_{\text{pol.}}}$$
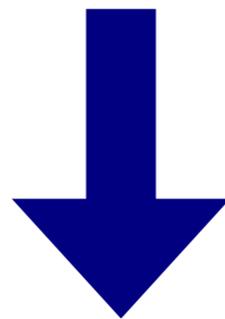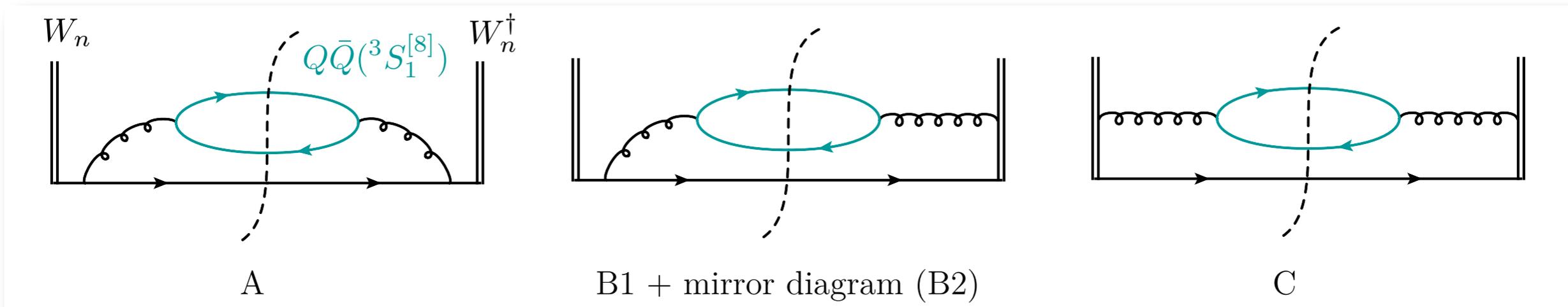
Interested in Jpsi production. Thus at LO:

$$D_{q \to \psi}(z, b; \mu, \zeta) = d_{q \to Q\bar{Q}(^3S_1^{[8]})}(z, b; \mu, \zeta) \frac{\langle \mathcal{O}^\psi(^3S_1^{[8]}) \rangle}{(d-1)(N_c^2 - 1)} \left(1 + O(\alpha_s) \right)$$

●**At LO there are no rapidity divergences**, and there is no need to calculate the soft function

$$Q\bar{Q}(^3S_1^{[8]})$$

$$W_n \qquad W_n^\dagger$$

A

B1 + mirror diagram (B2)

C

$$d_A = 4\pi\alpha_s^2 C_F \frac{I_0}{M^4 z^3}\left[(\epsilon-1)\left(2\bar{z}F_1(B\Delta) - 2z^2 F_0(B\Delta)\right) + z^2 F_{-1}(B\Delta)\right]\mu^{2\epsilon}$$

$$d_{B1+B2} = 8\pi\ \alpha_s^2\ C_F \frac{I_0}{M^4 z^3}\left[2\bar{z}F_0(B\Delta) - z^2 F_{-1}(B\Delta)\right]\mu^{2\epsilon}$$

$$d_C = 4\pi\alpha_s^2\ C_F \frac{I_0}{M^4 z}F_{-1}(B\Delta)\mu^{2\epsilon}$$

$$m_c \simeq M/2$$

Only one master integral needed:

$$\psi^\dagger \sigma T^A \chi$$

$$F_n(B\Delta) = M^{2n}\int \frac{d^{2-2\epsilon}k_T}{(2\pi)^{2-2\epsilon}}\frac{e^{i\mathbf{k_T}\cdot\mathbf{b}}}{(k_T^2+\Delta)^{n+1}} = \frac{2M^{2n}}{(4\pi)^{1-\epsilon}\Gamma[n+1]}\left(\frac{B}{\Delta}\right)^{\frac{n+\epsilon}{2}}K_{-n-\epsilon}\left(2\sqrt{B\Delta}\right)$$

$$B = b^2/4$$

- Putting everything together:

$$d_{q \to Q\bar{Q}(^3S_1^8)}(z, b; \mu, \zeta) = -16\pi \frac{\alpha_s^2 \, C_F}{M^3 z^3} \left[(1 - \epsilon)\bar{z} F_1(B\Delta) - (z^2(1 - \epsilon) + 2\bar{z}) F_0(B\Delta) \right] \mu^{2\epsilon}$$

Consistent with collinear fragmentation function:

$$D_{q \to \psi}(z) \Big|_{z \to 1} \simeq -4 \frac{\alpha_s^2 C_F}{M^3 z} \left[\frac{1}{\epsilon} - 2 - \ln \frac{\bar{z} M^2}{\mu^2} \right] \frac{\langle \mathcal{O}^\psi(^3S_1^{[8]}) \rangle}{(d - 1)(N_c^2 - 1)}$$

[Bodwin-Chung-Kim-Lee 1412.7106]

- For completeness, in momentum space (k_T=0 regulated by M$\neq$0 and z$\neq$1):

$$D_{q \to \psi}(z, \boldsymbol{k}_T; \mu, \zeta) = 2 \frac{\alpha_s^2 \, C_F}{\pi} \frac{\bar{z}}{M^3 z^3} \left[\frac{2z^2}{\bar{z}} \frac{\boldsymbol{k}_T^2}{(\boldsymbol{k}_T^2 + \bar{z} M^2/z^2)^2} + \frac{4}{\boldsymbol{k}_T^2 + \bar{z} M^2/z^2} \right] \frac{\langle \mathcal{O}^\psi(^3S_1^{(8)}) \rangle}{3(N_c^2 - 1)}$$

EVOLUTION OF LDMES (1/2)

●Need LDMEs at mu~1/b, but extracted at mu~M. Thus need their evolution!

$$\frac{d}{d\ln\mu}\langle\mathcal{O}_\psi(n)\rangle^{(\mu)} = \sum_m \gamma_{\mathcal{O}}^{nm}\langle\mathcal{O}_\psi(m)\rangle^{(\mu)} \qquad \text{Mixing of LDMEs}$$

$$\gamma_d = -\gamma_{\mathcal{O}}^T$$
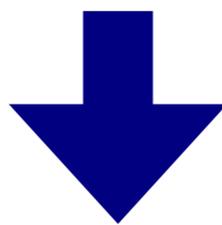
$$\frac{d}{d\ln\mu}d_{q\to Q\bar{Q}(n)}(z,b;\mu,\zeta) = \sum_m \Big(\gamma_D\delta^{nm} + \gamma_d^{nm})\Big)d_{q\to Q\bar{Q}(m)}(z,b;\mu,\zeta)$$

Would need TMDFF
at NLO to check it

●Assuming NRQCD factorization holds beyond LO:

$$D_{q\to\psi}(z,b;\mu,\zeta) = d_{q\to Q\bar{Q}(^3S_1^{[8]})}(z,b;\mu,\zeta)\frac{\langle\mathcal{O}^\psi(^3S_1^{[8]})\rangle}{N_{col.}N_{pol.}}\Big(1 + O(\alpha_s)\Big)$$
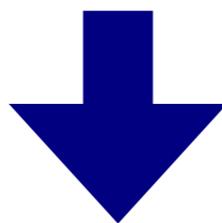
NLL evolution of LDMEs

$$D_{q\to\psi}(z,b;\mu,\zeta) = \frac{d_{q\to Q\bar{Q}(^3S_1^{[8]})}(z,b;\mu,\zeta)}{N_{col.}N_{pol.}}\Big[\langle\mathcal{O}^H(^3S_1^{[8]})\rangle^{(\mu_f)} - \frac{24B_F}{\beta_0}\ln\Big(\frac{\alpha_s(\mu)}{\alpha_s(\mu_f)}\Big)\frac{\langle\mathcal{O}^H(^3P_0^{[8]})\rangle^{(\mu_f)}}{m_Q^2}\Big]$$

EVOLUTION OF LDMES (2/2)

- Evolution of LDMEs has quite some impact (we use BCKL set of LDMEs)
- Effects disappear for $P_T \sim M \sim 3$ GeV, as expected.

$$\mathcal{N} \times \frac{d\sigma}{d\boldsymbol{P}_\perp^2} \ [\text{pb/GeV}^2], \ \sqrt{s} = 63 \text{ GeV} \ :$$

– – – $\gamma^* q \ (^3S_1^{[8]} : \text{NNLL/Res.-LDME})$ ——— $\gamma^* q \ (^3S_1^{[8]} : \text{NNLL/NoRes.-LDME})$

$z \in [0.1, 0.4], \ Q \in [10., 30.]$ $z \in [0.1, 0.4], \ Q \in [30., 50.]$ $z \in [0.4, 0.8], \ Q \in [10., 30.]$ $z \in [0.4, 0.8], \ Q \in [30., 50.]$

$x \in [0.1, 0.5]$

$x \in [0.5, 1.0]$

$\mathcal{N} = 5 \times 10^3$ $\mathcal{N} = 1 \times 10^6$ $\mathcal{N} = 2 \times 10^4$ $\mathcal{N} = 2 \times 10^6$

$\mathcal{N} = 2 \times 10^5$ $\mathcal{N} = 2 \times 10^6$ $\mathcal{N} = 2 \times 10^5$ $\mathcal{N} = 1 \times 10^7$

$P_\perp$ [GeV]
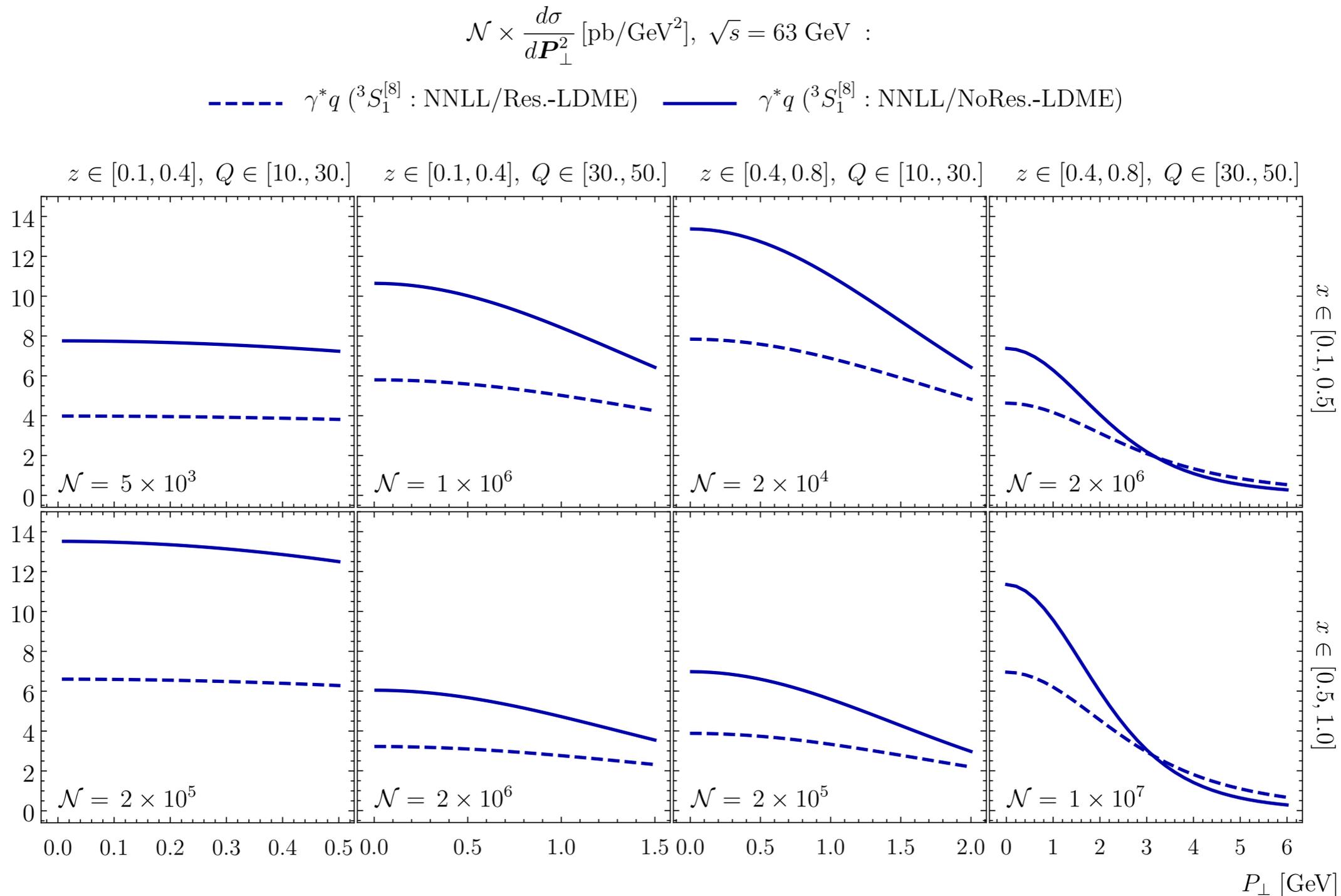
PHOTON-GLUON FUSION VS FRAGMENTATION

● Photon-gluon fusion at fixed-order analyzed long ago. We take the low P_T limit:

$$\frac{d\sigma(\gamma^* g)}{dxdzdQ^2d\boldsymbol{P}_\perp^2} \simeq \frac{\alpha_s^2(\mu)\,\alpha_{\rm em}^2\pi}{(1-\varepsilon)s^2}\frac{64\,e_H^2}{27z\bar{z}^3(2-z)^2}\frac{1}{x^2}\frac{\langle\mathcal{O}^\psi(^3S_1^{[1]})\rangle}{Q^2M^3}f_{g\leftarrow h}(x;\mu^2)F_g(\bar{z},\frac{P_\perp}{M})$$

$$F_g(a,b) = \frac{a^4+a^2+b^2}{(1+b^2/a^2)^2}$$

● For a back-of-the-envelope comparison, we also take the large P_T limit of the TMD cross-section:

$$\frac{d\sigma(\gamma^* q)}{dxdzdQ^2d\boldsymbol{P}_\perp^2} \simeq \frac{\alpha_s^2(\mu)\,\alpha_{\rm em}^2\,\pi}{(1-\varepsilon)s^2}\sum_f\frac{4\,e_f^2}{9x^2\,z}\frac{\langle\mathcal{O}^\psi(^3S_1^{[8]})\rangle}{M^5}f_{f\leftarrow h}(x;\mu^2)F_q(z,\frac{\boldsymbol{P}_\perp^2}{M^2\bar{z}})$$

$$F_q(a,b) = \frac{a^2b+2(1-a)(1+b)}{(1-a)(1+b)^2}$$

● An order of magnitude comparison of these two contributions to the cross-section:

$$\frac{d\sigma(\gamma^* g)}{d\sigma(\gamma^* q)} \sim \left(\frac{M}{Qv^2}\right)^2$$

$M^2/(Q^2v^4)$	$Q=10$ [GeV]	$Q=30$ [GeV]	$Q=50$ [GeV]	$Q=100$ [GeV]
charmonium ($v^2 \sim 0.3$)	1.0	0.1	0.04	0.01
bottomonium ($v^2 \sim 0.1$)	n.a.	10.0	3.8	1.0

Next we perform a proper comparison with full results

NUMERICS

- We make use of arTeMiDe code and modify it as needed   *[Scimemi, Vladimirov 1912.06532]*
- We use NNPDF31 PDF set and a recent fit of quark TMDPDFs at NNLO-N³LL
- TMD evolution implemented with the so-called zeta-prescription and at N³LL
- We take the full fixed-order result for photon-gluon fusion
- We go away from z~1 to suppress color-octet channels in photon-gluon fusion
- Chosen kinematical regions relevant for the EIC, with TMD constraint: $P_\perp \in [0, z_{\min} Q_{\min}/2]$

- LDMEs supposed to be universal, but not yet. We take 3 sets:

	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle$	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle/m_c^2$
BCKL	1.32 ± 0.20	$(1.1 \pm 1.0) \times 10^{-2}$	$(0.49 \pm 0.44) \times 10^{-2}$
B&K	1.32 ± 0.20	$(0.224 \pm 0.59) \times 10^{-2}$	$(-0.72 \pm 0.88) \times 10^{-2}$
CMSWZ	1.32 ± 0.20	$(0.30 \pm 0.12) \times 10^{-2}$	$(0.56 \pm 0.21) \times 10^{-2}$

[Bodwin-Chung-Kim-Lee, 1403.3612]

[Butenschoen-Kniehl, 1105.0820]

[Chao-Ma-Shao-Wang-Zhang, 1201.2675]

PREDICTIONS

●**Photon-gluon fusion** is in general **dominant at low values of Q and x**, while for some LDME sets quark fragmentation plays a major part in the opposite limit

$$\mathcal{N} \times \frac{d\sigma}{d\boldsymbol{P}_\perp^2} \; [\text{pb}/\text{GeV}^2], \; \sqrt{s} = 140 \text{ GeV} \; :$$

$\gamma^* g \; ({}^3S_1^{[1]} : \text{LO})$ $\gamma^* q \; ({}^3S_1^{[8]} : \text{NNLL/BCKL})$

$\gamma^* q \; ({}^3S_1^{[8]} : \text{NNLL/B\&K})$ $\gamma^* q \; ({}^3S_1^{[8]} : \text{NNLL/CMSW})$

$z \in [0.1, 0.4], \; Q \in [10., 30.]$ $z \in [0.1, 0.4], \; Q \in [30., 50.]$ $z \in [0.4, 0.8], \; Q \in [10., 30.]$ $z \in [0.4, 0.8], \; Q \in [30., 50.]$

$P_\perp \; [\text{GeV}]$

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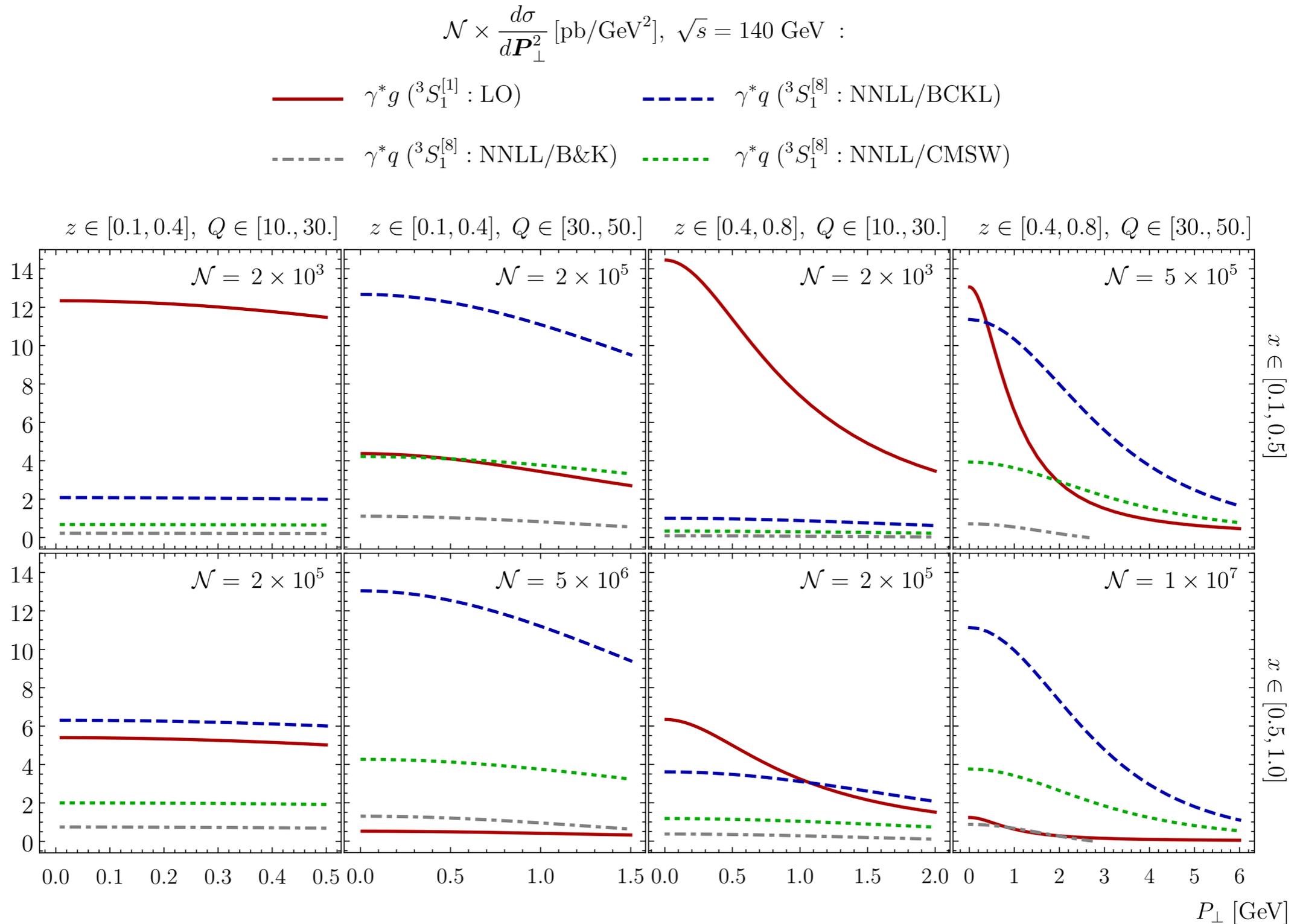
CONCLUSIONS & OUTLOOK

- We studied single-parton fragmentation into quarkonium, and defined and calculated quarkonium TMDFF at LO (1 loop) using NRQCD factorization in terms of coefficients and LDMEs.

- Applied the results to the EIC kinematics to explore the relevance of both photon-gluon fusion and quark fragmentation channels.

- Quarkonium production @EIC generally considered as a tool to probe gluon TMDs, but it is actually non-trivial: fragmentation process needs to be considered.

✤ Extend the calculation of the quarkonium TMDFF to NLO

✤ Apply this formalism to other processes (e.g. quarkonium TMDFF from a gluon)

✤ Obtain TMD factorization for photon-gluon fusion (twist-3 TMDs and TMD-shape functions…)

Thank you!

Backup

$\mathcal{N} \times \dfrac{d\sigma}{d\boldsymbol{P}_\perp^2}$ [pb/GeV^2], $\sqrt{s} = 63$ GeV :

— $\gamma^* g$ $({}^3S_1^{[1]} : \text{LO})$

- - - $\gamma^* q$ $({}^3S_1^{[8]} : \text{NNLL/BCKL})$

-·-·- $\gamma^* q$ $({}^3S_1^{[8]} : \text{NNLL/B\&K})$

···· $\gamma^* q$ $({}^3S_1^{[8]} : \text{NNLL/CMSW})$

$z \in [0.1, 0.4],\ Q \in [10., 30.]$ $z \in [0.1, 0.4],\ Q \in [30., 50.]$ $z \in [0.4, 0.8],\ Q \in [10., 30.]$ $z \in [0.4, 0.8],\ Q \in [30., 50.]$

$\mathcal{N} = 2 \times 10^3$ $\mathcal{N} = 2 \times 10^6$ $\mathcal{N} = 2 \times 10^3$ $\mathcal{N} = 5 \times 10^6$

$x \in [0.1, 0.5]$

$\mathcal{N} = 2 \times 10^5$ $\mathcal{N} = 5 \times 10^6$ $\mathcal{N} = 2 \times 10^5$ $\mathcal{N} = 2 \times 10^7$

$x \in [0.5, 1.0]$

$P_\perp$ [GeV]