X atom

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Based on:

Z.-H. Zhang, FKG, *X atom: a new key to revealing the X(3872) mystery*, arXiv:2012.08281

X(3872): mass

- PDG2020 average from the $J/\psi\rho$ and $J/\psi\omega$ modes

$\chi_{c1}(3872)$ MASS FROM $J/\psi X$ MODE

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VALUE (MeV)	EVTS		DOCUMENT ID		TECN	COMMENT
3871.69 ± 0.17	**OUR AVERAGE**					
$3871.9 \pm 0.7 \pm 0.2$	20 ± 5		ABLIKIM	2014	BES3	$e^+ e^- \to J/\psi \pi^+ \pi^- \gamma$
$3871.95 \pm 0.48 \pm 0.12$	0.6k		AAIJ	2012H	LHCB	$p p \to J/\psi \pi^+ \pi^- X$
$3871.85 \pm 0.27 \pm 0.19$	~ 170	1	CHOI	2011	BELL	$B \to K \pi^+ \pi^- J/\psi$
$3873\ ^{+1.8}_{-1.6} \pm 1.3$	27 ± 8	2	DEL-AMO-SANCH..	2010B	BABR	$B \to \omega J/\psi K$
$3871.61 \pm 0.16 \pm 0.19$	6k	3, 2	AALTONEN	2009AU	CDF2	$p \bar{p} \to J/\psi \pi^+ \pi^- X$
$3871.4 \pm 0.6 \pm 0.1$	93.4		AUBERT	2008Y	BABR	$B^+ \to K^+ J/\psi \pi^+ \pi^-$
$3868.7 \pm 1.5 \pm 0.4$	9.4		AUBERT	2008Y	BABR	$B^0 \to K^0_S J/\psi \pi^+ \pi^-$
$3871.8 \pm 3.1 \pm 3.0$	522	4, 2	ABAZOV	2004F	D0	$p \bar{p} \to J/\psi \pi^+ \pi^- X$

- Latest LHCb determination w/ Flatte

LHCb, PRD102(2020)092005

$$M_X = 3871.69^{+0.05}_{-0.14}\ \text{MeV}$$

- Coincides with the $D^0 \bar{D}^{*0}$ threshold:

PDG average: $M_{D^0} = (1864.84 \pm 0.05)$ MeV, $M_{D^{*0}} = (2006.85 \pm 0.05)$ MeV

- Precise measurements of $\delta = M_{D^0} + M_{\bar{D}^{*0}} - M_X$ and $\Gamma_X \Rightarrow$ probability of the $D^0 \bar{D}^{*0}$ component in the X wave function

X atom

- $X(3872)$: strong coupling to $D^0\bar{D}^{*0}$

  Unavoidably extended, large radius, $r_X \simeq \frac{1}{\sqrt{2\mu_0\delta}} \gtrsim 10\ \text{fm}$

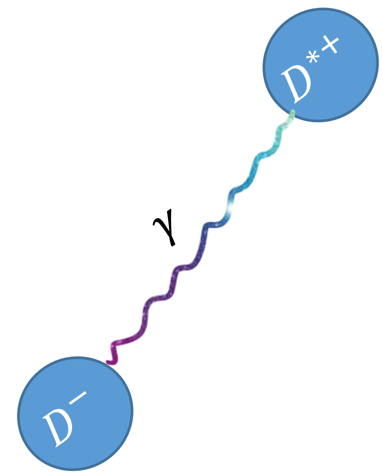
- The same order as the Bohr radius of Coulomb bound state of D^-D^{*+}, D^+D^{*-} : hadronic atoms $r_B = \frac{1}{\alpha\mu_c} = 27.86\ \text{fm}$

  $\mu_0 = \frac{m_{D^0}m_{D^{*0}}}{\Sigma_0}$ $\mu_c = \frac{m_D m_{D^*}}{\Sigma_c}$ thresholds: $\Sigma_{0,c}$

- Coulomb binding energies: $E_n = \frac{\alpha^2\mu_c}{2n^2} = \frac{25.81\ \text{keV}}{n^2}$

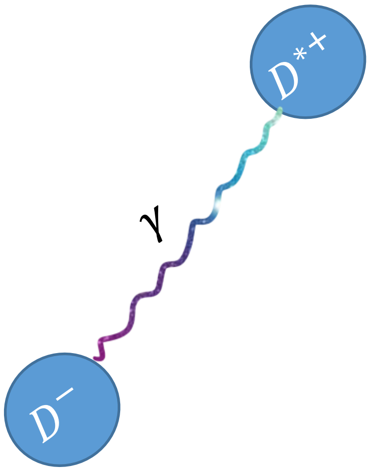
- For production: the more extended, the more difficult to be produced. ($\sigma \propto \delta^{1/2}$)

- X atom: The ground state $D^-D^{*+} - D^+D^{*-}$ atom with $C = +$



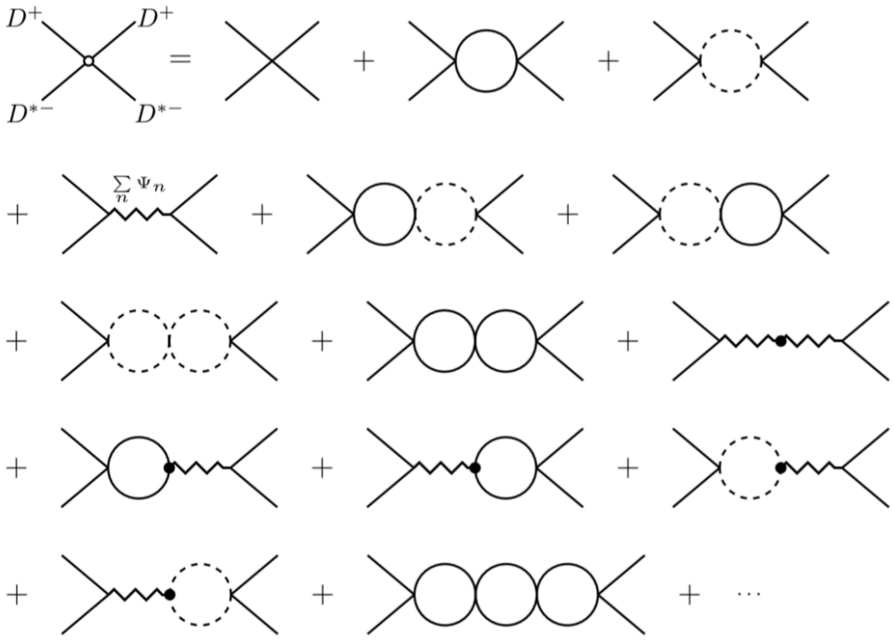
X atom

- Scale separation: $r_B\Lambda_{QCD} \gg 1$, strong interaction is a perturbation for hadronic atoms:
  - Correction to the binding energy: $\Delta E_n = O(\alpha^3)$
  - Decay modes: $D^0\bar{D}^{*0}, D^0\bar{D}^0\pi^0, J/\psi\pi\pi, \ldots$
- For X atom, strong interaction by itself is nonperturbative due to the existence of $X(3872)$



X atom

- Nonrelativistic effective field theory (NREFT) for coupled channels:
  - 1^{++} $D^0\bar{D}^{*0}$
  - 1^{++} D^+D^{*-}
  - The D^+D^{*-} Green function contains both Coulomb bound states and continuum



X atom

- Around the threshold, LO in NREFT: constant contact terms for strong interaction

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\phi=D^{\pm},D^{0},\bar{D}^{0}} \phi^{\dagger}\left(iD_t - m_{\phi} + \frac{\nabla^2}{2m_{\phi}}\right)\phi \\
& + \sum_{\phi=D^{*\pm},D^{*0},\bar{D}^{*0}} \phi^{\dagger}\left(iD_t - m_{\phi} + i\frac{\Gamma_{\phi}}{2} + \frac{\nabla^2}{2m_{\phi}}\right)\phi \\
& - \frac{C_0}{2}(D^{+}D^{*-} - D^{-}D^{*+})^{\dagger}(D^{+}D^{*-} - D^{-}D^{*+}) \\
& - \frac{C_0}{2}\left[(D^{+}D^{*-} - D^{-}D^{*+})^{\dagger}(D^{0}\bar{D}^{*0} - \bar{D}^{0}D^{*0}) + \text{h.c.}\right] \\
& - \frac{C_0}{2}(D^{0}\bar{D}^{*0} - \bar{D}^{0}D^{*0})^{\dagger}(D^{0}\bar{D}^{*0} - \bar{D}^{0}D^{*0}) + \cdots,
\end{aligned}
$$

- Approximation: Isospin-1 strong interaction neglected
  - No isovector state was found
  - Isospin breaking in the couplings is small: Hanhart et al., PRD85(2012)011501

$$
\frac{g_{X\rho}}{g_{X\omega}} = 0.26^{+0.08}_{-0.05}
$$

X atom

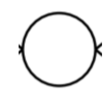
- The T-matrix for positive C parity channels: $T(E) = V[1 - G(E)V]^{-1}$

$$V = C_0 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad G(E) = \begin{pmatrix} J_0(E) & 0 \\ 0 & J_c(E) + J_{|\Psi\rangle}(E) \end{pmatrix}$$

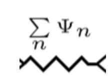
$$J_0(E) = \frac{\mu_0}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_0(E + \Delta + i\Gamma_0/2)}\right),$$

$\Delta = \Sigma_c - \Sigma_0$

$$J_c(E) = \frac{\mu_c}{2\pi}\left(-\frac{2\Lambda}{\pi} + \sqrt{-2\mu_c(E + i\Gamma_c/2)}\right),$$



$$J_{|\Psi\rangle}(E) = \sum_{n=1}^{\infty} \frac{\alpha^3\mu_c^3}{\pi n^3}\frac{1}{E + E_n + i\Gamma_c/2},$$



- The T-matrix has infinity of poles: $X(3872)$, hadronic atoms

$$T(E) = \frac{1}{C_0^{-1} - [J_0(E) + J_c(E) + J_{|\Psi\rangle}(E)]}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Renormalization: $C_{0R}^{-1} = C_0^{-1} + \Lambda(\mu_0 + \mu_c)/\pi^2$

X atom

- $X(3872)$ gives the renormalization condition: pole at $E = -\Delta - \delta - i\frac{\Gamma_0}{2}$

$$C_{0R}^{-1} = \frac{\mu_0}{2\pi}\sqrt{2\mu_0\delta} + \frac{\mu_c}{2\pi}\sqrt{2\mu_c\left(\Delta + \delta - i\frac{\delta\Gamma}{2}\right)} - \sum_{n=1}^{\infty}\frac{\alpha^3\mu_c^3}{\pi n^3}\frac{1}{\Delta + \delta - E_n - i\delta\Gamma/2}$$
$$= \frac{\mu_c}{2\pi}\sqrt{2\mu_c\Delta}\left[1 + \mathcal{O}\left(\frac{\delta}{\Delta}, \frac{\delta\Gamma}{\Delta}, \frac{\alpha^3\mu_c^{3/2}}{\Delta^{3/2}}\right)\right]$$

- S-wave hadronic atom poles: $E = -E_{An} - i\frac{\Gamma_c}{2}$

$$0 = C_{0R}^{-1} + i\frac{\mu_0}{2\pi}\sqrt{2\mu_0\left(\Delta - E_{An} - i\frac{\delta\Gamma}{2}\right)} - \frac{\mu_c}{2\pi}\sqrt{2\mu_c E_{An}} - \sum_{n=1}^{\infty}\frac{\alpha^3\mu_c^3}{\pi n^3}\frac{1}{-E_{An} + E_n}$$

- X atom binding energy and decay width (due to decays of D^{*-} and into $D^0\bar{D}^{*0}$)

$$\mathrm{Re}\,E_{A1} = E_1 - \frac{\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} \simeq 22.92\ \mathrm{keV}$$

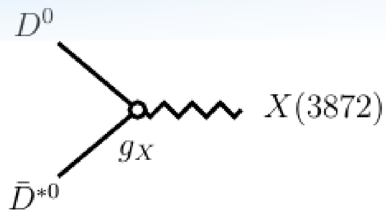
$M_{A1} = (3879.89 \pm 0.07)$ MeV

$$\Gamma_c + 2\,\mathrm{Im}\,E_{A1} = \Gamma_c + \frac{2\alpha^3\mu_c^2}{\sqrt{2\mu_c\Delta}} = (89.2 \pm 1.8)\ \mathrm{keV}$$

Γ_c || (83.4 ± 1.8) KeV

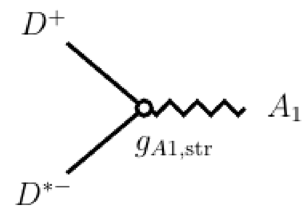
X atom

- Effective coupling for $D^0\bar{D}^{*0} \to X(3872)$

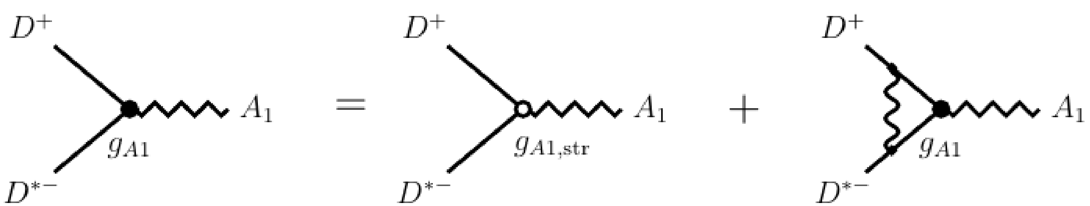


$$g_X^2 = \lim_{E\to-\Delta-\delta-i\frac{\Gamma_0}{2}} \left(E+\Delta+\delta+i\frac{\Gamma_0}{2}\right) T_{11}(E) = \frac{2\pi}{\mu_0^2}\sqrt{2\mu_0\delta}\left[1+\mathcal{O}\left(\frac{\delta^{1/2}}{\Delta^{1/2}}\right)\right]^{-1}$$

- Effective coupling for $D^+D^{*-} \to A_1$



$$g_{A1,\rm str}^2 = \lim_{E\to-E_{A1}-i\frac{\Gamma_c}{2}} \left(E+E_{A1}+i\frac{\Gamma_c}{2}\right) T_{22}(E) = -i\frac{\pi\alpha^3}{\Delta}\left[1+\mathcal{O}\left(\frac{\alpha^2\mu_c}{\Delta}\right)\right]^{-1}$$



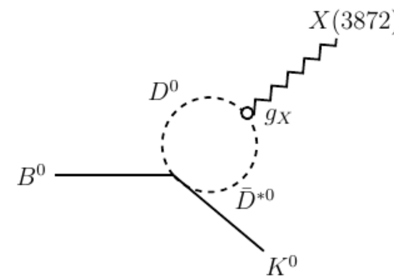
$$|g_{A1}|^2 = S_0(x)|g_{A1,\rm str}|^2 = \frac{\pi\alpha^3}{\Delta}\frac{2\pi x}{1-e^{-2\pi x}}$$

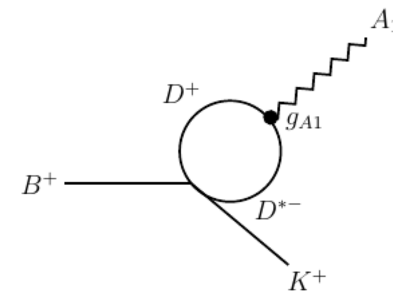
Sommerfeld factor

$$x = \frac{\alpha\mu_c}{p} = \frac{\alpha\mu_c}{\sqrt{2\mu_c E_{A1}}}$$

X atom

- Productions of the $X(3872)$ and the X atom
- Scale separation: factorization





Braaten, Kusunoki, PRD72(2005)014012

$$\mathcal{A}_{B^0\to XK^0} = \mathcal{A}^{\text{s.d.}}_{B^0\to(DD^*)^0_+K^0}\, g_X$$

$$\mathcal{A}_{B^+\to A_1K^+} = \mathcal{A}^{\text{s.d}}_{B^+\to(DD^*)_+K^+}\, g_{A1}$$

- Isospin symmetry: the short-distance parts are the same

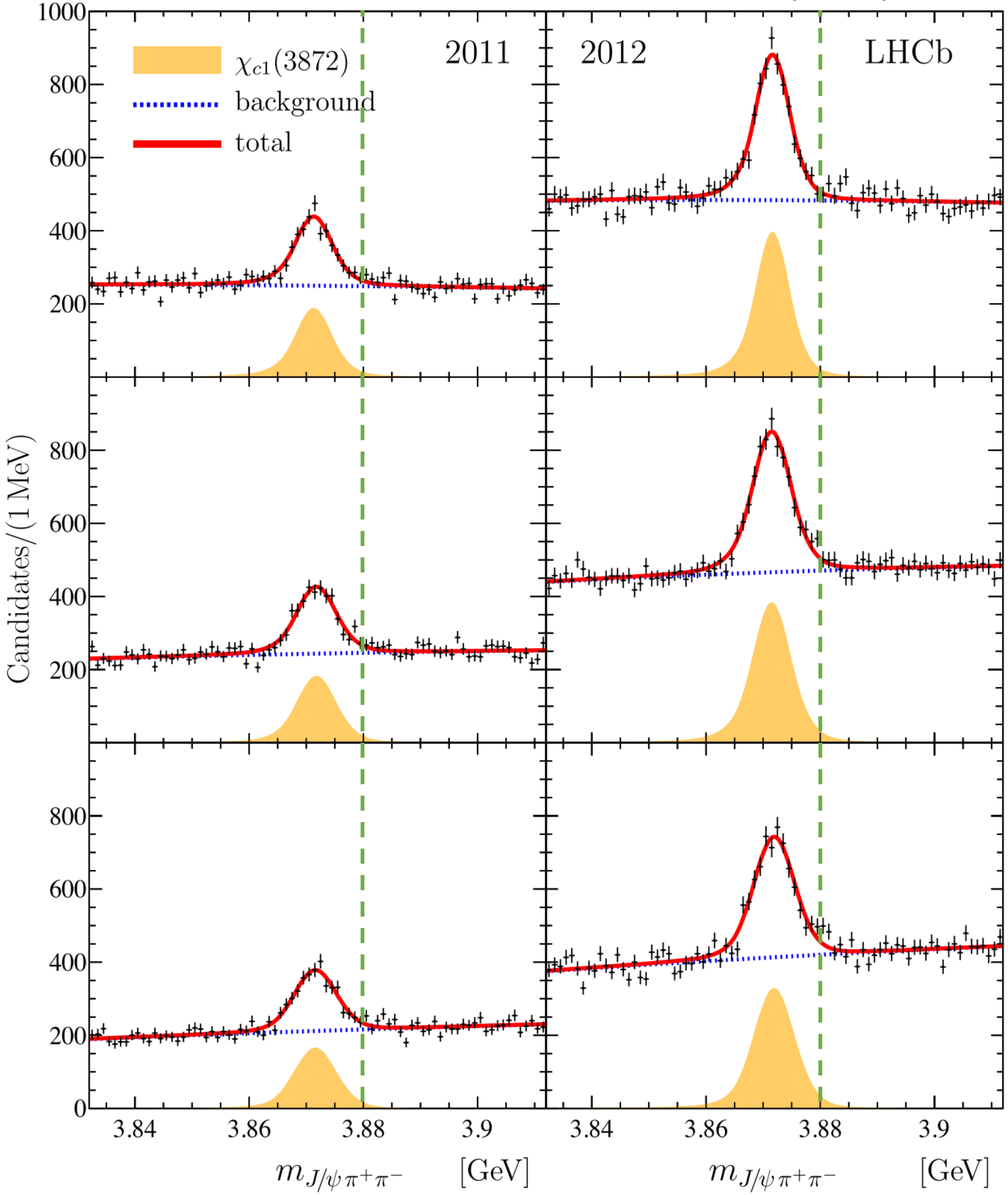
$$R_\Gamma \equiv \frac{\Gamma_{B^+\to A_1K^+}}{\Gamma_{B^0\to XK^0}} = \frac{|g_{A1}|^2}{|g_X|^2} \qquad R_\sigma \equiv \frac{d\sigma_{pp\to A_1+y}}{d\sigma_{pp\to X+y}} = \frac{|g_{A1}|^2}{|g_X|^2}$$

- Production rate for the X atom: $R_\Gamma \simeq R_\sigma \gtrsim 8\times 10^{-3}$
- Null signal leads to a lower bound on the $X(3872)$ binding energy

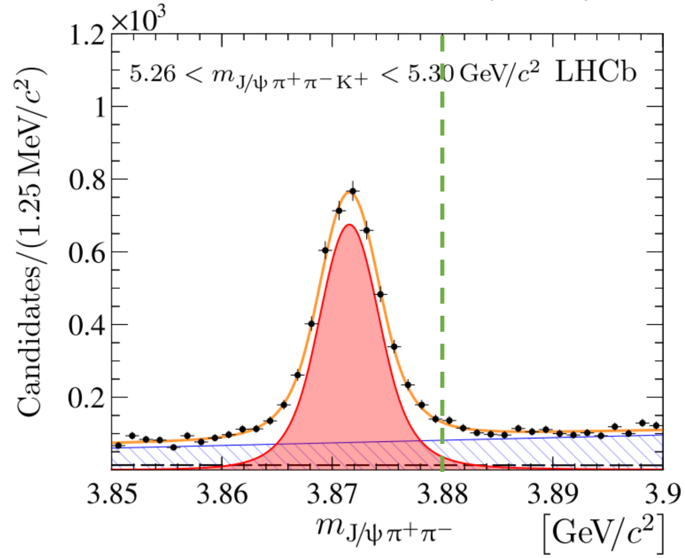
$$\delta \simeq \frac{11\ \text{eV}}{R_\Gamma^2} \simeq \frac{11\ \text{eV}}{R_\sigma^2}$$

X atom

LHCb, PRD102(2020)092005

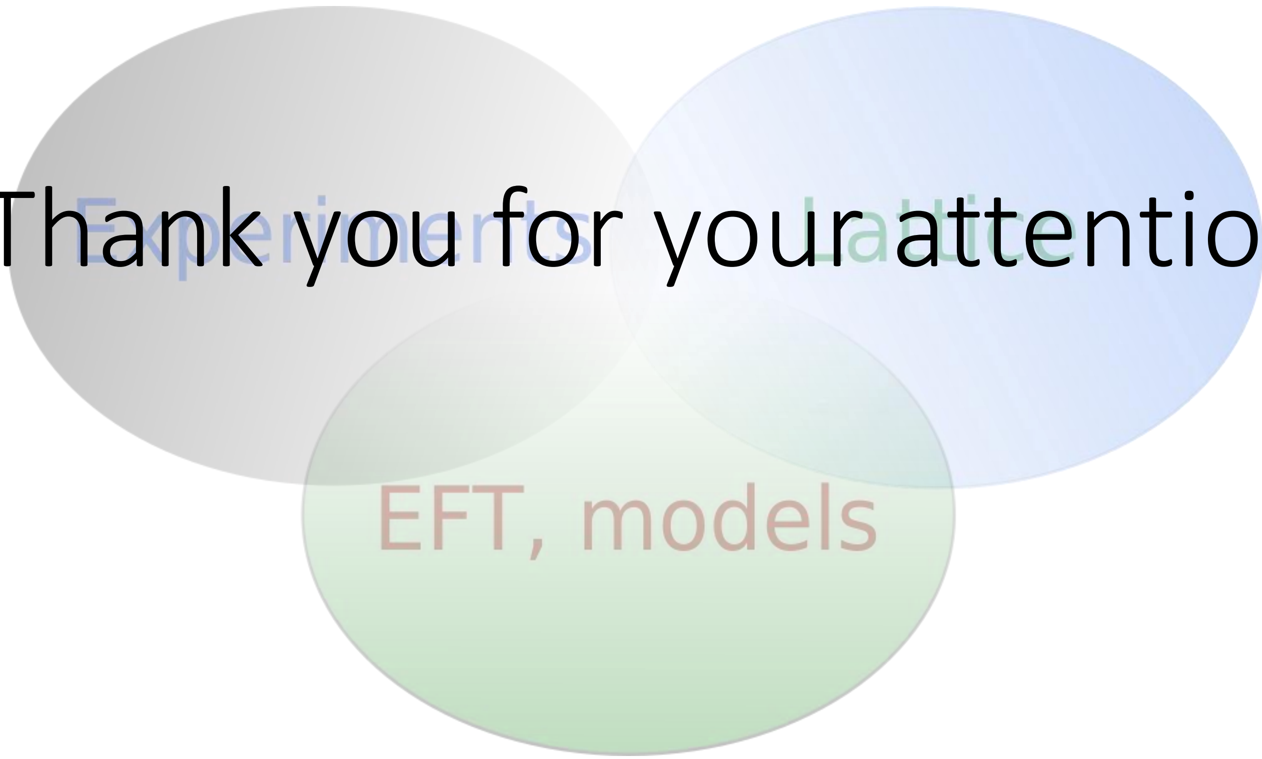


LHCb, JHEP08(2020)123



Conclusion

- The X(3872) has been discovered for 17 years, debates continue
- Important to precisely measure the binding energy and width of X(3872):
  - X atom can be used to set a lower limit on the X(3872) binding energy and to understand the debates regarding the production



Thank you for your attention!