

Exotic quarkonium and double heavy baryons in EFT

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**Institut de Física
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Introduction and Motivation

- ▶ A large number of exotic quarkonium states have been discovered in the past 20 years.
- ▶ Many new structures with a heavy quark antiquark pair have been proposed.
- ▶ All double heavy hadrons have some common characteristics.



- ▶ $Q\bar{Q}$ + light quarks and gluons.
 - Heavy quarkonium.
 - Heavy hybrids $Qg\bar{Q}$.
 - Tetraquarks $Q\bar{Q}q\bar{q}$.
 - Pentaquarks $Q\bar{Q}qqq$.
 - ...
- ▶ QQ + light quarks and gluons.
 - Double heavy baryons QQq .
 - Hybrids baryons $QQgq$.
 - Tetraquarks $QQq\bar{q}$.
 - Pentaquarks $QQqqq$.
 - ...
- ▶ We propose an **unified EFT framework** to describe any **doubly heavy hadron**.

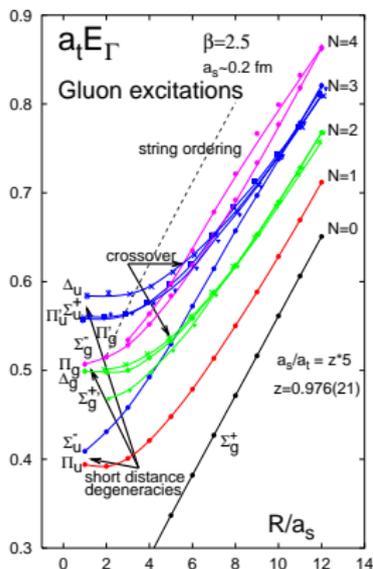
Static Limit

- ▶ The natural starting point when studying hadrons with heavy quarks is NRQCD.
- ▶ At leading order in NRQCD the heavy quarks are static.
- ▶ In this limit the spectrum of the theory is formed by the static energies

$$E_{\kappa^p \Lambda_\eta}(\mathbf{r}) = \lim_{t \rightarrow \infty} \frac{i}{t} \ln \langle 0 | \text{Tr} \left[\mathcal{P}_{\kappa \Lambda} \mathcal{O}_{\kappa^p}(t/2, \mathbf{r}, \mathbf{R}) \mathcal{O}_{\kappa^p}^\dagger(-t/2, \mathbf{r}, \mathbf{R}) \right] | 0 \rangle$$

- The heavy quark pair distance r .
 - Light-degrees of freedom quantum numbers encoded in \mathcal{O}_{κ^p} : spin κ , parity p , flavor...
 - Representation Λ_η^σ of the cylindrical symmetry group $D_{\infty h}$.
 - $\mathcal{P}_{\kappa \Lambda}$ projects the κ^p representation of the light degrees of freedom into Λ_η^σ representation.
- ▶ Static energies are nonperturbative and should be computed on the lattice.

Static Limit: Hybrids



• Quenched lattice NRQCD.

Juge, Kuti, Morningstar Phys.Rev.Lett.90
(2003)

$O(3)$	$D_{\infty h}$
1^{+-}	Σ_u^-, Π_u
1^{--}	$\Sigma_g^{+'}, \Pi_g$
2^{+-}	$\Sigma_u^+, \Pi_u', \Delta_u$
2^{--}	$\Sigma_g^-, \Pi_g', \Delta_g$

$$\mathcal{P}_{10} = \mathbb{1}_3 - (\hat{r} \cdot \mathbf{S}_1)^2,$$

$$\mathcal{P}_{11} = (\hat{r} \cdot \mathbf{S}_1)^2,$$

$$\mathcal{P}_{20} = \mathbb{1}_5 - \frac{5}{4} (\hat{r} \cdot \mathbf{S}_2)^2 + \frac{1}{4} (\hat{r} \cdot \mathbf{S}_2)^4,$$

$$\mathcal{P}_{21} = \frac{4}{3} (\hat{r} \cdot \mathbf{S}_2)^2 - \frac{1}{3} (\hat{r} \cdot \mathbf{S}_2)^4,$$

$$\mathcal{P}_{22} = -\frac{1}{12} (\hat{r} \cdot \mathbf{S}_2)^2 + \frac{1}{12} (\hat{r} \cdot \mathbf{S}_2)^4.$$

$$\mathcal{O}_{\kappa\rho}^{Q\bar{Q}}(t, \mathbf{r}, \mathbf{R}) = \chi_c^\top(t, \mathbf{x}_2) \phi(t, \mathbf{x}_2) \mathcal{Q}_{Q\bar{Q}\kappa\rho}(t, \mathbf{R}) \phi(t, \mathbf{R}, \mathbf{x}_1) \psi(t, \mathbf{x}_1)$$

$$\mathcal{Q}_{1^{+-}}^\alpha(t, \mathbf{x}) = \left(\mathbf{e}_\alpha^\dagger \cdot \mathbf{B}(t, \mathbf{x}) \right),$$

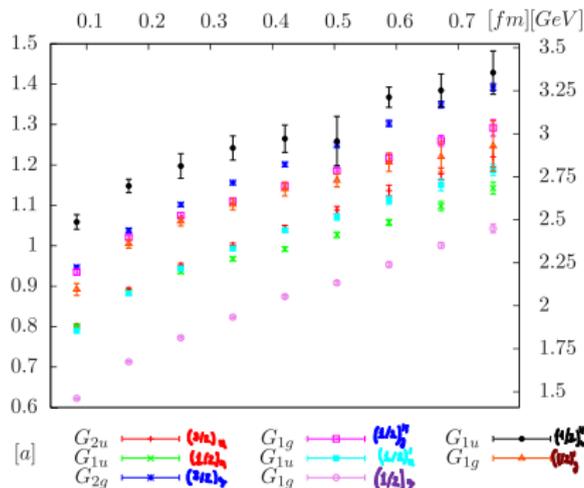
$$\mathcal{Q}_{1^{--}}^\alpha(t, \mathbf{x}) = \left(\mathbf{e}_\alpha^\dagger \cdot \mathbf{E}(t, \mathbf{x}) \right),$$

$$\mathcal{Q}_{2^{+-}}^\alpha(t, \mathbf{x}) = C_{1 m_1 1 m_2}^{2 \alpha} \left(\mathbf{e}_{m_1}^\dagger \cdot \mathbf{D}(t, \mathbf{x}) \right) \left(\mathbf{e}_{m_2}^\dagger \cdot \mathbf{E}(t, \mathbf{x}) \right),$$

$$\mathcal{Q}_{2^{--}}^\alpha(t, \mathbf{x}) = C_{1 m_1 1 m_2}^{2 \alpha} \left(\mathbf{e}_{m_1}^\dagger \cdot \mathbf{D}(t, \mathbf{x}) \right) \left(\mathbf{e}_{m_2}^\dagger \cdot \mathbf{B}(t, \mathbf{x}) \right),$$

...

Static Limit: Double heavy baryons



$O(3)$	$D_{\infty h}$
$(1/2)^+$	$(1/2)_g$
$(3/2)^-$	$(1/2)_u, (3/2)_u$
$(1/2)^-$	$(1/2)'_u$
$(3/2)^+$	$(1/2)'_g, (3/2)_g$

$$P_{\frac{1}{2}\frac{1}{2}} = \mathbb{1}_2$$

$$P_{\frac{3}{2}\frac{1}{2}} = \frac{9}{8}\mathbb{1}_4 - \frac{1}{2}(\hat{r} \cdot \mathbf{S}_{3/2})^2$$

$$P_{\frac{3}{2}\frac{3}{2}} = -\frac{1}{8}\mathbb{1}_4 + \frac{1}{2}(\hat{r} \cdot \mathbf{S}_{3/2})^2$$

Najjar, Bali PoS LAT2009 (2009) $N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV.

$$Q_{\kappa\rho}^{QQ}(t, \mathbf{r}, \mathbf{R}) = \psi^\dagger(t, \mathbf{x}_2)\phi^\dagger(t, \mathbf{R}, \mathbf{x}_2) Q_{QQ\kappa\rho}(t, \mathbf{R}) \phi(t, \mathbf{R}, \mathbf{x}_1)\psi(t, \mathbf{x}_1)$$

$$Q_{(1/2)^+}^\alpha(t, \mathbf{x}) = \underline{T}^I [P_+ q^I(t, \mathbf{x})]^\alpha$$

$$Q_{(1/2)^-}^\alpha(t, \mathbf{x}) = \underline{T}^I [P_+ \gamma^5 q^I(t, \mathbf{x})]^\alpha$$

...

$$Q_{(3/2)^-}^\beta(t, \mathbf{x}) = C_{1m1/2\alpha}^{3/2\beta} \underline{T}^I [(\mathbf{e}_m^\dagger \cdot \mathbf{D})(P_+ q(t, \mathbf{x}))^\alpha]^I$$

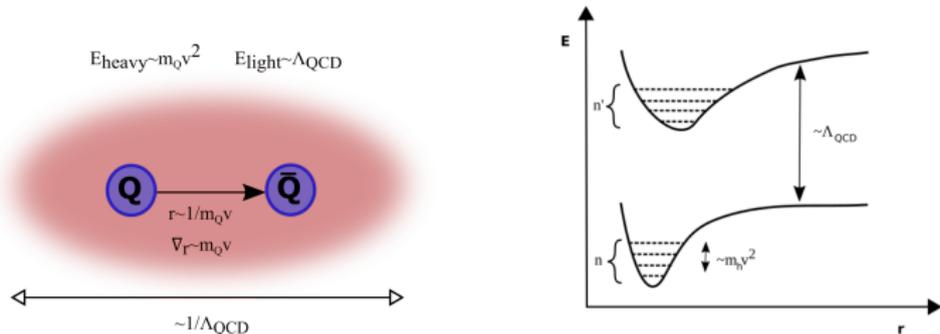
$$Q_{(3/2)^+}^\beta(t, \mathbf{x}) = C_{1m1/2\alpha}^{3/2\beta} \underline{T}^I [(\mathbf{e}_m^\dagger \cdot \mathbf{D})(P_+ \gamma^5 q(t, \mathbf{x}))^\alpha]^I$$

...

- \underline{T}^I are the $\bar{3}$ tensor invariants.

EFT for double heavy hadrons

- ▶ We aim to build an EFT for the heavy quark-quark or heavy quark-antiquark bound states in the minima of the static energies.



- ▶ Heavy quarks are non relativistic $m_Q \gg \Lambda_{\text{QCD}}$.
- ▶ The binding energy is small compared to the characteristic hadronic scale $\Lambda_{\text{QCD}} \gg E_B \sim m_Q v^2$.
- ▶ No assumption on the scale of the interquark distance $r \sim 1/(m_Q v)$ respect to Λ_{QCD} as in strongly coupled pNRQCD. [Brambilla, Pineda, Soto, Vairo Phys.Rev.D 63 \(2001\)](#); [Pineda, Vairo Phys.Rev.D 63 \(2001\)](#)

EFT for double heavy hadrons

- ▶ The fields $\Psi_{\kappa P}^{\alpha}$ have quantum numbers matching those of $\mathcal{O}_{\kappa P}^{\alpha}$.

$$\mathcal{L} = \sum_{\kappa P} \Psi_{\kappa P}^{\dagger} [i\partial_t - h_{\kappa P}] \Psi_{\kappa P}$$

- ▶ $\Psi_{\kappa P}$ has spin components corresponding to the heavy quarks and light degrees of freedom.
- ▶ The **heavy quark spin is suppressed** by $1/m_Q$ but the light degrees of freedom spin is **not**.

$$h_{\kappa P} = \frac{\mathbf{p}^2}{m_Q} + \frac{\mathbf{P}^2}{4m_Q} + V_{\kappa P}^{(0)}(\mathbf{r}) + \frac{1}{m_Q} V_{\kappa P}^{(1)}(\mathbf{r}, \mathbf{p}) + \mathcal{O}(1/m_Q^2)$$

- ▶ The potentials are $(2\kappa + 1) \times (2\kappa + 1)$ matrices in the light degrees of freedom spin space.
- ▶ The static potential can be decomposed into representations of $D_{\infty h}$ with the projectors $\mathcal{P}_{\kappa\Lambda}$

$$V_{\kappa P}^{(0)}(\mathbf{r}) = \sum_{\Lambda} V_{\kappa P \Lambda}^{(0)}(\mathbf{r}) \mathcal{P}_{\kappa\Lambda}$$

EFT for double heavy hadrons

- ▶ Up to leading order the EFT is equivalent to the Born-Oppenheimer approximation.

$$V_{\kappa P}^{(1)}(\mathbf{r}) = V_{\kappa P \text{SI}}^{(1)}(\mathbf{r}) + V_{\kappa P \text{SD}}^{(1)}(\mathbf{r})$$

- ▶ $V_{\kappa P \text{SI}}^{(1)}$ decomposes as the static potential.
- ▶ At next-to-leading order the first heavy-quark spin dependent potentials appear.

$$V_{\kappa P \text{SD}}^{(1)}(\mathbf{r}) = \sum_{\Lambda\Lambda'} \mathcal{P}_{\kappa\Lambda} \left[V_{\kappa P \Lambda\Lambda'}^{sa}(r) \mathbf{S}_{QQ} \cdot (\mathcal{P}_{10}^{\text{c.r.}} \cdot \mathbf{S}_{\kappa}) + V_{\kappa P \Lambda\Lambda'}^{sb}(r) \mathbf{S}_{QQ} \cdot (\mathcal{P}_{11}^{\text{c.r.}} \cdot \mathbf{S}_{\kappa}) \right. \\ \left. + V_{\kappa P \Lambda\Lambda'}^l(r) (\mathbf{L}_{QQ} \cdot \mathbf{S}_{\kappa}) \right] \mathcal{P}_{\kappa\Lambda'}$$

- ▶ $2\mathbf{S}_{QQ} = \boldsymbol{\sigma}_{Q_1} \mathbb{1}_2 \mathbb{1}_2 \mathbb{1}_2 \mathbb{1}_2 \mathbb{1}_2 \boldsymbol{\sigma}_{Q_2} + \mathbb{1}_2 \mathbb{1}_2 \mathbb{1}_2 \mathbb{1}_2 \boldsymbol{\sigma}_{Q_1} \boldsymbol{\sigma}_{Q_2}$, $(\mathcal{P}_{10}^{\text{c.r.}})^{ij} = \hat{r}^i \hat{r}^j$, $(\mathcal{P}_{11}^{\text{c.r.}})^{ij} = \delta^{ij} - \hat{r}^i \hat{r}^j$.
- ▶ The potentials, $V_{\kappa P}^{(0)}$, $V_{\kappa P}^{(1)}$ are nonperturbative and should be computed with lattice QCD.

Matching to NRQCD

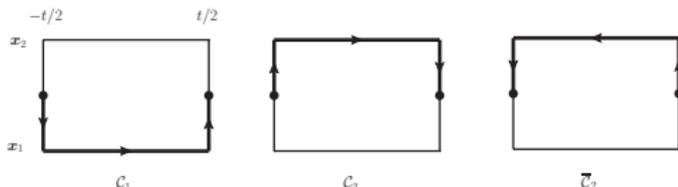
- ▶ The matching condition is

$$\mathcal{O}_{\kappa\rho}^h(t, \mathbf{r}, \mathbf{R}) = \sqrt{Z_{h\kappa\rho}} \Psi_{h\kappa\rho}(t, \mathbf{r}, \mathbf{R}), \quad h = Q\bar{Q}, QQ.$$

- ▶ We match the NRQCD and heavy exotic hadron EFT correlators,

$$\langle 0 | T \{ \mathcal{O}_{\kappa\rho}^h(t/2) \mathcal{O}_{\kappa\rho}^{h\dagger}(-t/2) \} | 0 \rangle = \sqrt{Z_{h\kappa\rho}} \langle 0 | T \{ \Psi_{h\kappa\rho}(t/2) \Psi_{h\kappa\rho}^\dagger(-t/2) \} | 0 \rangle \sqrt{Z_{h\kappa\rho}^\dagger}$$

- ▶ The left hand side can be expanded in terms of Wilson loops with operator insertions.



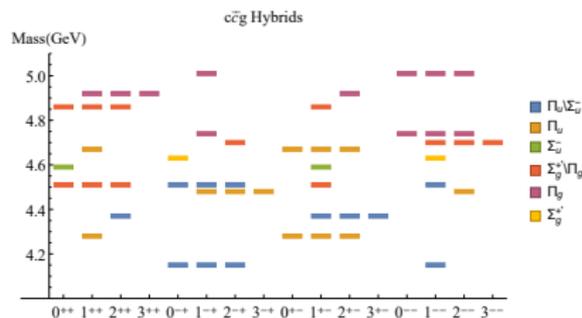
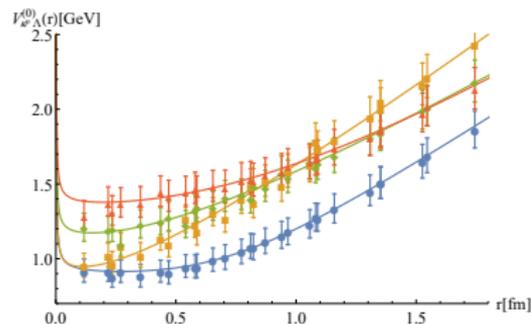
- ▶ At leading order no insertion is needed

$$V_{\kappa\rho\Lambda}^{(0)}(\mathbf{r}) = E_{\kappa\rho\Lambda\eta}(\mathbf{r})$$

Application to heavy hybrids

► Spectrum computed including nonadiabatic mixing for:

- $\kappa = 1^{+-}$ Berwein, Brambilla, JTC, Vairo Phys.Rev.D92 (2015); Oncala, Soto Phys.Rev.D96 (2017)
- $\kappa = 1^{--}$ Pineda, JTC, Phys.Rev.D100 (2019)

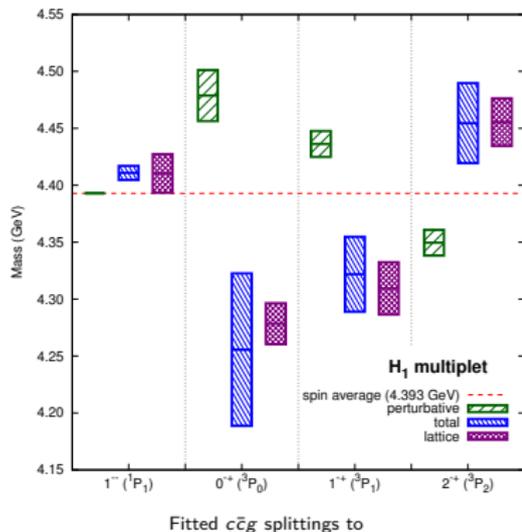


► $b\bar{b}g$ $b\bar{c}g$ also available.

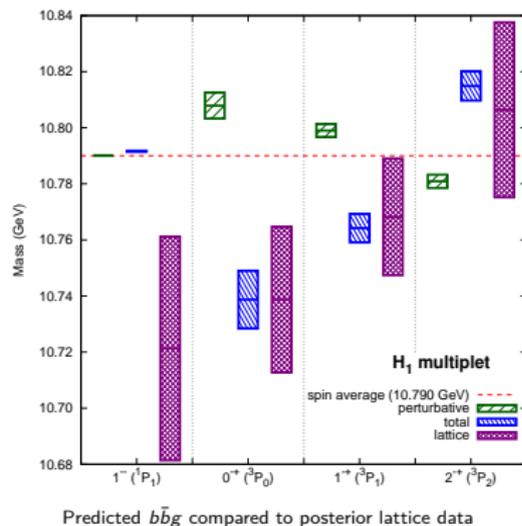
Application to heavy hybrids

► Hyperfine contributions

- LO [Soto Nucl.Part.Phys.Proc. 294 \(2018\)](#)
- LO+NLO and predictions for $b\bar{b}g$ [Brambilla, Lai, Segovia, JTC, Vairo, Phys.Rev.D99 \(2019\)](#)
- Short distance Matching [Brambilla, Lai, Segovia, JTC, Phys.Rev.D101 \(2020\)](#)



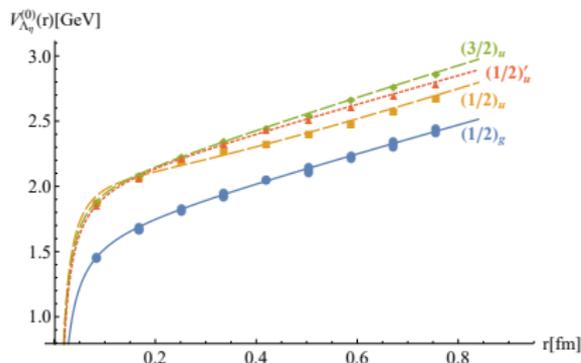
[Cheung et al JHEP12 \(2016\)](#)



[Ryan et al JHEP 02 \(2021\)](#)

Application to double heavy baryons

- ▶ We have applied our formalism to double heavy baryons.



- ▶ We consider the 4 lowest lying static energies.
- ▶ $(3/2)_u$ and $(1/2)_u$ are coupled since both correspond to $\kappa = 3/2^-$.
- ▶ Lattice data: $N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV. [Najjar, Bali PoS LAT2009 \(2009\)](#)

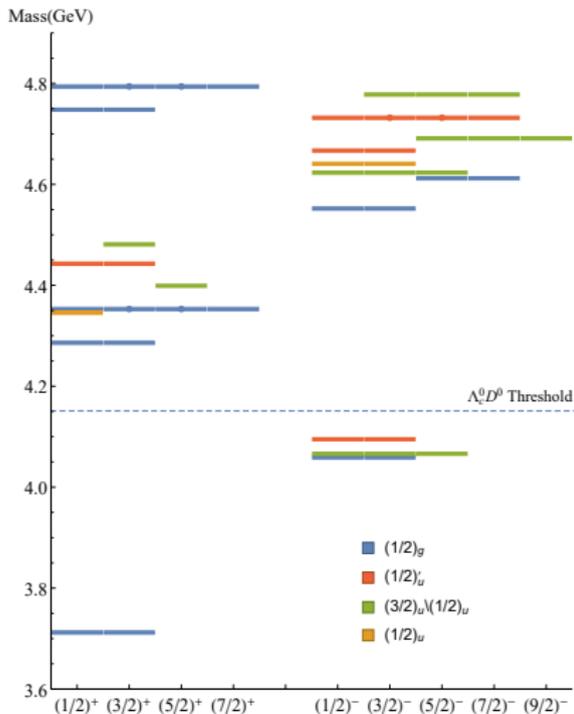
- The potentials are obtained from constrained fits to the lattice data:

- ▶ Short distance:

- * The potentials must reproduce the perturbative quark-quark Coulombic potential plus an offset constant, i.e. should match to weakly coupled pNRQCD. [Brambilla, Vairo Rösch, Phys. Rev. D72 \(2005\)](#)
- * Energy gaps between static energies consistent with heavy-quark-diquark symmetry. [Savage, Wise, Phys. Lett. B248,\(1990\)](#)
- * The offsets are shifted so the lowest one is $\bar{\Lambda}_{(1/2)^+} = 0.555(31)$ GeV from lattice studies of D and B mesons masses. [Bazavov et al. Phys. Rev. D98 \(2018\)](#)

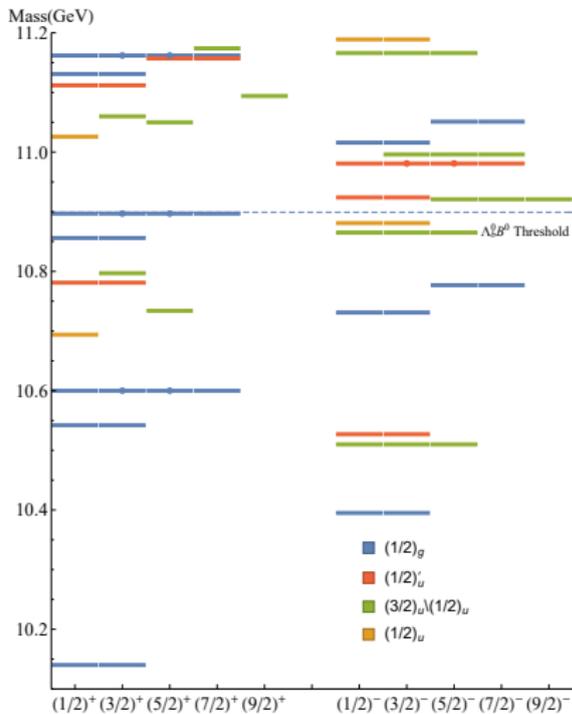
- ▶ Long distance: fixed string-like potential σr with a fitted offset constant, $\sigma = 0.21$ GeV².

Ξ_{cc} spectrum



Soto, JTC, Phys.Rev.D 102 (2020)

Ξ_{bb} spectrum



Double heavy baryons hyperfine splittings

- ▶ At $\mathcal{O}(1/m_Q)$ we have heavy-quark spin dependent operators responsible for hyperfine splittings.
- ▶ There is no lattice data for the potentials but some model independent relations can be derived.
- ▶ For $\kappa = 1/2$ there are only 3 operators:

$$V_{(1/2)\pm\text{SD}}^{(1)}(\mathbf{r}) = V_{(1/2)\pm}^{s_1}(\mathbf{r}) \mathbf{S}_{QQ} \cdot \mathbf{S}_{1/2} + V_{(1/2)\pm}^{s_2}(\mathbf{r}) \mathbf{S}_{QQ} \cdot (\mathcal{T}_2 \cdot \mathbf{S}_{1/2}) + V_{(1/2)\pm}^l(\mathbf{r}) (\mathbf{L}_{QQ} \cdot \mathbf{S}_{1/2})$$

$$\text{with } (\mathcal{T}_2)^{ij} = \hat{r}^i \hat{r}^j - \delta^{ij}/2$$

- ▶ Let us label the mass of the states as $M_{nj\ell} = M_{nl}^{(0)} + M_{nj\ell}^{(1)} + \dots$
 - * n principal quantum number.
 - * l orbital angular momentum.
 - * ℓ sum of orbital angular momentum and light-quark spin.
 - * j total angular momentum.
 - * Heavy quark spin state fixed by Pauli principle, $s_{QQ} = 1$ for l even and $s_{QQ} = 0$ for l odd.

Double heavy baryons hyperfine splittings

- ▶ $l = 0$ only hyperfine contributions from $V_{(1/2)^+}^{s1}$

$$M_{nj0\frac{1}{2}}^{(1)} = \frac{1}{2} \left(j(j+1) - \frac{11}{4} \right) \frac{\langle V_{(1/2)^+}^{s1} \rangle_{n0}}{m_Q}, \Rightarrow 2M_{n\frac{3}{2}0\frac{1}{2}} + M_{n\frac{1}{2}0\frac{1}{2}} = 3M_{n0}^{(0)}$$

- ▶ $l = 1$ only hyperfine contributions from $V_{(1/2)^+}^l$

$$M_{nj1j}^{(1)} = \frac{1}{2} \left(j(j+1) - \frac{11}{4} \right) \frac{\langle V_{(1/2)^+}^l \rangle_{n1}}{m_Q}, \Rightarrow 2M_{n\frac{3}{2}1\frac{3}{2}} + M_{n\frac{1}{2}1\frac{1}{2}} = 3M_{n1}^{(0)}$$

- ▶ $l = 2$ all 3 operators contribute, but since we have 6 states we can find:

$$M_{n\frac{3}{2}2\frac{3}{2}} = \frac{1}{8} \left(5M_{n\frac{1}{2}2\frac{3}{2}} + 3M_{n\frac{5}{2}2\frac{3}{2}} \right)$$
$$M_{n\frac{5}{2}2\frac{5}{2}} = \frac{1}{12} \left(7M_{n\frac{3}{2}2\frac{5}{2}} + 5M_{n\frac{7}{2}2\frac{5}{2}} \right)$$

Double heavy baryons hyperfine splittings: Comparison with lattice QCD

- ▶ Ground state: 1^3S doublet $(1/2, 3/2)^+$. $M_{\Xi_{cc}} \equiv M_{1\frac{1}{2}0\frac{1}{2}}$, $M_{\Xi_{cc}^*} \equiv M_{1\frac{3}{2}0\frac{1}{2}}$,
 $\delta_{hf} = M_{\Xi_{cc}^*} - M_{\Xi_{cc}}$.

Ref.	δ_{hf} [MeV]	spin avg.
Briceno et al Phys.Rev.D86 (2012)	53(94)	3630(50)
Namekawa et al Phys.Rev.D87 (2013)	101(36)	3672(20)
Brown et al Phys.Rev.D90 (2014)	82.8(9.2)	3665(36)
Alexandrou et al Phys.Rev.D90 (2014)	84(58)	3624(33)
Bali et al Phys.Rev.D92 (2015)	85(9)	3666(13)
Padmanath et al Phys.Rev.D91 (2015)	94(12)	3700(6)
Alexandrou et al Phys.Rev.D96 (2017)	76(41)	3657(25)
Our values	136(44)	3712(63)

- ▶ First excitation: 1^1P doublet $(1/2, 3/2)^-$. Spin average agreement with Padmanath et al Phys.Rev.D91 (2015); Bali et al Phys.Rev.D92 (2015)
- ▶ Higher excitations below $\Lambda_c - D$ threshold: $1^3(S \setminus D)$ ($\ell = 3/2$) triplet of the $(3/2)_u - (1/2)_u$ BO potentials $(1/2, 3/2, 5/2)^-$; 1^3S doublet of the $(1/2)'_u$ BO potential $(1/2, 3/2)^-$. Qualitative agreement with Padmanath et al Phys.Rev.D91 (2015).
- ▶ $l = 2$ hyperfine splitting formulas for $n = 1$ can be compared with Padmanath et al Phys.Rev.D91 (2015) and are fulfilled within 2 MeV.
- ▶ Bottomonium ground state doublet: our value $M_{10}^{(0)} = 10.140(77)$ MeV. Agreement with $M_{\text{s.a.}} = 10.166(40)$ MeV Lewis et al Phys.Rev.D79(2009) and $M_{\text{s.a.}} = 10.143(29)$ MeV Brown et al Phys.Rev.D90 (2014), compatible with $M_{\text{s.a.}} = 10.099(17)$ MeV Mohanta et al Phys.Rev.D101 (2020).

Conclusions

- We have developed an EFT framework to describe doubly heavy hadrons (either QQ or $Q\bar{Q}$) containing light degrees of freedom (gluons or light quarks) with any quantum numbers.
 - ▶ The EFT incorporates the heavy quark mass and adiabatic expansions.
 - ▶ We have worked out the EFT up to $\mathcal{O}(1/m_Q)$.
 - ▶ The potentials, which are nonperturbative, are given by NRQCD Wilson loops.
 - ▶ No assumption on the interquark distance is made. In the short distance we can incorporate heavy-quark-diquark symmetry or perturbative computations; in the long distance string models.
- We have applied the framework to double heavy baryons
 - ▶ We have obtain the spectrum for Ξ_{cc} and Ξ_{bb} for the four lowest static energies including the mixing.
 - ▶ Provided model independent formulas for the hyperfine splittings of multiplets with $\kappa = 1/2$ and $l = 0, 1, 2$.
 - ▶ Results compatible with lattice QCD results.

Thank you for your attention

Multiplets

κ^P	Λ_η	l	ℓ	s_{QQ}	j	η_P
$(1/2)^\pm$	$(1/2)_{g/u'}$	0	1/2	1	(1/2, 3/2)	\pm
		1	(1/2, 3/2)	0	(1/2, 3/2)	\mp
		2	(3/2, 5/2)	1	((1/2, 3/2, 5/2), (3/2, 5/2, 7/2))	\pm
		3	(5/2, 7/2)	0	(5/2, 7/2)	\mp
$(3/2)^-$	$(3/2)_u \setminus (1/2)_u$	0\2	3/2	1	(1/2, 3/2, 5/2)	-
		1\3	5/2	0	5/2	+
		2\4	7/2	1	(5/2, 7/2, 9/2)	-
		3\5	9/2	0	9/2	+
		1\3	3/2	0	3/2	+
		2\4	5/2	1	(3/2, 5/2, 7/2)	-
	$(1/2)_u$	3\5	7/2	0	7/2	+
		4\6	9/2	1	(7/2, 9/2, 11/2)	-
		1	1/2	0	1/2	+
		2	1/2	1	(1/2, 3/2)	-