

# Charmonium-like resonances in $D\bar{D}$ scattering

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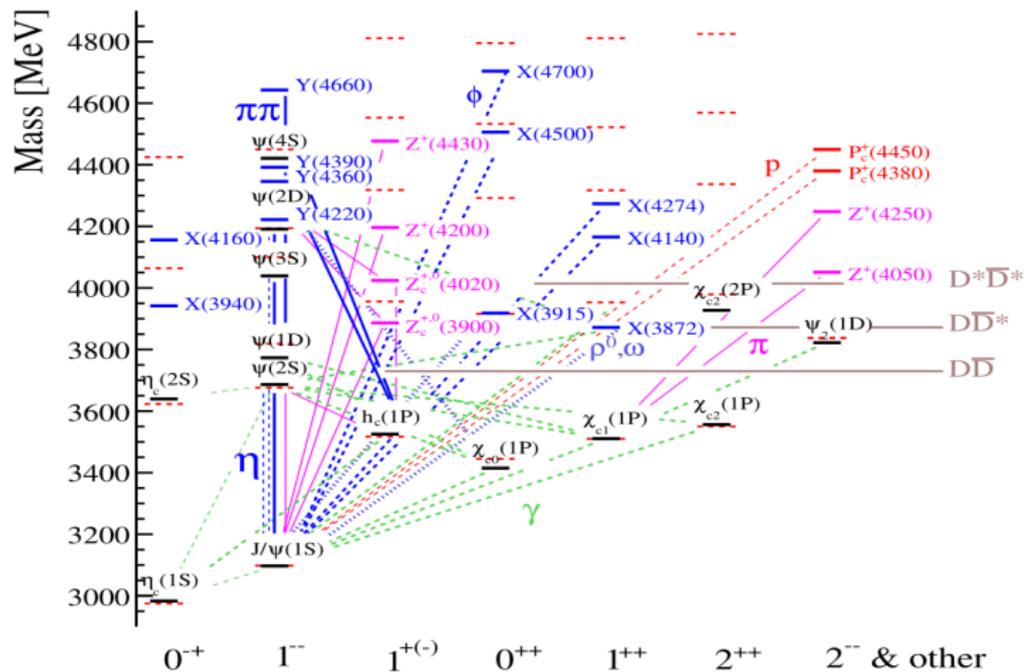
QWG 2021

16<sup>th</sup> March, 2021

with S. Collins, D. Mohler, S. Piemonte and S. Prelovsek

Based on articles [1811.04116](#), [1905.03506](#), [2011.02542](#)

# Why charmonium?



Rich energy spectrum. XYZ states.

Olsen *et al* 1708.04012

$\bar{c}c$  picture works well for states below open charm threshold.

No single description for states above the open charm threshold.

# What do we intend to do?

- Investigate what can lattice QCD educate us in this regard?
- Focus on vector and scalar charmonium spectra.  
Calculations in the past [ [Lang, et al. 1503.05363](#) ]
- Progressively increase the rigor in the investigation.
  - Multiple inertial frames assuming the  $\bar{c}c$  picture.
  - Amplitude analysis for excitations above the open charm threshold.  
Start with elastic pseudoscalar-pseudoscalar scattering.
  - Coupled channel studies.
  - Remarks and future plans ...
- Working assumptions:
  - Effects from charm annihilation to be small.
  - Three hadron scattering not to be important.
  - Unphysically heavy pion mass  $m_{\pi}^{lat} \sim 2m_{\pi}^{phys}$ .
  - Neglected 'irrelevant' scattering channels, discretization effects, ....

# How do we do?

- Ensemble : CLS
  - U101  $N_f = 2 + 1$ ,  $L \sim 2$  fm,  $N_{ev} = 90$
  - H105  $N_f = 2 + 1$ ,  $L \sim 2.7$  fm,  $N_{ev} = 150$
  - $m_\pi \sim 280$  MeV,  $m_K \sim 467$  MeV
  - Two charm quark masses:  $m_D \sim 1762$  MeV and  $1927$  MeV
- Multiple excited state extraction  
Correlation matrices using a large basis of interpolating operators

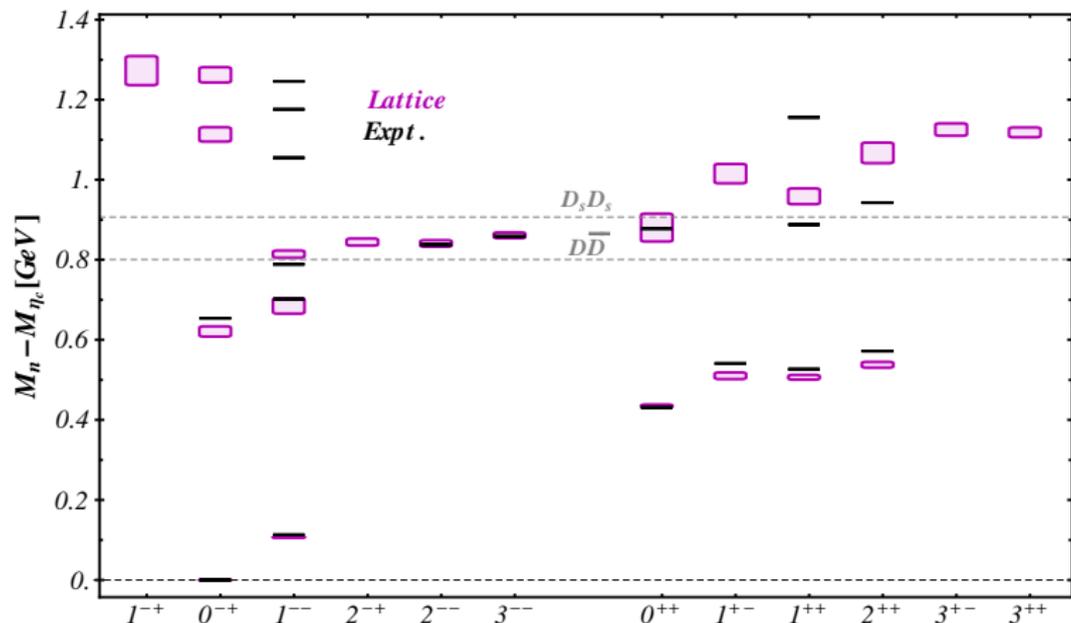
$$C_{ji}(t_f - t_i) = \langle 0 | O_j(t_f) \bar{O}_i(t_i) | 0 \rangle = \sum_n \frac{Z_n^{i*} Z_n^j}{2E_n} e^{-E_n(t_f - t_i)}$$

Operator state overlap factors :  $Z_n^j = \langle 0 | O_j | n \rangle$ .

Physical state information from variational analysis of  $C_{ji}(t_f - t_i)$ .

- Finite volume spectrum including all relevant two-meson operators  
 $\mathcal{O} = \bar{Q}\Gamma Q, (\bar{Q}\Gamma_1 q)_{1_c} (\bar{q}\Gamma_2 Q)_{1_c}, (\bar{Q}\Gamma_1 Q)_{1_c} (\bar{q}\Gamma_2 q)_{1_c}$ .
- Determination of scattering amplitudes:  
Utilize “TwoHadronsInBox” toolbox. [Morningstar et al. 1707.05817](#)

# Charmonium spectrum assuming $\bar{c}c$ picture

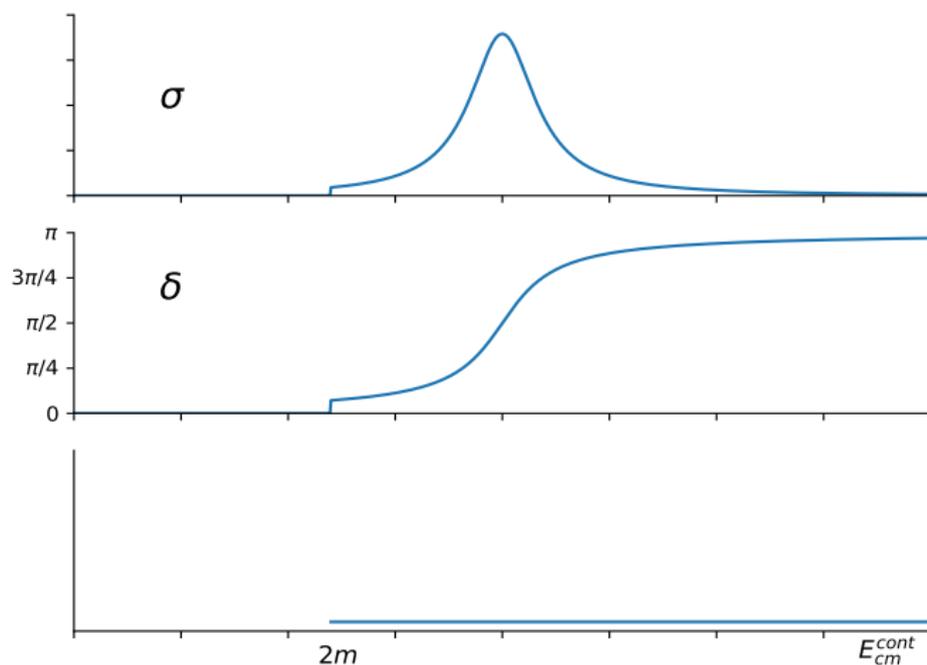


Analysis utilizing only  $\bar{c}c$  type operators.

MP *et al*, 1811.04116

# Resonances in the infinite volume continuum

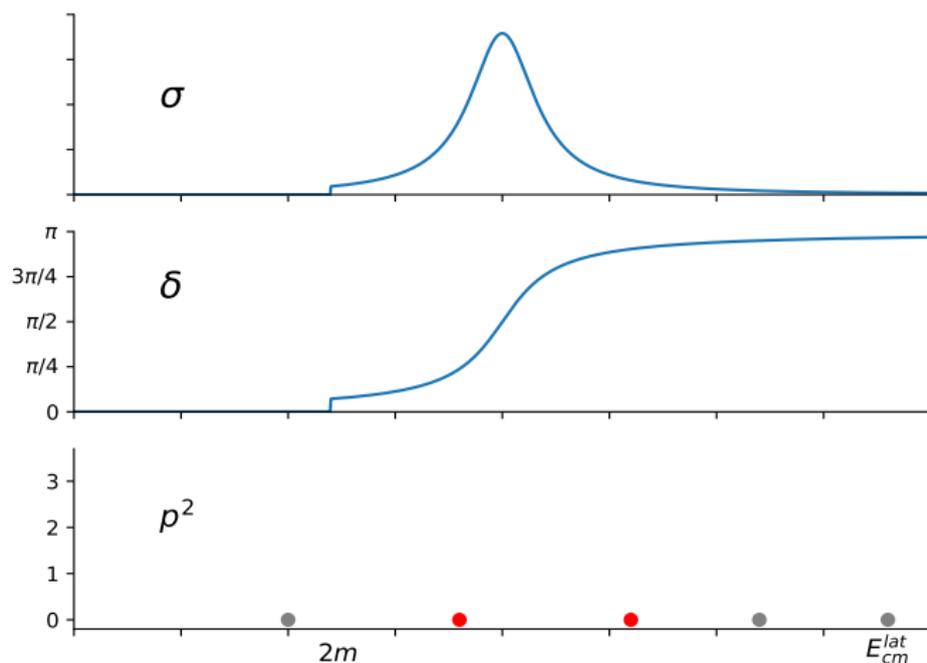
Scattering cross sections, phase shifts, branch cuts, Riemann sheets.



Schematic picture for illustration. Do not take it quantitatively.

# Resonances on the lattice (elastic) : ??

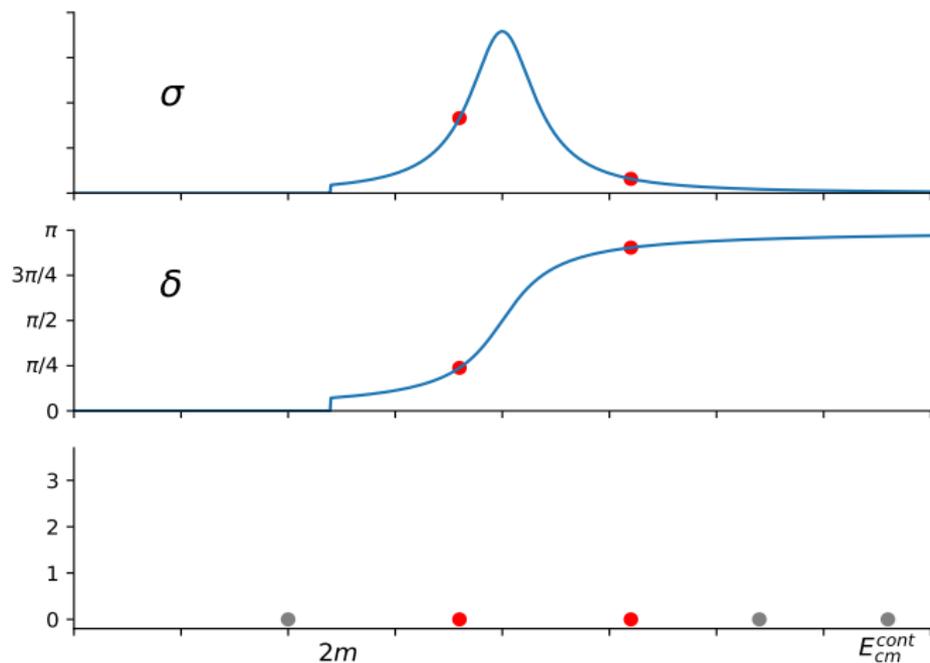
Discrete spectrum: No branch cuts, no Riemann sheets, no resonances!



Maiani-Testa no-go theorem [1990]

# Resonances on the lattice (elastic) : Lüscher (1991)

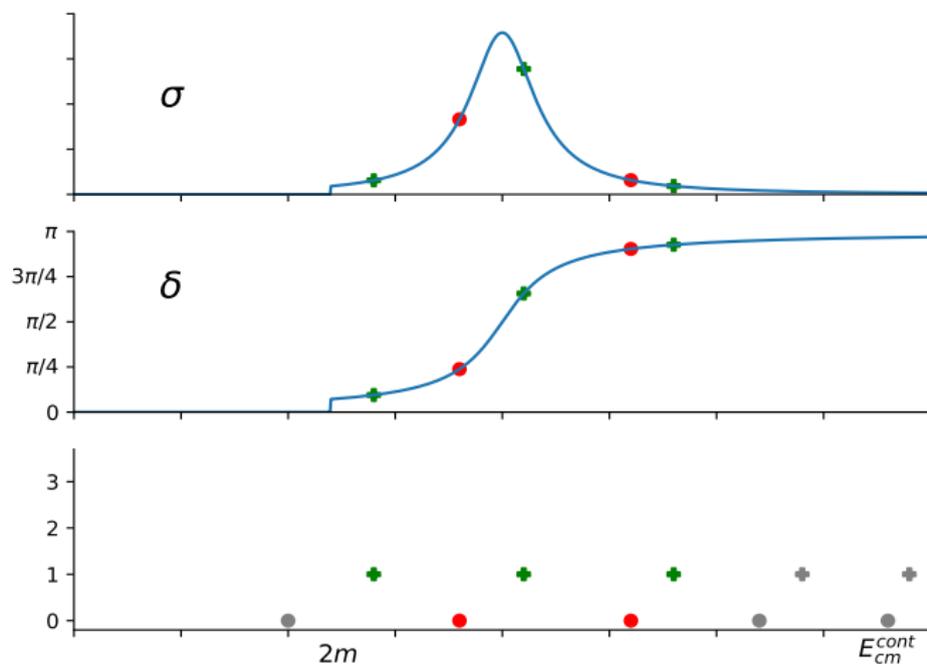
Infinite volume scattering amplitudes  $\Leftrightarrow$  Finite volume spectrum



Lüscher [1991]

# Resonances on the lattice (elastic) : Lüscher (1991)

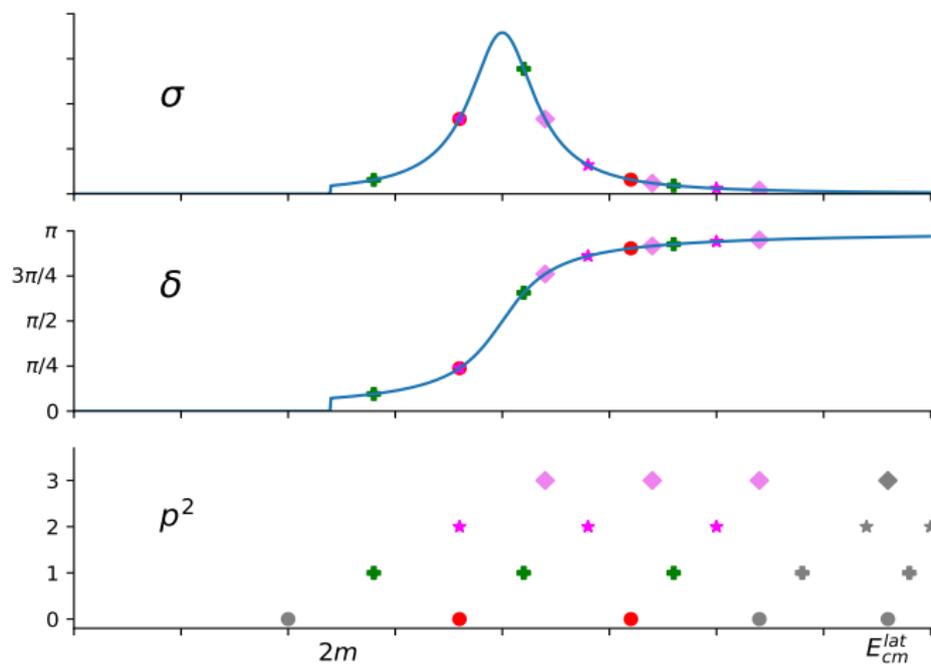
Infinite volume scattering amplitudes  $\Leftrightarrow$  Finite volume spectrum



Different inertial frames can be utilized to extract more information

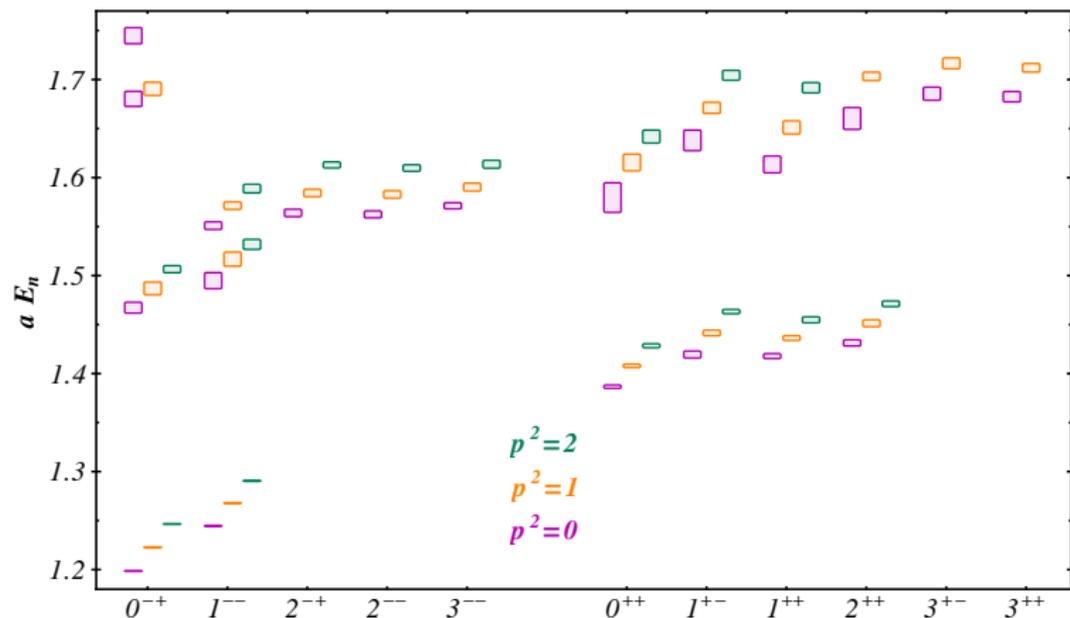
# Resonances on the lattice (elastic) : Lüscher (1991)

Infinite volume scattering amplitudes  $\Leftrightarrow$  Finite volume spectrum



Different inertial frames can be utilized to extract more information

# $\bar{c}c$ spectrum: multiple inertial frames



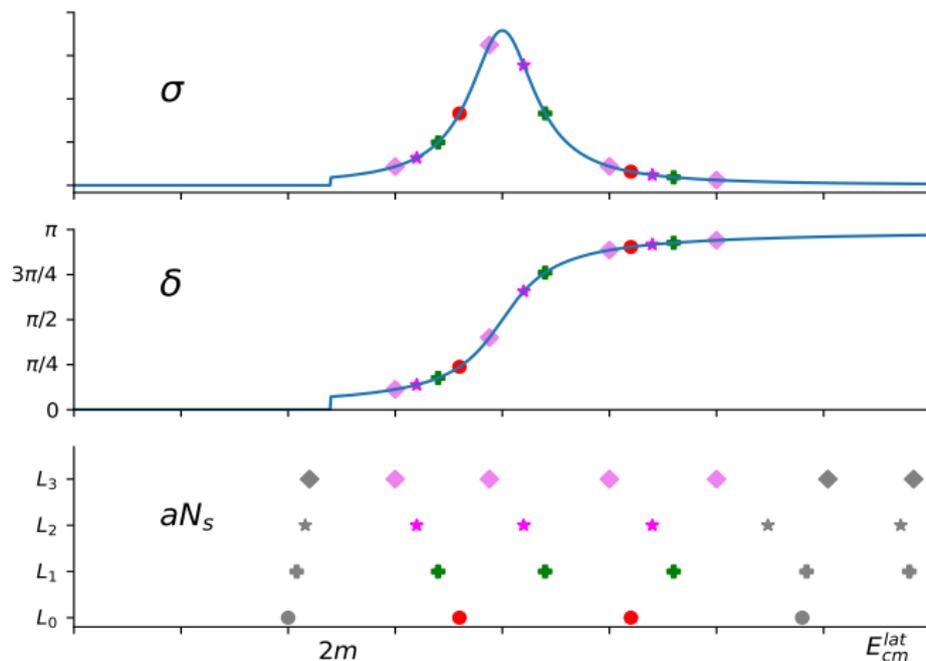
We utilize three inertial frames.  $p^2$  in units of  $(2\pi/L)^2$ .

MP *et al*, 1811.04116

Above results are using only  $\bar{c}c$  interpolators.

# Resonances on the lattice (elastic) : Lüscher (1991)

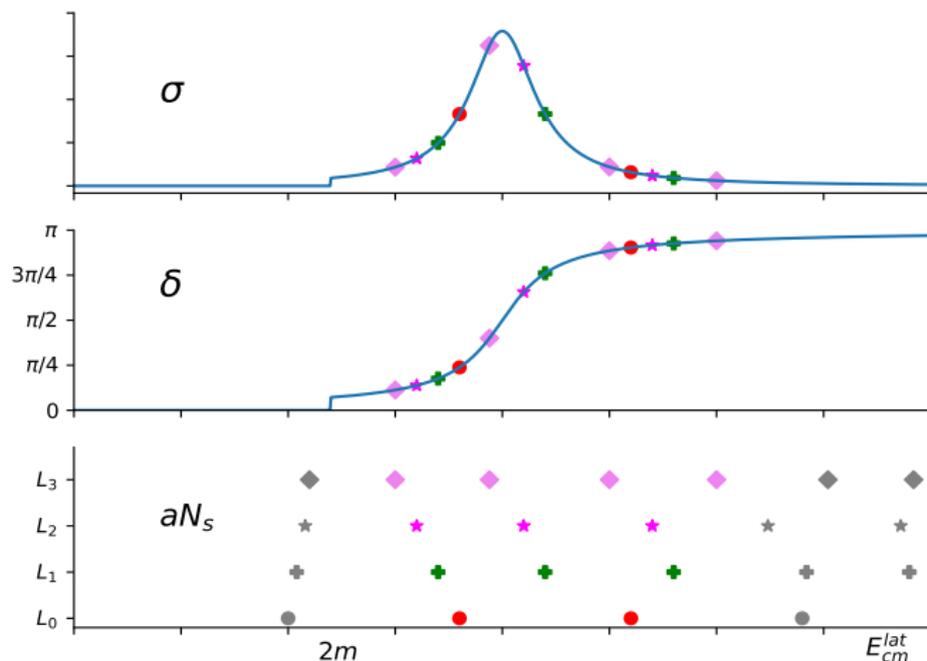
Infinite volume scattering amplitudes  $\Leftrightarrow$  Finite volume spectrum



Multiple physical volumes can also be utilized to extract more information

# Resonances on the lattice (elastic) : Lüscher (1991)

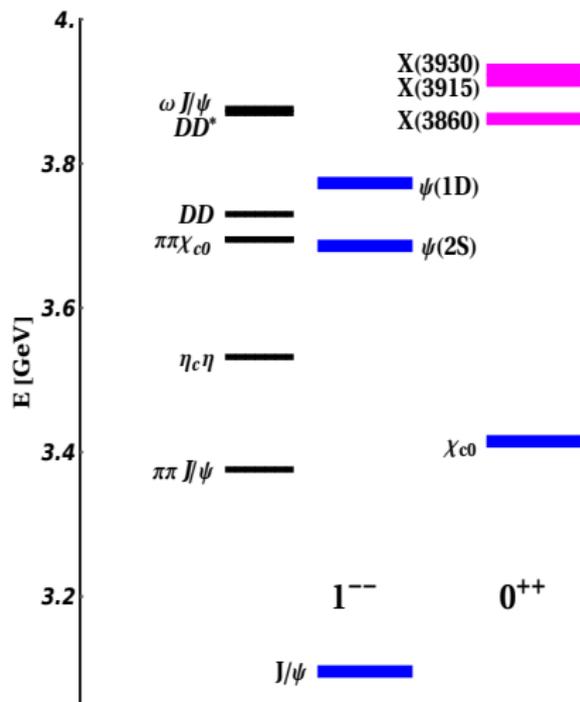
Infinite volume scattering amplitudes  $\Leftrightarrow$  Finite volume spectrum



We utilize two volumes.

For generalizations of Lüscher framework, c.f. Briceño, Hansen 2014-15

# Our current focus



Currently we ignore  $\eta_c \eta$  and all three particle channels in our entire analysis.

$D\bar{D}$  scattering.

Involves only spinless scattering particles.

$J^{PC} = 1^{--}$ , vector charmonia.

$\Gamma_{\psi(1D) \rightarrow D\bar{D}} / \Gamma \sim 93\%$ .

Well understood below 4 GeV.

As a proof of concept.

$J^{PC} = 0^{++}$ , scalar charmonia.

In search of the  $\chi_{c0}(2P)$ .

Other excitations, if any exist!

Three experimentally observed states.

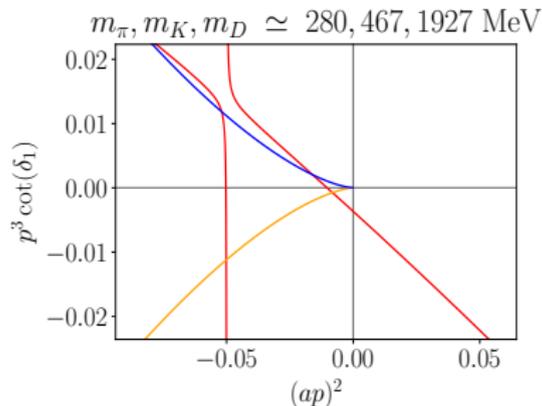
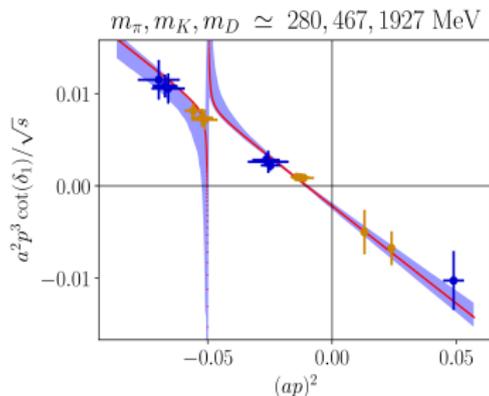
X(3860): [Belle 1704.01872](#)

X(3915): [Belle hep-ex/0408126](#)

X(3930): [LHCb 2009.00026](#)

Results are exploratory in nature, considering the assumptions involved.

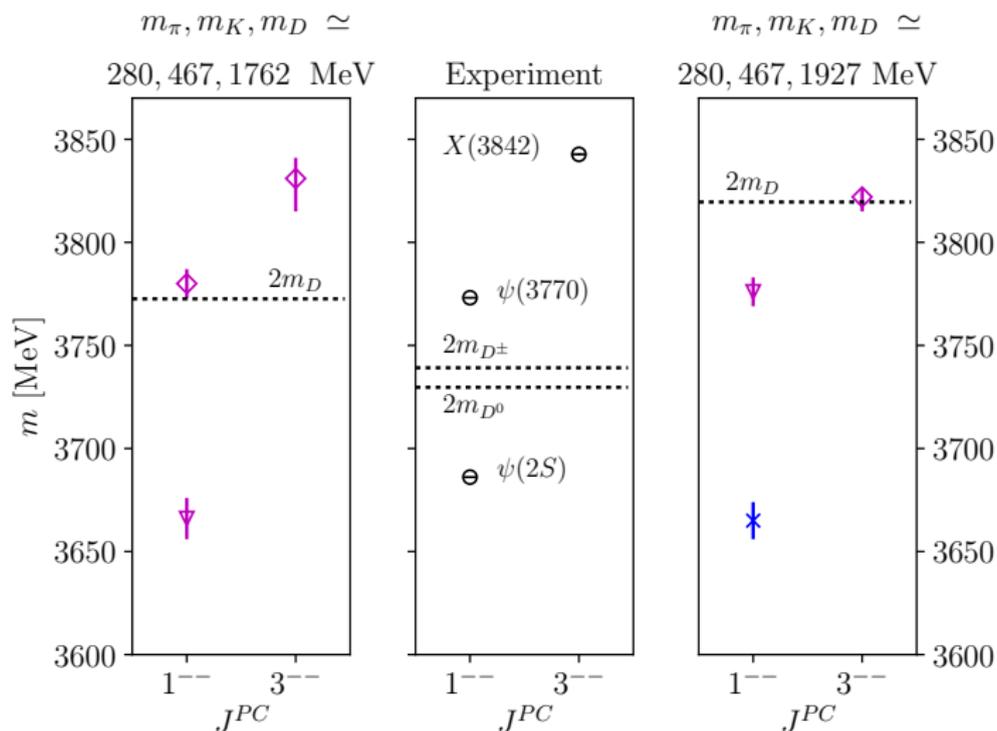
# Elastic analysis: Vector charmonia



- $1^{--}$  channel in  $\bar{D}D$  scattering in  $p$ -wave.
- Reaction matrix ( $K: S = (1 + iK)(1 - iK)^{-1}$ ):  $K_1^{-1} = \left( \frac{G_1^2}{m_1^2 - s} + \frac{G_2^2}{m_2^2 - s} \right)^{-1}$
- (Virtual) bound state constraint:  $p^3 \cot \delta_1 = (-)(p^2) \sqrt{-p^2}$ .
- Results presented for heavier than physical charm quark mass.

Piemonte *et al*, 1905.03506

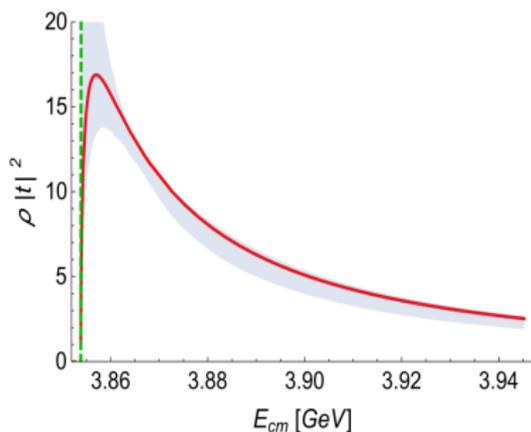
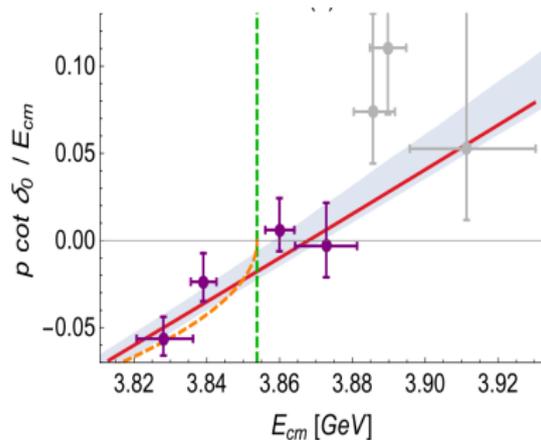
# Spectrum summary: Vector charmonia



The coupling  $g_{\psi(1D) \rightarrow \bar{D}D} = 18.9^{+0.8}_{-0.7}$  for light  $m_c$ :  $g_{exp} = 18.7(9)$

Piemonte *et al*, 1905.03506

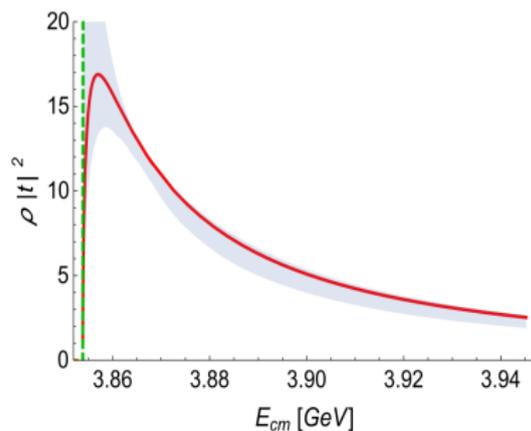
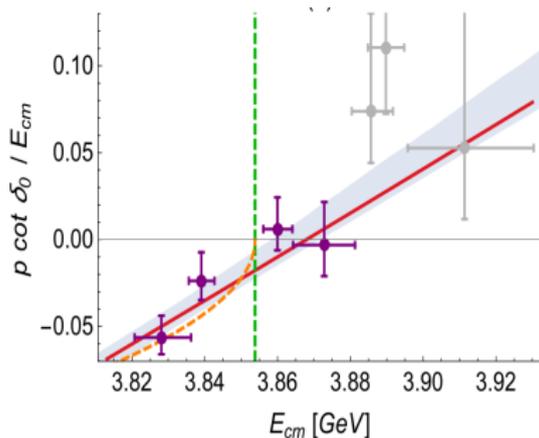
# Scalar charmonia around the $D\bar{D}$ threshold



- $0^{++}$  channel in  $\bar{D}D$  scattering in  $s$ -wave.
- Reaction matrix ( $K$ :  $S = (1 + iK)(1 - iK)^{-1}$ ):  $K_1^{-1}/\sqrt{s} = a + b s$
- Bound state constraint:  $p \cot \delta_0 = -\sqrt{-p^2}$ .
- A shallow bound state with B.E.:  $m - 2m_D = -4.0^{+3.7}_{-5.0}$  MeV
- Results presented for heavier than physical charm quark mass.

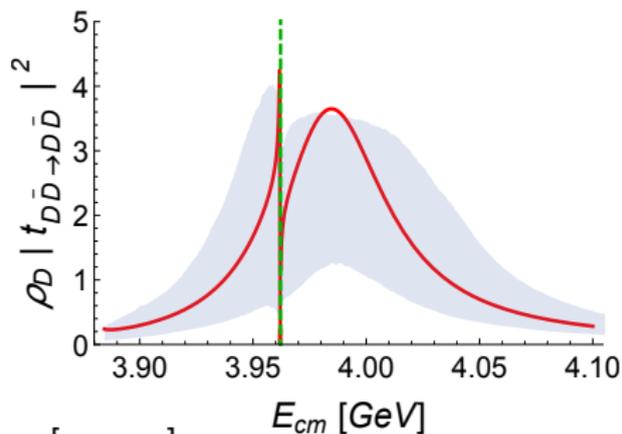
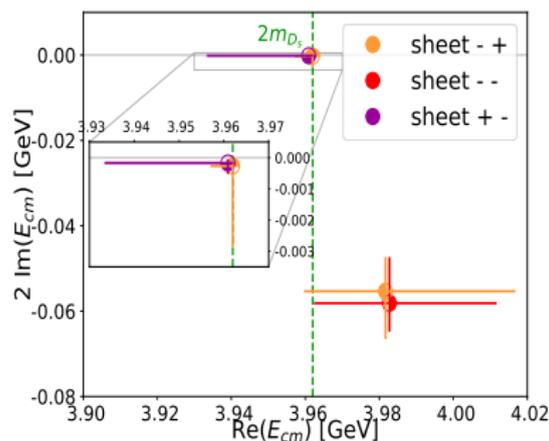
Prelovsek *et al*, 2011.02542

# Scalar charmonia around the $D\bar{D}$ threshold



- Signal: An enhancement in the  $D\bar{D}$  rate just above threshold.  
 $N_{D\bar{D}} \propto \rho |t|^2$
- Physics in a simulation with physical quark masses is yet to be performed.
- Predictions from phenomenological models.  
[hep-ph/0612179](#), [1305.4487](#), [1605.09649](#)
- Evidences in the experiments  
[0708.3812](#), [0712.1758](#)

# Scalar charmonia around the $D_s^+ D_s^-$ threshold



- Coupled channel  $\bar{D}D - D_s^+ D_s^-$  scattering [*s*-wave].  
 $J\psi - \omega$  and  $\eta_c \eta$  are not included in the amplitude analysis.

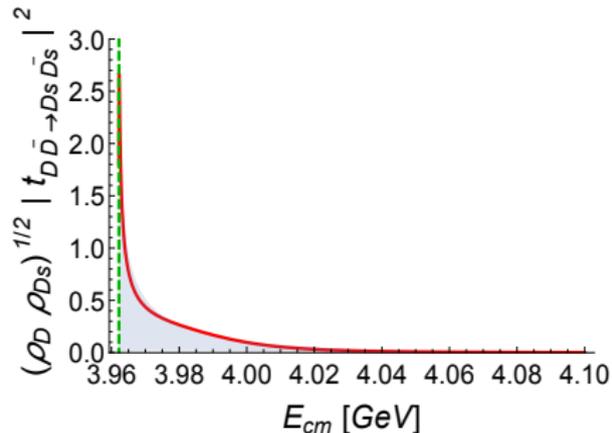
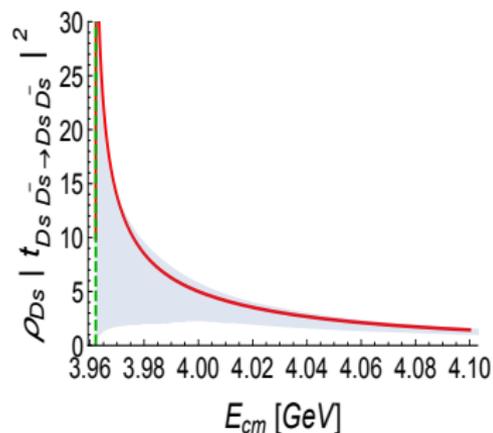
- Reaction matrix ( $K$ :  $S = (1 + iK)(1 - iK)^{-1}$ ):

$$\frac{(\tilde{K}^{-1})^{l=0}}{\sqrt{s}} = \begin{pmatrix} a_{11} + b_{11}s & a_{12} \\ a_{12} & a_{22} + b_{22}s \end{pmatrix}$$

- The near threshold pole leads to a dip in  $D\bar{D} \rightarrow D\bar{D}$  rate and an enhancement in  $D_s^+ D_s^- \rightarrow D_s^+ D_s^-$  and  $D\bar{D} \rightarrow D_s^+ D_s^-$  rates.

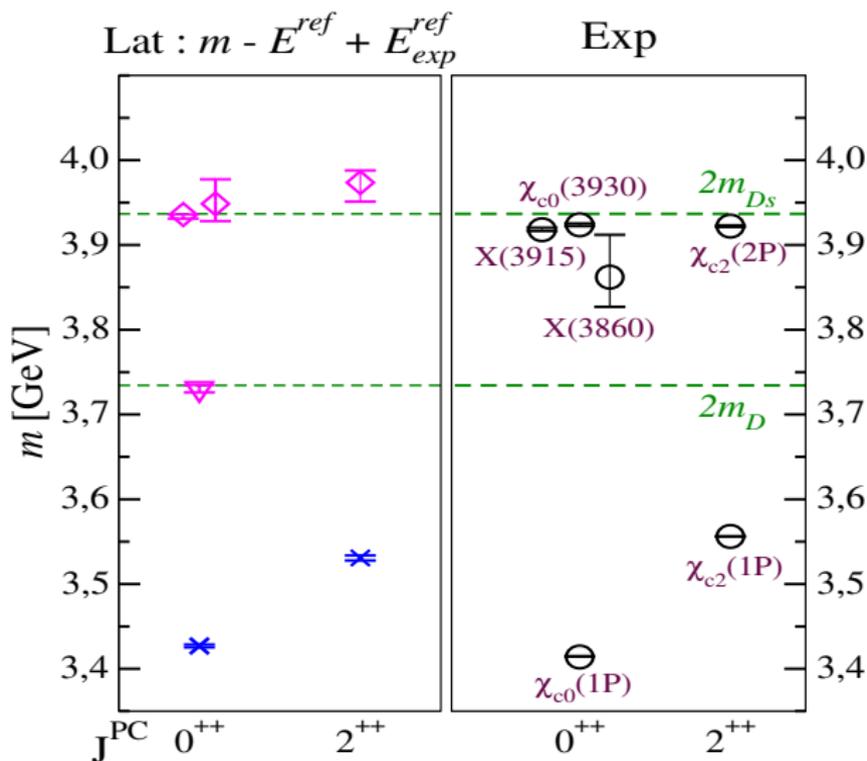
Prelovsek et al, 2011.02542

## Scalar charmonia around the $D_s^+ D_s^-$ threshold



- The near threshold pole couples strongly (weakly) to  $D_s^+ D_s^-$  ( $D\bar{D}$ ). Possibly related to X(3915) and/or X(3930).
- Another pole in sheet — with strong coupling to  $D\bar{D}$ . Responsible for the peak structure in  $D\bar{D} \rightarrow D\bar{D}$  rate above the threshold.
- This pole could be related to the X(3860) observed by Belle.

# Spectrum summary: $0^{++}$ charmonia



Results presented for the heavier than physical charm quark mass.

[Prelovsek et al, 2011.02542](#)

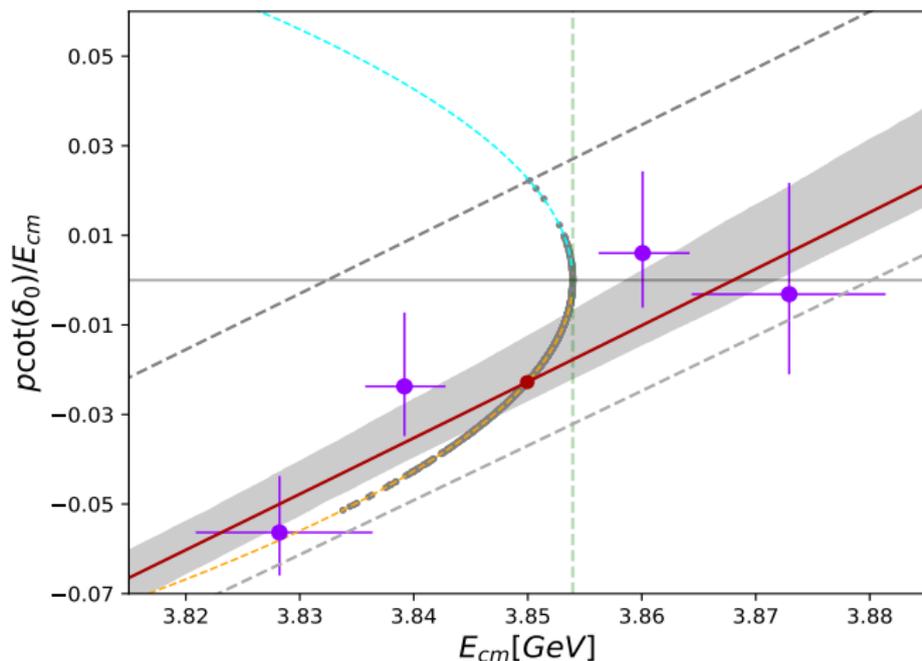
## Remarks and future plans

- We investigate charmonium bound state and resonances with  $J^{PC} = 0^{++}$  and  $1^{--}$ .
- Vector channel as a proof of concept.
- Scalar channel:
  - Suggests three  $s$ -wave states.
  - A shallow (virtual) bound state pole near  $D\bar{D}$  threshold.
  - A pole near  $D_s^+ D_s^-$  threshold leading to a dip in  $D\bar{D}$  rate. Possibly related to X(3915) and/or X(3930).
  - A resonance pole with strong coupling to  $D\bar{D}$  channel. Possibly related to X(3860).
- Future directions :
  - Include effects from other scattering channels in the analysis.
  - Light and charm quark mass dependence.
  - Discretization effects.
  - Explore other quantum numbers, e.g.  $1^+$ .

Thank you

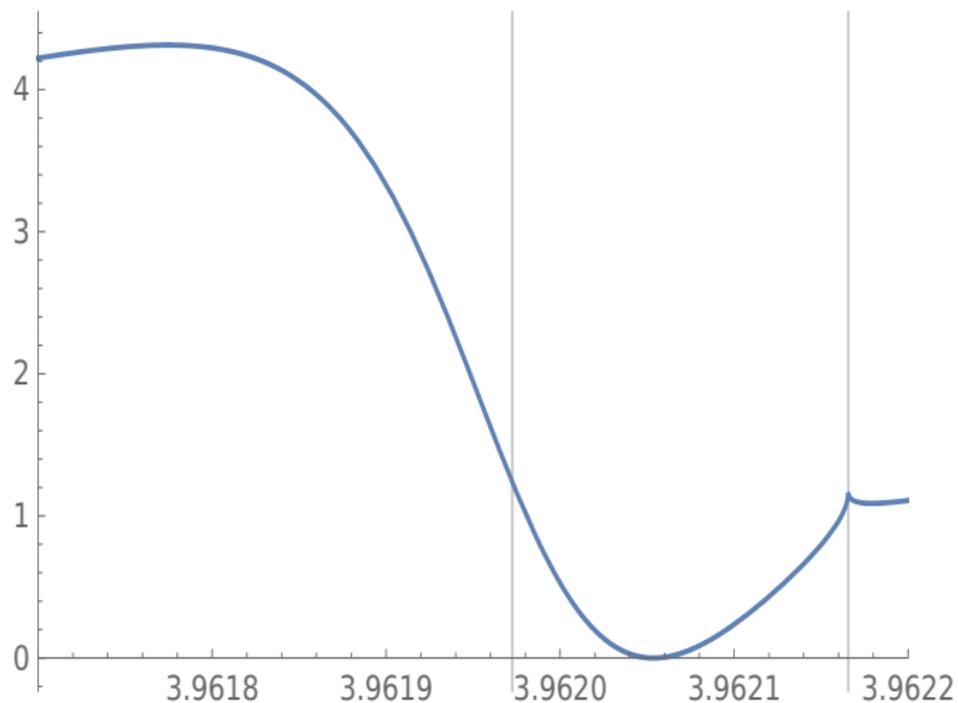
Back up slides

# Scalar charmonia around the $D\bar{D}$ threshold



Pole distribution of shallow (virtual) bound state in  $D\bar{D}$ . [Prelovsek et al, 2011.02542](#)

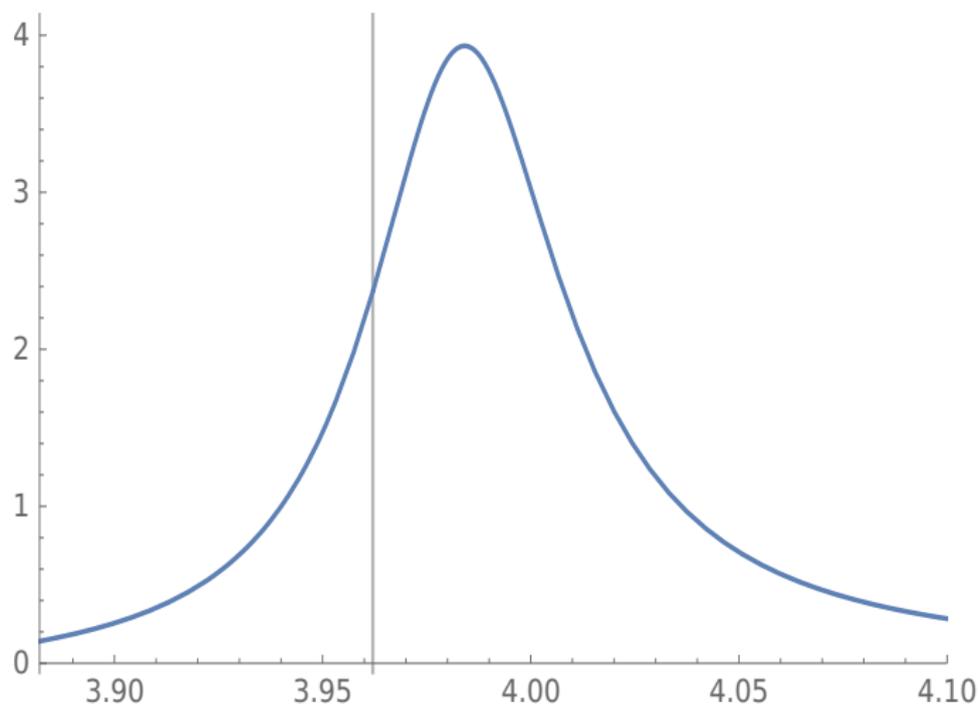
## Dip in the $\rho|t_{D\bar{D}\rightarrow D\bar{D}}|^2$



Dip in the  $D\bar{D} \rightarrow D\bar{D}$  rate.

Prelovsek *et al*, 2011.02542

No dip in the  $\rho |t_{D\bar{D} \rightarrow D\bar{D}}|^2$

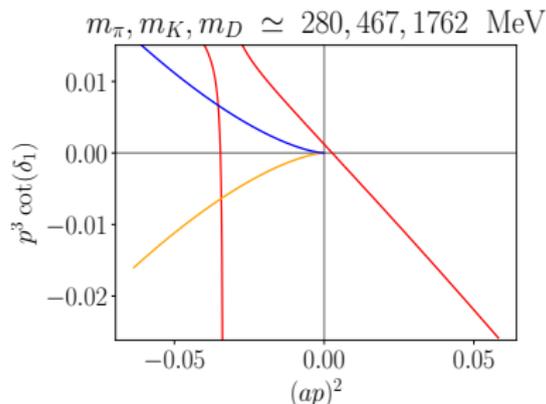
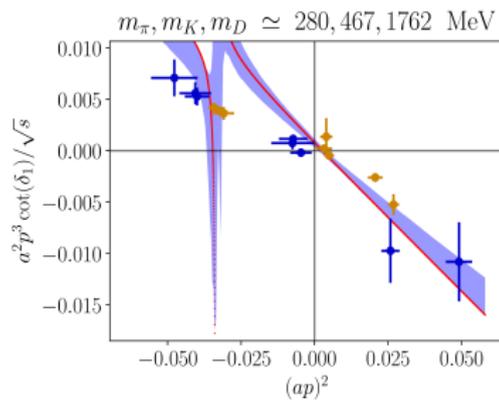


No dip in the  $D\bar{D} \rightarrow D\bar{D}$  rate, when  $K_{12} = 0$ .

Prelovsek *et al*, 2011.02542



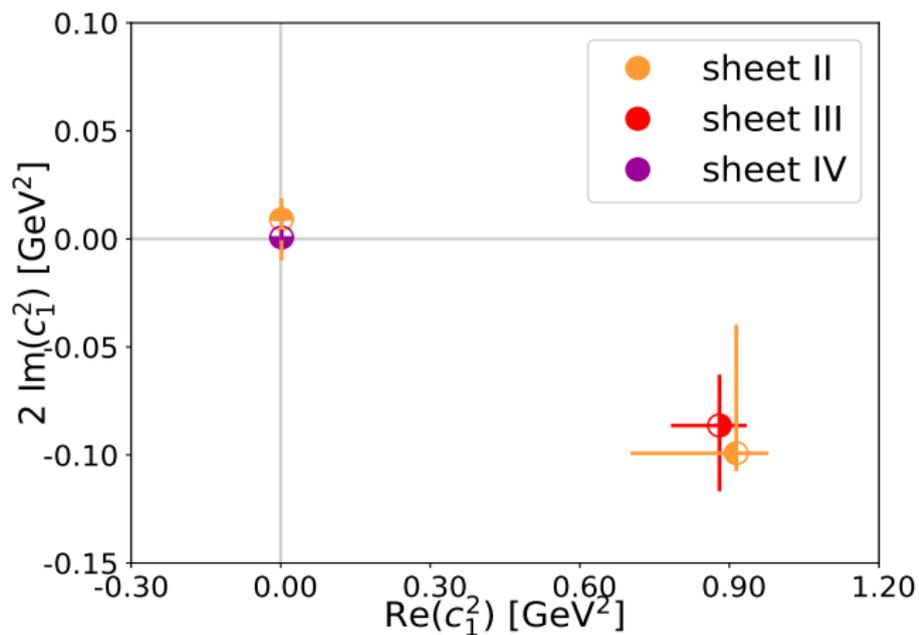
# Elastic analysis: vector charmonia



- (Virtual) bound state constraint:  $p^3 \cot \delta_l = (-)(p^2) \sqrt{-p^2}$
- Two bound states at heavier charm quark mass.
- A bound state and a resonance at lighter charm quark mass.
- An inevitable contamination  $J^{PC} = 3^{--}$  excitation and  $l = 3$  partial wave in the lattice spectra.
- Reaction matrix,  $K = \begin{pmatrix} K_1 & 0 \\ 0 & K_3 \end{pmatrix}$ , with  $K_3 = \frac{G_3^2}{m_3^2 - s}$

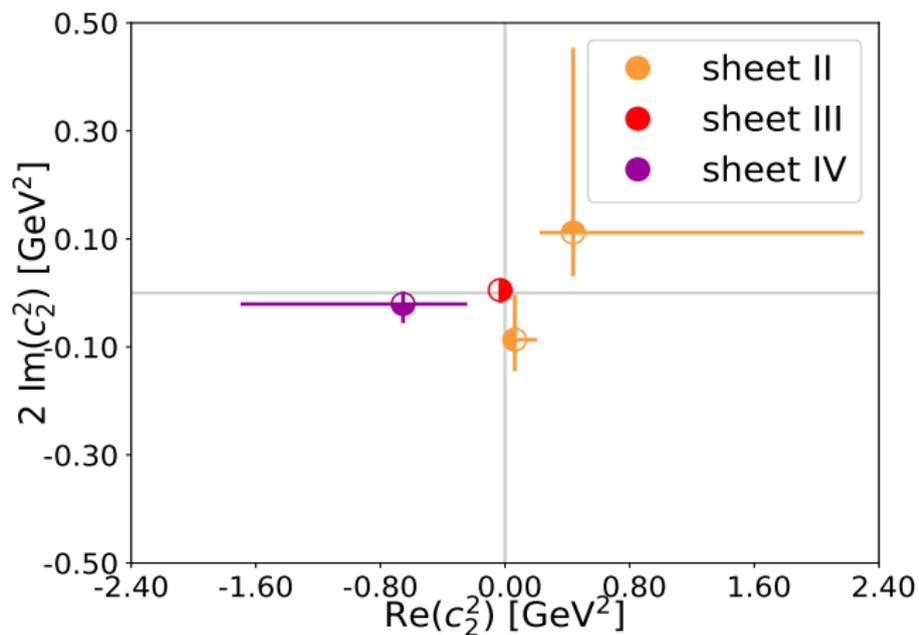
Piemonte *et al*, 1905.03506

## Residues (c1): scalar charmonia



Prelovsek *et al*, 2011.02542

## Residues ( $c_2$ ): scalar charmonia



Prelovsek *et al*, 2011.02542