Z_b tetraquarks

from lattice QCD

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in collaboration with: H. Bahtiyar, J. Petkovic and M. Sadl

update arXiv:1912.02656v4 of arXiv:1912.02656

+

some new results

Z_b in experiment

$$Z_b^+ \to \Upsilon \;\; \pi^+$$

$\bar{b}b\,\bar{d}u$ $\quad \bar{b}\,b\,\bar{d}\,u$

discovered by Belle in 2011 [PRL 108 (2012) 122001]

$Z_b^+(10610)$, $Z_b^+(10650)$

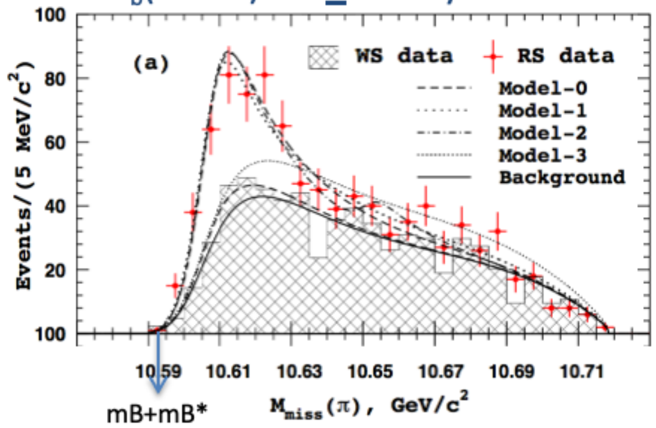
I=1, $J^{PC}=1^{+-}$

Z_b observed in decays $\Upsilon(1S)\,\pi$, $\Upsilon(2S)\,\pi$, $\Upsilon(3S)\,\pi$
$h_b(1S)\,\pi$, $h_b(2S)\,\pi$
$B\,\underline{B}^*$, $B^*\,\underline{B}^*$

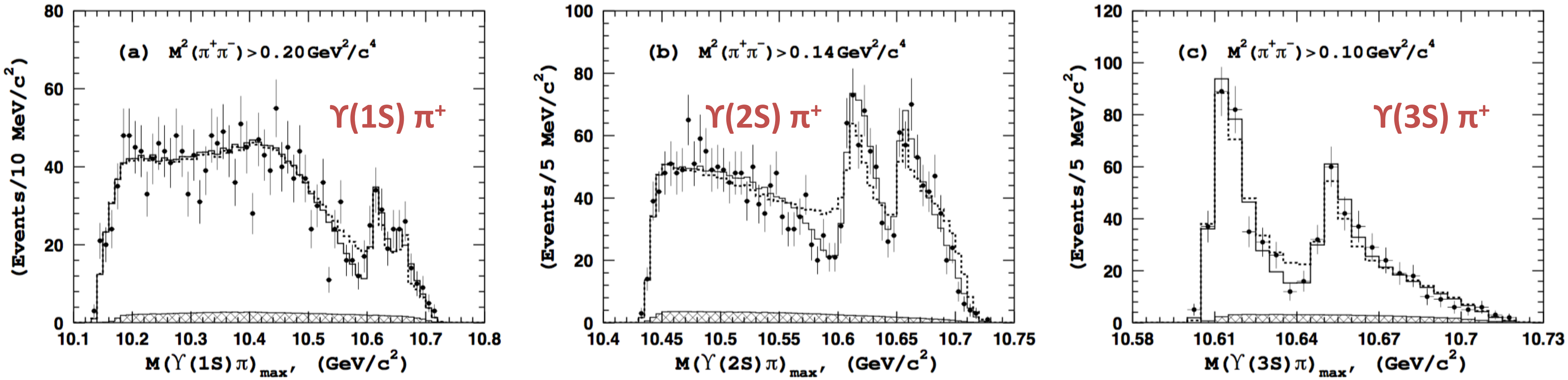
allowed (unobserved) $\eta_b\,\rho$

[Belle, 1512.07419, PRL 2016]

$Z_b(10610)$ in $B\underline{B}^*$ decay mode Br≈85 %



Belle PRD 91 (2015) 072003



↑ m_B+m_{B*}

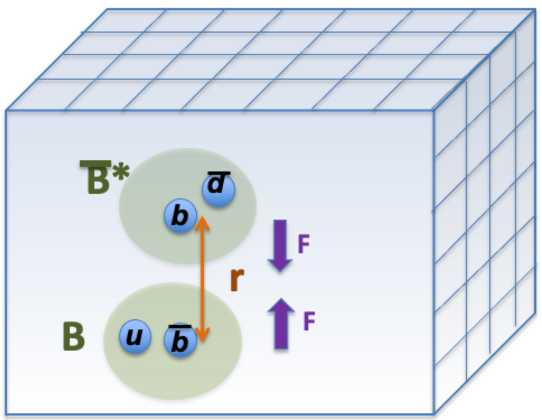
Theory in breif for Zb(10610) near BB*

Phenomenology + analysis of exp data

- many works, to many to cite [Belle's PRL 2012 on Zb discovery has ~700 citations ..]
- many works find : threshold BB* important for existence, BB* molecular nature
- example: Zb(10650) found as virtual BB* bound st. 1-3 MeV below threshold when small couplings to lower channels are omitted [Wang, Baru, Filin, Hanhart, Nefediev, Wynen, 1805.07453, PRD 2018]

Lattice QCD

[S.P., Bahtiyar, Petkovic arXiv:1912.02656v4; Wagner, Peres, Bicudo, Cichy 1602.07621]



- attractive potential between B and B* at small distance r
- this attraction leads to one bound state not far below m_B+m_{B*}
- this bound state is likely related to experimental Zb(10610)

[This conclusion applies for channel S_h=1]

- this system is (to) challenginh for rigorous treatment
- several simplifying assumptions have to be made : this talk

Lattice study of Zb channel

Study with non-static b quarks and rigorous Luscher's approach: (to) challenging

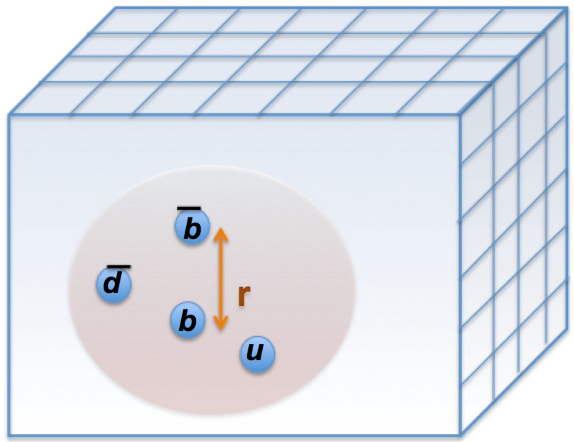
Reason:

- at least 7 two-particle channels coupled
- very dense $B\underline{B}^*$ and $B^*\underline{B}^*$ energy levels
- see back-up slides

→

$Y(1S)\,\pi$, $Y(2S)\,\pi$, $Y(3S)\,\pi$
$h_b(1S)\,\pi$, $h_b(2S)\,\pi$
$B\,\underline{B}^*$, $B^*\,\underline{B}^*$

Z_b with static b and $\underline{b}$



Idea and the only previous lat study

Bicudo, Cichy, Peters, Wagner [proceedings : Lat16: 1602.07621 , Lat17: 1709.03306]

Born-Oppenheimer approach

h = heavy: b, $\underline{b}$ l=light:u,d,gluons

Step 1: fix static b and $\underline{b}$ at distance r: determine $E_n(r)$ for light d.o.f.: lattice QCD

Step 2: consider motion of heavy d.o.f. in the potential determined in Step 1 : non-relativistic Schrodinger equation

[Braaten et al PRD 1402.0438 , Brambilla et al PRD 1707.09647, Bali et al. hep-lat/0505012 PRD, Bicudo & Wagner 1209.6274 + many others ..]

$m_b = \infty$

static b → b quark can not flip spin via gluon exchange

$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$ conserved : S_h=1 and S_h=0 are seperate channels

$\Upsilon_b \not\leftrightarrow \eta_b$, h_b

$\Upsilon_b \pi \not\leftrightarrow \eta_b \rho$, $h_b \pi$

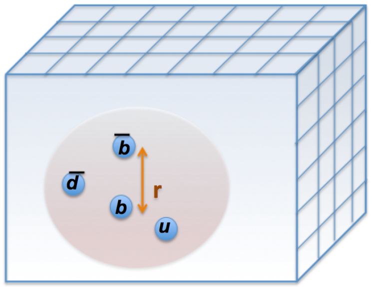
$m_b = \text{finite}$

$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$ not conserved : S_h=1 and S_h=0 couple

$\Upsilon_b \leftrightarrow \eta_b$, h_b

$\Upsilon_b \pi \leftrightarrow \eta_b \rho$, $h_b \pi$

Z_b channel with S_h=1

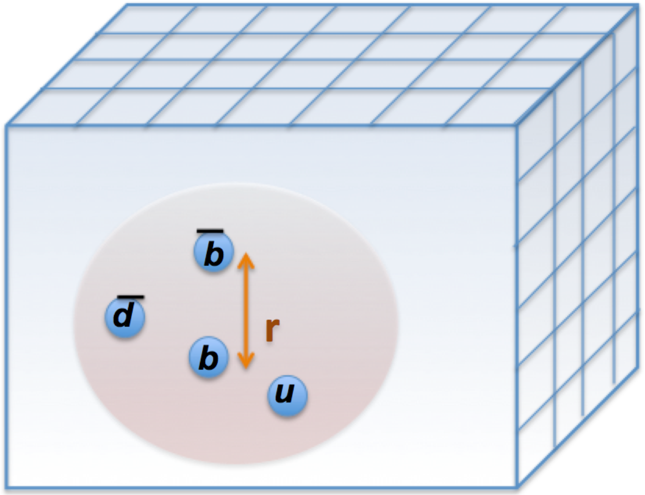


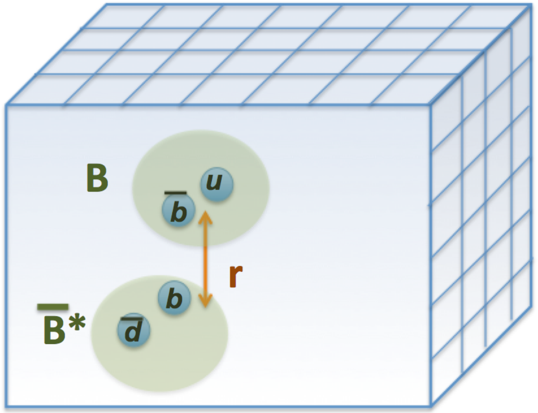
$$\bar{b}\, b\, \bar{d}\, u$$

$$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$$

[S.P., H. Bahtiyar, J. Petkovic: arXiv:1912.02656v4]

Fock components incorporated

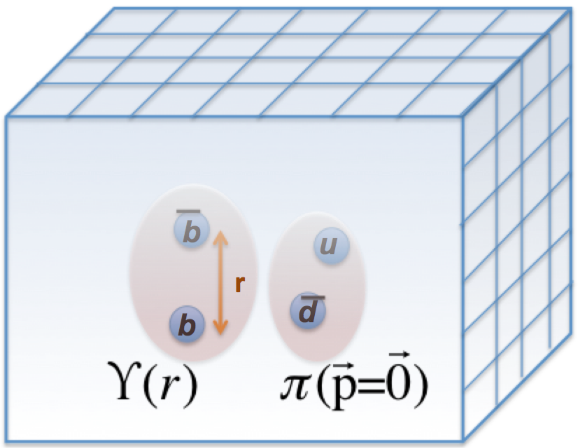


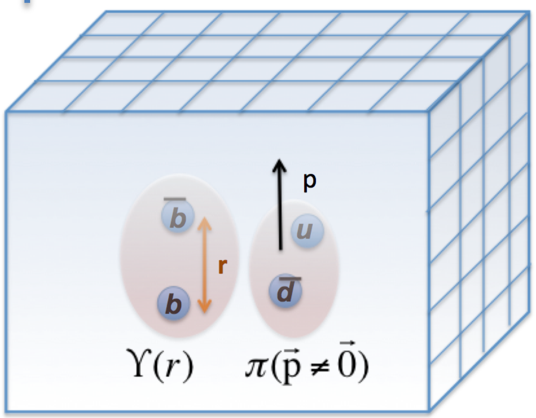


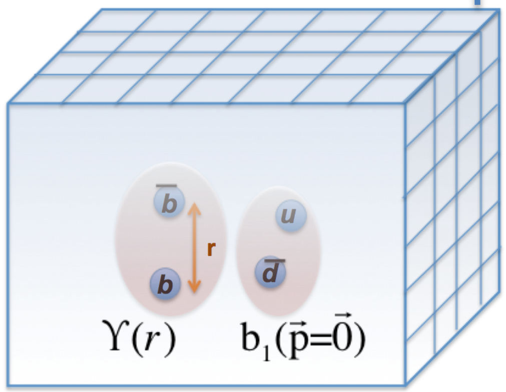
- main aim: extract static potential V(r) between B and B*
- momentum of light degrees of freedom not conserved

in presence of static quarks

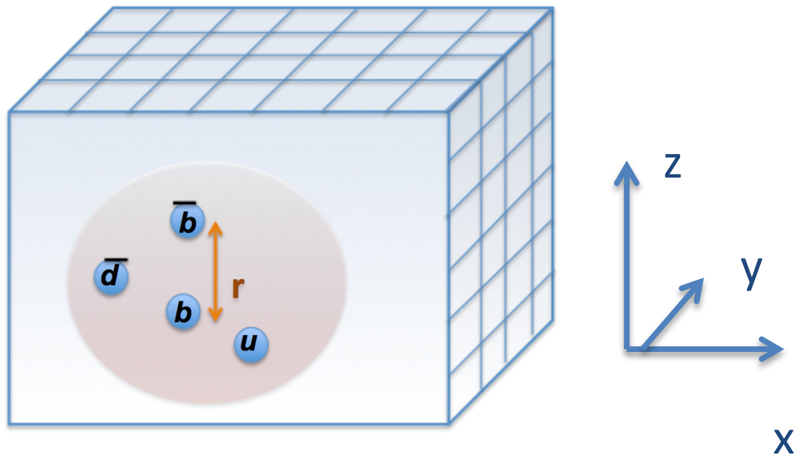
not incorporated before







Symmetries and quantum numbers



$I=1 \quad I_3=0$ (consider neutral Z_b)

$S_h=1 \quad (Sz)_h=0$

$(J_z)_l=0$ [J_x and J_y not conserved]

$$\vec{J} = \vec{S}_h + \vec{J}_l$$

h = heavy: b, <u>b</u>

l = light:u,d,gluons

C•P= -1 (P= inversion over midpoint between b and <u>b</u>)

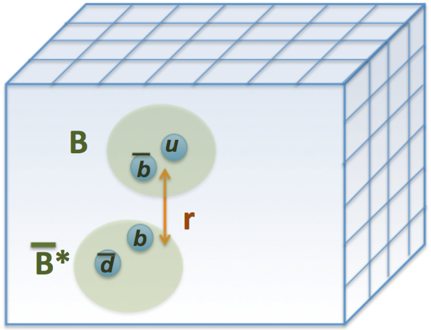
R_l= reflection over yz plane = $P_l * R_l(y,\pi)$: $\varepsilon=-1$

momentum of light degrees of freedom: not conserved

Determine E_n with operators O that annihilate/create the system

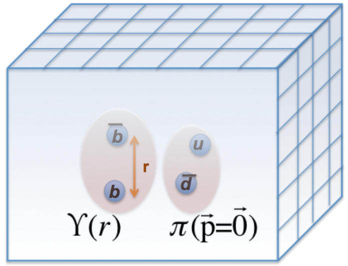
in this way S_h=1 and $(Sz)_h$=0 and $(Jz)_l$=0 and are indeed separately good quantum num. (inspired by Wagner et al.)

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}^a_C(0) q^a_A(0)\ \bar{q}^b_B(r) b^b_D(r)\ , \quad \Gamma = P_-\gamma_5\ \tilde{\Gamma} = \gamma_z P_+,$$



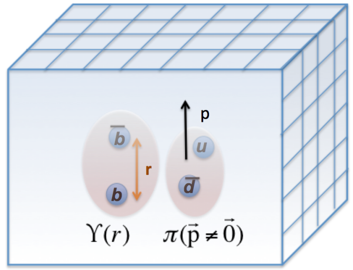
[..] indicate color singlets

$$O^{\Upsilon\pi(0)} = \Upsilon_z\ \pi_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\ [\bar{q}\gamma_5 q]_{p=000}$$



$$O^{\Upsilon\pi(1)} = \Upsilon_z\ (\pi_{p=001} + \pi_{p=00-1}) = [\bar{b}(0)\gamma_z P_+ b(r)]\ \left([\bar{q}\gamma_5 q]_{p=001} + [\bar{q}\gamma_5 q]_{p=00-1}\right)$$

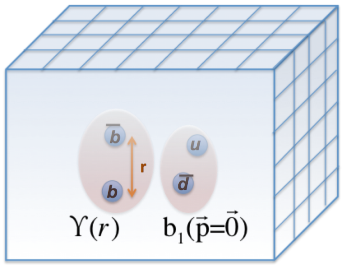
p=n 2π/L

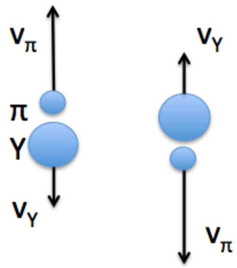


$$O^{\Upsilon\pi(2)} = \Upsilon_z\ (\pi_{p=002} + \pi_{p=00-2}) = [\bar{b}(0)\gamma_z P_+ b(r)]\ \left([\bar{q}\gamma_5 q]_{p=002} + [\bar{q}\gamma_5 q]_{p=00-2}\right)$$

- momentum in z direction ensures that $(Jz)_{light}$=0
- the sum ensures that CP is good q.n.

$$O^{\Upsilon\ b1(0)} = \Upsilon_z\ (b1_z)_{p=000} = [\bar{b}(0)\gamma_z P_+ b(r)]\ [\bar{q}\gamma_x\gamma_y q]_{p=000}$$





For each r : correlation matrix (6x6) renders E_n and Z_i^n

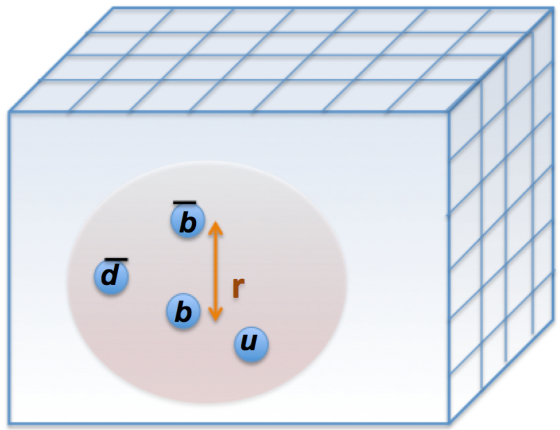
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t} \qquad Z_i^n \equiv \langle 0 | \mathcal{O}_i | n \rangle$$

overlaps

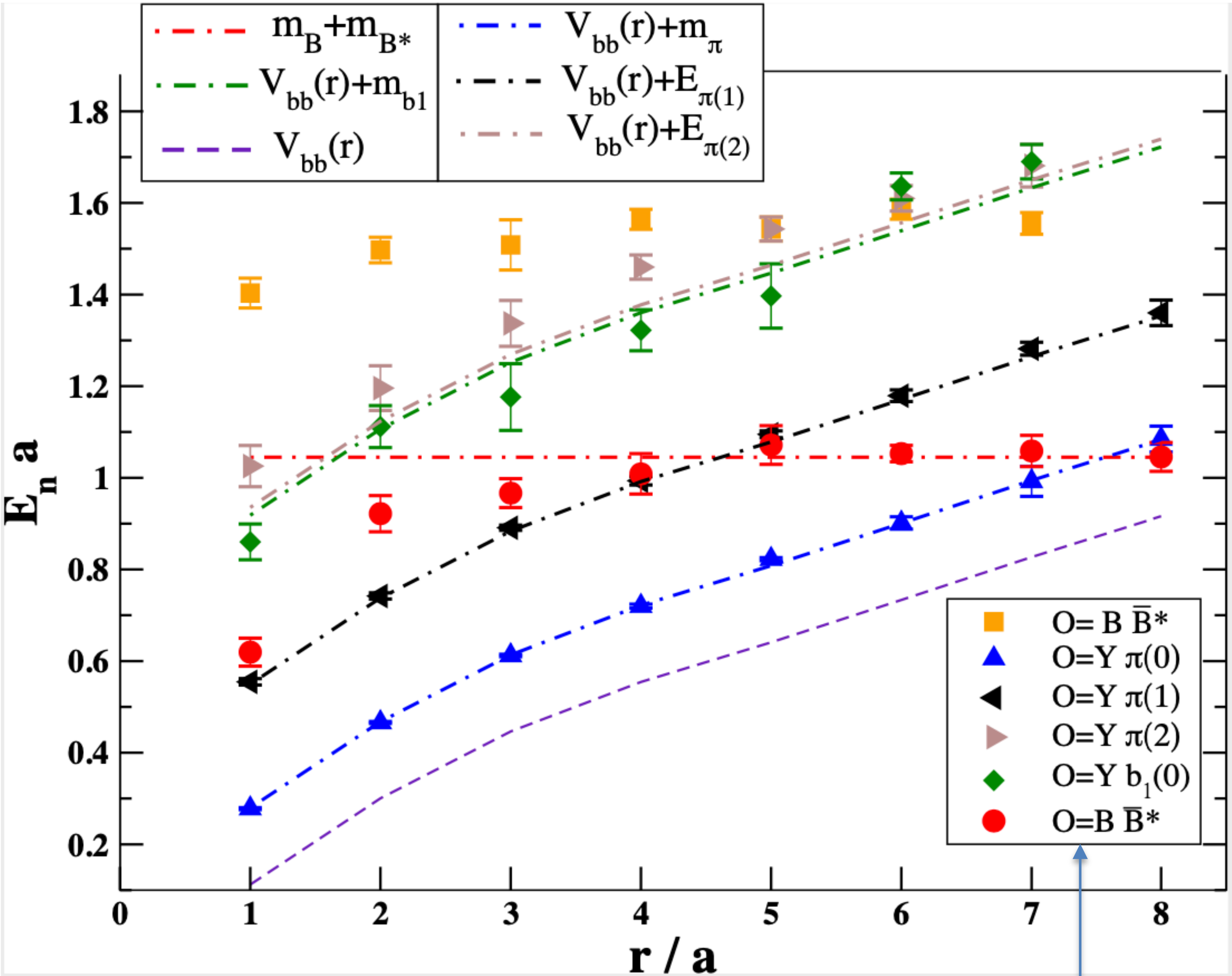
ensemble used: Nf=2, $m_\pi \approx 266$ MeV, a≈0.124 fm, L≈2 fm

[larger L would require Υπ(p=003) ...]

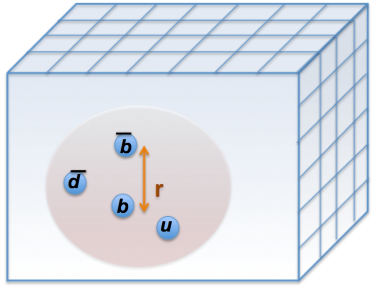
full distillation method to compute Wick contractions



Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1

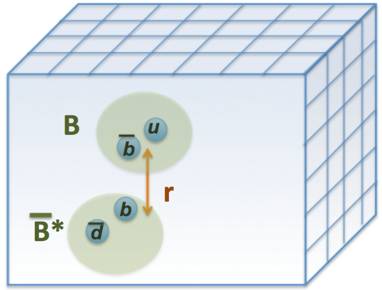


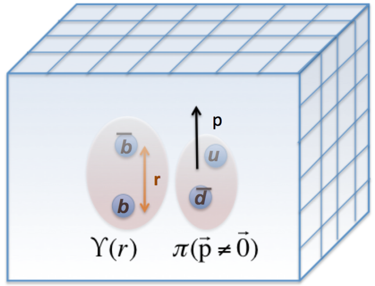
m_B+m_{B*}

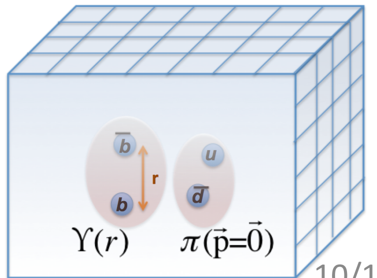


dot-dashed-lines:

$E_n^{\text{non-int}}$





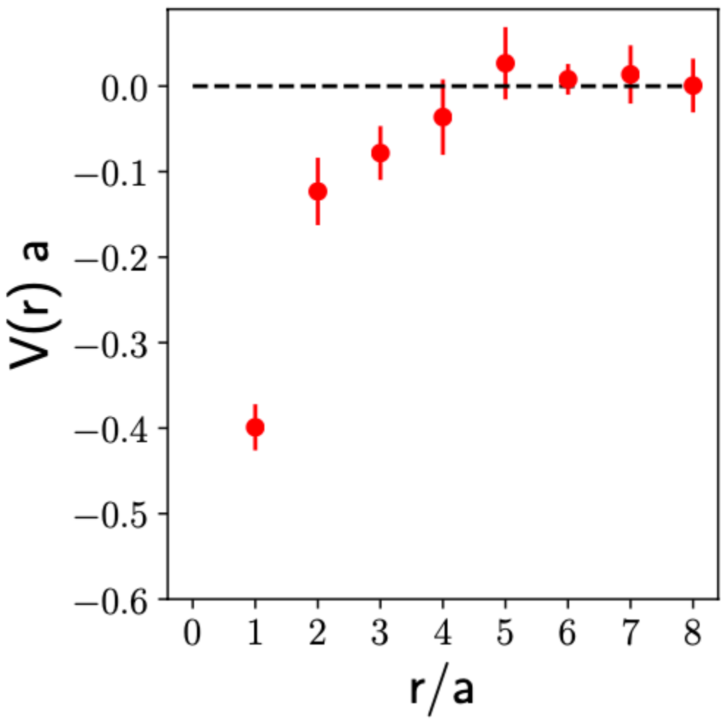


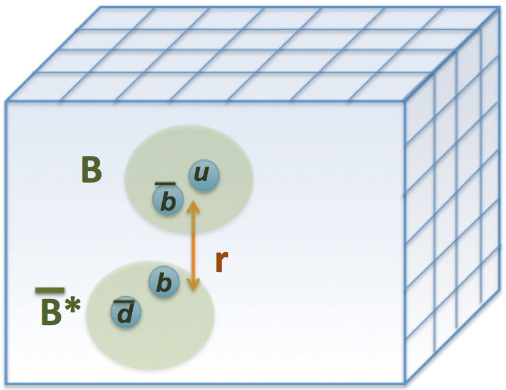
dominant operator

in each |n>

according to $\langle O_i | n \rangle$

Static potential V(r) for interaction between B and $\underline{B}$*





We assume that B $\underline{B}$* eigenstate is decoupled from ϒπ and ϒb_1 channels (overlaps support that).

Born-Oppenheimer approach: B and $\underline{B}$* move in

$V(r)= E_n(r) - m_B - m_{B^*} - E_{B\underline{B}^*}^{kin}$ $\quad (m_{B^*}=m_B)$

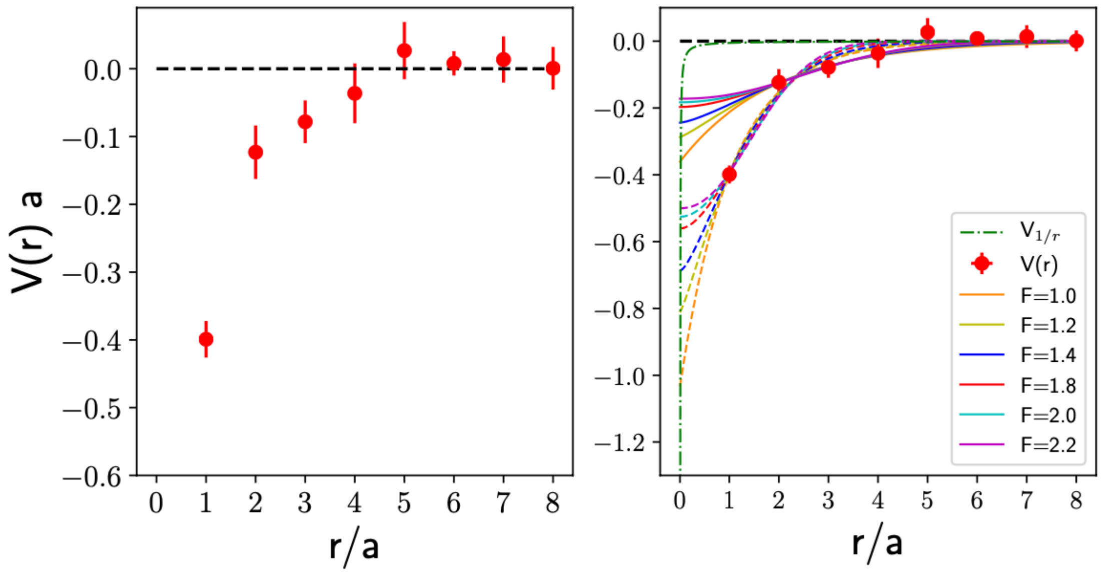
$E_{B\underline{B}^*}^{kin} \to 0$

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2 L(L+1)}{2\mu r^2}+V(r)\right]u(r)=Eu(r)$$

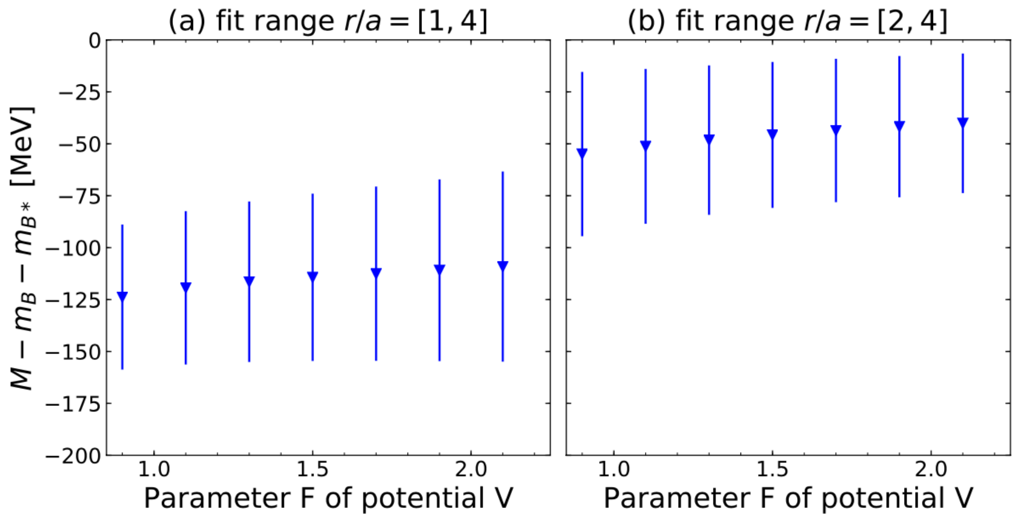
$$\mu=\tfrac{1}{2}m_B^{\exp},\quad \psi\propto\frac{u}{r}Y_{LM}$$

We focus on most relevant : s-wave (L=0)

V(r) for interaction between B and $\underline{B}$*



binding enery



Open problems

- V(r/a<1) = ???

reqest to the community: determine V(r) for small r in this and similar channels

perturbative estimate at very small r
supp. S4 1912.02656 & backup-slides
green line : negligible effect

$$V_{1/r}(r) = -C\ \alpha_s^3/r$$

- V(r/a>>1) = ???
- analytic fit form for V(r) = ???

reqest to the community:

determine analytic form for V(r)

We employ form below for various choices F

$$V(r) = -A\ e^{-(r/d)^F}$$

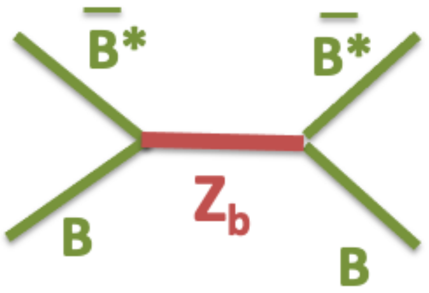
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{\hbar^2 L(L+1)}{2\mu r^2} + V(r)\right]u(r) = Eu(r)$$

we find one bound state with binding enery

$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \text{MeV}$$

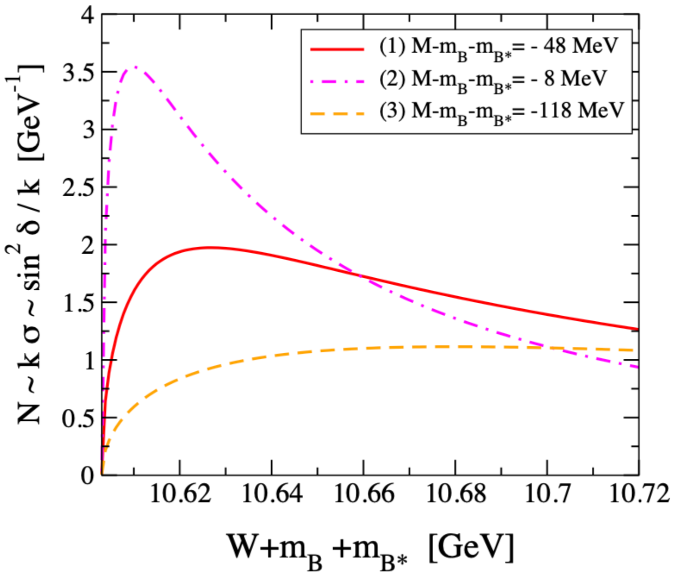
Peak above $B\underline{B}^*$ for shallow bound state Z_b

Schrodinger equation for BB* motion -> scattering phase shift $\boldsymbol{\delta}$ -> cross section $\boldsymbol{\sigma}$



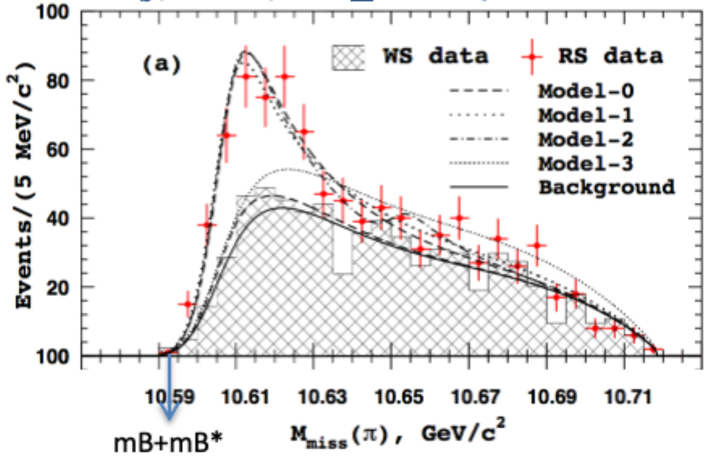
$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \mathrm{MeV}$$

$N_{B\underline{B}^*}$ for three parametrizations consistent with lattice potential



[Belle, 1512.07419, PRL 2016]

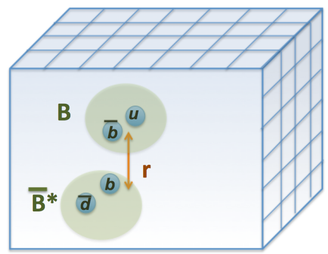
$Z_b(10610)$ in $B\underline{B}^*$ decay mode



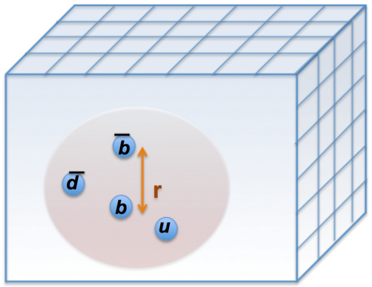
$$E_B = (-58 \pm 71)\,\mathrm{MeV}$$

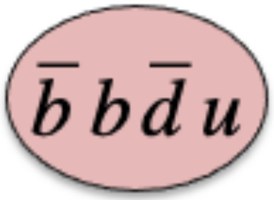
Conclusion from our lattice study [in agreement with conclusion from Wagner & Bicudo & Peters]

- Most robust conclusion: attraction between B and $\underline{B}^*$ at small distances
- attraction renders one bound state
- binding energy depends on parametrization of the potential
- for certain parametrizations bound state is close below threshold
  and renders peak in BB* cross-section above threshold



Z_b channel with S_h=0

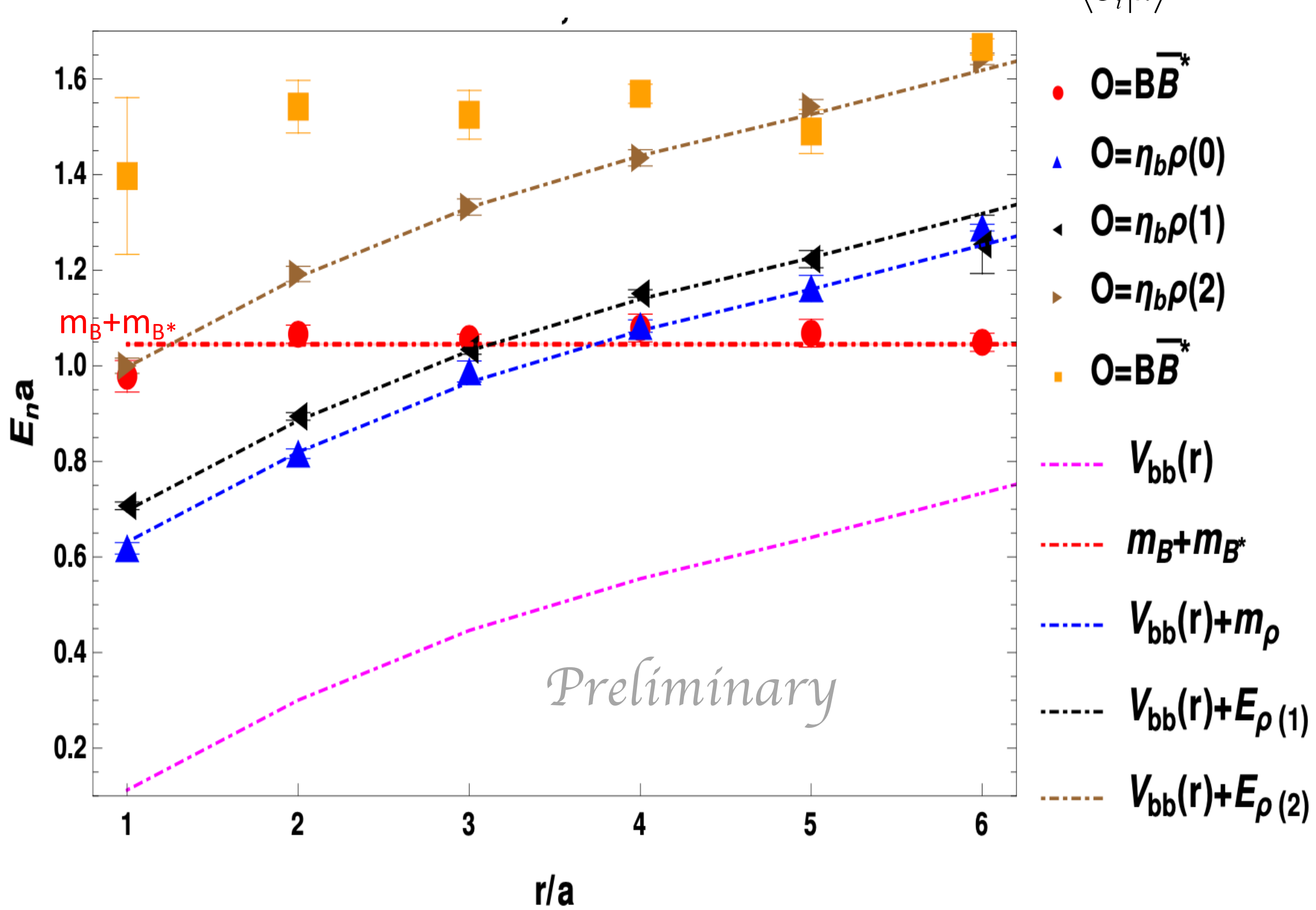


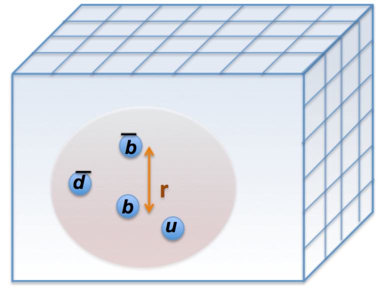


$$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$$

Mitja Sadl and S.P., in progress, preliminary result

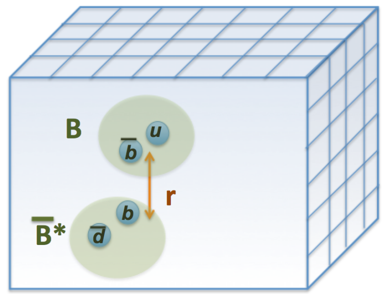
Eigen-energies $E_n(r)$: channel S_h=0, CP=1, ε=1

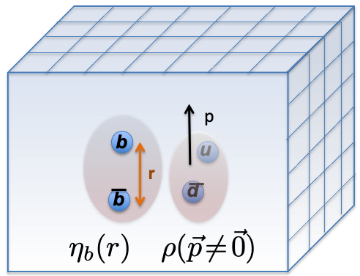


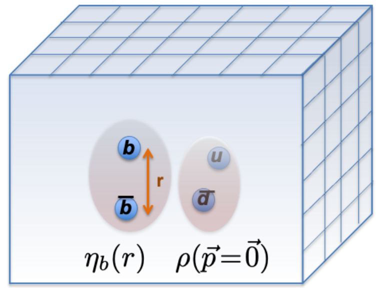


dot-dashed-lines:

$E_n^{\text{non-int}}$







No sizable attraction between B and B*
is found in this channel.

B $\underline{B}$* and B*$\underline{B}$* are superpositions of S_h=1 and S_h=0

$$\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$$

h=heavy=b,$\underline{b}$ l=light=u,d,gluons

Static limit: heavy (S_h and Sz_h) and light (J_l) angular momomentum are separately conserved,
because heavy quark can not flip direction of spin under interaction with gluon.
Physical b quarks: S_h is not conserved, S_h=1 and S_h=0 couple

Zb as a molecule B$\underline{B}$* or B*$\underline{B}$*

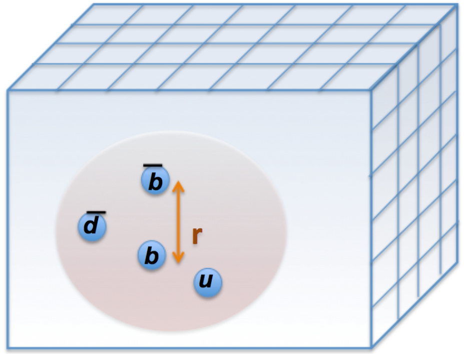
B $\underline{B}$* B* $\underline{B}$

$$\bar{b}\gamma_5 q \;\; \bar{q}\gamma_z b + \bar{b}\gamma_z q \;\; \bar{q}\gamma_5 b \propto (S_h\!=\!1)(J_l\!=\!0) + (S_h\!=\!0)(J_l\!=\!1)$$

B* $\underline{B}$* B* $\underline{B}$*

$$\bar{b}\gamma_1 q \;\; \bar{q}\gamma_2 b - \bar{b}\gamma_2 q \;\; \bar{q}\gamma_1 b \propto (S_h\!=\!1)(J_l\!=\!0) - (S_h\!=\!0)(J_l\!=\!1)$$

Conclusions



Study of Zb channel with lattice QCD

- study with non-static b-quarks and rigorous Luscher's approach to challenging

- study with static b, b :

  attraction between B and B* at small distances in channel S_h=1

  this attraction is most likely responsible for the existence of Zb exotic hadron

  no sizable attraction between B and B* in channel S_h=0

Request to the community: determine BO potentials at very small r

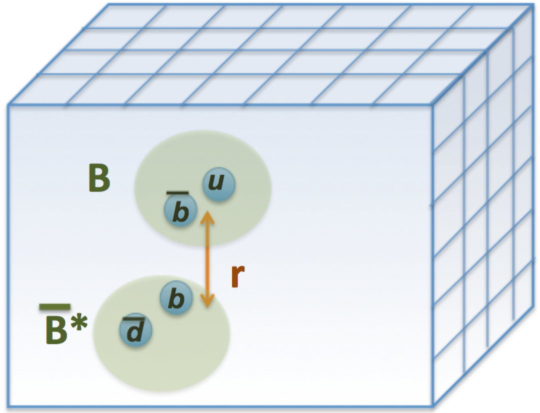
determine analytic "fit" form for BO potentials

Much more work within lattice QCD is required to overcome all the simplifications ...

It would be great to see experimental confirmation of Belle's Zb at another exp (LHCb)

Backup

Operators O_{BB*} with given quantum numbers



in this way $(Jz)_{light}$, S_{heavy} and $(Sz)_{heavy}$

are indeed separately good quantum num.

inspired by Wagner et al.

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA} \tilde{\Gamma}_{CD} \; \bar{b}^a_C(0) q^a_A(0) \; \bar{q}^b_B(r) b^b_D(r) \;, \quad \Gamma = P_- \gamma_5 \; \tilde{\Gamma} = \gamma_z P_+,$$

$$= \sum_{a,b} \sum_{A,B,C,D} \bar{q}^b_B(r) \Gamma_{BA} q^a_A(0) \; \bar{b}^a_C(0) \tilde{\Gamma}_{CD} b^b_D(r)$$

$$\propto [\bar{b}(0) P_- \gamma_5 q(0)] \; [\bar{q}(r) \gamma_z P_+ b(r)] + [\bar{b}(0) P_- \gamma_z q(0)] \; [\bar{q}(r) \gamma_5 P_+ b(r)]$$

$$- [\bar{b}(0) P_- \gamma_x q(0)] \; [\bar{q}(r) \gamma_y P_+ b(r)] + [\bar{b}(0) P_- \gamma_y q(0)] \; [\bar{q}(r) \gamma_x P_+ b(r)]$$

$$O^{B\bar{B}*} = \sum_{a,b} \sum_{A,B,C,D} \Gamma_{BA} \tilde{\Gamma}_{CD} \; \bar{b}^a_C(0) \nabla^2 q^a_A(0) \; \bar{q}^b_B(r) \nabla^2 b^b_D(r) \;, \quad \Gamma = P_- \gamma_5 \; \tilde{\Gamma} = \gamma_z P_+,$$

I=1 I_3=0 : $\bar{q}q \to \bar{u}u - \bar{d}d$

Position of pole in S-matrix for bound state and virtual bund state

$$W_B = \sqrt{m_B^2 + p^2} + \sqrt{m_{B^*}^2 + p^2}$$

p= + i |p| for bound state

p= - i |p| for virtual bound state

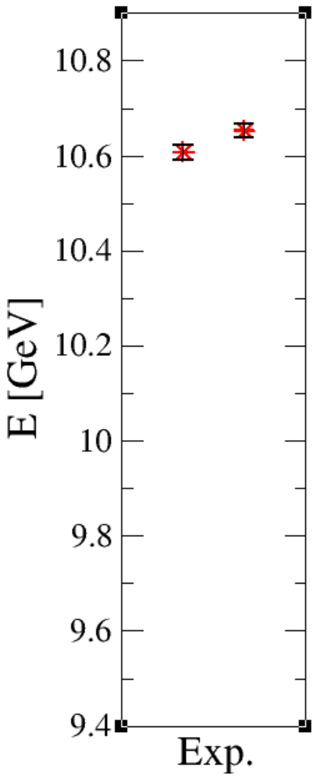
a=0.12 fm

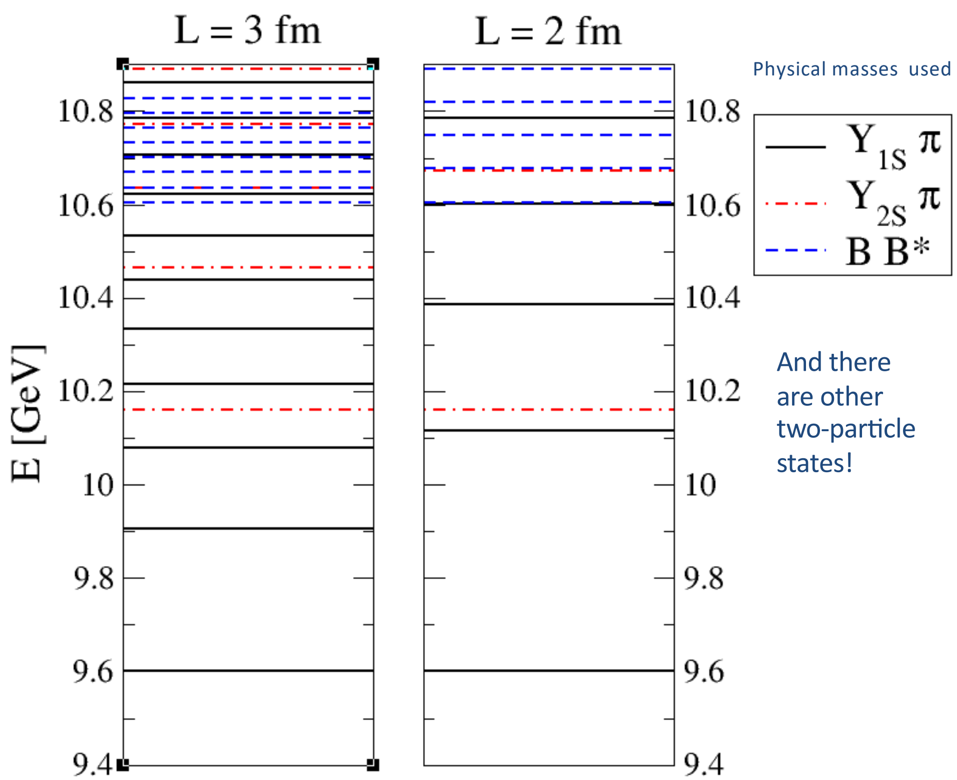
Z_b^+ with non-static b and Luscher's approach: (to) challenging

Eigen-energies in non-interacting limit

$$E^{n.i.}(L) = \sqrt{m_1^2 + \vec{p}^2} + \sqrt{m_2^2 + (-\vec{p})^2}$$

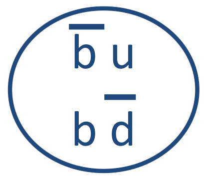
$$\vec{p} = \frac{2\pi}{L}\vec{n}$$





Physical masses used

And there are other two-particle states!

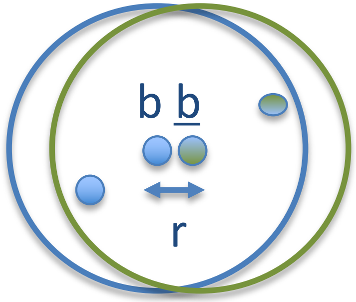


Rigorous treatment very challenging:

- at least 7 two-particle channels coupled
- very dense BB* and B*B* energy levels

Potential between B and B* at very small r

Suppelement S4 [S.P., H. Bahtiyar, J. Petkovic: arXiv:1912.02656v4]



Let us consider the potential between B and $\bar{B}^*$ analytically, where b and $\bar{b}$ are separated by a very small distance $r \ll r_B$, such that r is much smaller than average distance r_B between b and $\bar{q}$ in $B^{(*)}$ meson (i.e. average radius r_B of a static $B^{(*)}$ meson). We address the question whether this potential has a singular form $V_{1/r}(r) = \frac{K}{r}$ for $r \to 0$ and determine prefactor K, while we omit all sub-leading contributions that are finite at $r \to 0$. Among all pairs of the four quarks $\bar{b}b\bar{q}q$, only the interaction between b and $\bar{b}$ at very small r could give potential proportional to $1/r$. All other pairs are at average distance of the order of $O(r_B)$, which is finite for $r \to 0$; these pairs do not lead to infinite potential for $r \to 0$ and we therefore omit their contribution to $V_{1/r}$.

$$|B\bar{B}^*\rangle = \tfrac{1}{\sqrt{3}}(\bar{b}q)\tfrac{1}{\sqrt{3}}(\bar{q}b) = \tfrac{1}{3}\sum_{a=1,3}\sum_{b=1,3}\bar{b}_a q_a\ \bar{q}_b b_b$$

$|B\bar{B}^*\rangle$ is expressed in terms of color singlets and octets

$$|B\bar{B}^*\rangle = \tfrac{1}{3}\left\{(\tfrac{1}{\sqrt{3}}\bar{b}b)(\tfrac{1}{\sqrt{3}}\bar{q}q) + \sum_{A=1,..,8}(\tfrac{1}{\sqrt{2}}\bar{b}\lambda_A b)(\tfrac{1}{\sqrt{2}}\bar{q}\lambda_A q)\right\}.$$

$$\langle\tfrac{1}{\sqrt{3}}\bar{b}b|\tfrac{1}{\sqrt{3}}\bar{b}b\rangle \to V_0(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \mathcal{O}(\frac{\alpha_s^2}{r}), \qquad \text{(S4)}$$

$$\langle\tfrac{1}{\sqrt{2}}\bar{b}\lambda_A b|\tfrac{1}{\sqrt{2}}\bar{b}\lambda_A b\rangle \to V_8(r) = \frac{1}{6}\frac{\alpha_s}{r} + \mathcal{O}(\frac{\alpha_s^2}{r}),\ A = 1,..,8,$$

V0 and V8 cancel at the leading order

$$V_{1/r}(r) = \tfrac{1}{9}[V_0(r) + 8V_8(r)], \quad V_{1/r}^{\mathcal{O}(\alpha_s)}(r) = 0,$$

$$V_{1/r}(r) = \frac{1}{9}\frac{4}{3}\frac{\alpha_s}{r}\left(\frac{\alpha_s}{4\pi}\right)^2 \delta a_2\ .$$

V0, V8 at third order in alpha$_s$
Kniehl, hep-ph/0412083

$$\delta a_2 = -189.2.$$

$$V_{1/r}(r) \simeq -0.0051/r.$$

this contribution is negligible on or lattice