Excited bottomonium spectroscopy from Lattice QCD

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Outline

- Motivation and (some) lattice details.
- Relativistic heavy quarks on a lattice
- Results, from JHEP 02 (2021) 214
  - The spectrum of excited and exotic $b\bar{b}$ states up to J=4.
  - Focus on hybrids mesons - a lightest supermultiplet identified.
- Conclusions, ongoing work & future prospects.

Motivation

- The renaissance in heavy quark spectroscopy continues and will flourish with many experimental results to come.
- New and unexpected results in charmonium have prompted extensive lattice studies.
- Open question whether a similarly rich spectrum is predicted/discovered in bottomonium.
- Motivates this exploratory study of excited and exotic quarkonia.

Heavy quarks & anisotropic lattices:

- The b quark presents a well-known problem: potentially large discretisation errors which must be removed or controlled.
- Solutions include EFTs approach or extremely fine lattices such that $am_Q < 1$.
- Anisotropic lattices, $a_t \ll a_s$, already proven very useful for charm - explore their effectiveness at bottom quark masses.

Heavy quarks and anisotropic lattices

- Not a new story. Hashimoto, Onogi, Kronfeld and collaborators investigated the practicalities in early 2000s. [PRD64 114503(2001); PRD64 074501 (2001); PRD66 014509 (2002)].
- Investigate discretisation effects via dispersion relations and

$$I = \frac{2\delta M_{hl} - (\delta M_{hh} + \delta M_{ll})}{2M_{2hl}}, \quad \delta M = M_2 - M_1$$

  $I \neq 0 \Rightarrow$ inconsistent binding energies.
- Isolates $\mathcal{O}((a_s p)^2)$ discretisation effects in quarkonium. [NPB (proc suppl) 47 (1996); NPB (proc suppl) 53 (1997)]
- Strategy: monitor dispersion relations and I (via differences in M_2, M_1).
- Noting improvements on earlier studies include stout smearing of spatial links and mass-dependent anisotropy tuning.

Lattice details

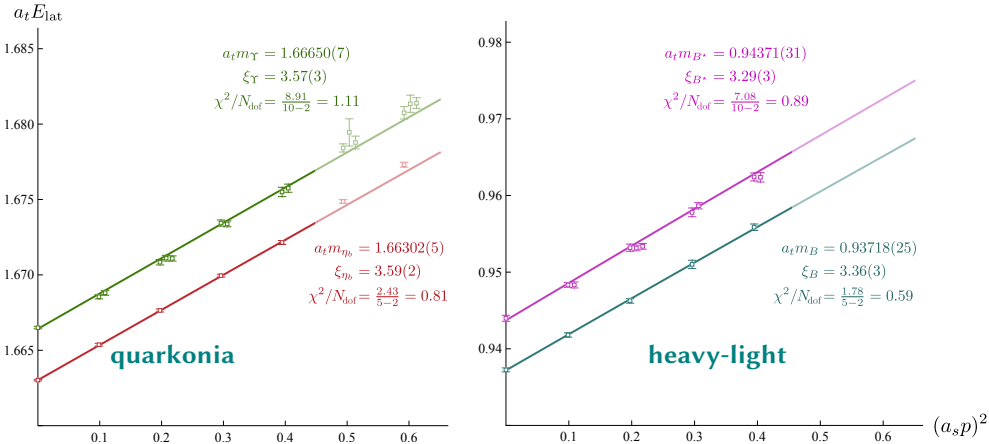
- Dynamical, anisotropic lattices with stout-smeared spatial links.
- $\xi = a_s/a_t = 3.5$.
- $20^3 \times 128$ volume; and volume dependence investigated with $24^3 \times 128$.
- $m_\pi \sim 400$ MeV
- Distillation for quark propagation.
- Operators of definite momenta constructed from up to 3 derivatives in each channel. Form a large bases (up to 26 operators in e.g. T_1) for variational analysis.

Related work:

- Charmonium: JHEP1207 (2012) 126, JHEP 1612 (2016) 089.
- Open charm: JHEP 05 (2013) 021.

The lattice dispersion relation

- Fermion action has 2 parameters to tune: m_q and ξ. Demand $M_{\eta_b}^{latt} = M_{\eta_b}^{expt}$ *and* a relativistic dispersion relation for the η_b (red line below).
- Everything else a prediction.



- Excellent fits to relativistic behaviour, agreement in measured ξ.

The "usual" recipe for (meson) spectroscopy

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

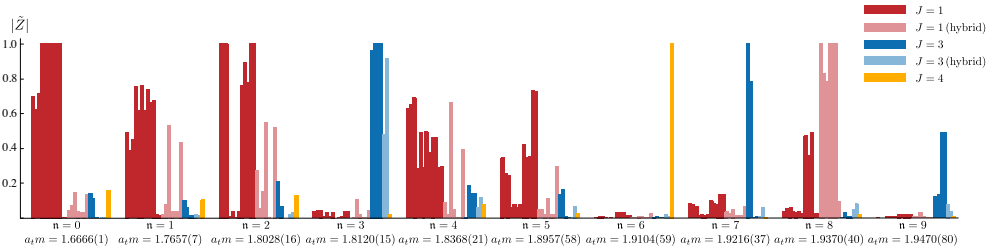
- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem $C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- operators of definite J^{PC} are subduced into relevant lattice irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subdduced - it **overlaps** predominantly with states of this J.

J	0	1	2	3	4
A_1	x				x
A_2				x	
E			x		x
T_1		x		x	x
T_2			x	x	x

- Use overlaps, $Z = \langle 0|\Phi|k\rangle$, to identify continuum spins
- For *Jgeq2*, look for agreement between irreps
- Example: T_1^{--} irrep, with **Spin 1**, **Spin 3** and **Spin 4**.

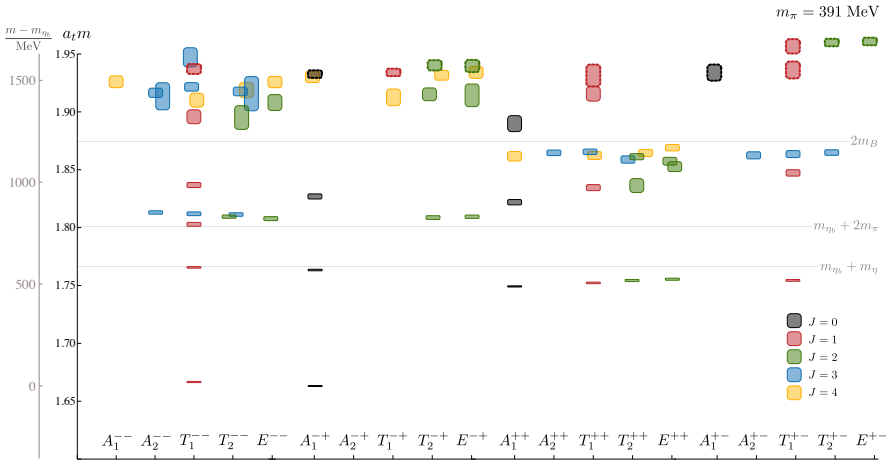


The spectrum of $b\bar{b}$ mesons

Caveat Emptor

- Spectra determined from single-hadron operators. Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- Relatively heavy ($\sim$400MeV) pions

Lattice spectrum of bottomium ($M - M_{\eta_b}$)



- Broadly similar to pattern of states in charmonium
- Use overlaps to make continuum spin identifications
- Includes exotic channels: 0^{+-}, 1^{-+}, 2^{+-}.

Bottomonium: the J^{PC} spectrum

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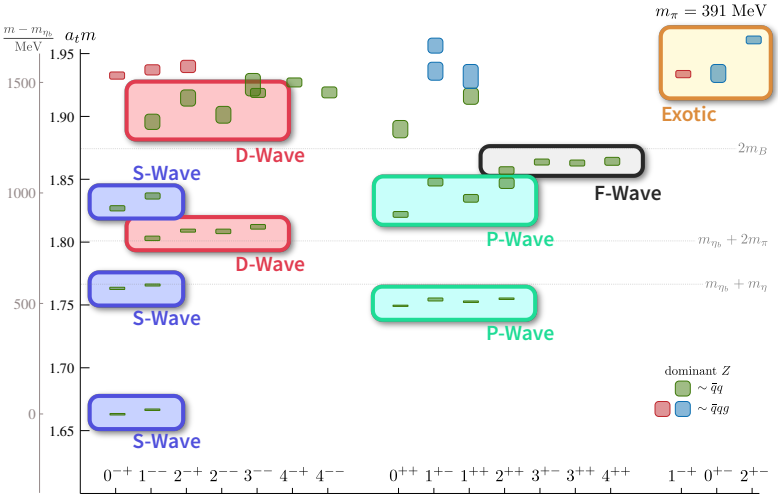
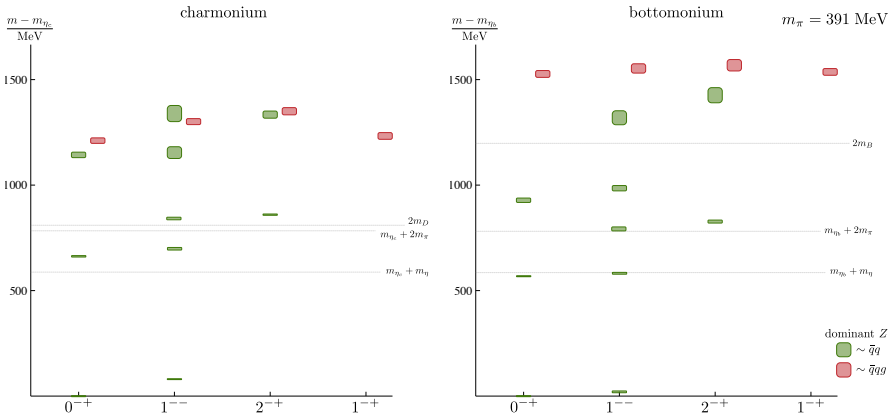
- Single-hadron operators
- No continuum extrapolation
- Heavy pions



Improvements possible with methods established

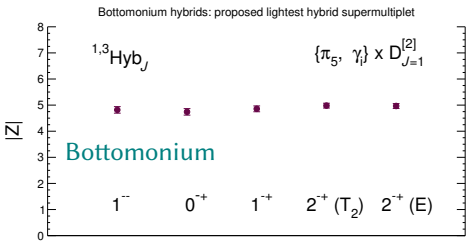
Hybrids in bottomonium

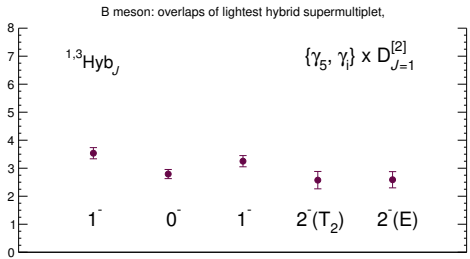


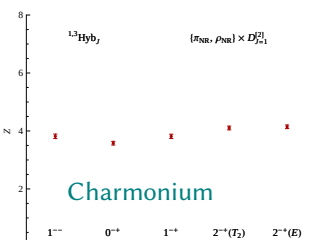
- Spin exotic and non-exotic hybrids determined
- Similar result in charmonium and in agreement with Brambilla et al PRD101 (2020).

Hybrids

Expect a large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$. Used to identify hybrid multiplets.



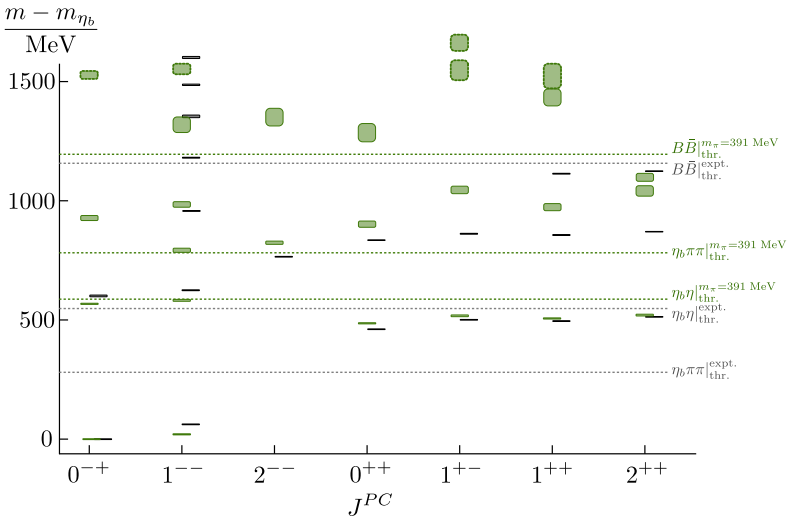




- Similar pattern & structure of hybrids seen in light, open-charm and charm (and baryons).
- Energy scale $\sim$ 1.5GeV - as previously.

Looking to experiment

- Emphasise the pattern of states in the spectrum is reliable and broadly compatible with known experimental states.



Summary & outlook

- An exploratory study of relativistic quarkonia following a mass-dependent anisotropy tuning is encouraging.
- Following the by-now well-established HadSpec Collaboration recipe a large basis of operators can be constructed, enabled by distillation.
  - Dispersion relations are consistent between heavy and heavy-light sectors, to large(ish) lattice momenta, for parameters tuned once at η_b.
  - Evidence of a hybrid supermultiplet is found in bottomonium, with similar characteristics to those in charmonium, open-charm and light mesons as well as baryons.
- The B spectrum including the B_c underway.
- The study can be extended to larger volumes and lighter pion masses (all available).
- Paves the way for spectroscopy and decays of b-quark hadrons.

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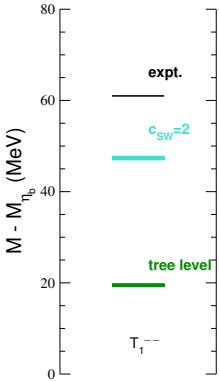
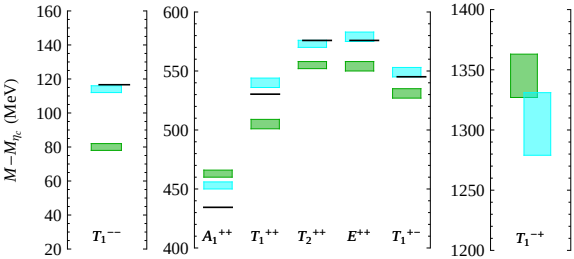
Thanks for listening!

Backup: (brief) investigation of the effect of c_{SW}

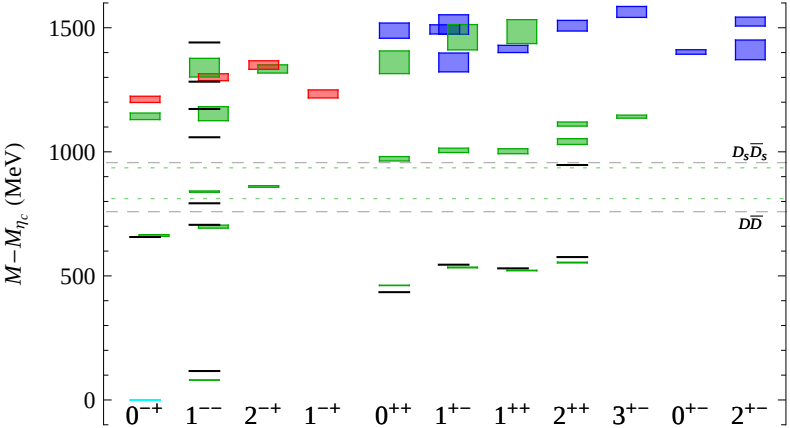




Note 1: different statistics for the tree-level and $c_{SW} = 2$ results in bottomonium.

Note 2: Same "improved" value of $c_{SW} = 2$ used for charm and bottom.

Backup: (exotic) hybrid meson in charmonium



- Spin exotic and non-exotic hybrids determined.
- Same pattern & energy scale in mesons and baryons, light and heavy.

[Hadron Spectrum Collaboration, 2012]