Muonium-Antimuonium Oscillations in Effective Field Theory

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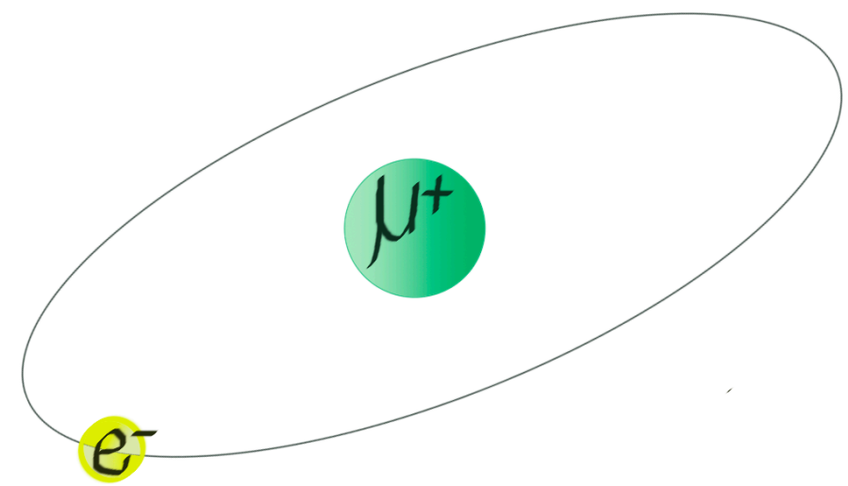
Outline

- ✳Motivation
- ✳Muonium Oscillation Formalism
- ✳Calculation of mixing parameters Δm and $\Delta\Gamma$
- ✳Results
- ✳Summary

Muonium

- Muonium is a non-relativistic QED Coulombic bound state
  - Comparable to B_c , but with computable non-perturbative matrix elements
- Simpler than atomic hydrogen
  - Spin-0 (singlet): para-muonium
  - Spin-1 (triplet): ortho-muonium
  - Electrically neutral $\Rightarrow$ flavor oscillations are possible
  - Ideal system for NP seaches



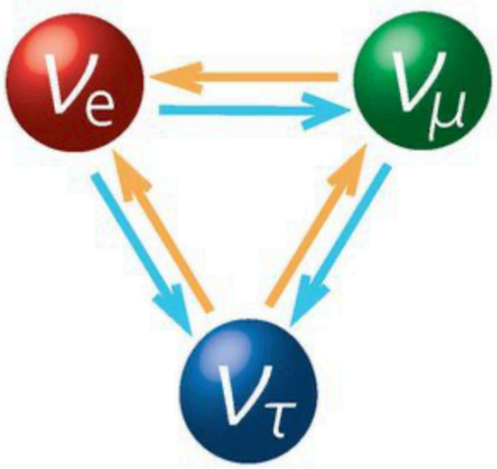
Muonium (μ^+e^-)

- An oscillation is the process $M_\mu(\mu^+e^-) \to \overline{M}_\mu(\mu^-e^+)$
  - It violates muon lepton number by two units $\Delta L_\mu = 2$, i.e.: it can probe different types of NP than $\mu \to e\gamma$ or $\mu + N \to e + N$
  - Conversion rate was calculated in several NP models with heavy DOF

New Physics Searches with Muonium

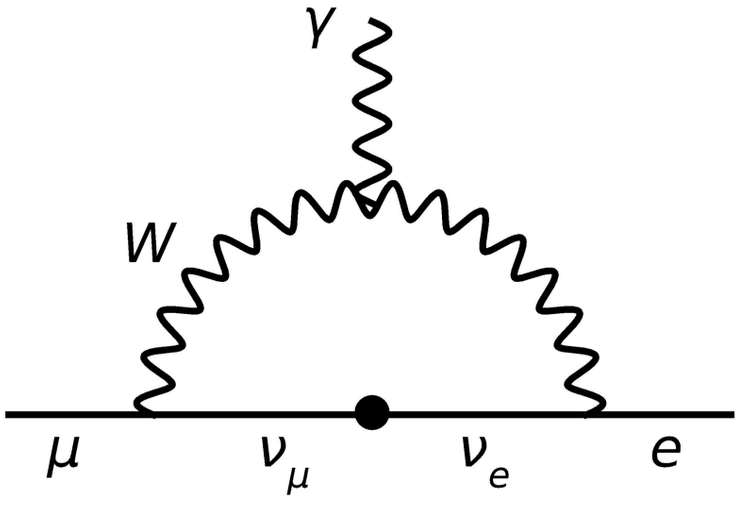
- From experiment neutrino's mix and change flavor
  - Flavor violating processes in the charged lepton sector
  - Highly suppressed in the standard model (SM)
    - $\mu \to e\gamma$ Br$(\mu \to e\gamma)_{SM} \sim 10^{-54}$

      $\Rightarrow$ no background for NP!



https://j-parc.jp/Neutrino/en/intro-t2kexp.html

- LFV searched for in various processes:
  - $\Delta L_\mu = 1$: $\mu \to e\gamma$, $\mu \to eee$ and $\mu + N \to e + N$
  - $\Delta L_\mu = 2$: Muonium anti-muonium oscillations
    - In some models we can expect $\Delta L_\mu = 2$ contributions to be the dominant ones, e.g. doubly charged Higgs has a tree level contribution to $\Delta L_\mu = 2$, but not to $\mu \to e\gamma$



Abdallah, W. 2108 arXiv:1105.1047 [hep-ph]

Muonium Oscillation Formalism

- Similar to meson-antimeson oscillations, but unlike $K\overline{K}$ or $B\overline{B}$ oscillations both spin-0 and spin-1 states can oscillate

- Time development of M_μ and $\overline{M}_\mu$ is given by a Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} |M_\mu(t)\rangle \\ |\overline{M}_\mu(t)\rangle \end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix} |M_\mu(t)\rangle \\ |\overline{M}_\mu(t)\rangle \end{pmatrix}$$

matrix Hamiltonian

m and Γ are 2×2 Hermitian matrices: the *mass* matrix and the *decay* matrix

- Assume CPT invariance, then the diagonal and off diagonal elements

$$m_{11} = m_{22}, \quad \Gamma_{11} = \Gamma_{22} \qquad\qquad m_{12} = m_{21}^*, \quad \Gamma_{12} = \Gamma_{21}^*$$

Muonium Oscillation Formalism

- Off diagonal element of this matrix

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu = 2} \middle| M_\mu \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu = 1} \middle| n \right\rangle \left\langle n \middle| \mathscr{H}^{\Delta L_\mu = 1} \middle| M_\mu \right\rangle}{M_M - E_n + i\epsilon}$$

- M_μ and $\overline{M}_\mu$ are not mass eigenstates: diagonalize!

$$\left(m - i\frac{\Gamma}{2}\right)_{12} \neq 0$$

  - Mass eigenstates, $M_{\mu 1}$ and $M_{\mu 2}$ (assume CP conservation)

$$\boxed{\left|M_{\mu 1,2}\right\rangle = \frac{1}{\sqrt{2}}(|M_\mu\rangle \mp |\overline{M}_\mu\rangle)}$$

- $M_{\mu 1}$ and $M_{\mu 2}$ have mass difference and width difference
  - Mass difference (Δm)

$$\Delta m \equiv M_1 - M_2,$$

  - Width difference ($\Delta\Gamma$)

$$\Delta\Gamma \equiv \Gamma_2 - \Gamma_1$$

Muonium Oscillation Time Evolution

- Time development of M and $\overline{M}$

$$\left|M_\mu(t)\right\rangle = g_+(t)\left|M_\mu\right\rangle + g_-(t)\left|\overline{M}_\mu\right\rangle \qquad \left|\overline{M}_\mu(t)\right\rangle = g_-(t)\left|M_\mu\right\rangle + g_+(t)\left|\overline{M}_\mu\right\rangle$$

Where, $g_\pm(t) = \frac{1}{2}e^{-\Gamma_1 t/2}e^{-iM_1 t}\left[1 \pm e^{-\Delta\Gamma t/2}e^{i\Delta m t}\right]$

- Mass eigenstates $M_{\mu 1}$ and $M_{\mu 2}$ mass (Δm), and width difference ($\Delta\Gamma$)

$$\Delta m \equiv M_1 - M_2, \qquad \Delta\Gamma \equiv \Gamma_2 - \Gamma_1$$

$$x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

but $x, y \ll 1$

- Probability of M_μ decaying as $\overline{M}_\mu$ at $t > 0$
  - Dependence on x and y!

$$\Gamma(M_\mu \to \bar{f})(t) = N_f\left|\left\langle \bar{f}\left|S\right|M_\mu(t)\right\rangle\right|^2,$$

$$\Gamma(M_\mu \to f)(t) = N_f\left|\left\langle f\left|S\right|M_\mu(t)\right\rangle\right|^2$$

$$P\left(M_\mu \to \overline{M}_\mu\right) = \frac{\Gamma(M_\mu \to \bar{f})}{\Gamma(M_\mu \to f)} = R_M(x,y), \qquad R_M(x,y) = \frac{1}{2}(x^2 + y^2)$$

Need to calculate x and y!

Effective Theory of Oscillations

- An effective field theory (EFT) approach: all possible heavy NP models

- Classify effective operators by lepton quantum numbers

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{eff}}^{\Delta L_\mu = 0} + \mathscr{L}_{\text{eff}}^{\Delta L_\mu = 1} + \mathscr{L}_{\text{eff}}^{\Delta L_\mu = 2}$$

SM and NP contributions

$$\mathscr{L}_{eff} = -\frac{1}{\Lambda^2} \sum_i c_i(\mu) Q_i$$

- Heavy degrees of freedom are "integrated out" only operators local at the scale of muonium mass are contained in $\mathscr{L}_{\text{eff}}$

Effective Theory of Oscillations

- Remember we want x and y
- Our $\Delta L_\mu = 2$ Lagrangian

$$\mathscr{L}_{eff}^{\Delta L_\mu = 2} = -\frac{1}{\Lambda_1^2}\sum_i C_i(\mu) Q_i(\mu)$$

- Most general set of operators in terms of the electron and muon degrees of freedom, $Q_1 - Q_5$, relevant for x

$$Q_1 = (\bar{\mu}\gamma_\alpha P_L e)(\bar{\mu}\gamma^\alpha P_L e), \quad Q_2 = (\bar{\mu}\gamma_\alpha P_R e)(\bar{\mu}\gamma^\alpha P_R e), \quad Q_3 = (\bar{\mu}\gamma_\alpha P_L e)(\bar{\mu}\gamma^\alpha P_R e),$$

$$Q_4 = (\bar{\mu}P_R e)(\bar{\mu}P_R e), \quad Q_5 = (\bar{\mu}P_L e)(\bar{\mu}P_L e)$$

  - Other possible structures can be Fierz'd into the operators above

- $\Delta L_\mu = 2$ operators that will be important, relevant for y

$$Q_6 = (\bar{\mu}_L\gamma_\alpha e_L)\left(\overline{\nu_{\mu_L}}\gamma^\alpha \nu_{eL}\right), \quad Q_7 = (\bar{\mu}_R\gamma_\alpha e_R)\left(\overline{\nu_{\mu_L}}\gamma^\alpha \nu_{eL}\right)$$

Matrix Elements of Effective Operators

- Rewrite off diagonal element in terms of x

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu = 2} \middle| M_\mu \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu = 1} \middle| n \right\rangle \left\langle n \middle| \mathscr{H}^{\Delta L_\mu = 1} \middle| M_\mu \right\rangle}{M_M - E_n + i\epsilon}$$

$$|\Delta m| = 2\left|\mathrm{Re}\, m_{12}\right| \longrightarrow x = \frac{\Delta m}{\Gamma}$$

$$\Longrightarrow \quad x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\left\langle \overline{M}_\mu \middle| \mathscr{H}_{\mathrm{eff}} \middle| M_\mu \right\rangle + \left\langle \overline{M}_\mu \middle| i\int d^4x \mathrm{T}\left[\mathscr{H}_{\mathrm{eff}}(x)\mathscr{H}_{\mathrm{eff}}(0)\right] \middle| M_\mu \right\rangle\right]$$

- Dominant contribution

$$\left\langle \overline{M}_\mu \middle| \mathscr{H}_{\mathrm{eff}} \middle| M_\mu \right\rangle = \left\langle \overline{M}_\mu \middle| \mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu = 2} \middle| M_\mu \right\rangle$$

  - Suppressed by Λ^2

- Muonium is a non-relativistic Coulombic bound state

$$|M_\mu\rangle = \sqrt{\frac{2M_M}{2m_\mu 2m_e}}\int \frac{d^3p}{(2\pi)^3}\tilde{\varphi}(p)\,|p, p'\rangle$$

  - Perturbative QED bound state: can calculate!

Calculation of Matrix Elements

- Direct computation is equivalent to vacuum saturation (factorization)
- Factorization approach,

$$\left\langle 0 \left| \bar{\mu}\gamma^{\alpha}\gamma^{5}e \right| M_{\mu}^{P} \right\rangle = i f_P p^{\alpha}, \qquad \left\langle 0 \left| \bar{\mu}\gamma^{\alpha}e \right| M_{\mu}^{V} \right\rangle = f_V M_M \epsilon^{\alpha}(p),$$

$$\left\langle 0 \left| \bar{\mu}\sigma^{\alpha\beta}e \right| M_{\mu}^{V} \right\rangle = i f_T \left(\epsilon^{\alpha}p^{\beta} - \epsilon^{\beta}p^{\alpha}\right)$$

- Where in the non-relativistic limit f_M

$$f_P = f_V = f_T = f_M$$

and

$$f_M^2 = 4\frac{|\varphi(0)|^2}{M_M}$$

Coulombic bound state wave function

$$\varphi(r) = \frac{1}{\sqrt{\pi a_{M_\mu}^3}} e^{-\frac{r}{a_{M_\mu}}}$$

Then, $|\varphi(0)|^2 = \frac{\left(m_{red}\alpha\right)^3}{\pi} = \frac{1}{\pi}\left(m_{red}\alpha\right)^3$

Calculation of Matrix Elements
Para-muonium and Ortho-muonium

- Matrix elements from operators - spin-singlet states

$$\left\langle \overline{M}^P_\mu \middle| Q_1 \middle| M^P_\mu \right\rangle = f_M^2 M_M^2, \quad \left\langle \overline{M}^P_\mu \middle| Q_2 \middle| M^P_\mu \right\rangle = f_M^2 M_M^2,$$

$$\left\langle \overline{M}^P_\mu \middle| Q_3 \middle| M^P_\mu \right\rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \left\langle \overline{M}^P_\mu \middle| Q_4 \middle| M^P_\mu \right\rangle = -\frac{1}{4} f_M^2 M_M^2,$$

$$\left\langle \overline{M}^P_\mu \middle| Q_5 \middle| M^P_\mu \right\rangle = -\frac{1}{4} f_M^2 M_M^2$$

- x_P for para-muonium state in terms of the Wilson coefficients

$$x_P = \frac{4\left(m_{red}\alpha\right)^3}{\pi \Lambda^2 \Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} - \frac{3}{2} C_3^{\Delta L=2} - \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]$$

- Matrix elements from operators - spin-triplet state

$$\left\langle \overline{M}^V_\mu \middle| Q_1 \middle| M^V_\mu \right\rangle = -3 f_M^2 M_M^2, \quad \left\langle \overline{M}^V_\mu \middle| Q_2 \middle| M^V_\mu \right\rangle = -3 f_M^2 M_M^2$$

$$\left\langle \overline{M}^V_\mu \middle| Q_3 \middle| M^V_\mu \right\rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \left\langle \overline{M}^V_\mu \middle| Q_4 \middle| M^V_\mu \right\rangle = -\frac{3}{4} f_M^2 M_M^2$$

$$\left\langle \overline{M}^V_\mu \middle| Q_5 \middle| M^V_\mu \right\rangle = -\frac{3}{4} f_M^2 M_M^2$$

- x_V for ortho-muonium state in terms of the Wilson coefficients

$$x_V = -\frac{12\left(m_{red}\alpha\right)^3}{\pi \Lambda^2 \Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} + \frac{1}{2} C_3^{\Delta L=2} + \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]$$

Results hold true for any NP model that can be matched onto a set of local $\Delta L_\mu = 2$ interactions

Δm calculated for Q_2
[Feinberg and Weinberg, (1961)]

Width Difference $\Delta\Gamma$

- Calculated Δm from operators that change the lepton quantum flavor number by two units, $\Delta L_\mu = 2$
- Particles oscillate by two insertions of operators that change lepton flavor number by one unit, $\Delta L_\mu = 1$, highly suppressed by Λ^4
- Oscillations occur when a SM $\Delta L_\mu = 0$ decay interferes with a LFV $\Delta L_\mu = 2$ process
- Mass eigenstates have width difference ($\Delta\Gamma$)
- Leading contribution to $\Delta\Gamma$ - SM decay interfering with the LFV $\Delta L_\mu = 2$ process

$\Delta\Gamma$ calculated for the first time!

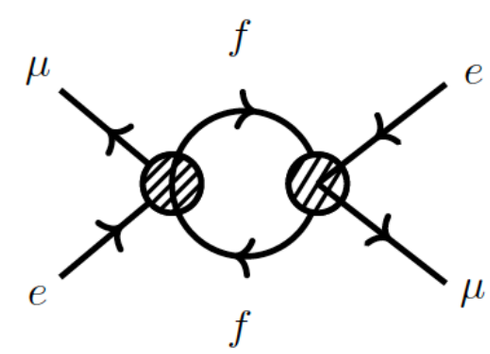
Width Difference $\Delta\Gamma$

- Contribution from $\Delta L_\mu = 1$ and $\Delta L_\mu = 2$ operators
  - Two insertions of $\Delta L_\mu = 1$ operators

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = -\frac{1}{\Lambda^2}\sum_f \Big[\left(C_{VR}^f\,\overline{\mu}\gamma^\alpha P_R e + C_{VL}^f\,\overline{\mu}\gamma^\alpha P_L e\right)\,\overline{f}\gamma_\alpha f$$
$$+ \left(C_{AR}^f\,\overline{\mu}\gamma^\alpha P_R e + C_{AL}^q\,\overline{\mu}\gamma^\alpha P_L e\right)\,\overline{f}\gamma_\alpha\gamma_5 f$$
$$+ m_e m_f G_F \left(C_{SR}^f\,\overline{\mu}P_L e + C_{SL}^f\,\overline{\mu}P_R e\right)\,\overline{f}f$$
$$+ m_e m_f G_F \left(C_{PR}^f\,\overline{\mu}P_L\, e + C_{PL}^f\,\overline{\mu}P_R e\right)\,\overline{f}\gamma_5 f$$
$$+ m_e m_f G_F \left(C_{TR}^f\,\overline{\mu}\sigma^{\alpha\beta}P_L e + C_{TL}^f\,\overline{\mu}\sigma^{\alpha\beta}P_R e\right)\,\overline{f}\sigma_{\alpha\beta} f \;+\; h.c.\Big]$$

Remember $\boxed{\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm eff}^{\Delta L_\mu=0} + \mathscr{L}_{\rm eff}^{\Delta L_\mu=1} + \mathscr{L}_{\rm eff}^{\Delta L_\mu=2}}$

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu=2} \middle| M_\mu \right\rangle + \frac{1}{2M_M}\sum_n \frac{\left\langle \overline{M}_\mu \middle| \mathscr{H}^{\Delta L_\mu=1} \middle| n \right\rangle \left\langle n \middle| \mathscr{H}^{\Delta L_\mu=1} \middle| M_\mu \right\rangle}{M_M - E_n + i\epsilon}$$



$$|\Delta\Gamma| = 2\left|\Gamma_{12}\right| \longrightarrow y = \frac{\Delta\Gamma}{2\Gamma}$$

$$\Longrightarrow \boxed{y = \frac{1}{\Gamma}\sum_n \rho_n \left\langle \overline{M}_\mu \middle| \mathscr{H}_{\rm eff} \middle| n \right\rangle \left\langle n \middle| \mathscr{H}_{\rm eff} \middle| M_\mu \right\rangle}$$

$\Delta\Gamma^{ff}$ is generated by on-shell degrees of freedom, $f = e, \nu$

Where, $\mathscr{H}_{\rm eff} = \mathscr{H}_{\rm eff}^{\Delta L_\mu=1}$

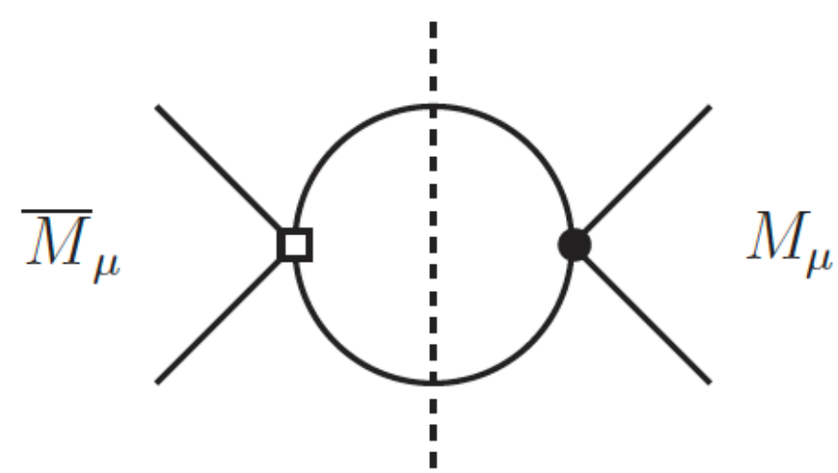
Suppressed by a factor of $\frac{1}{\Lambda^4}$, i.e. $y \ll x$

Width Difference $\Delta\Gamma$

- Contribution from $\Delta L_\mu = 2$ operators?

- Standard model $\Delta L_\mu = 0$ decay interferes with $\Delta L_\mu = 2$ process

$$M_\mu \to \overline{\nu_\mu}\nu_e \to \overline{M}_\mu$$

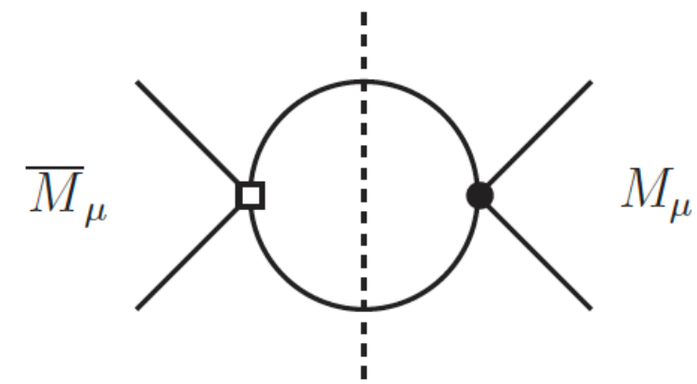


- Only suppressed by $\Lambda^2 M_W^2$
  - Leading contribution to y!

Width Difference $\Delta\Gamma$

- Writing y in terms of the correlation function

$$y = \frac{1}{2M_M\Gamma}\,\mathrm{Im}\left[\left\langle \bar{M}_\mu \left| i\int d^4x\,\mathrm{T}\left[\mathscr{H}_{\mathrm{eff}}(x)\mathscr{H}_{\mathrm{eff}}(0)\right]\right| M_\mu \right\rangle\right]$$
$$= \frac{1}{M_M\Gamma}\,\mathrm{Im}\left[\left\langle \bar{M}_\mu \left| i\int d^4x\,\mathrm{T}\left[\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=2}(x)\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=0}(0)\right]\right| M_\mu \right\rangle\right]$$



- Here, $\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=0} = -\mathscr{L}_{\mathrm{eff}}^{\Delta L_\mu=0}$ is the ordinary SM Lagrangian

$$\mathscr{L}_{\mathrm{eff}}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}}\left(\bar{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu}_{e_L}\gamma^\alpha\nu_{\mu_L}\right)$$

- And $\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=2}$ only contributes through the operators Q_6 and Q_7

$$Q_6 = \left(\bar{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu}_{\mu_L}\gamma^\alpha\nu_{eL}\right), \quad Q_7 = \left(\bar{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\nu}_{\mu_L}\gamma^\alpha\nu_{eL}\right)$$

Width Difference $\Delta\Gamma$

- operator product expansion

$$T = i\int d^4x \mathrm{T}\left[\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=2}(x)\mathscr{H}_{\mathrm{eff}}^{\Delta L_\mu=0}(0)\right]$$
$$= i\int d^4x \mathrm{T}\left[\left(\bar{\mu}\Gamma_\alpha e\right)\left(\overline{\nu_{\mu_L}}\gamma^\alpha\nu_{eL}\right)(x)\left(\bar{\mu}\gamma_\beta P_L e\right)\left(\overline{\nu_{e_L}}\gamma^\beta\nu_{\mu_L}\right)(0)\right]$$

- Contract neutrino fields into propagators
- Cutkoski rules to calculate the discontinuity (imaginary part) of T

$$\mathrm{Disc}\, T = \frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{3\pi}\left[C_6^{\Delta L=2}\left(Q_1+Q_5\right)+\frac{1}{2}C_7^{\Delta L=2}Q_3\right]$$

- T and the matrix elements we found earlier, compute y

Width Difference $\Delta\Gamma$

- Para-muonium,

$$y_P = \frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}\left(m_{red}\alpha\right)^3\left(C_6^{\Delta L=2} - C_7^{\Delta L=2}\right)$$

- Ortho-muonium,

$$y_V = -\frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}\left(m_{red}\alpha\right)^3\left(5C_6^{\Delta L=2} + C_7^{\Delta L=2}\right)$$

Leading contributions to muonium lifetime difference, only suppressed by powers of Λ^2

Experimental Constraints

- Have expressions for x and y for both para and ortho-muonoium
- Want to place constraints on the BSM scale Λ
- Can do this using experimental constraints on the muonium anti-muonium parameters
- Oscillation probability from an experiment performed at PSI [L. Willmann et. al. (1999)]

$$P\left(M_\mu \to \bar{M}_\mu\right) \leq 8.3 \times 10^{-11}/S_B\left(B_0\right)$$

- $S_B(B_0)$ describes the suppression of the oscillation due to the external magnetic field and depends on the interaction type, or the Lorentz structure of the operators

Interaction type	2.8 μT	0.1 T	100 T
SS	0.75	0.50	0.50
PP	1.0	0.9	0.50
$(V \pm A) \times (V \pm A)$ or $(S \pm P) \times (S \pm P)$	0.75	0.35	0.0
$(V \pm A) \times (V \mp A)$ or $(S \pm P) \times (S \mp P)$	0.95	0.78	0.67

We will use $S_B(B_0) = 2.8\mu\text{T}$

Experimental Constraints - Results

- Oscillation probability is averaged over number of polarization degrees of freedom

$$P\left(M_\mu \to \bar{M}_\mu\right)_{\text{exp}} = \sum_{i=P,V} \frac{1}{2S_i+1} P\left(M_\mu^i \to \bar{M}_\mu^i\right)$$

- Recall, $P\left(M_\mu \to \overline{M}_\mu\right) = R_M(x,y)$

- Constraints on Λ probed by different $\Delta L_\mu = 2$ operators, we set the corresponding Wilson coefficient $C_i = 1$

Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

Summary

- ✳LFV muonium oscillations are a highly effective way to search for NP
- ✳Set up muonium oscillation formalism
- ✳Calculated the parameters x and y for spin-0 and spin-1, here y was calculated for the first time!
- ✳Used expressions for x and y and the current experimental bounds on muonium oscillations to put a constraint on the NP scale Λ

Questions?

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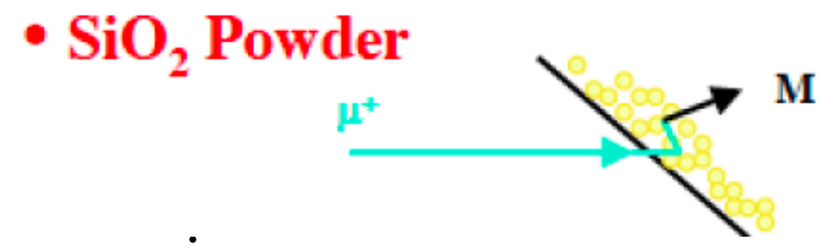
Searches for Muonium Oscillations

- Most recent search for muonium oscillations was at the Paul Scherrer Institute (PSI)
  - Probability of $P_{M\overline{M}} \leq 8.2 \times 10^{-11}$ @ 90 % C.L.
  - $P(M \to \bar{M}) = \int_0^\infty \frac{dt}{\tau} e^{-t/\tau} |\langle \bar{M} | M(t) \rangle|^2$



- Beam of μ^+ at SIO_2 powder target, electron capture to form muonium
- If $M(\mu^+e^-) \to \overline{M}(\mu^-e^+)$ then an energetic e^- and an e^+ would be detected, with the e^- from the $\mu^- \to e^-\overline{\nu}_e\nu_\mu$ decay
- Background
  - Bhabha scattering of the e^- and a e^+ from the μ^+ decay
  - $\mu^+ \to e^-e^+e^+\nu_e\overline{\nu}_\mu$

Muonium Oscillation Formalism

- In the Schrödinger picture start with a flavor eigenstate

$$|\psi\rangle = |\psi, t = 0\rangle \qquad\qquad |\psi, t\rangle = U(t,0)|\psi\rangle = e^{-iHt}|\psi\rangle$$

- We get the Schrödinger equation

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$

- Generalizing to a two state system describing muonium oscillations

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix}$$

- With matrix Hamiltonian

$$H = \left(m - i\frac{\Gamma}{2}\right)$$

Calculation of Matrix Elements

Example: $$Q_1 = \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_L e\right)$$

$$\left\langle \overline{M}_\mu \left| Q_1 \right| M_\mu \right\rangle = 4(\bar{u}\gamma^\alpha P_L v)(\bar{v}\gamma_\alpha P_L u)\left| \int \frac{d^3p}{(2\pi)^3}\tilde{\psi}(p)\right|^2$$

Where, $\left| \int \frac{d^3q}{(2\pi)^3}\tilde{\psi}(q)\right|^2 = \left|\psi(0)\right|^2$

spatial wavefuntion at the origin

non-relativistic spinors

$$u = \sqrt{m_e}\begin{pmatrix}\xi\\ \xi\end{pmatrix}, \qquad v = \sqrt{m_e}\begin{pmatrix}\eta\\ -\eta\end{pmatrix},$$

$$\bar{u} = \sqrt{m_\mu}\left(\xi^\dagger, \xi^\dagger\right)\gamma^o, \qquad \bar{v} = \sqrt{m_\mu}\left(\eta^\dagger, -\eta^\dagger\right)\gamma^o$$

spinor products: spin-0, spin-1
Spin-1 with 3 possible polarizations

$$\xi\eta^\dagger = \frac{1}{\sqrt{2}}\mathbf{1}_{2\times 2} \qquad \xi\eta^\dagger = \frac{1}{\sqrt{2}}\vec{\epsilon}^{\,*}\cdot\vec{\sigma}$$

$$\left\langle \bar{M}_\mu \left| Q_1 \right| M_\mu \right\rangle_{spin-0} = 2\left|\psi(0)\right|^2$$

$$\left\langle \bar{M}_\mu \left| Q_1 \right| M_\mu \right\rangle_{spin-1} = -6\left|\psi(0)\right|^2$$

Matrix Element - Vacuum Insertions

- Operator Q_1 as example

$$Q_1 = \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_L e\right)$$

- Matrix element

$$\left\langle \overline{M}_\mu \left| -\mathscr{L}_{\text{eff}}^{\Delta L_\mu = 2} \right| M_\mu \right\rangle = \left\langle \overline{M}_\mu \left| \frac{1}{\Lambda_1^2} \sum_i C_i(\mu) Q_i(\mu) \right| M_\mu \right\rangle$$

$$\left\langle \overline{M}_\mu \left| Q_1 \right| M_\mu \right\rangle = \left\langle \overline{M}_\mu \left| \left(\bar{\mu}\gamma_\alpha P_L e\right)\left(\bar{\mu}\gamma^\alpha P_L e\right) \right| M_\mu \right\rangle$$

$$\simeq \left\langle \overline{M}_\mu \left| \left(\bar{\mu}\gamma_\alpha P_L e\right) \right| 0 \right\rangle \underbrace{\left\langle 0 \left| \left(\bar{\mu}\gamma^\alpha P_L e\right) \right| M_\mu \right\rangle}$$

Projection operators

$$P_{R,L} = \frac{1}{2}\left(1 \pm \gamma^5\right)$$

$$\left\langle 0 \left| \left(\bar{\mu}\gamma^\alpha P_L e\right) \right| M_\mu \right\rangle = \frac{1}{2}\left\langle 0 \left| \left(\bar{\mu}\gamma^\alpha \left(1 - \gamma^5\right) e\right) \right| M_\mu \right\rangle$$

$$= \frac{1}{2}\left\langle 0 \left| \left(\bar{\mu}\gamma^\alpha e\right) \right| M_\mu \right\rangle - \frac{1}{2}\left\langle 0 \left| \left(\bar{\mu}\gamma^\alpha \gamma^5 e\right) \right| M_\mu \right\rangle$$