

Charmonium and Bottomonium properties from lattice QCD + QED

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References

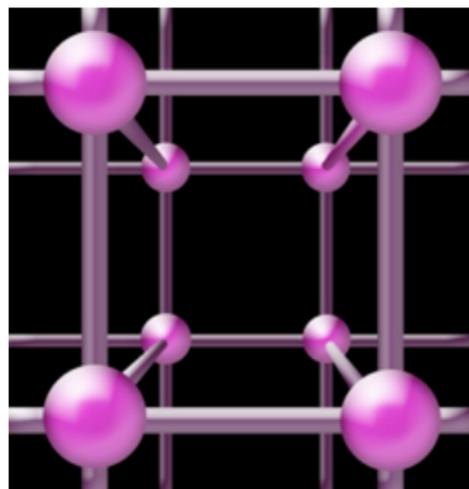
- 1 Charmonium: PRD **102** (2020) 054511, [ArXiv 2005.01845](#)
- 2 Bottomonium: PRD (in press), [ArXiv 2101.08103](#)
- 3 b quark mass [ArXiv 2102.09609](#)

- 1 Motivation
- 2 Lattice calculation
 - QED on the lattice
- 3 Charmonium and bottomonium
 - Hyperfine splitting
 - Decay constants
 - Quark masses

Motivation: precise and accurate SM predictions

Lattice QCD calculations for many quantities have reached, or surpassed, statistical precision of 1%

- masses and decay constants of ground-state pseudoscalar mesons are good examples
- meson masses used for tuning, and determining quark masses



The precision of modern lattice QCD results means that sources of small **SYSTEMATIC UNCERTAINTY** that could appear at the percent level need to be understood. Here we focus on QED effects.

We use lattice configurations generated by the MILC collaboration

- 17 gluon field ensembles
- six different lattice spacings from very coarse ($a \approx 0.15$ fm) to exafine ($a \approx 0.03$ fm)
- $n_f = 2+1+1$: light, s and c quarks in the sea, $m_l = (m_u + m_d)/2$
- one ensemble with $n_f = 1+1+1+1$ (physical m_u and m_d)
- various light sea quark masses, including (close to) physical masses, to control chiral extrapolation
- Highly Improved Staggered Quark (HISQ) action for both sea and valence quarks
 - removes tree-level a^2 discretisation errors
 - For heavy quarks 'Naik' term is adjusted to remove $(am)^4$ errors at tree-level
- heavy-HISQ: use several heavy valence quark masses $m_h > m_c$ to extract quantities at physical b mass

Lattice QCD with quenched QED

- quenched QED = include effects from the valence quarks having electric charge (the largest QED effect) but neglect effects from the electric charge of the sea quarks
- generate a random momentum space photon field $A_\mu(k)$ for each QCD gluon field configuration and set zero modes to zero using the QED_L formulation (QED in finite box)
- Fourier transform A_μ into position space
- $\exp(i e Q A_\mu)$ gives the desired $U(1)$ field (Q =quark electric charge in units of the proton charge e)
- this approach is known as the stochastic approach to quenched QED, in contrast to the perturbative approach
- c and b lattice quark masses tuned separately in pure QCD and QCD+QED so that J/ψ and Υ masses match experiment

Meson correlators

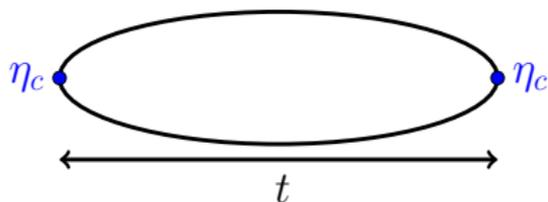
We calculate the pseudoscalar and vector meson correlators on each ensemble and use a multi-exponential fit to extract amplitudes and energies:

$$C_{2\text{-point}}(t) = \sum_i A_i \left(e^{-E_i t} + e^{-E_i(L_t - t)} \right)$$

The decay constants are related to the ground state amplitude and meson mass:

$$f_P = 2m_q \sqrt{\frac{2A_0^P}{(M_0^P)^3}}, \quad f_V = Z_V \sqrt{\frac{2A_0^V}{M_0^V}}.$$

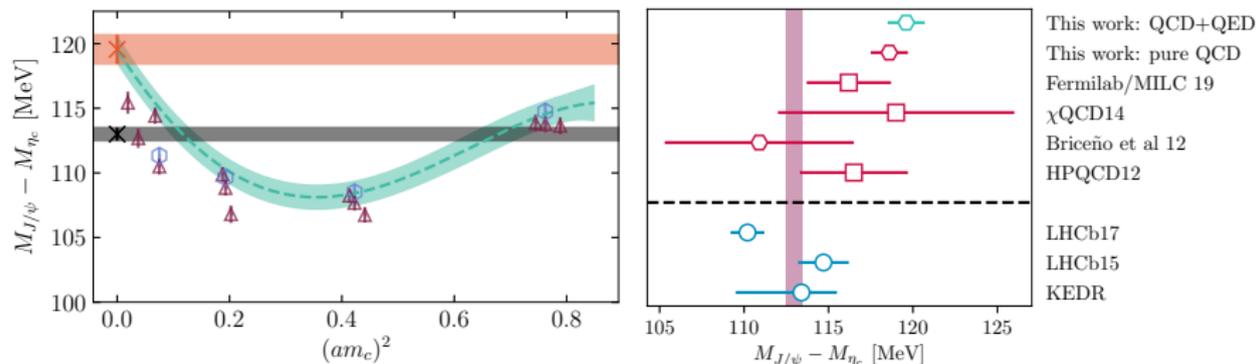
Z_V is a renormalisation factor for the lattice vector current.



We then take the results at different lattice spacings and extrapolate to continuum, taking into account $(am)^{2n}$ and $(a\Lambda)^{2n}$ discretisation effects. Terms that take into account mistuned sea quark masses are also included. For bottomonium, we map out the dependence in quark mass to extract the result at physical m_b .

Charmonium hyperfine splitting

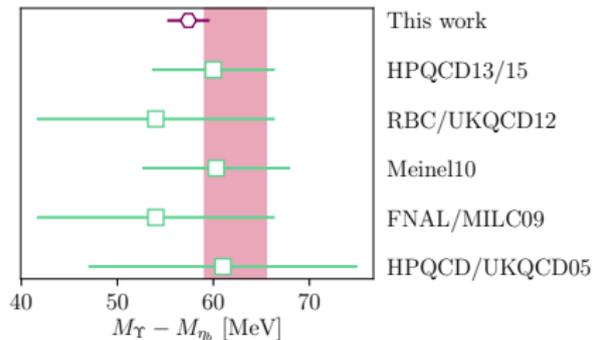
Our QCD+QED result for the charmonium hyperfine splitting is $M_{J/\psi} - M_{\eta_c} = 120.3(1.1)$ MeV.



Quark-line disconnected correlation functions are not included in the lattice calculation. The difference in the results is taken to be the effect of the η_c decay to two gluons (prohibited in the lattice calculation):

$$\Delta M_{\eta_c}^{\text{annihln}} = +7.3(1.2) \text{ MeV}$$

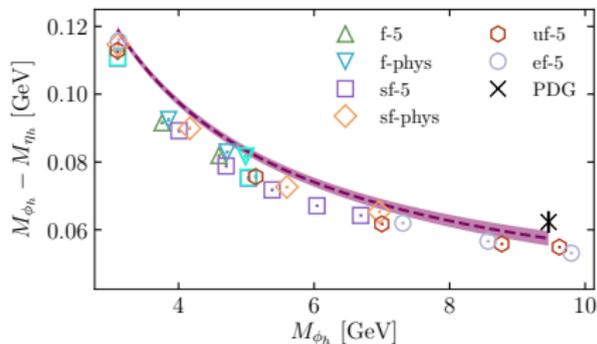
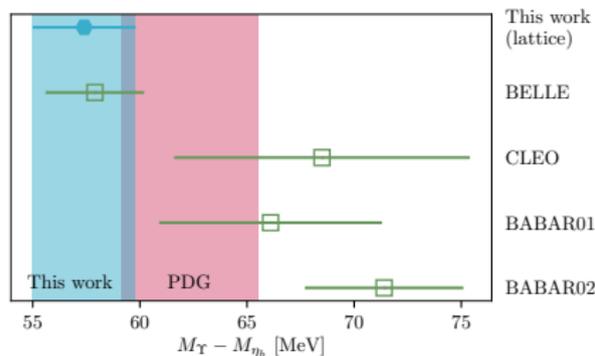
Bottomonium hyperfine splitting



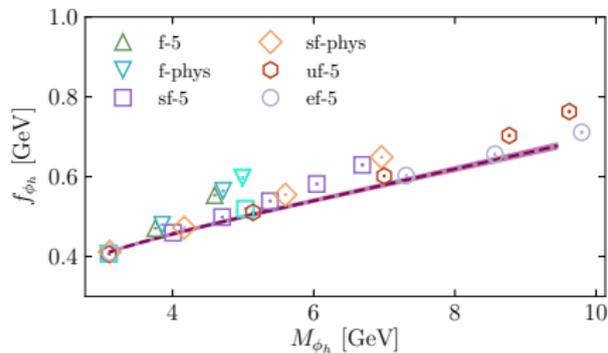
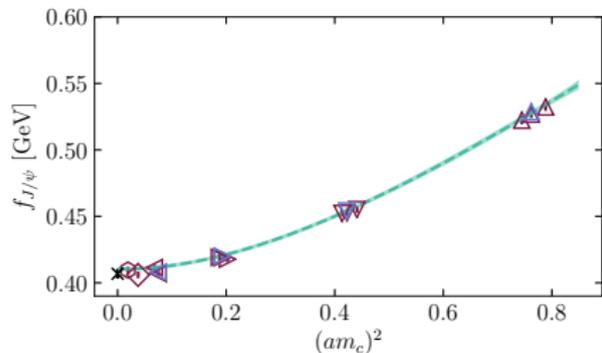
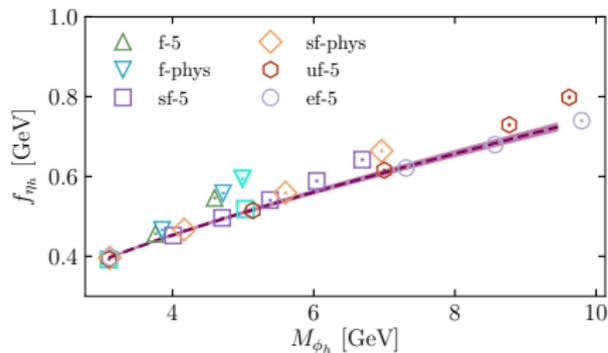
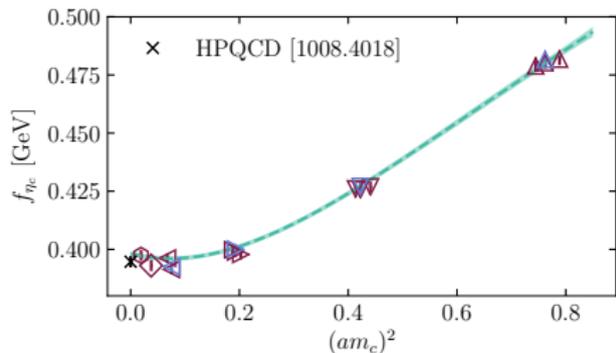
Map out the dependence in m_h to extract the result at physical m_b .

Our QCD+QED result is
 $M_\Upsilon - M_{\eta_b} = 57.5(2.3)(1.0)$ MeV.

The second uncertainty takes into account missing quark-line disconnected contributions.



Decay constants f_{η_c} , $f_{J/\psi}$, f_{η_b} and f_{Υ}



Comparison of decay constant results

Vector and pseudoscalar meson decay constants and their ratios:

$$f_{J/\psi} = 410.4(1.7) \text{ MeV}$$

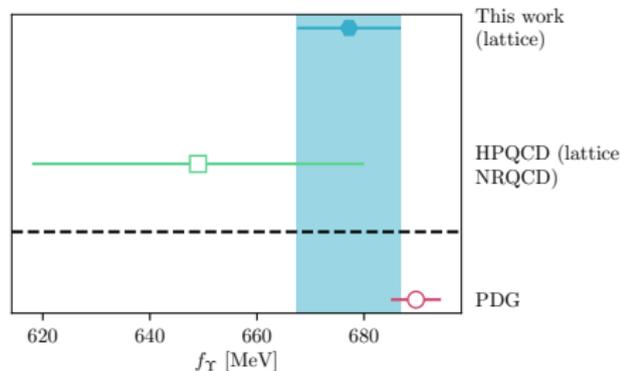
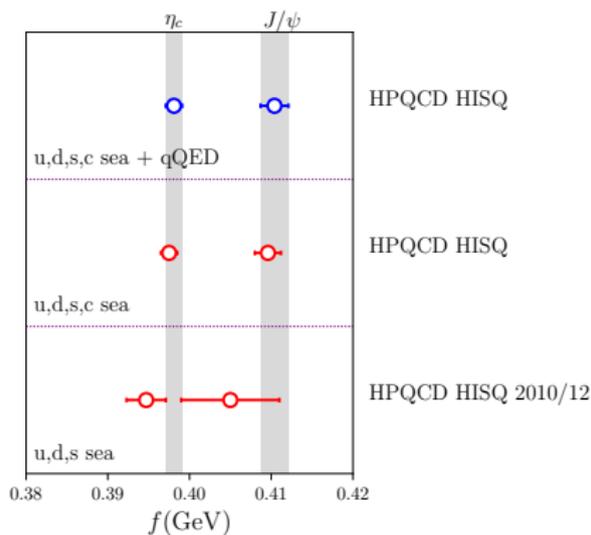
$$f_{\eta_c} = 398.1(1.0) \text{ MeV}$$

$$\frac{f_{J/\psi}}{f_{\eta_c}} = 1.0284(19)$$

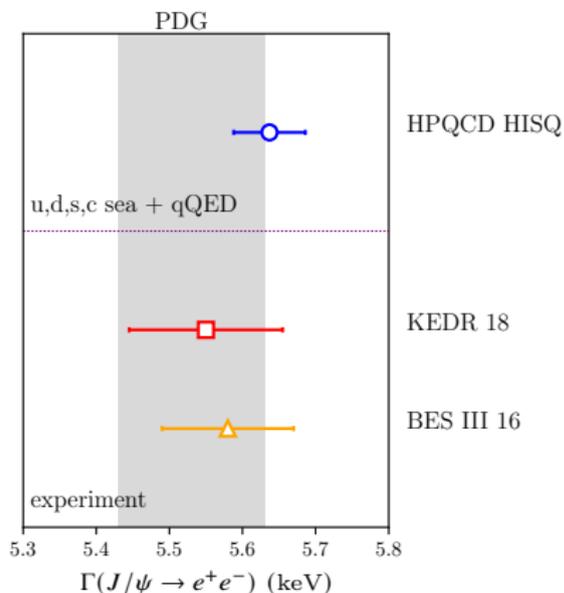
$$\frac{f_{\Upsilon}}{f_{\eta_b}} = 0.9454(99)$$

$$f_{\Upsilon} = 677.2(9.7) \text{ MeV}$$

$$f_{\eta_b} = 724(12) \text{ MeV}$$



Leptonic widths



The vector meson decay constant is related to leptonic width by

$$\Gamma(\phi_h \rightarrow e^+e^-) = \frac{4\pi}{3} \alpha_{\text{QED}}^2 e^2 \frac{f_{\phi_h}^2}{M_{\phi_h}}.$$

Our results are:

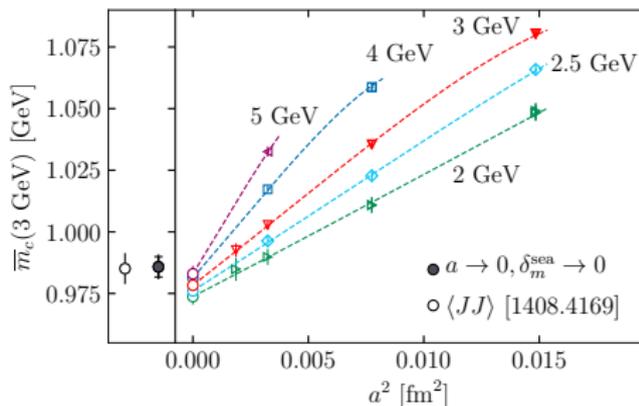
$$\Gamma(J/\psi \rightarrow e^+e^-) = 5.637(47)(13) \text{ keV}$$

$$\Gamma(\Upsilon \rightarrow e^+e^-) = 1.292(37)(3) \text{ keV}$$

There is no experimental decay rate that can be directly compared with the pseudoscalar decay constant.

Quark masses: Recipe

- 1 Tune the lattice charm mass so that the J/ψ mass takes its experimental value on each ensemble.
- 2 Calculate the mass renormalisation factor Z_m^{SMOM} . Convert the mass to the \overline{MS} scheme at multiple scales μ using RI-SMOM scheme as an intermediate scheme on each ensemble.
- 3 Run all the \overline{MS} quark masses at a range of scales μ to a reference scale of 3 GeV using the \overline{MS} β function.



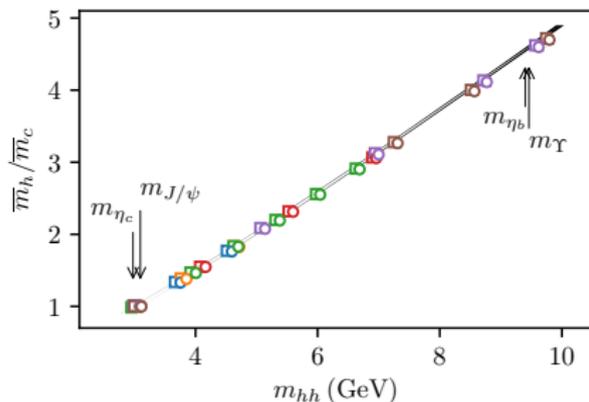
- 4 Fit all of the results for the \overline{MS} mass at 3 GeV to a function that allows for discretisation effects and condensate contamination from Z_m^{SMOM} .
- 5 Repeat all steps for QCD+QED.

Ratio of quark masses

Ratio of quark masses: in pure QCD

$$\frac{\bar{m}_b(\mu)}{\bar{m}_c(\mu)} = \frac{m_b^{\text{tuned}}}{m_c^{\text{tuned}}} + \mathcal{O}(\alpha_s(\pi/a)a^2)$$

We have very few data points at physical b mass.



Instead we define functions

$$m_{hh}^P = f_{hh}^P(\bar{m}_h/\bar{m}_c),$$

$$m_{hh}^V = f_{hh}^V(\bar{m}_h/\bar{m}_c),$$

for pseudoscalar and vector meson masses, where

$$f_{hh}^P(1) = M_{\eta_c}^{\text{cont}},$$

$$f_{hh}^V(1) = M_{J/\psi}^{\text{cont}}.$$

The ratio can be obtained by solving

$$M_{\eta_b}^{\text{cont}} = f_{hh}^P(\bar{m}_b/\bar{m}_c) \text{ or}$$

$$M_{\Upsilon}^{\text{cont}} = f_{hh}^V(\bar{m}_b/\bar{m}_c).$$

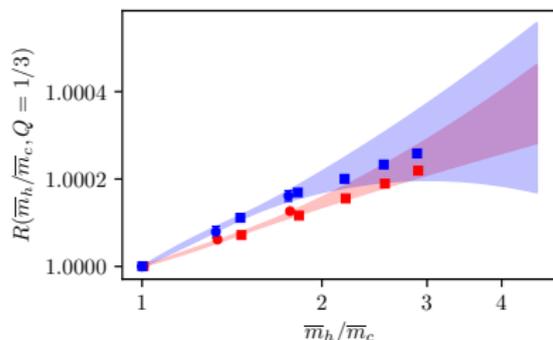
Ratio of quark masses: adding QED

Adding QED can be done in steps:

$$\left. \frac{\bar{m}_b(\mu)}{\bar{m}_c(\mu)} \right|_{**} = \frac{R(\bar{m}_b/\bar{m}_c, Q_{c,b}=0 \rightarrow \frac{1}{3})}{R(\bar{m}_c(\mu), Q_c = \frac{1}{3} \rightarrow \frac{2}{3})} \times \left. \frac{\bar{m}_b}{\bar{m}_c} \right|_*$$

* is pure QCD and ** QCD+QED.

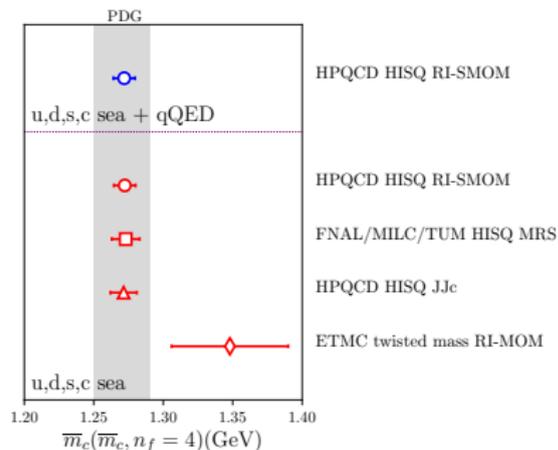
- Consider the mass ratio in theory where c and b have the same Q . $R(\bar{m}_b/\bar{m}_c, Q_{c,b}=0 \rightarrow \frac{1}{3})$ is the factor needed to change that charge from 0 (pure QCD) to $\frac{1}{3}$.
- $R(\bar{m}_c(\mu), Q_c = \frac{1}{3} \rightarrow \frac{2}{3})$ is the factor of changing charm quark electric charge Q from $\frac{1}{3}$ to $\frac{2}{3}$. Get from charm mass calculation.



Ratio of quark masses in \overline{MS} scheme in QCD+QED:

$$\frac{\bar{m}_b(3 \text{ GeV}, n_f = 4)}{\bar{m}_c(3 \text{ GeV}, n_f = 4)} = 4.586(12)$$

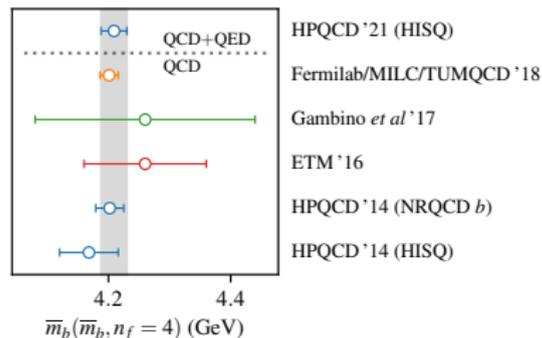
Quark masses



Charm quark mass in \overline{MS} scheme in QCD+QED:

$$\bar{m}_c(3 \text{ GeV}, n_f = 4) = 0.9841(51) \text{ GeV}$$

$$\bar{m}_c(\bar{m}_c, n_f = 4) = 1.2719(78) \text{ GeV}$$



Bottom quark mass in \overline{MS} scheme in QCD+QED:

$$\bar{m}_b(3 \text{ GeV}, n_f = 4) = 4.513(26) \text{ GeV}$$

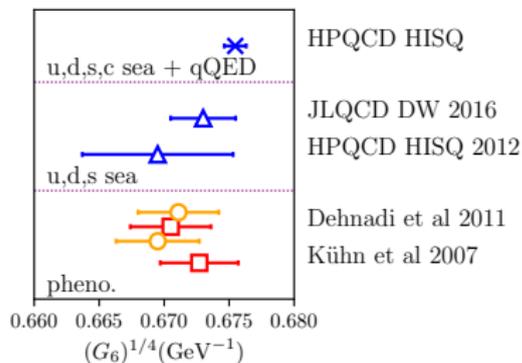
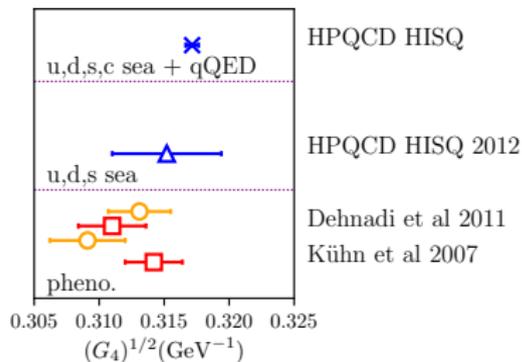
$$\bar{m}_b(\bar{m}_c, n_f = 4) = 4.209(21) \text{ GeV}$$

$$\bar{m}_b(\bar{m}_c, n_f = 5) = 4.202(21) \text{ GeV}$$

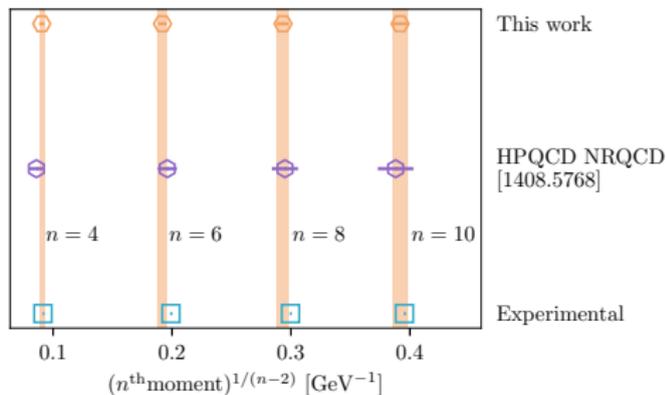
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Backup slides

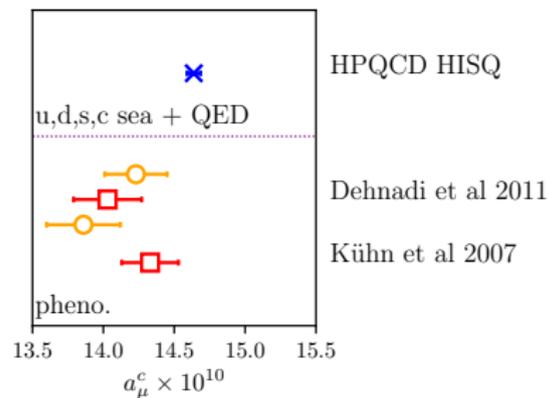
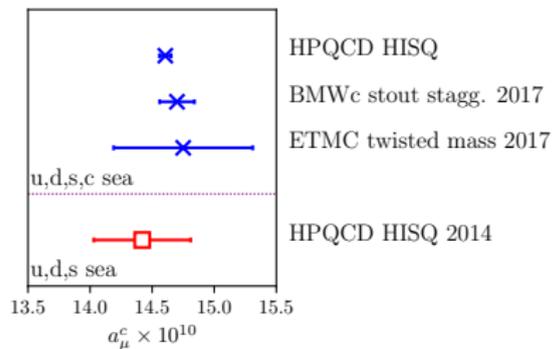
Time moments of vector current-current correlators



The time moments of the quarkonium vector current-current correlators can be compared with phenomenological results derived from $R(e^+e^- \rightarrow \text{hadrons})$. As the results are very accurate, this provides a stringent test.



Hadronic vacuum polarisation contribution to a_μ



The time moments of the quarkonium vector current-current correlators can also be used to calculate the quark-line connected HVP contribution to muon anomalous magnetic moment, a_μ . This is of particular interest, as new results are expected soon from the Muon $g - 2$ experiment at Fermilab.

Our QCD+QED results are:

$$a_\mu^c = 14.638(47) \times 10^{-10}$$

$$a_\mu^b = 0.300(15) \times 10^{-10}$$

Error budgets: Charmonium

Error budget for our final result for the charmonium hyperfine splitting and decay constants including quenched QED corrections. The uncertainties shown are given as a percentage of the final result.

	ΔM_{hyp}	$f_{J/\psi}$	f_{η_c}
$a^2 \rightarrow 0$	0.13	0.09	0.03
Z_V	-	0.05	-
Pure QCD statistics	0.24	0.12	0.05
QCD+QED statistics	0.08	0.05	0.02
w_0/a	0.24	0.11	0.08
w_0	0.87	0.34	0.24
Valence mistuning	0.02	0.05	0.01
Sea mistuning	0.06	0.01	0.00
Total (%)	0.96	0.40	0.26

Error budgets: Bottomonium

	$M_Y - M_{\eta_b}$	f_Y	f_{η_b}
statistics	2.40	0.77	0.38
SVD cut	1.48	0.44	0.67
w_0	0.55	0.61	0.59
w_0/a	0.66	0.23	0.18
Z_V	-	0.29	-
M_{ϕ_h} dependence	0.03	0.01	0.00
$1/M_{\phi_h}$ dependence	0.05	0.02	0.01
$(am_h)^{2k}$ discretisation effects	1.14	0.17	0.18
$(a\Lambda)^{2k}$ discretisation effects	0.48	0.24	0.31
$(am_h)^2(a\Lambda)^2$ discretisation effects	0.42	0.28	0.45
light and strange sea quark mistuning	1.45	0.73	0.98
charm sea quark mistuning	1.08	0.29	0.27
QED M_{ϕ_h} dependence	0.29	0.07	0.08
QED $1/M_{\phi_h}$ dependence	0.19	0.01	0.00
Total (%)	3.99	1.43	1.59

Error budgets: Charm quark mass

	$\overline{m}_c(3 \text{ GeV})$
$a^2 \rightarrow 0$	0.23
Missing α_s^4 term	0.10
Condensate	0.21
m_{sea} effects	0.00
$Z_m^{\overline{\text{MS}}/\text{SMOM}}$ and r	0.07
Z_m^{SMOM}	0.12
Uncorrelated m^{tuned}	0.15
Correlated m^{tuned}	0.30
Gauge fixing	0.09
μ error from w_0	0.12
QED effects	0.02
Total (%)	0.52

Error budgets: c and b quark mass ratio

	$\bar{m}_b/\bar{m}_c[m_{hh}^P]$	$\bar{m}_b/\bar{m}_c[m_{hh}^V]$	$\bar{m}_b/\bar{m}_c[\text{avg}]$
$(am_h)^2 \rightarrow 0$	0.20	0.21	0.20
$w_0, w_0/a$	0.10	0.18	0.12
σ_u	0.12	0.12	0.09
g_m, ζ	0.05	0.05	0.05
m_{cc}	0.06	0.01	0.04
m_{bb}	0.03	0.00	0.02
$(am_h)^2 \delta m_{uds}^{\text{sea}} \rightarrow 0$	0.06	0.07	0.06
$\delta m_c^{\text{sea}} \rightarrow 0$	0.03	0.03	0.03
$d\tilde{m}_c/dm_{cc}$	0.03	0.02	0.02
Total (%)	0.27	0.32	0.27

Error budgets: Charm quark HVP contribution to a_μ^c

	a_μ^c
$a^2 \rightarrow 0$	0.15
Z_V	0.07
Pure QCD Statistics	0.08
QCD+QED Statistics	0.01
w_0/a	0.16
w_0	0.18
Sea mistunings	0.09
Valence mistunings	0.03
$M_{J/\psi}^{\text{exp.}}$	0.05
Total (%)	0.32