# Determination of the charm quark mass in lattice QCD with 2 + 1 flavors on fine lattices

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# Introduction

- The calculation of quark masses is a truly non-perturbative problem.
- We employ $2+1$ flavor lattice QCD to determine the charm quark mass with carefully controlled statistical and systematic uncertainties.
- This talk is based on [ALPHA, 2101.02694].

# Quark masses on the lattice

## Lattice QCD

Solve the path integral of QCD in Euclidean space via a Monte Carlo process

- Choose a suitable discretization prescription.
- Regularization: UV cutoff proportional to inverse lattice spacing $a^{-1}$.
- Calibrate the bare quark mass parameters to reproduce physical observables (meson masses, decay constants).
- Determine bare quantities from large scale computations.
- Renormalize in scheme of choice.

# Quark masses on the lattice

- Determine meson and quark masses from axial $A_\mu$ and pseudoscalar $P$ quark bilinears.

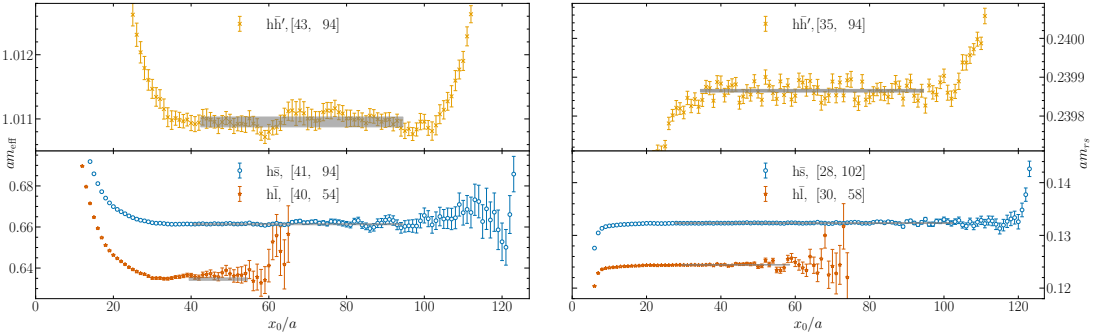
- Meson masses from the time dependence of two-point functions, e.g.,

$$\sum_{\vec{x},\vec{y}} \langle P^{\mathrm{sc}}(x_0,\vec{x}) P^{\mathrm{cs}}(y_0,\vec{y})\rangle \propto \mathrm{e}^{-\mathbf{m}_{\mathbf{D}_\mathrm{s}}(y_0-x_0)} + \mathrm{O}\left(\mathrm{e}^{-E_2(y_0-x_0)}\right)$$

- Bare quark masses from the PCAC relation:

$$\partial_\mu \langle (A_\mathrm{I})_\mu^{ij} P^{ji}\rangle = 2m_{ij}\langle P^{ij}P^{ji}\rangle + \mathrm{O}(a^2)$$

# Meson and quark masses on the lattice




- Investigate heavy-light, heavy-strange and heavy-heavy correlation functions: Calibration via $m_D$ and $m_{D_{\rm s}}$ or $\eta_{\rm c}$.
- Obtain bare quark masses with a relative error of $\mathrm{O}(10^{-3})$ to $\mathrm{O}(10^{-5})$.

# Heavy quark masses on the lattice

- Heavy quarks require special attention: We encounter large cutoff effects.
- Fine lattice spacings required to ensure $am_\mathrm{c} \ll 1$.
- Wilson (charm) quarks:

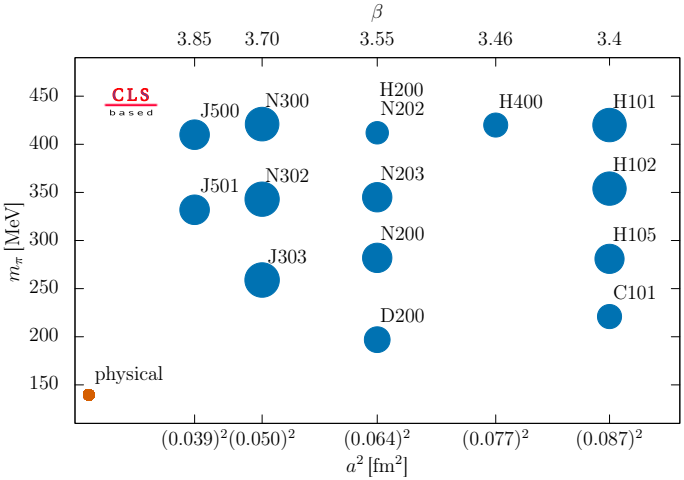
$$M_\mathrm{c} = Z_\mathrm{M}\, m_\mathrm{cc'} \left[1 + (b_\mathrm{A} - b_\mathrm{P})\, am_\mathrm{q,c} + (\bar{b}_\mathrm{A} - \bar{b}_\mathrm{P})\, a\mathrm{Tr}[M_\mathrm{q}]\right] + \mathrm{O}(a^2)$$

  improve quark masses (subtract $\mathrm{O}(am_\mathrm{c})$ effects) non-perturbatively to approach the continuum limit with $\mathrm{O}(a^2)$, cf. [Bhattacharya et al, hep-lat/0511014], [ALPHA, 1502.04999] [ALPHA, 1906.03445] [ALPHA, 2101.10969].

- Employ several discretization prescriptions for the quark mass (including PCVC and appropriate flavor combinations).
  **These have to coincide in the continuum.**

# Our calculation

# Gauge ensembles




- Ensembles by the **Coordinated Lattice Simulations** (CLS) initiative [Bruno et al., 1411.3982].
- $2+1$ flavors of clover improved Wilson fermions (2 light + 1 strange) and tree-level Lüscher-Weisz improved gluons.
- Open boundaries in time direction.

# Need to control all systematic effects

## Discretization effects

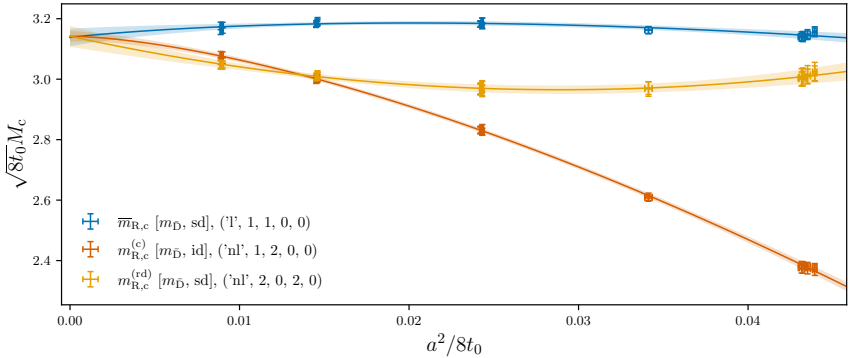
## Physical point extrapolation

## Renormalization and running

## Finite volume

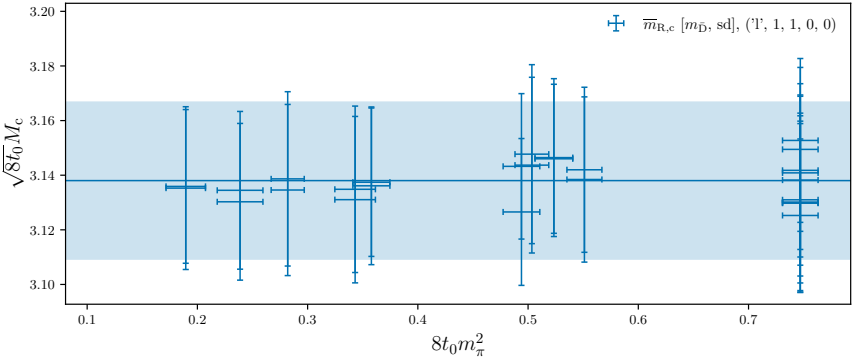
- Negligible in our volumes $L^3$ with $L \geq 2.4$ fm [Colangelo et al., 1005.1485], checked in our study.

# Continuum extrapolation




- We find significant $\mathrm{O}(a^3), \mathrm{O}(a^4)$ contributions for $a > 0.06$ fm.
- Different definitions coincide in the continuum limit (unconstrained fits!).
- Quantify systematic uncertainties via model averaging procedure.

# Dependence on light quark masses




- We keep $8t_0(m_{\mathrm{K}}^2 + \frac{1}{2}m_\pi^2) \propto (m_{\mathrm{u}} + m_{\mathrm{d}} + m_{\mathrm{s}})$ constant on all ensembles.
- No dependence on the pion mass resolved.

# Non-perturbative renormalization and running

- Our central result is the non-perturbatively defined *scale and scheme independent* **Renormalization Group Invariant** (RGI) value of the charm quark mass (cf. [APLHA, hep-lat/9810063]).

- Non-perturbative renormalization and running up to $\mu \approx m_{\mathrm{W}}$: $M/m_{\mathrm{R}}(\mu_{\mathrm{had}})Z_{\mathrm{m}}$ from [ALPHA, 1802.05243] using the renormalization of the axial current from [Dalla Brida et al., 1808.09236].

$$M_{\mathrm{c}}(N_{\mathrm{f}}=3) = 1486(14)_{\mathrm{stat}}(14)_{\mathrm{run}}(6)_{\mathrm{sys}}\,\mathrm{MeV} = 1486(21)\,\mathrm{MeV}$$

# Control over all systematic effects

## Discretization effects

- Improved action and observables: Approach continuum limit $\propto a^2$.
- Employ five lattice spacings down to $a^{-1} \approx 5\,\mathrm{GeV}$.
- Universality of the continuum limit provides strong test.

## Physical point extrapolation

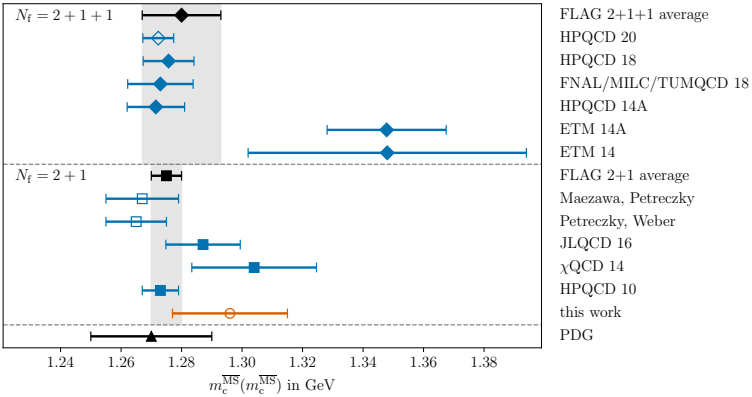
- No dependence of $M_\mathrm{c}$ on light quark masses for $200\,\mathrm{MeV} \leq m_\pi \leq 420\,\mathrm{MeV}$.

## Renormalization and running

- Fully non-perturbative renormalization and renormalization group running.

# Lattice results for $m_c(m_c)$ in $\overline{\mathrm{MS}}$

- Perturbative running in $\overline{\mathrm{MS}}$: $m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}}) = 1296(19)\,\mathrm{MeV}$



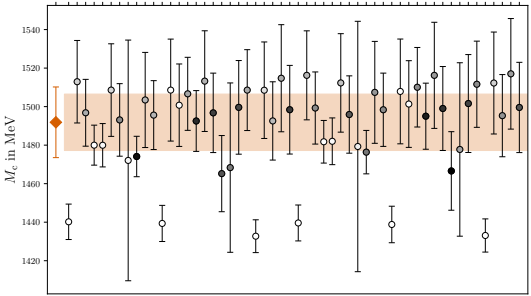

- No significant difference between three and four flavor results.

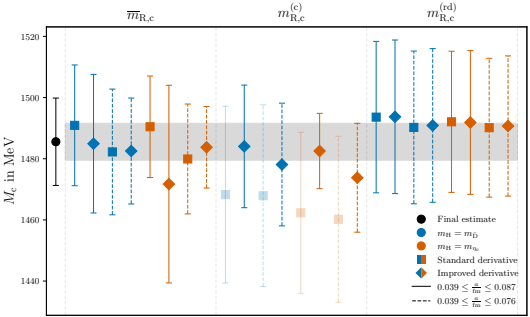
# Conclusions

- We have presented the ALPHA Collaborations determination of the charm quark mass in $2+1$ flavor lattice QCD.
- No uncontrolled sources of systematic errors due to fully non-perturbative renormalization and renormalization group running.
- Very fine lattices below $0.06$ fm are vital for full control over the continuum extrapolation.
- Our precision is limited by the statistical uncertainty of **external quantities**:
  - Renormalization and running (55% of the total variance)
  - Scale setting (20% of the total variance)
  - $\mathrm{O}(a)$ improvement (14% of the total variance)

# Backup

# Model average

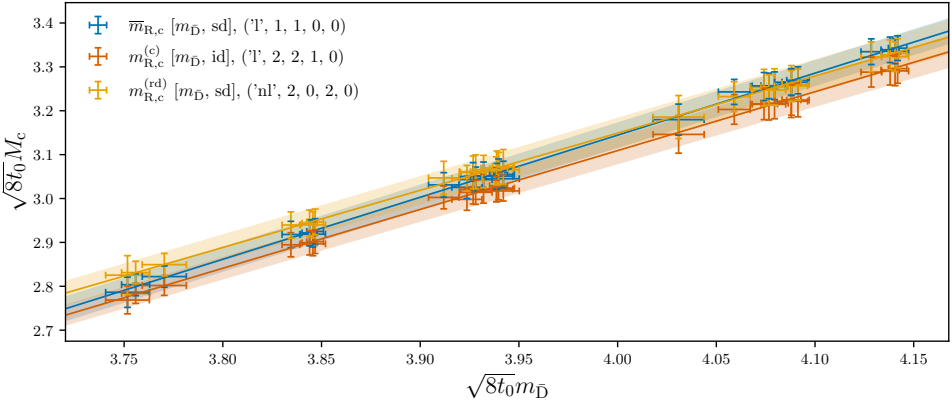



- Use full information to extract the final result together with statistical and systematic uncertainties [Jay and Neil, 2008.01069]:

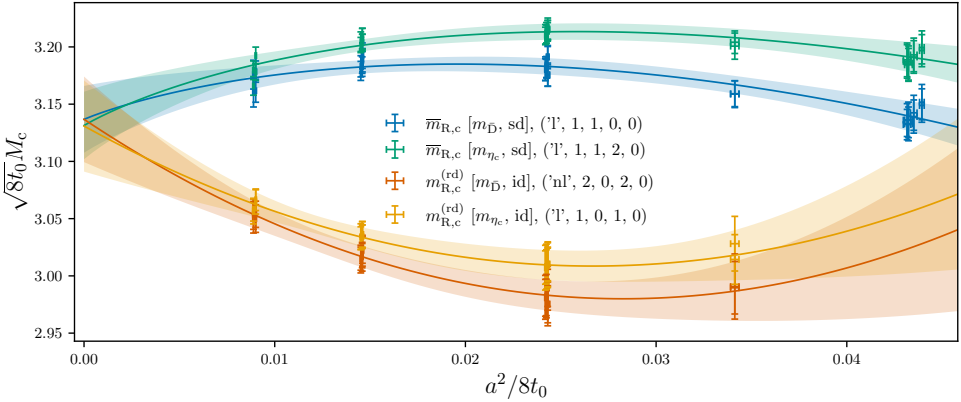
$$\langle M_c \rangle = \sum_{k=1}^{K} w_k \langle M_c \rangle_k \,, \qquad \text{systematic error:} \quad \sigma^2_{M_c} = \sum_{k=1}^{K} w_k \langle M_c \rangle_k^2 - \left( \sum_{k=1}^{K} w_k \langle M_c \rangle_k \right)^2$$

# Calibration of input parameters: Charm quark mass




- Linear dependence of $m_{\mathrm{h}}$ on the heavy meson mass in the considered regime.

# Calibration of input parameters: Charm quark mass




- No significant difference in the continuum limit when comparing the calibration via $\eta_c$ or via a combination of $m_D$ and $m_{D_s}$ (as expected).

# Comparison of lattice results - References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