The Report is dedicated to Gridnev K. A.

Chiral Imbalance Medium in Linear Sigma Model and Chiral Perturbation Theory

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The goal of the report

A possible manifestation of phase Local Parity Breaking (LPB) in the presence of chiral imbalance medium in the sector of light mesons

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Particles **2020**, 3, 15–22; doi:10.3390/particles3010002

Outline of the report

i. Introducing chiral imbalance

ii. Linear Sigma Model for light pions and scalar mesons in the presence of chiral imbalance

iii. Extended Chiral Lagrangian with chiral chemical potential

vi. Conclusions and outlook

Topological charge fluctuations

Local large fluctuations in the topological charge presumably exist in a hot environment.

Chiral Magnetic Effect

- For *peripheral* heavy ion collisions they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

  D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998)

  K. Buckley, T. Fugleberg, & A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814

  D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

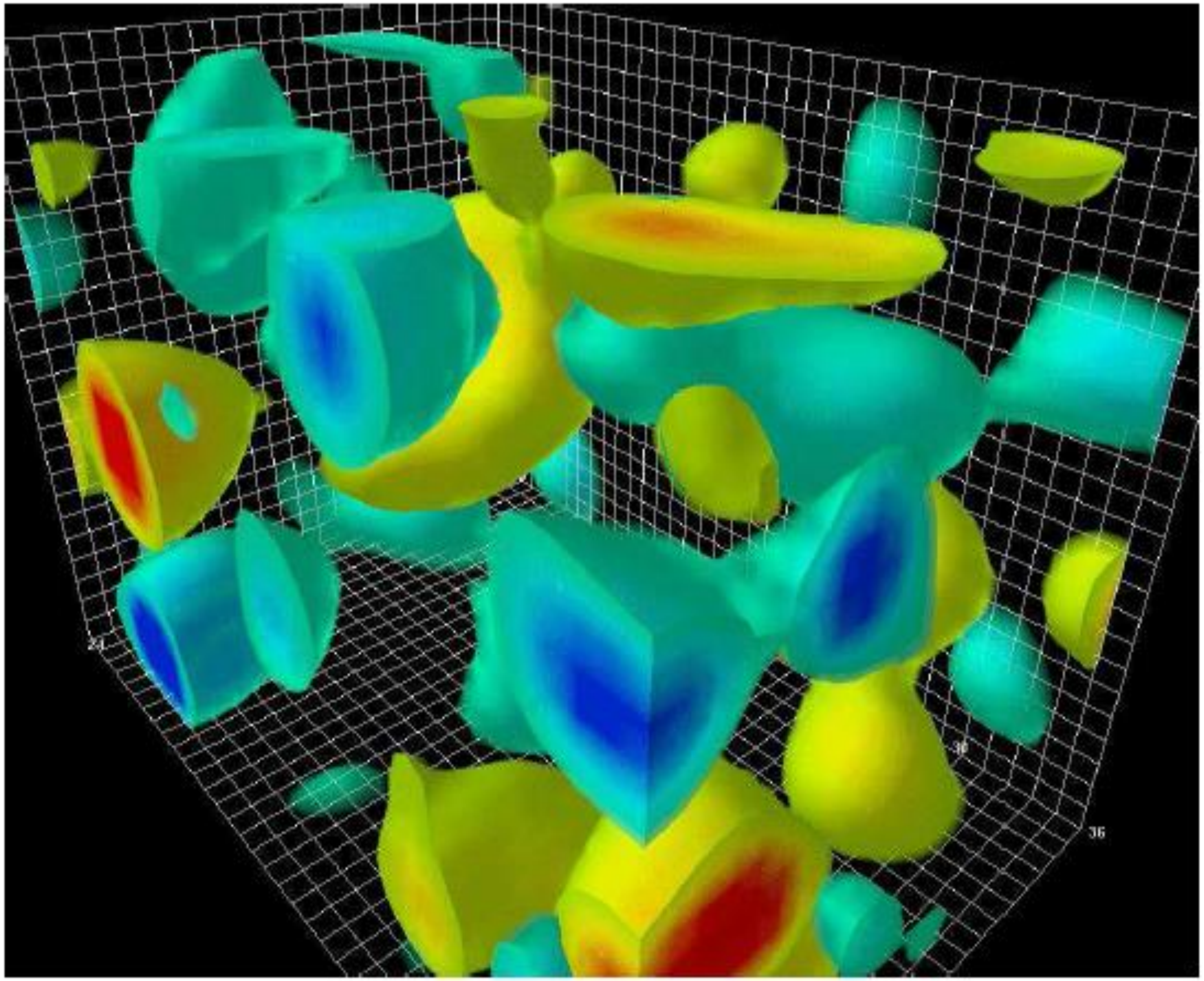
- For *central* collisions (and light quarks) they correspond to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$ located in "fluctons" of few-Fermi size. Isosinglet pseudoscalar condensate

  A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.

Topological charge

Lattice simulations of local topological fluctuations of gluon fields in QCD vacuum

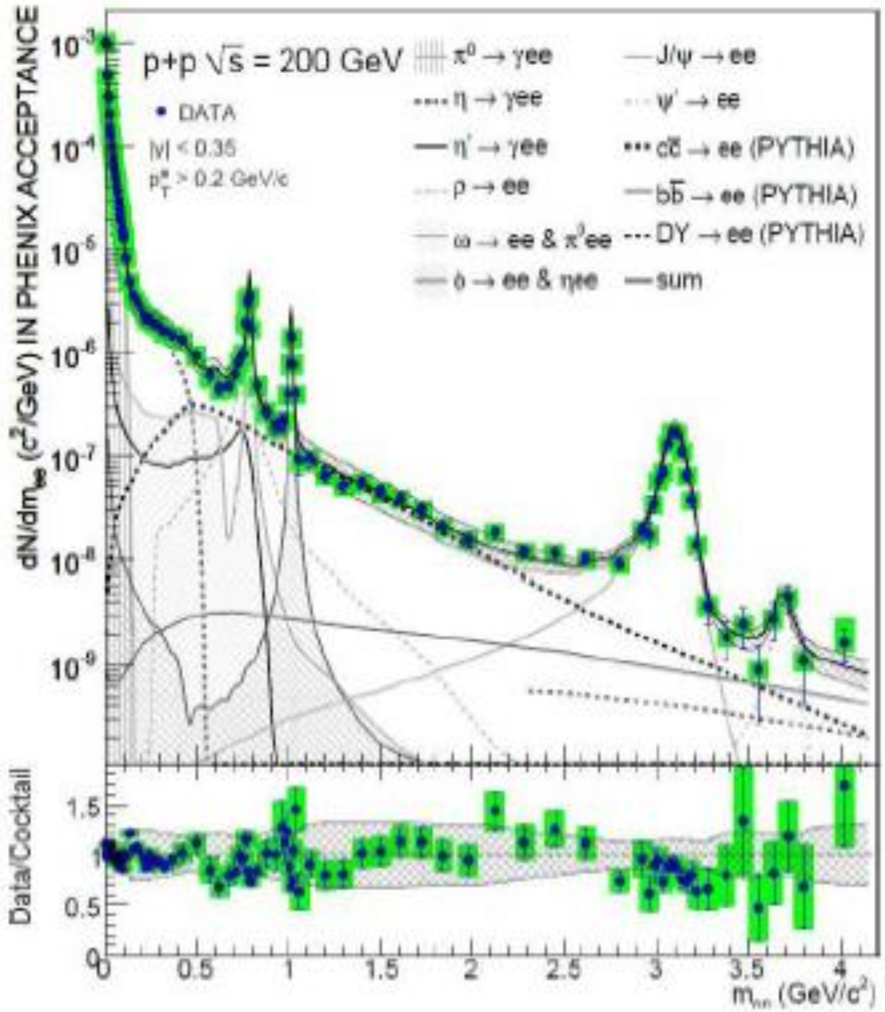
Axial baryon charge and axial chemical potential

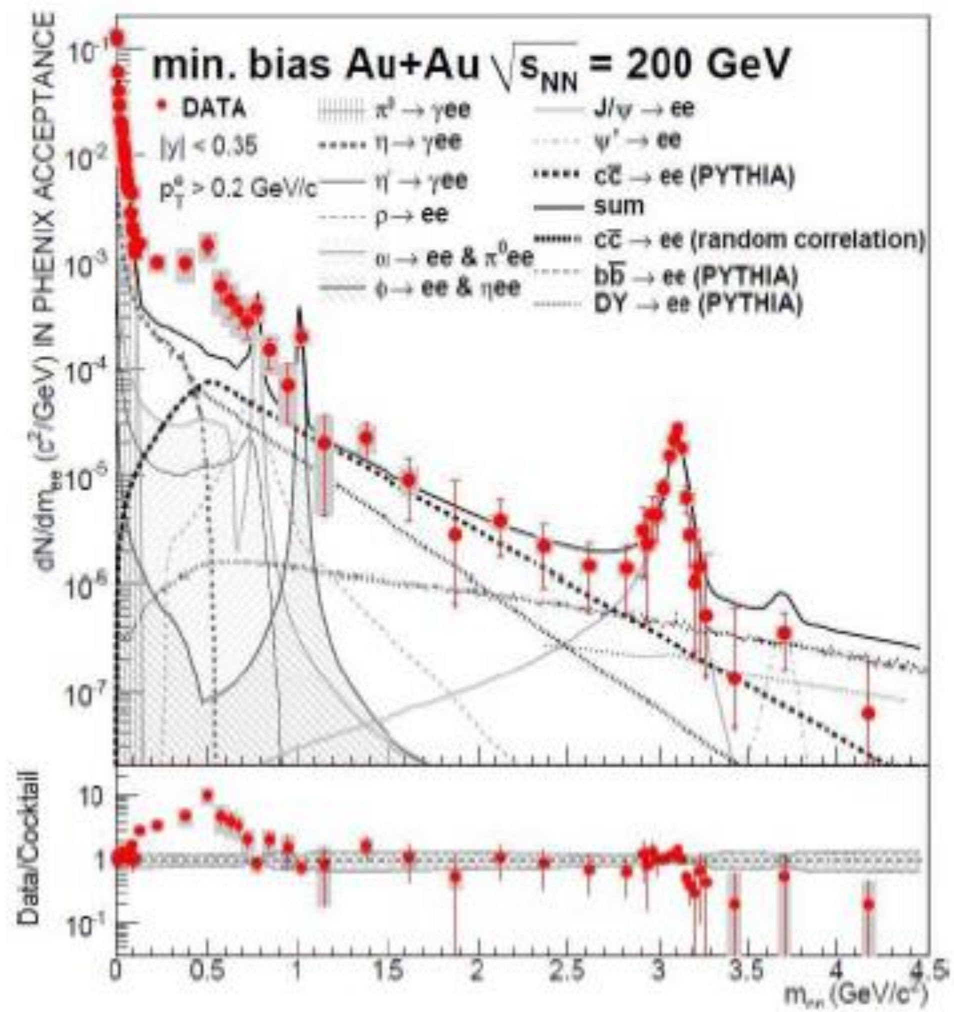


Lattice simulation of local fluctuations of the topological charge in the QCD vacuum [Leinweber].

anifestation of LPB in heavy ion collisions

ENIX/STAR anomaly - Abnormal e^+e^- excess in central HIC





omalous dilepton yield in Au+Au collisions in PHENIX as compared with

Topological charge and Chern-Pontryagin density

Topological charge T_{5} may arise in a fnite volume due to quantum fuctuations in a hot medium due to sphaleron transitions (Klinkhamer, Manton, Kuzmin, Rubakov, Shaposhnikov) as a consequence of a HIC, and survives for a lifetime in the fireball $\Delta t \simeq \tau_{\text{fireball}} \simeq 5\text{–}10 \text{ fm}$

$$T_5(t) = \frac{1}{4\pi^2} \int\limits_{\text{vol.}} d^3x \, K_0,$$

$$K_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} \operatorname{Tr}\left(G^\nu \partial^\rho G^\sigma - i\frac{2}{3} G^\nu G^\rho G^\sigma \right)$$

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{16\pi^2} \int\limits_0^{t_f} dt \int\limits_{\text{vol.}} d^3x \operatorname{Tr}\left(G^{\mu\nu}\widetilde{G}_{\mu\nu}\right)$$

$$= \frac{1}{4\pi^2} \int\limits_0^{t_f} dt \int\limits_{\text{vol.}} d^3x \, \partial^\mu K_\mu .$$

PCAC and Chiral Imbalance

For the fireball lifetime one can control the value of $\langle \Delta T_5 \rangle$ introducing into the QCD Lagrangian a topological chemical potential μ_θ in a gauge invariant way via $\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2} \int_0^{t_f} dt \int_{\text{vol.}} d^3x \, \text{Tr}\left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right).$$

The partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{8\pi^2} \text{Tr}\left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right)$$

predicts the induced axial charge (in the chiral limit $m_q \simeq 0$)

$$\frac{d}{dt}\left(Q_5^q - 2N_f T_5\right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \, \bar{q}\gamma_0\gamma_5 q = \langle N_L - N_R \rangle$$

to be conserved during $\tau_{fireball}$.

$$\Delta(N_L - N_R) = N_f Q$$

Chiral imbalance !

Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing winding number!

Quark-hadron continuity

Sizeable chiral imbalance

Initial Stage
After
HI collision
QGP

Hadronization:
Hadron vacuum with chiral imbalance
inherited from quark vacuum

< 1 fm/c

1 fm/c < 7-10 fm/c

Quark-hadron continuity during hadronization through crossover (Fukushima et al.)

In the hadron phase we shall assume that the environment in central HIC acquires a linear in time pseudoscalar background and search for possible manifestation of LPB in electromagnetic and lepton probes.

Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5\ \mathrm{MeV}^{-1} \sim 40\ \mathrm{fm} \gg \tau_{\mathrm{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150\ \mathrm{MeV}^{-1} \sim 1\ \mathrm{fm} \ll \tau_{\mathrm{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential μ_θ or by axial chemical potential μ_5

A.A., V,A.Andrianov, D.Espriu, X.Planells, Phys. Lett. B 710 (2012) **230**

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

In the Lorentz invariant form the field of fluctons is described by means of the classical pseudoscalar field a(t), so that

$$\Delta \mathcal{L}_a = \frac{N_f}{2\pi^2} K_\mu \partial^\mu a(x) = \frac{1}{4\pi^2} \mu_\theta K_0 \iff \mu_5 \bar{q} \gamma_0 \gamma_5 q; \quad \mu_5 \simeq \dot{a}(t) \simeq \text{constant}$$

Axial (chiral) chemical potential

- This background a(t), is associated with constant axial vector which zero component is identified to chiral (axial) chemical potential.

$$D_\nu \Longrightarrow \bar{D}_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \star\} = \mathbf{I}_q \partial_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu},$$

Scalar (and pseudoscalar) SU(2)- sector mesons

Linear Sigma Model model with chiral chemical potential

Linear Sigma Model for light pions and scalar mesons in the presence of chiral imbalance

current quark mass

$$L = \frac{1}{4} Tr\,(D_\mu H\,(D^\mu H)^\dagger) + \frac{b}{2} Tr\,[m(H + H^\dagger)] + \frac{M^2}{2} Tr\,(HH^\dagger)$$
$$- \frac{\lambda_1}{2} Tr\,[(HH^\dagger)^2] - \frac{\lambda_2}{4} [Tr\,(HH^\dagger)]^2 + \frac{c}{2} (\det H + \det H^\dagger)$$

$$\boxed{H = \xi\,\Sigma\,\xi}$$

pions

$$U = \xi\xi = \exp\left(i\,\frac{\vec{\pi}\vec{\tau}}{f_\pi}\right) \qquad \vec{\pi}\vec{\tau} = \begin{bmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} v + \sigma + a_0^0 & \sqrt{2}\,a^+ \\ \sqrt{2}\,a_0^- & v + \sigma - a_0^0 \end{bmatrix}$$

scalar mesons (isosingles + isotriplets)

$$\boxed{D_\mu H = \partial_\mu H - i\mathcal{L}_\mu H + iH\mathcal{R}_\mu}$$

$$\mathcal{R}_\mu = e\,Q_{em} A_\mu - \mu_5 \delta_{\mu,0} \cdot 1_{2\times 2}; \quad \mathcal{L}_\mu = e\,Q_{em} A_\mu + \mu_5 \delta_{\mu,0} \cdot 1_{2\times 2}$$

$$Q_{em} = \tfrac{1}{2}\tau_3 + \tfrac{1}{6} 1_{2\times 2}.$$

Chiral chemical potential
= time-component of axial field

Mass spectrum in vacuum

Take

$$\mu_5 = 0,\ M = 300\ \text{MeV},\ v = 92\ \text{MeV}$$

Obtain

$$\boxed{m_\pi = 139\ \text{MeV},\ m_a = 980\ \text{MeV},\ m_\sigma = 500\ \text{MeV},\ m = 5.5\ \text{MeV},}$$

for

$$\boxed{\lambda_1 = 16.4850,\ \lambda_2 = -13.1313,\ c = -4.46874 \times 10^4\ \text{MeV}^2,\ b = 1.61594 \times 10^5\ \text{MeV}^2}$$

Mass spectrum in chiral medium

σ meson,

$$\tfrac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma - \tfrac{1}{2}\,m_\sigma^2\sigma^2$$

Neutral meson sector,

$$\tfrac{1}{2}\,\partial_\mu a_0^0\,\partial^\mu a_0^0 + \tfrac{1}{2}\,\partial_\mu\pi^0\partial^\mu\pi^0 - \tfrac{1}{2}m_a^2\,(a_0^0)^2 - \tfrac{1}{2}m_\pi^2\,(\pi^0)^2 - 4\mu_5\,\dot{\pi}^0 a_0^0$$

Charged meson sector,

Parity breaking mixture

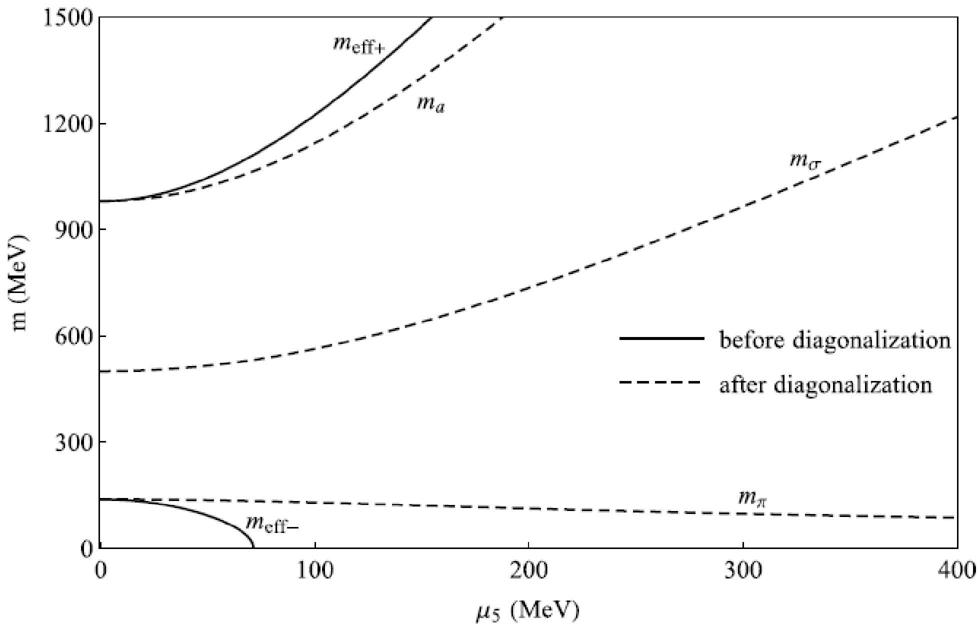
$$\partial_\mu a_0^-\,\partial^\mu a_0^+ + \partial_\mu\pi^-\partial^\mu\pi^+ - m_a^2 a_0^- a_0^+ - m_\pi^2\pi^-\pi^+ - 4\mu_5\,\dot{\pi}^+ a_0^- - 4\mu_5\,\dot{\pi}^- a_0^+$$

mass matrix
and
chiral condensate

$$\left\{\begin{aligned} m_\sigma^2 &= -2\left(M^2 - 6\,(\lambda_1+\lambda_2)v^2 + c + 2\mu_5^2\right) \\ m_a^2 &= -2\left(M^2 - 2\,(3\lambda_1+\lambda_2)v^2 - c + 2\mu_5^2\right) \\ m_\pi^2 &= \frac{2\,b\,m}{v} \\ v(\mu_5) &= \sqrt{\frac{M^2 + 2\mu_5^2 + c}{2(\lambda_1+\lambda_2)}} + \frac{b}{2(M^2+2\mu_5^2+c)}\,m \end{aligned}\right. ,$$

$$F_\pi^2(\mu_5) \approx \frac{M^2 + c}{2(\lambda_1+\lambda_2)} + \frac{\mu_5^2}{(\lambda_1+\lambda_2)}$$

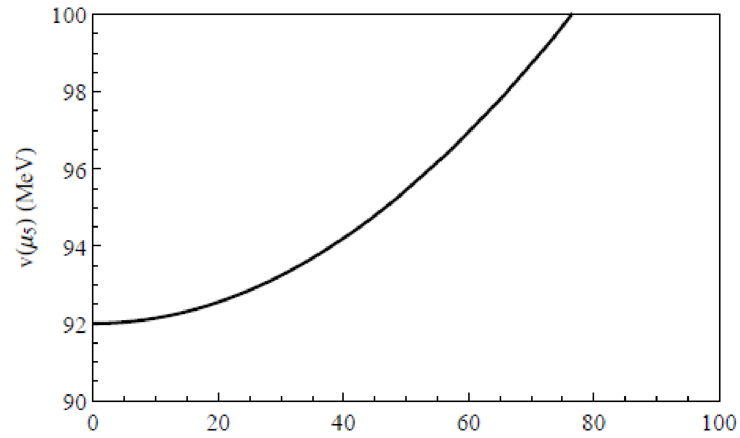
Mass spectrum in chiral imbalanced medium



$$??\pi^{\pm} \rightarrow \mu^{\pm}\bar{\nu}$$

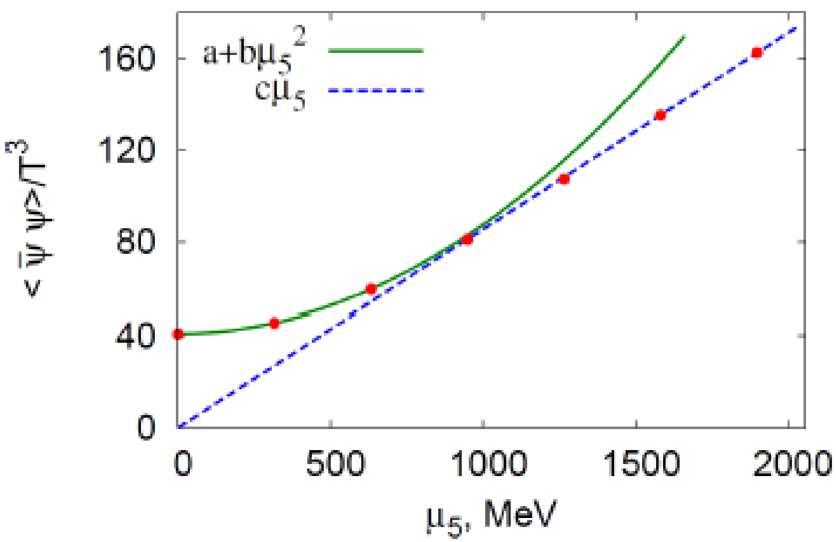
Quark condensate

T = 0



SU(2) effective Lagrangian
this talk

T = 158 MeV



NJL model
V.Braguta, A.Kotov.
PRD, 93,105025(2016)

SU(2) QCD
V.Braguta et al
JHEP 06 (2015) 094)

Mass spectrum for moving mesons

$$m^2_{eff-} = \frac{1}{2}\left(16\,\mu_5^2 + m_a^2 + m_\pi^2 - \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4\left(m_a^2\, m_\pi^2 - 16\mu_5^2\,|\vec{p}|^2\right)}\right)$$

$$m^2_{eff+} = \frac{1}{2}\left(16\,\mu_5^2 + m_a^2 + m_\pi^2 + \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4\left(m_a^2\, m_\pi^2 - 16\mu_5^2\,|\vec{p}|^2\right)}\right)$$

For small $\mu_5^2, m_\pi^2 \ll m_a^2 \simeq 1GeV^2$ $\quad m^2_{eff-} \simeq m_\pi^2 - 16\mu_5^2\frac{|\vec{p}|^2}{m_a^2}$

To increase predictability LPB in the proposed approach, we extend the vacuum Chiral Lagrangians with the phenomenological low-energy structural constants taking into account the chiral medium in the fireball.

General form of the Chiral Lagrangian GL

ANNALS OF PHYSICS **158**, 142–210 (1984) J.Gasser, H. Leutwyler

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{tr}\left[d_\mu U^\dagger d^\mu U + \chi^\dagger U + \chi U^\dagger\right] + C\mathrm{tr}\left[Q_R U Q_L U^\dagger\right]$$

$$d_\mu U = \partial_\mu U - i(v_\mu + Q_R A_\mu + a_\mu)U + iU(v_\mu + Q_L A_\mu - a_\mu)$$

External e.m.charges breaking isospin symmetry (R.Urech)

$$\begin{aligned}\mathcal{L}_{p^4} = {} & \frac{l_1}{4}\langle d^\mu U^+ d_\mu U\rangle^2 + \frac{l_2}{4}\langle d^\mu U^+ d^\nu U\rangle\langle d_\mu U^+ d_\nu U\rangle \\ & + \frac{l_3}{16}\langle \chi^+ U + U^+\chi\rangle^2 + \frac{l_4}{4}\langle d^\mu U^+ d_\mu \chi + d^\mu \chi^+ d_\mu U\rangle \\ & + l_5\langle G^{\mathrm{R}}_{\mu\nu} U G^{\mathrm{L}\,\mu\nu} U^+\rangle + \frac{il_6}{2}\langle G^{\mathrm{R}}_{\mu\nu} d^\mu U d^\nu U^+ + G^{\mathrm{L}}_{\mu\nu} d^\mu U^+ d^\nu U\rangle \\ & - \frac{l_7}{16}\langle \chi^+ U - U^+ \chi\rangle^2 + \tfrac{1}{4}(h_1 + h_3)\langle \chi^+ \chi\rangle \\ & + \tfrac{1}{2}(h_1 - h_3)\,\mathrm{Re}\,(\det\chi) - h_2\langle G^{\mathrm{R}}_{\mu\nu} G^{\mathrm{R}\,\mu\nu} + G^{\mathrm{L}}_{\mu\nu} G^{\mathrm{L}\,\mu\nu}\rangle .\end{aligned}$$

Chiral Lagrangian with chiral chemical potential

$$D_\nu \Longrightarrow \bar{D}_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \star\} = \mathbf{I}_q \partial_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu},$$

$$\mathcal{L}_2 = \frac{F_0^2}{4} < -j_\mu j^\mu + \chi^\dagger U + \chi U^\dagger >$$

where $< ... >$ denotes the trace in flavor space, $j_\mu \equiv U^\dagger \partial_\mu U$, the chiral field $U = \exp(i\hat{\pi}/F_0)$, $F_0 \simeq 92MeV$, $\chi(x) = 2B_0 s(x)$ and $M_\pi^2 = 2B_0 \hat{m}_{u,d}$, the tree-level neutral pion mass. The constant B_0 is related to the chiral quark condensate $< \bar{q}q >$ as $F_0^2 B_0 = \ < \bar{q}q >$. Taking now the covariant derivative in (1) it yields

$$\mathcal{L}_2(\mu_5) = \mathcal{L}_2(\mu_5 = 0) + \mu_5^2 N_f F_0^2$$

$$\mathcal{L}_4 = \bar{L}_3 < j_\mu j^\mu j_\nu j^\nu > + L_0 < j_\mu j_\nu j^\mu j^\nu > - L_5 < j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) >,$$

Extended Chiral lagrangian in the chiral medium

$$\begin{aligned}\mathcal{L}_4 &\to \frac{l_1}{4}\mathrm{tr}^2\left[\left(\partial^\mu - 2i\mu_5\delta^{\mu 0}\right)U^\dagger\left(\partial_\mu + 2i\mu_5\delta_{\mu 0}\right)U\right]\\ &+ \frac{l_2}{4}\mathrm{tr}\left[\left(\partial^\mu - 2i\mu_5\delta^{\mu 0}\right)U^\dagger\left(\partial^\nu + 2i\mu_5\delta^{\nu 0}\right)U\right]\mathrm{tr}\left[\left(\partial_\mu - 2i\mu_5\delta_{\mu 0}\right)U^\dagger\left(\partial_\nu + 2i\mu_5\delta_{\nu 0}\right)U\right]\\ &+ \frac{l_4}{8}\mathrm{tr}\left[\left(\partial^\mu - 2i\mu_5\delta^{\mu 0}\right)U^\dagger\left(\partial_\mu + 2i\mu_5\delta_{\mu 0}\right)U\right]\mathrm{tr}\left[\left(\chi^\dagger U + \chi U^\dagger\right]\end{aligned}$$

$$\mathcal{L}_4 \to \mathcal{L}_4 + \mu_5^2\left[4l_1\mathrm{tr}\left(\partial_\mu U^\dagger\partial^\mu U\right) + 4l_2\mathrm{tr}\left(\partial_0 U^\dagger\partial^0 U\right) + l_4\mathrm{tr}\left(\chi^\dagger U + \chi U^\dagger\right)\right] + 4\mu_5^4(l_1 + l_2)$$

Comparison of the ChPT and Sigma Model

In the rest frame of pion

$$F_\pi^2(\mu_5) \approx F_0^2 + 4\mu_5^2\left[4l_1 + 4l_2\right]$$

whereas in the sigma model with chiral chemical potential

$$F_\pi^2(\mu_5) \approx \frac{M^2 + c}{2(\lambda_1 + \lambda_2)} + \frac{\mu_5^2}{(\lambda_1 + \lambda_2)}$$

Numericaly they have the same sign but differ in magnitude

Chiral Lagrangian with chiral chemical potential

For $\mathrm{SU}(3)$ and $\mathrm{SU}(2)$ $< j_\mu >= 0$

$$< j_\mu j_\nu j^\mu j^\nu >= -2 < j_\mu j^\mu j_\nu j^\nu > + \frac{1}{2} < j_\mu j^\mu >< j_\nu j^\nu > + < j_\mu j_\nu >< j^\mu j^\nu >,$$

$\mathrm{SU}(2)$ $$2 < j_\mu j^\mu j_\nu j^\nu >=< j_\mu j^\mu >< j_\nu j^\nu >$$

$\mathrm{SU}(3)$ Gasser-Leutwyler (GL)

$$\begin{aligned} \mathcal{L}_4 \; = \; & L_1 < j_\mu j^\mu >< j_\nu j^\nu > + L_2 < j_\mu j_\nu >< j^\mu j^\nu > + L_3 < j_\mu j^\mu j_\nu j^\nu > \\ & - L_5 < j_\mu j^\mu (\chi^\dagger U + \chi U^\dagger) > \end{aligned}$$

$$L_1 = \frac{1}{2} L_0; \;\; L_2 = L_0; \;\; L_3 = \bar{L}_3 - 2L_0.$$

Chiral Lagrangian with chiral chemical potential (the response of the ChL on chiral medium)

For SU(2) dim=4

$$\mathcal{L}_4 = \frac{1}{4} l_1 < j_\mu j^\mu >< j_\nu j^\nu > + \frac{1}{4} l_2 < j_\mu j_\nu >< j^\mu j^\nu > - \frac{1}{4} l_4 < j_\mu j^\mu (\chi^\dagger U + U^\dagger \chi) >$$

$$l_1 = 2L_0 + 2\bar{L}_3,\ l_2 = 4L_2 = 4L_0,\ (l_1 + l_2) = 2\bar{L}_3 + 6L_0; \quad l_4 = 4L_5.$$

Chiral Lagrangian on chiral imbalance

$$\Delta\mathcal{L}_4(\mu_5) = -\mu_5^2\{12(l_1 + l_2) < j^0 j^0 > -4(l_1 + l_2) < j_k j_k > -l_4 < \chi^\dagger U + U^\dagger \chi >\}$$

Dispersion low (inverse propagator for pions)

$$\mathcal{D}^{-1}(\mu_5) = (F_0^2 + 48\mu_5^2(l_1 + l_2))p_0^2 - (F_0^2 + 16\mu_5^2(l_1 + l_2))|\vec{p}|^2 - (F_0^2 + 4l_4\mu_5^2)m_\pi^2(0) \to 0.$$

Chiral Lagrangian with chiral chemical potential

In the leading order of large N_c expansion (neglecting the RG logarithm contribution)

$$l_1^r = (-0.4 \pm 0.6) \times 10^{-3};\ l_2^r = (8.6 \pm 0.2) \times 10^{-3};$$
$$l_1^r + l_2^r = (8.2 \pm 0.8) \times 10^{-3};\ l_4^r = (2,64 \pm 0.01) \times 10^{-2}$$

RG scale $\mu \simeq M_\pi \simeq 140 MeV, \qquad \log\Big(m_\pi/\mu\Big) \simeq 0$

In the pion rest frame

$$F_\pi^2(\mu_5^2) \simeq F_0^2 + 48\mu_5^2(l_1 + l_2); \quad m_\pi^2(\mu_5^2) \simeq \Big(1 - 4\frac{\mu_5^2}{F_0^2}(12(l_1 + l_2) - l_4)\Big)m_\pi^2(0),$$

i.e. the pion decay constant is growing and its mass is decreasing in the chiral media.

$$< \bar{q}q > (\mu_5) =< \bar{q}q > (0)\Big(1 + 4l_4\frac{\mu_5^2}{F_0^2}\Big),$$ for $l_4 > 0$ quark condensate increases with growing the chiral chemical potential

Comparison in two approaches

The change of pion coupling constant vs. the ChPT

$$\frac{\Delta F_\pi^2}{\mu_5^2} = \frac{1}{\lambda_1 + \lambda_2} \approx 0.3 \quad vs \quad 48(l_1 + l_2),$$

i.e. $(l_1 + l_2) \approx 6.2 \times 10^{-3}$. It is a satisfactory correspondence to the pion phenomenology [14] .

Analogously, in the rest frame using the pion mass correction, $m_\pi^2(\mu_5) F_\pi^2(\mu_5) \simeq 2 m_q b F_\pi(\mu_5)$ it is easy to find the estimation for

$$l_4 = \frac{1}{8(\lambda_1 + \lambda_2)}) \approx 3.7 \times 10^{-2},$$

wherefrom one can also guess the relation $6(l_1 + l_2) = l_4$ following from the LSM. It is again a satisfactory correspondence to the pion phenomenology [14] .

The mass of a_0 - meson

$$m_{eff+}^2 = \frac{1}{2}\left(16\,\mu_5^2 + m_a^2 + m_\pi^2 + \sqrt{(16\mu_5^2 + m_a^2 + m_\pi^2)^2 - 4\left(m_a^2 m_\pi^2 - 16\mu_5^2\,|\vec{p}|^2\right)}\,\right)$$

For small $\mu_5^2, m_\pi^2 \ll m_a^2 \simeq 1GeV^2 \quad m_{eff-}^2 \simeq m_\pi^2 - 16\mu_5^2 \frac{|\vec{p}|^2}{m_a^2}$

Comparing one establishes the relationship of isotriplet scalar mass and GL constants

$$\mathcal{D}^{-1}(\mu_5) = (F_0^2 + 48\mu_5^2(l_1 + l_2))p_0^2 - (F_0^2 + 16\mu_5^2(l_1 + l_2))|\vec{p}|^2 - (F_0^2 + 4l_4\mu_5^2)m_\pi^2(0) \to 0.$$

$$m_a = \frac{F_0}{\sqrt{2(l_1 + l_2)}} \simeq 0.9GeV,$$

which is close to the Particle Data Group value within the experimental error bars [18].

Decay of charged pion in Chiral medium

(Possible experimental detection of Chiral Imbalance in the charged pion decays)

$$\pi^+ \to \mu^+ \nu$$

$$\mathcal{D}^{-1}(\mu_5) = (F_0^2 + 48\mu_5^2(l_1+l_2))p_0^2 - (F_0^2 + 16\mu_5^2(l_1+l_2))|\vec{p}|^2 - (F_0^2 + 4l_4\mu_5^2)m_\pi^2(0) \to 0.$$

The threshold for the decay of charged pion

$$\left(1 - 32(l_1 + l_2)\frac{\mu_5^2}{F_0^2}\right)|\vec{p}|^2 + \left(1 - 24(l_1 + l_2)\frac{\mu_5^2}{F_0^2}\right)m_{0,\pi}^2 \geq |\vec{p}|^2 + m_\mu^2,$$

The decay channel is closed for $|\vec{p}|^2 \simeq 0 \text{ if } \mu_5 \simeq 160 MeV$

It must be detected as a substantial decrease of muon flow originated from pion decays In fireball.!!!

Conclusions and outlook

- **Above two descriptions of the spectrum of light mesons in a medium provide a solid basis for the searching of a medium with a Chiral imbalance and, as a result, a phase with Local Parity Breaking in heavy-ion collisions.**

The characteristic features of the manifestation of a LPB can serve modificationsof the dispersion laws for scalar and vector mesons:

lightest "pseudoscalar" mesons tend to massless states in flight, vector meson polarizations split with different in-flight masses.

Signature of LPB in reactions of fireball

There exist observables unambiguously indicating LPB
(STAR, ALICE LHC?):
suppression of charged pion decays into leptons,
exotic scalar/pseudoscalar meson decays,
asymmetry in photon polarizations,
mass splitting of vector mesons and quarks/nucleons etc.
AAA+AVA+ Kovalenko…

Signature of LPB in reactions of fireball

LPB modifies dispersion laws for scalar and vector mesons: lightest “pseudoscalar” mesons tend to massless states in flight, vector meson polarizations split with different in-flight masses LPB changes the shape, position of light resonances and also kinematics of decays of light mesons.

The asymmetry of the longitudinal and transverse polarized states for the different values of the invariant mass light mesons.

Measurement of photon polarization asymmetry in the processes with light mesons.

Abnormal excess of dileptons pairs with different circular polarizations outside the resonance region of the invariant mass for \rho and \omega- mesons

(details in A. A. Andrianov, V. A. Andrianov, D. Espriu, and X. Planells, Phys. Rev. D, 90 (2014), 034024).

Thanks for your attention!

Results

- For light mesons in the chiral imbalance medium we compared the chiral perturbation theory (ChPT) and the linear sigma model (LSM) as realizations of low energy QCD. The relations between the low-energy constants of the chiral Lagrangian and the corresponding constants of the linear sigma model are established and expressions for the decay constant of the pion in the medium and the mass of the a_0 meson are found.

- The resulting dispersion law for pions in the medium allows us reveal the threshold of decay of a charged pion into a muon and neutrino which can be suppressed by increasing chiral chemical potential.

- Thus a possible experimental detection of chiral imbalance in medium (and therefore a phase with LPB) in the charged pion decays and vector meson polarizations inside of the fireball can be realized.

- We would like to mention the recent proposal to measure the photon polarization asymmetry in $\pi\gamma$ scattering [19, 16, 20] as a way to detect LPB due to chiral imbalance. This happens in the ChPT including electromagnetic fields due to the Wess–Zumino–Witten operators.

Linear Sigma Model

$$
\begin{aligned}
L &= N_c\Big\{\frac{1}{4} < (D_\mu H\,(D^\mu H)^\dagger > + \frac{B_0}{2} < m(H + H^\dagger > + \frac{M^2}{2} < HH^\dagger > \\
&- \frac{\lambda_1}{2} < (HH^\dagger)^2 > - \frac{\lambda_2}{4} < (HH^\dagger) >^2 + \frac{c}{2}\,(\det H + \det H^\dagger)\Big\},
\end{aligned}
$$

where $H = \xi\,\Sigma\,\xi$ is an operator for meson fields, N_c is a number of colours, m is an average mass of current u, d quarks, M is a "tachyonic" mass generating the spontaneous breaking of chiral symmetry, $B_0, c, \lambda_1, \lambda_2$ are real constants.

The matrix Σ includes the singlet scalar meson σ, its vacuum average v and the isotriplet of scalar mesons a_0^0, a_0^-, a_0^+, the details see in [16, 17]. The covariant derivative of H including the chiral chemical potential μ_5 is defined in (1). The operator ξ realizes a nonlinear representation (see (2)) of the chiral group $SU(2)_L \times SU(2)_R$, namely, $\xi^2 = U$.

$$\mathcal{R}_\mu = e\,Q_{em}A_\mu - \mu_5\delta_{\mu,0}\cdot 1_{2\times 2};\quad \mathcal{L}_\mu = e\,Q_{em}A_\mu + \mu_5\delta_{\mu,0}\cdot 1_{2\times 2}$$

$$D_\mu H = \partial_\mu H - i\mathcal{L}_\mu H + iH\mathcal{R}_\mu \qquad Q_{em} = \tfrac{1}{2}\tau_3 + \tfrac{1}{6}1_{2\times 2}.$$

Effective meson theory in a medium with LPB

- Vector mesons

Low energy QCD can be described with the help of Vector Meson Dominance

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2} g_\omega \omega_\mu \mathbb{I} + \frac{1}{2} g_\rho \rho^0_\mu \tau_3,$$

$$(V_{\mu,a}) \equiv \left(A_\mu,\, \omega_\mu,\, \rho^0_\mu\right)$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6$.
In this framework, the following term is generated in the effective lagrangian for vector mesons

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \operatorname{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{}\ \equiv$ isospin content) and $\zeta \propto \mu_5$.
Two different cases of isospin structure for μ_5: $\quad \zeta = N_c g^2 \mu_5 / 8\pi^2 \qquad \zeta \simeq 1.5\mu_5$

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Pion-like (isotriplet) background ($\mu \gg T$) [FAIR, NICA]

Massive MCS electrodynamics for vector mesons

$$\mathcal{L}_{MCS} = -\frac{1}{4}F^{\alpha\beta}(x)F_{\alpha\beta}(x) + \frac{1}{2}m^2 A_\nu(x)A^\nu(x) + \frac{1}{2}\zeta_\mu A_\nu(x)\tilde{F}^{\mu\nu}(x) + \mathrm{g.f.}$$

In momentum space wave Eqs.

$$\begin{cases} \left[g^{\lambda\nu}\left(k^2 - m^2\right) - k^\lambda k^\nu + i\,\varepsilon^{\lambda\nu\alpha\beta}\,\zeta_\alpha\,k_\beta\right]\mathbf{a}_\lambda(k) = 0 \\ k^\lambda\,\mathbf{a}_\lambda(k) = 0 \end{cases}$$

Energy spectrum:

Transversal polarizations

$$\begin{aligned} K_\nu^\mu\,\varepsilon_\pm^\nu(k) &= \left(k^2 - m^2 \pm \sqrt{\mathrm{D}}\right)\varepsilon_\pm^\mu(k); \\ \omega_{\mathbf{k},\pm} &= \sqrt{\mathbf{k}^2 + m^2 \pm \zeta_0|\,\mathbf{k}\,|}; \quad \zeta_\mu = (\zeta_0, 0, 0, 0) \end{aligned}$$

Longitudinal polarization

$$\omega_{\mathbf{k},L} = \sqrt{\mathbf{k}^2 + m^2}$$

Vector Meson spectrum in PB medium

After diagonalization of mass matrix

$$m_{V,\epsilon}^2 = m_V^2 - \epsilon\zeta|\vec{k}| \implies |\zeta|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.
The photon itself happens to be unaffected by a **singlet** $\hat{\zeta}$.
The position of the poles for $\pm$ polarized mesons is changing with wave vector $|\vec{k}|$.
Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$.

This splitting unambiguously signifies LPB. Can it be measured?

$\rightarrow$ dilepton production in HIC from the decays $\rho, \omega \rightarrow e^+e^-$

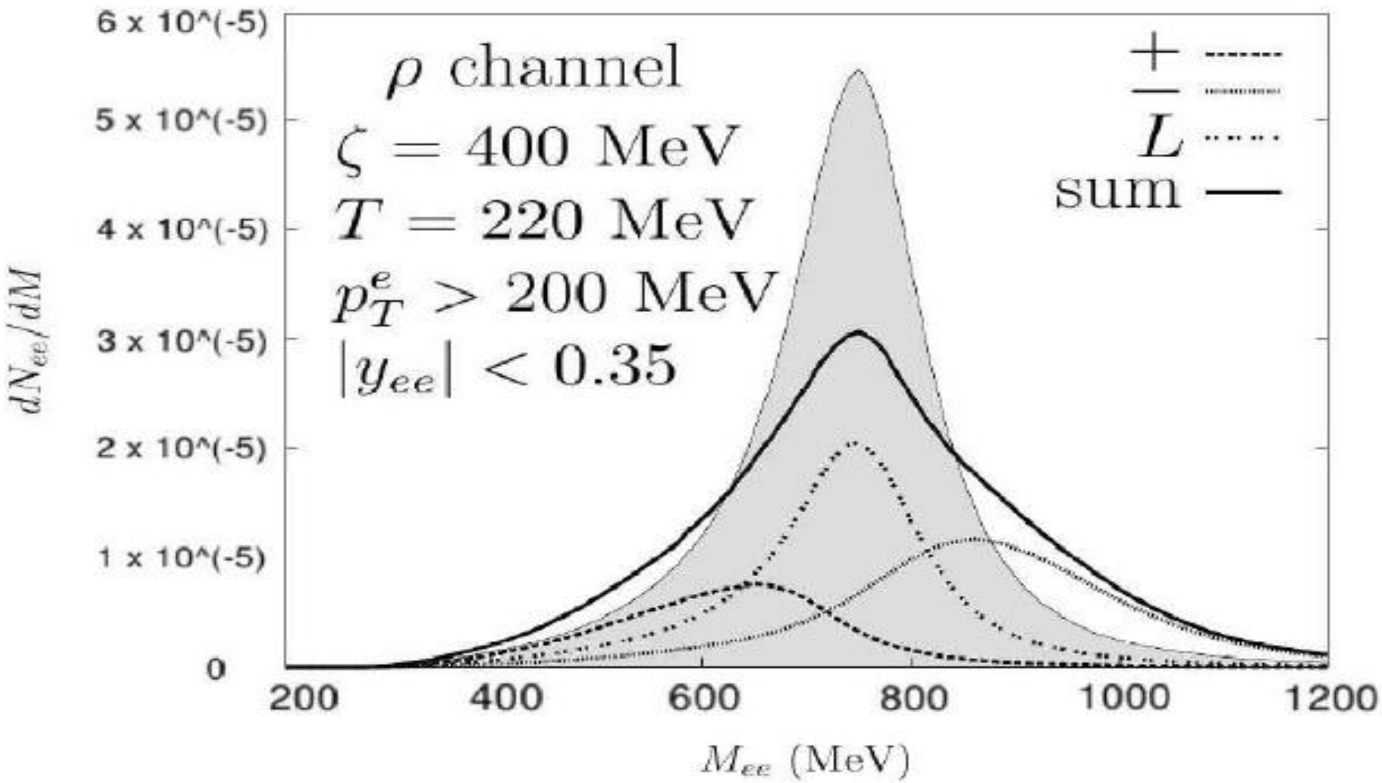
Thus the question arises:

can these effects be registered somehow in experiments at heavy ion collisions and thereby can one assert that the existence of the local space parity breaking phase is possible?

The graphic results of the calculations for the abnormal excess of the dilepton pairs in the vicinity of the resonance of the polarized vector \rho meson

Manifestation of LPB in heavy ion collisions

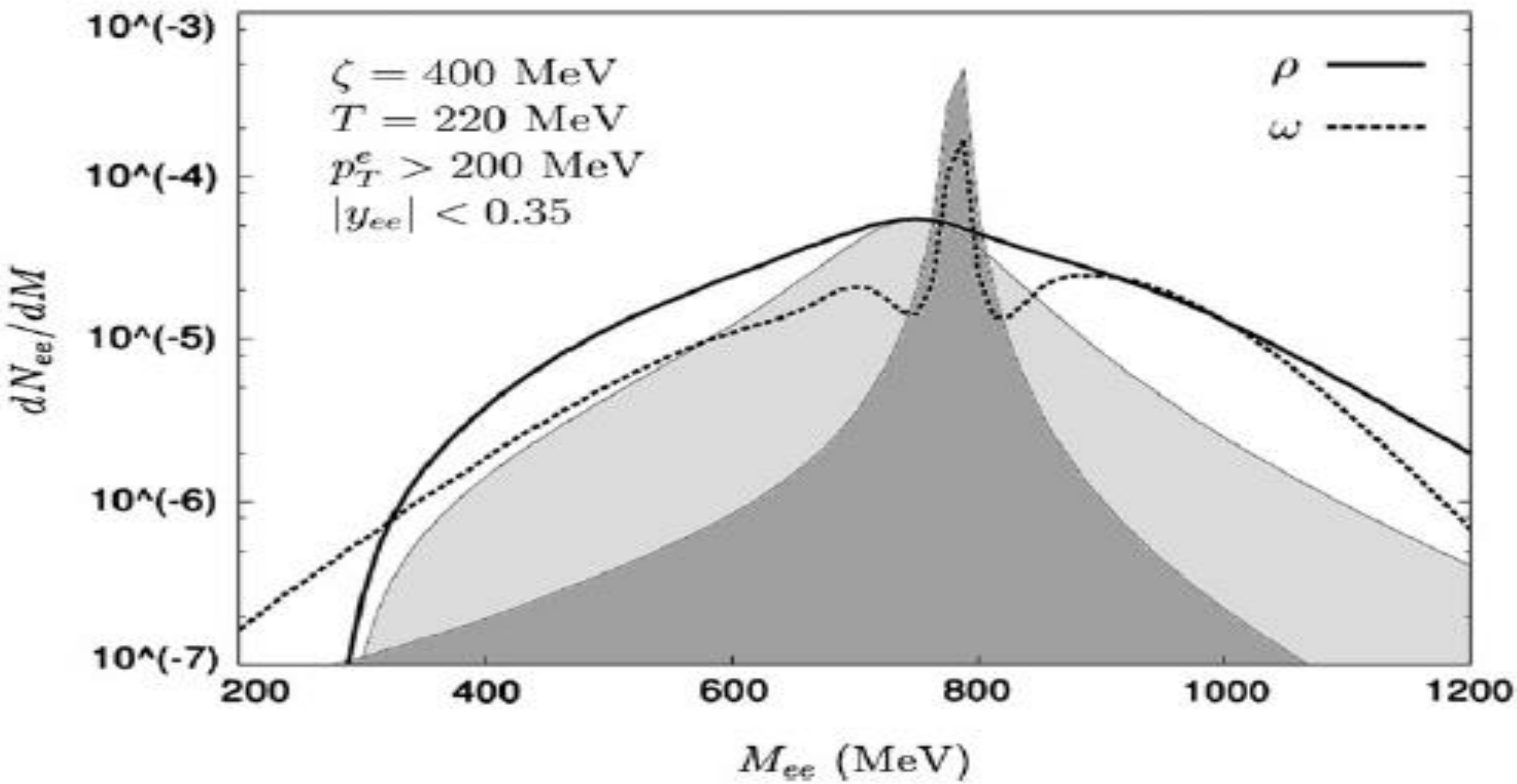
ρ spectral function



Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

POLARIZATION ASYMMETRY!!

The graphic results of the calculations for the abnormal excess of the dilepton pairs in the vicinity of the resonance of the polarized vector \rho and \omega meson



The in-medium contribution in the ρ and ω channels (solid and dashed line, respectively) is presented for $\zeta = 400$ MeV together with their vacuum contributions (light and dark shaded regions, respectively). The in-medium ρ yield is enhanced by a factor 1.8 (see text). The vertical units are taken to coincide with PHENIX experimental data [9], as well as experimental detector cuts and temperature.

Exotic decays of mesons,

asymmetry in the polarizability of pions

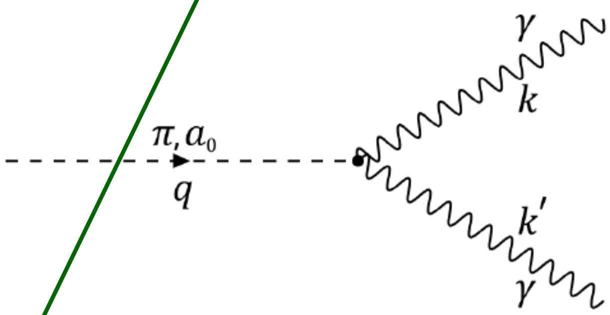
as an indication of LPB

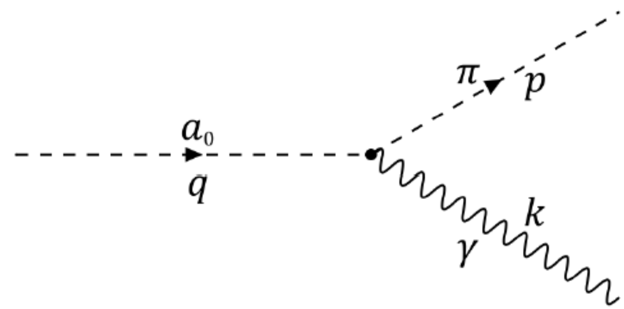
Wess-Zumino-Witten action

Describing of anomalous decay of strong interaction $\pi \to \gamma\gamma$
and other interaction: $\gamma\pi^- \to \pi^0\pi^-$ and $\gamma \to \pi\pi\pi$

$$-\frac{e^2 N_c}{24\pi^2 f_\pi}\epsilon^{\nu\sigma\lambda\rho}\partial_\sigma A_\lambda \partial_\nu A_\rho \pi^0 \quad (1)$$

$$-\frac{ie\mu_5 N_c}{6\pi^2 f_\pi^2}\epsilon_0^{\sigma\lambda\rho}A_\rho \partial_\sigma \pi^+ \partial_\lambda \pi^- \quad (2)$$





$\pi^0 \to \gamma\gamma, \quad a_0^0 \to \gamma\gamma$

$a_0^\pm \to \pi^\pm\gamma$

M. Kawaguchi, M. Harada, S. Matsuzaki, R. Ouyang , PHYS. REV. C 95, 065204 (2017)

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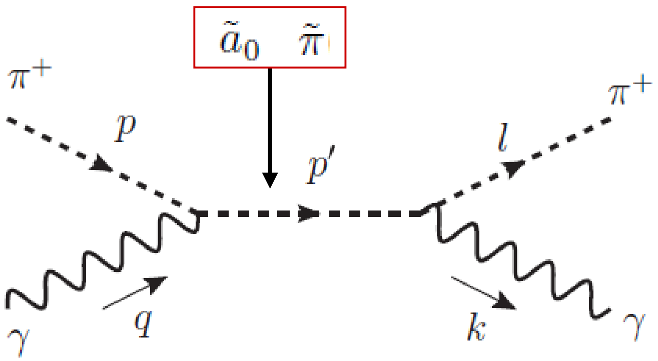
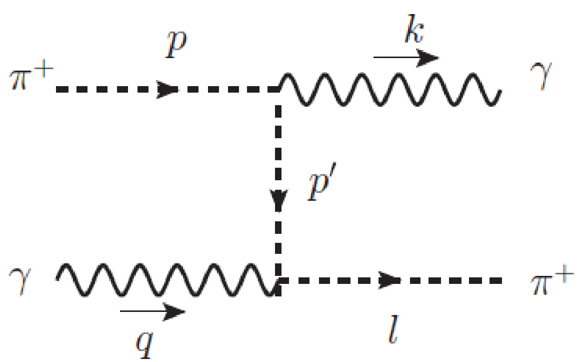




processes are parity conjugate:

$$\pi^{\pm}(\vec{p}) + \gamma(\vec{q}) \to \pi^{\pm}(\vec{l}) + \gamma_{+}(\vec{k}),$$
$$\pi^{\pm}(-\vec{p}) + \gamma(-\vec{q}) \to \pi^{\pm}(-\vec{l}) + \gamma_{-}(-\vec{k}),$$

where $\pm$ attached on photons in the final state denote photon helicities.

asymmetry ($\mathcal{A}$) can be evaluated as

$$\mathcal{A} = \left| \frac{\mathcal{N}_{+} - \mathcal{P}[\mathcal{N}_{+}]}{\sum_{\lambda}\{\mathcal{N}_{\lambda} + \mathcal{P}[\mathcal{N}_{\lambda}]\}} \right|,$$

where $\mathcal{N}_{\lambda}$ stands for the number of events per the phase space, $d\,E_{\gamma}\,d\cos\theta\,d\phi$, for the parity conjugate processes with the helicity λ and the photon energy E_{γ} in the final state. The symbol $\mathcal{P}$ acts as the parity conjugation projection. The denominator represents the total number of the $\pi^{\pm}\gamma$ emission events with unpolarized photons per the phase space.

Our prediction:
Scalar resonance enhancement!

$$\mathcal{A}^{s\text{-channel}}\big|_{\max} = \frac{\mu_5 E_{\pi} N_c}{6\pi^2 f_{\pi}^2} \simeq 0.2 \times \left(\frac{\mu_5}{200\ \mathrm{MeV}}\right)\left(\frac{E_{\pi}}{1\ \mathrm{GeV}}\right)$$

Asymmetry in photon polarizations

$\pi^+\gamma \to a_0^{+*} \to \pi^+\gamma$

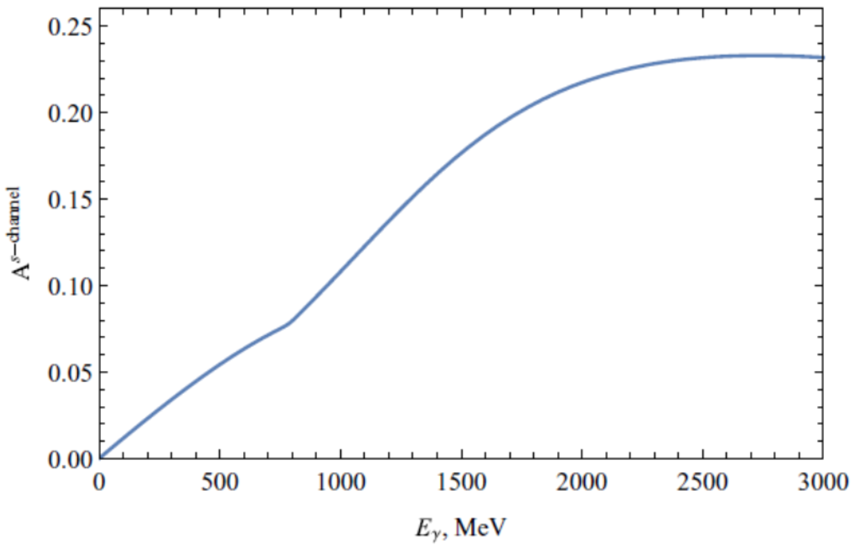


Figure 7: Asymmetry, $\mu_5 = 200$ MeV, $E_{\pi 2} = 1$ GeV

Resonance enhancement at energies comparable with the mass of a_0 scalars !

The lagrangian of the vector meson dominance model in the matter

After the bosonization of the QCD quark sector

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu} + \phi_{\mu\nu}\phi^{\mu\nu}\right) + \frac{1}{2}V_{\mu,a}m^2_{ab}V^{\mu}_{b}$$

$$m^2_{ab} = m^2_V \begin{pmatrix} \frac{4e^2}{3g^2} & -\frac{e}{3g} & -\frac{e}{g} & \frac{\sqrt{2}eg_\phi}{3g^2} \\ -\frac{e}{3g} & 1 & 0 & 0 \\ -\frac{e}{g} & 0 & 1 & 0 \\ \frac{\sqrt{2}eg_\phi}{3g^2} & 0 & 0 & \frac{g_\phi^2}{g^2} \end{pmatrix}, \quad \det\left(m^2\right) = 0,$$

$$\left(V_{\mu,a}\right) \equiv \left(A_\mu, \omega_\mu, \rho^0_\mu \equiv \rho_\mu, \phi_\mu\right) \qquad m_V^2 = m_\rho^2 = 2g_\rho^2 f_\pi^2 \simeq m_\omega^2$$

The quark-meson interaction is described by

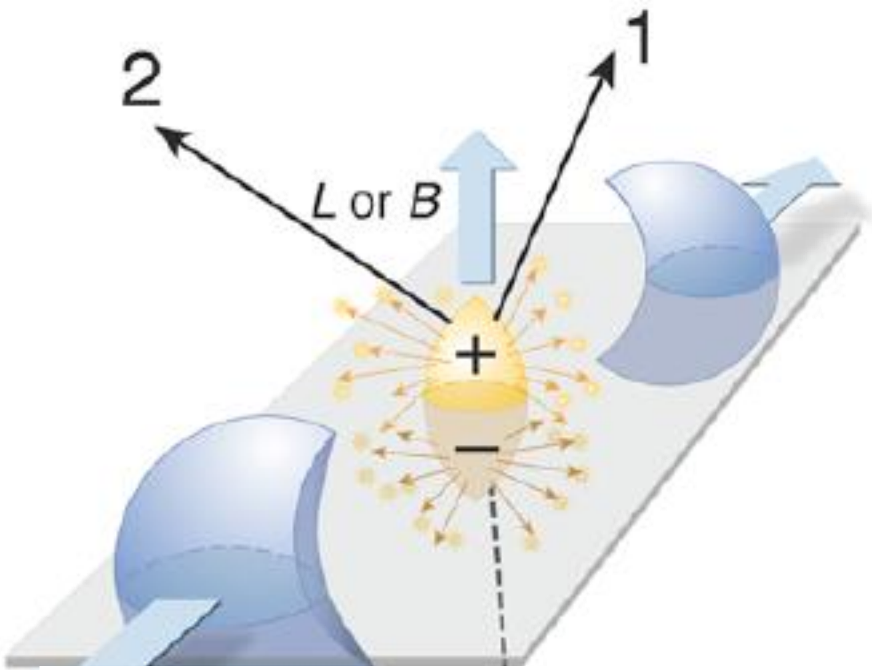
$$\mathcal{L}_{\rm int} = \bar{q}\gamma_\mu V^\mu q; \quad V_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega\omega_\mu \mathrm{I}_q + \frac{1}{2}g_\rho\rho_\mu\lambda_3 + \frac{1}{\sqrt{2}}g_\phi\phi_\mu \mathrm{I}_s \quad Q = \frac{\lambda_3}{2} + \frac{1}{6}\mathrm{I}_q - \frac{1}{3}\mathrm{I}_s,\ g_\omega \simeq g_\rho \equiv g \simeq 6 < g_\phi \simeq 7.8$$

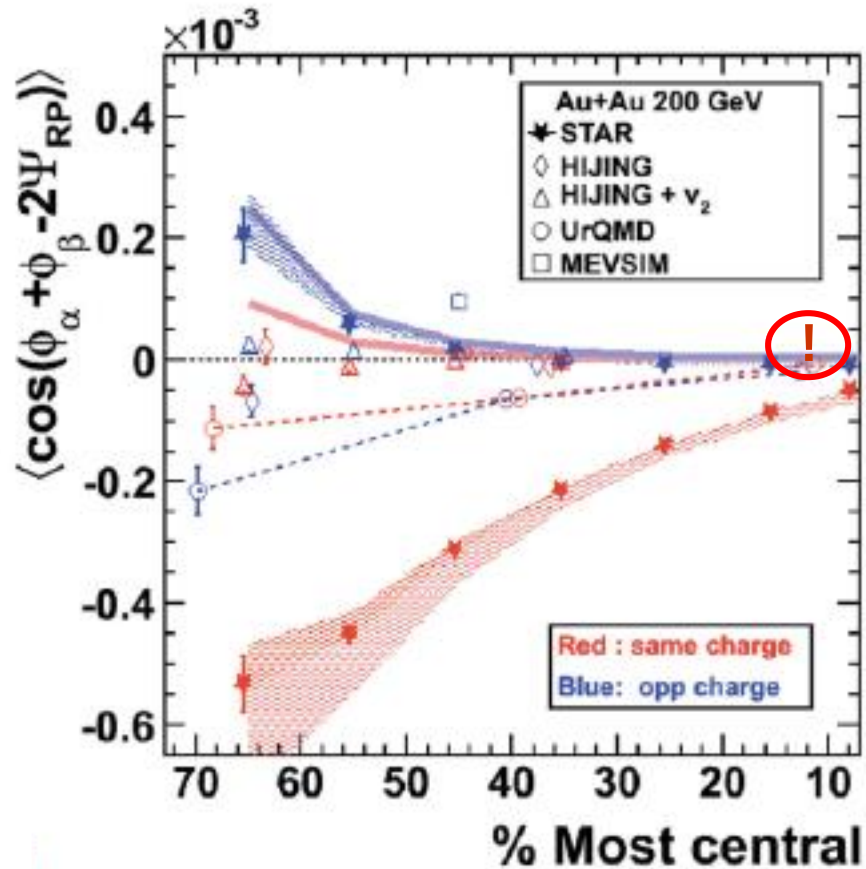
* Chiral Magnetic Effect (CME)
[Vilenkin, 1980; Kharzeev 2004;

$$\boxed{\vec{J} = \sigma_5 \mu_5 \vec{B}}$$

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

(STAR Collaboration)





CME disappears in central collisions
but chiral imbalance NOT!

One loop renlormalization of chiral lagrangian of G-L

$$l_i = l_i^r + \gamma_i \lambda, \qquad i = 1, \ldots, 7$$

$$h_i = h_i^r + \delta_i \lambda, \qquad i = 1, 2, 3$$

$$\lambda = (4\pi)^{-2} \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2} (\ln 4\pi + \Gamma'(1) + 1) \right\}$$

$$\gamma_1 = \tfrac{1}{3}, \qquad \gamma_2 = \tfrac{2}{3}, \qquad \gamma_3 = -\tfrac{1}{2}, \qquad \gamma_4 = 2, \qquad \gamma_5 = -\tfrac{1}{6}, \qquad \gamma_6 = -\tfrac{1}{3}, \qquad \gamma_7 = 0$$

$$\delta_1 = 2, \qquad \delta_2 = \tfrac{1}{12}, \qquad \delta_3 = 0.$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \ln \frac{M_\pi^2}{\mu^2}.$$

For the subtraction scale $\mu = 0.77$ GeV,

$$-\ln \frac{M_\pi^2}{\mu^2} = 3.42$$

Fits of constants

G. Colangelo, J. Gasser, H. Leutwyler Nuclear Physics B 603 (2001) 125–179

$$\bar{\ell}_1 = -0.4 \pm 0.6, \qquad \bar{\ell}_2 = 4.3 \pm 0.1. \qquad \bar{\ell}_4 = 4.4 \pm 0.2.$$

Aoki S. et al FLAG working group arXiv:1607.00299 [hep-lat]
give similar results for $\bar{l}_4$

From resonance saturation G.~Ecker, J.~Gasser, A.~Pich and E.~de Rafael, Nucl.\ Phys.\ B {\bf 321}, 311 (1989).

$$\bar{\ell}_1 \simeq -0.7, \qquad \bar{\ell}_2 \simeq 5.0, \qquad \bar{\ell}_3 \simeq 1.9, \qquad \bar{\ell}_4 \simeq 3.7,$$

ANNALS OF PHYSICS **158**, 142–210 (1984) J.Gasser, H. Leutwyler

One loop renlormalization of chiral lagrangian

$$\begin{aligned}\mathcal{L}_2 = {} & l_1(\nabla^\mu U^T \nabla_\mu U)^2 + l_2(\nabla^\mu U^T \nabla^\nu U)(\nabla_\mu U^T \nabla_\nu U) \\ & + l_3(\chi^T U)^2 + l_4(\nabla^\mu \chi^T \nabla_\mu U) + l_5(U^T F^{\mu\nu} F_{\mu\nu} U) \\ & + l_6(\nabla^\mu U^T F_{\mu\nu} \nabla^\nu U) + l_7(\tilde{\chi}^T U)^2 + h_1 \chi^T \chi + h_2 \operatorname{tr} F_{\mu\nu} F^{\mu\nu} \\ & + h_3 \tilde{\chi}^T \tilde{\chi}\end{aligned}$$

$$l_i = l_i^r + \gamma_i \lambda, \qquad i = 1, \ldots, 7$$

$$h_i = h_i^r + \delta_i \lambda, \qquad i = 1, 2, 3$$

$$\lambda = (4\pi)^{-2} \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2}(\ln 4\pi + \Gamma'(1) + 1) \right\}$$

$$\gamma_1 = \tfrac{1}{3}, \qquad \gamma_2 = \tfrac{2}{3}, \qquad \gamma_3 = -\tfrac{1}{2}, \qquad \gamma_4 = 2, \qquad \gamma_5 = -\tfrac{1}{6}, \qquad \gamma_6 = -\tfrac{1}{3}, \qquad \gamma_7 = 0$$

$$\delta_1 = 2, \qquad \delta_2 = \tfrac{1}{12}, \qquad \delta_3 = 0. \tag{9.6}$$

$$\bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \ln \frac{M_\pi^2}{\mu^2}.$$

For the subtraction scale $\mu = 0.77$ GeV,

$$-\ln \frac{M_\pi^2}{\mu^2} = 3.42$$

Signatures and proposals to detect of LPB in HIC

Based on the generalized lagrangian for vector mesons with Chern-Simons interaction in medium the phenomenology of LPB in fireball is described.

In particular:

- **Analysis spectra of constituents (\rho, \omega – mesons) showed that, the spectrum of the massless photons is not distorted when they are mixed with the massive of the vector mesons.**

  Meanwhile the spectrum of the massive vector mesons is split to three components with different polarizations and effective masses.

- **There is the broadening of resonances that leads to increasing of spectral contribution to the dilepton production as compared to the situation when the resonances are in the vacuum states.**

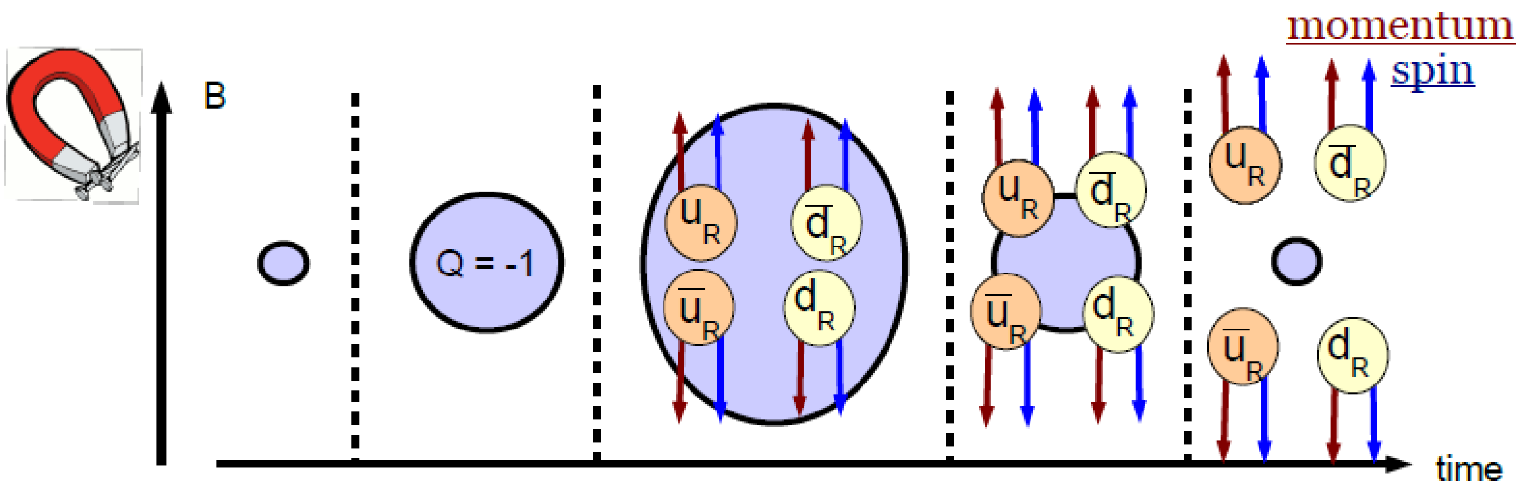
- **Thus the search of a signal (a phase) with space parity breaking in heavy ion collisions (in fireball) can be performed in experiments on abnormal excess of dileptons pairs with different circular polarizations outside the resonance region of the invariant mass \rho and \omega- mesons.**

- **The characteristic indicating on the possibility of the existence of LPB in this experiments may serve as the asymmetry of the longitudinal and transverse polarized states for the different values of the invariant mass.**

The proposed mechanism of generation of the LPB helps us qualitatively and quantitatively to explain the abnormal excess of the dilepton pairs in the experiments of the collaborations CERES, PHENIX, STAR, NA60, LHC, ALICE

Chiral magnetic effect

Topological Charge + Magnetic field = Chirality + Polarization =



Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

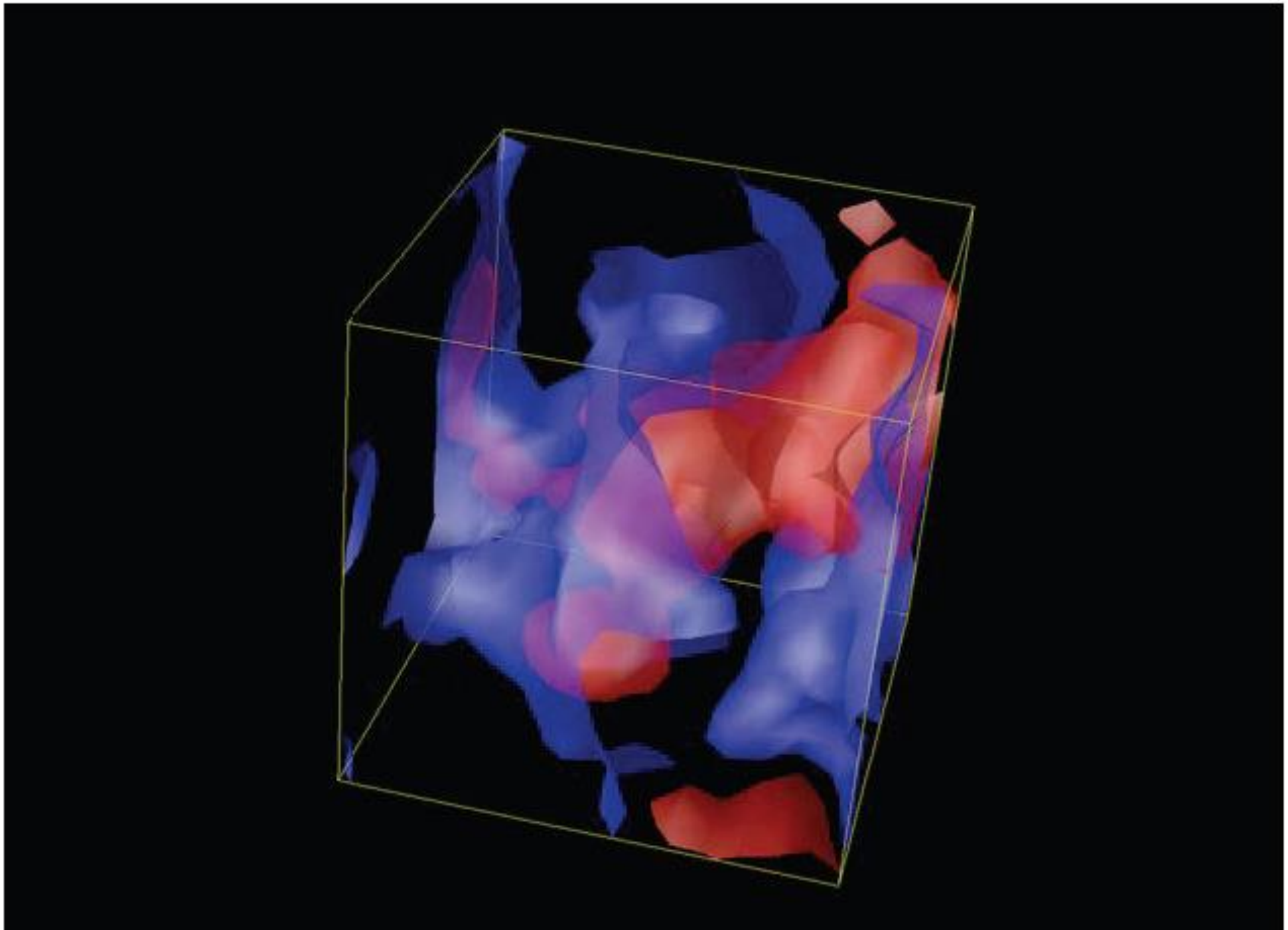
… = Electromagnetic Current

P- and CP-odd effect --> Chiral Magnetic Effect:

D.Kharzeev, L.McLerran, K.Fukushima, H.Warringa,…

Topological number fluctuations in QCD vacuum

ITEP Lattice Group



P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov

Eur. Phys. J. C 17, 623–649 (2000)

Large N_c in chiral perturbation theory*

R. Kaiser[a], H. Leutwyler[b]

The covariant derivatives and the field strength te
are defined by

$$
\begin{aligned}
D_\mu U &= \partial_\mu U - \mathrm{i}(v_\mu + a_\mu)U + \mathrm{i}U(v_\mu - a_\mu),\\
D_\mu \psi &= \partial_\mu \psi - 2\langle a_\mu\rangle, \quad D_\mu\theta = \partial_\mu\theta + 2\langle a_\mu\rangle,\\
R_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - \mathrm{i}[r_\mu, r_\nu],\\
L_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - \mathrm{i}[l_\mu, l_\nu],
\end{aligned}
$$

where $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$. The somewhat

$$
U = \mathrm{e}^{(\mathrm{i}/3)\bar\psi}\bar U, \qquad \bar\psi = \psi + \theta, \qquad \det\bar U = \mathrm{e}^{-\mathrm{i}\theta}.
$$

convenient to define the covariant derivative of $\bar U$

$$
\begin{aligned}
D_\mu \bar U &= \partial_\mu \bar U - \mathrm{i}(v_\mu + \bar a_\mu)\bar U + \mathrm{i}\bar U(v_\mu - \bar a_\mu),\\
\bar a_\mu &= a_\mu - \frac{1}{3}\langle a_\mu\rangle - \frac{1}{6}\partial_\mu\theta = a_\mu - \frac{1}{6}D_\mu\theta,
\end{aligned}
$$

at $\langle \bar U^\dagger D_\mu \bar U\rangle = 0$. In this notation, the deriv
ıds

$$
D_\mu U = \mathrm{e}^{(\mathrm{i}/3)\bar\psi}\left\{D_\mu\bar U + \frac{\mathrm{i}}{3}(\partial_\mu\bar\psi - D_\mu\theta)\bar U\right\},
$$

$$
\begin{aligned}
\mathcal{L}_B^{\mathrm{SU}_3} = & -\mathrm{i}L_{11}^{\mathrm{SU}_3} D_\mu\theta\langle \bar{U}^\dagger D^\mu \bar{U}\, D_\nu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{12}^{\mathrm{SU}_3} D_\mu\theta D^\mu\theta \langle D_\nu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{13}^{\mathrm{SU}_3} D_\mu\theta D_\nu\theta \langle D^\mu \bar{U}^\dagger D^\nu \bar{U}\rangle \\
& + L_{14}^{\mathrm{SU}_3} D_\mu\theta D^\mu\theta \langle \bar{U}^\dagger\chi + \chi^\dagger \bar{U}\rangle \\
& - \mathrm{i}L_{15}^{\mathrm{SU}_3} D_\mu\theta \langle D^\mu \bar{U}^\dagger \chi - D^\mu \bar{U}\chi^\dagger\rangle \\
& + \mathrm{i}L_{16}^{\mathrm{SU}_3} \partial_\mu D^\mu\theta \langle \bar{U}^\dagger\chi - \chi^\dagger \bar{U}\rangle \qquad (\\
& + \mathrm{i}L_{17}^{\mathrm{SU}_3} \epsilon^{\mu\nu\rho\sigma} D_\mu\theta \langle \bar{R}_{\nu\rho} D_\sigma \bar{U}\bar{U}^\dagger - \bar{L}_{\nu\rho}\bar{U}^\dagger D_\sigma \bar{U}\rangle
\end{aligned}
$$

$$
\begin{aligned}
L_{11}^{\mathrm{SU}_3} &= -4\left(L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{12}^{\mathrm{SU}_3} &= \frac{2}{3}\left(L_1 + \frac{1}{2}L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{13}^{\mathrm{SU}_3} &= \frac{4}{3}\left(L_2 + \frac{1}{3}L_3\right) + O(1), \\
L_{14}^{\mathrm{SU}_3} &= \frac{1}{3}(L_4 + 3L_5 + L_{18}) + O(1), \\
L_{15}^{\mathrm{SU}_3} &= \frac{1}{3}(2L_5 + 3L_{18}) + O(1), \\
L_{16}^{\mathrm{SU}_3} &= -F^4(1+\Lambda_1)(1+\Lambda_2)(72\tau)^{-1} + O(1), \\
L_{17}^{\mathrm{SU}_3} &= N_c(288\pi^2)^{-1} + \frac{1}{2}\tilde{L}_4 + O(N_c^{-1}),
\end{aligned}
$$

ant part $\bar{r}, \bar{l}$ and a remainder reads $r = \bar{r} + (1/6)D\theta$, $l = \bar{l} - (1/6)D\theta$. Using the identity $\langle \mathrm{d}\bar{U}\bar{U}^\dagger \rangle = -\mathrm{i}\mathrm{d}\theta$, which follows from $\det \bar{U} = \mathrm{e}^{-\mathrm{i}\theta}$, we then obtain

$$S_{\mathrm{WZW}}\{U, v, a\} = S_{\mathrm{WZW}}\{\bar{U}, v, \bar{a}\} + \int (A + B + P_1 + P_2),$$

$$A = -\frac{N_c}{144\pi^2}\bar{\psi}\{\langle \mathrm{i}\bar{F}_{\mathrm{R}} D\bar{U} D\bar{U}^\dagger + \bar{F}_{\mathrm{R}}\bar{U}\bar{F}_{\mathrm{L}}\bar{U}^\dagger + 2\bar{F}_{\mathrm{R}}^2 + (\mathrm{R} \leftrightarrow \mathrm{L})\rangle + \frac{1}{2}\langle F_{\mathrm{R}} + F_{\mathrm{L}}\rangle^2 + \frac{1}{6}\langle F_{\mathrm{R}} - F_{\mathrm{L}}\rangle^2\}.$$

$$\boxed{B = \frac{N_c}{144\pi^2}\mathrm{i}D\theta\langle \bar{F}_{\mathrm{R}} D\bar{U}\bar{U}^\dagger - \bar{F}_{\mathrm{L}}\bar{U}^\dagger D\bar{U}\rangle,}$$

$$\bar{F}_{\mathrm{R}} = F_{\mathrm{R}} - \frac{1}{3}\langle F_{\mathrm{R}}\rangle, \qquad \bar{F}_{\mathrm{L}} = F_{\mathrm{L}} - \frac{1}{3}\langle F_{\mathrm{L}}\rangle.$$

Note that terms proportional to $D\theta\langle (D\bar{U}\bar{U}^\dagger)^3\rangle$ cancel out on account of charge conjugation invariance. The term

$$\frac{N_c \epsilon^{\mu\nu\rho\sigma}}{288\pi^2}\left\{ -\mathrm{i}D_\mu\theta\langle \bar{R}_{\nu\rho}D_\sigma\bar{U}\bar{U}^\dagger - \bar{L}_{\nu\rho}D_\sigma\bar{U}^\dagger\bar{U}\rangle \right\}.$$

$$\partial_\mu\theta. \longrightarrow \mu_5\,\delta_{\mu 0}$$