Study of strongly intense quantities and robust variances in multi-particle production at LHC energies

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Outline:

- development of the quark-gluon string model approach and the MC algorithm for the analysis of high energy pp collisions;
- calculations of strongly internsive variable $\Sigma(n_F,\ n_B)$, scaled variance ω_n and robust variance R_n and investigations of their behavior with energy for pp collisions at the LHC;

each cut pomeron — formation of two string

The event by event distribution of the number of pomerons around this mean value at a given value of the impact parameter b at $N \geq 1$ was chosen in the following form with some parameter $\overline{N}(b)$:

$$\widetilde{P}(N,b) = \frac{e^{-\overline{N}(b)}\overline{N}(b)^N}{N![1-P(0,b)]}, \tag{1}$$

The difference of our distribution $\widetilde{P}(N,b)$ (1) from the poissonian one is only in excluding of the point $N=0$ from it: $\widetilde{P}(0,b)=0$, which corresponds to the absence of the non-diffractive scattering at $N=0$.

According to **Vechernin V.V., Lakomov I.A., The dependence of the number of pomerons on the impact parameter and the long-range rapidity correlations in pp collisions, PoS (Baldin ISHEPP XXI) 072.**, we suppose that in the case of proton-proton collision at the impact parameter b the string density in transverse plane at a point $\vec{s}$ is proportional to

$$w_{str}(\vec{s},\vec{b}) \sim \frac{1}{\sigma_{pp}(b)} T(\vec{s}-\vec{b}/2)\, T(\vec{s}+\vec{b}/2)\ , \qquad (2)$$

where now the $T(\vec{s})$ is the partonic profile function of nucleon. We will use for the partonic profile function of nucleon the simplest gaussian distribution:

$$T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\ . \qquad (3)$$

Substituting (3) in (2) one gets

$$w_{str}(\vec{s},\vec{b}) \sim \frac{1}{\sigma_{pp}(b)} e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}\ . \qquad (4)$$

Simultaneously we have $\overline{N}(b) = N_0 e^{-b^2/2\alpha^2}$, where the parameter N_0 depends on initial energy.

As has been shown in **Vechernin V.V., Lakomov I.A., The dependence of the number of pomerons on the impact parameter and the long-range rapidity correlations in pp collisions, PoS (Baldin ISHEPP XXI) 072.**, in the framework of this assumptions the average number of pomerons $\langle N_{pom}(E)\rangle$, the scaled variance of number of pomerons $\omega_{N_{pom}}(E)$, the cross-section of non-diffractive pp interaction σ_{pp} and the probability $P(N)$ to have N cut pomerons in a non-diffractive pp collision has the following form:

$$\langle N_{pom}(E)\rangle = \frac{N_0}{E_1(N_0)+\gamma+\ln N_0}\ ,\ \ E_1(x) = \int_x^{\infty}\frac{e^{-t}}{t}dt, \tag{5}$$

$$\omega_{N_{pom}}(E) = 1+\frac{N_0}{2}-\langle N\rangle_{pom}(E)\ , \tag{6}$$

$$\sigma_{pp} = 2\pi\alpha^2\left[E_1(N_0)+\gamma+\ln N_0\right], \tag{7}$$

$$P(N) = \frac{2\pi\alpha^2}{\sigma_{pp}N}\left[1-e^{-N_0}\sum_{l=0}^{N-1}N_0^l/l!\right]. \tag{8}$$

Two approaches, the string fusion model and the Gribov-Regge approach, are equivalent. This enables to connect the parameters N_0 and α of string fusion model, which describe the dependence of the mean number of pomerons on the impact parameter b with the parameters of the pomeron trajectory and its couplings to hadrons:

$$\alpha = \sqrt{\frac{2\lambda}{C}}/5.05\ fm,\ N_0 = \frac{2\gamma_{pp}C}{\lambda}\exp(\Delta\xi),\ \lambda = R_{pp}^2 + \alpha'\xi,\ \xi = \ln(s/1GeV^2)\,. \quad (9)$$

Here Δ and α' are the intercept and the slope of the pomeron trajectory. The parameters γ and R_{pp} characterize the coupling of the pomeron trajectory with the initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons. For the case of pp collisions the following numerical values of the parameters were chosen to describe the multiplicity and the non-diffractive cross section (Figure 1):

$$\Delta = 0.2, \quad \alpha' = 0.05\ GeV^{-2}, \quad \gamma_{pp} = 1.035\ GeV^{-2}, \quad R_{pp}^2 = 3.3\ GeV^{-2}, \quad (10)$$

$$C = 1.5.$$

In the framework of the string fusion model the dependence of the average number of particles formed from decay of the fused strings in the cell on the number of strings in the rapidity observation window of width δy have the following form:

$$\overline{n}\left(\eta_i\right) = \mu_0 \delta y \sqrt{\eta_i}, \tag{11}$$

where μ_0 is the average number of a particles produced from the hadronizations of the one string in the window of width $\delta y = 1$.

We assume that the number of particles produced from the hadronizations of the strings in i-th cell in the rapidity observation window of width δy is distributed over the negative binomial distribution (NBD) with mean value (11) and scaled variance [14]:

$$\omega_\mu(\delta y,\ \eta) = 1 + \delta y \mu_0^\eta J_{FF}^\eta, \tag{12}$$

where

$$J_{FF}^\eta = \frac{1}{(\delta y_F)^2} \int_{\delta y_F} dy_1 \int_{\delta y_F} dy_2 \Lambda_\eta(y_1 - y_2) \tag{13}$$

and $\Lambda_\eta(\Delta y)$ is the two-particle (pair) correlation function, which was chosen in the simplest way

$$\Lambda_\eta(\Delta y) = \Lambda_0^\eta e^{-\frac{|\Delta y|}{y_{corr}^\eta}}, \tag{14}$$

y_{corr}^η is a characteristic correlation length in the rapidity space. We assume that the dependence of the parameters on the string density is as follows

$$y_{corr}^\eta = \frac{y_1}{\sqrt{\eta}},\ \ \mu_0^\eta = \mu_0 \sqrt{\eta}. \tag{15}$$

The correlation function was chosen in a simplest way (14), and in this case integral J^{η}_{FF} can be calculated:

$$J^{\eta}_{FF} = \frac{2\Lambda^{\eta}_{0}}{(\delta y)^2} y^{\eta}_{corr} \left(\delta y - y^{\eta}_{corr} \left(1 - e^{-\frac{\delta y}{y^{\eta}_{corr}}} \right) \right). \qquad (16)$$

Parameters y_1 and Λ^{η}_{0} was chosen to obtain a correspondence with the results for $\Sigma(n_F,\ n_B)$ obtained in **V.V. Vechernin, EPJ Web Conf. 191, 04011 (2018)** using the pair correlation function extracted in [14] in the approximation of identical strings from ALICE [13] experimental data , the value of parameter μ_0 was chosen to describe dN/dy distribution at different energies:

$$\mu_0 = 0.7, \quad y_1 = 2.7, \quad \Lambda^{\eta}_{0} = 0.8. \qquad (17)$$

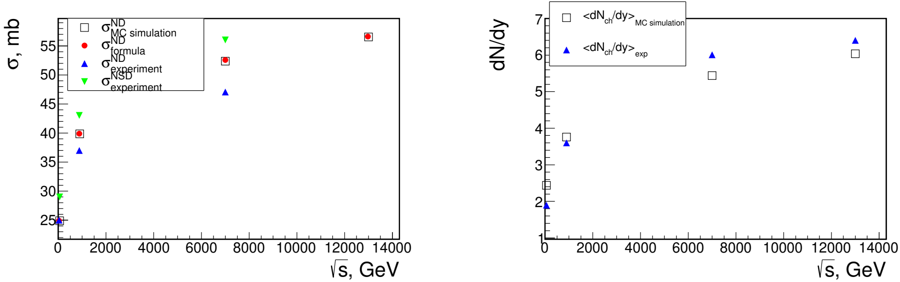

Figure: The cross-section of non-diffractive *pp* interaction σ_{pp} and multiplicity $\langle dN_{ch}/dy \rangle$ as a function of energy $\sqrt{s}$. $\sigma^{ND}_{statistical}$ and $\langle dN_{ch}/dy \rangle_{statistical}$ was calculated from the data generated accordingly to the algorithm described earlier; $\sigma^{ND}_{formula}$ was calculated according (7); σ^{ND}_{exp} and $\langle dN_{ch}/dy \rangle_{exp}$ are the experimental results taken from [15, 17, 18, 19, 20] .

Generation algorithm:

- value of impact parameter b are generated: b_x and b_y are generated independently with uniform distribution on the interval $(-3R_p,\ 3R_p),\ R_p = 0.7$ fm, $b = \sqrt{b_x^2 + b_y^2}$;
- for given energy $\sqrt{s} = E$ pomeron parameters (9) are calculated;
- lattice parameters at a fixed energy are calculated: lattice constant $a = R_{str}\sqrt{\pi}$, lattice size $L = 10\sigma$, number of cells $M = m^2,\ m = [L/a] + 1$, where r_{srt} — string radius, $\sigma = \alpha/2$ — parameter of the gaussian distributed density of the strings (4), square brackets $[x]$ mean integer part of x;
- average number of the pomerons at a fixed energy $\langle N_{pom}(b)\rangle$ (5) are calculated;

Generation algorithm:

- N_{pom} was generated by Poisson distribution (2). If it occurs equal 0, than we go back to 1. Note that we store the number N^0_{sim} of such simulations with $N_{pom} = 0$, what enables to calculate $\sigma^{ND}_{statistical} = \frac{N_{sim} - N^0_{sim}}{N_{sim}} 36 R_p^2$, where N_{sim} is the total number of simulations;
- number of pomerons N_{pom} in this event are generated, number of strings = 2 * number of pomerons;
- position $\overrightarrow{s}$ of each string are generated, after that number of cell are calculated;
- number of strings in every cell is calculated;
- number of particles n on rapidity observation window δy produced from hadronisation of the strings in every cell are generated with NBD: $NB\left(\frac{\overline{n}}{\omega_\mu - 1},\ p = \frac{1}{\omega_\mu}\right)$.

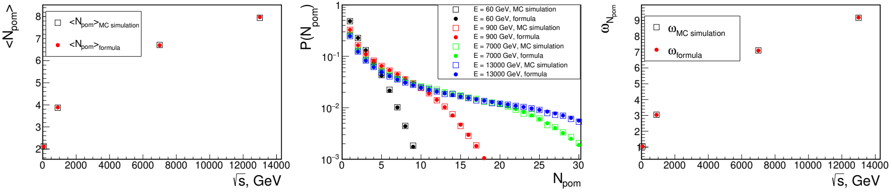

Figure: Comparison of the numerical values of the average number of pomerons $\langle N_{pom}(E)\rangle$, the scaled variance of number of pomerons $\omega_{N_{pom}}(E)$ and the probability $P(N)$ to have N cut pomerons in a non-diffractive *pp* collision calculated for MC generated events and calculated by formulas (5), (6), (8) for 60 Gev, 900 Gev, 7 TeV, 13 TeV.

This is the definitions of the strongly intensive variable $\Sigma(n_F,\ n_B)$, scaled variance ω_n and robust variance R_n:

$$\Sigma(n_F,\ n_B) \equiv \frac{\langle n_F\rangle\,\omega_{n_B} + \langle n_B\rangle\,\omega_{n_F} - 2cov(n_F,\ n_B)}{\langle n_F\rangle + \langle n_B\rangle}, \tag{18}$$

$$\omega_n \equiv \frac{\left\langle n^2\right\rangle - \langle n\rangle^2}{\langle n\rangle},\ R_n \equiv \frac{\omega_n - 1}{\langle n\rangle}. \tag{19}$$

As it shown in **S. N. Belokurova, V. V. Vechernin, Theoretical and Mathematical Physics, 200(2): 1094–1109 (2019)**, the definition of the strongly intensive variable $\Sigma(n_F,\ n_B)$ (18) can be rewritten in the following form:

$$\Sigma(n_F,\ n_B) = \sum_{\eta=1}^{\infty} \frac{\langle n \rangle_\eta}{\langle n \rangle} \Sigma_\eta(\mu_F,\ \mu_B),\ \Sigma_\eta(\mu_F,\ \mu_B) = 1 + \mu_0^\eta \delta y \left[J_{FF}^\eta - J_{FB}^\eta \right], \tag{20}$$

where $\Sigma_\eta(\mu_F,\ \mu_B)$ is for cells with η strings, $\langle n \rangle_\eta$ is the average numbers of particles produced from the decay of a string cluster with η strings, $\langle n \rangle$ — multiplicity,

$$J_{FB}^\eta = \frac{1}{\delta y_F \delta y_B} \int_{\delta y_F} dy_1 \int_{\delta y_B} dy_2\ \Lambda_\eta(y_1 - y_2). \tag{21}$$

For the correlation function of the simplest form (14), we have

$$J_{FB}^{\eta} = \frac{\Lambda_0^{\eta} \left(y_{corr}^{\eta}\right)^2}{(\delta y)^2} e^{\frac{-\Delta y}{y_{corr}^{\eta}}} \left(e^{\frac{\delta y}{y_{corr}^{\eta}}} - 2 + e^{\frac{-\delta y}{y_{corr}^{\eta}}} \right), \tag{22}$$

where $\delta y = \delta y_F = \delta y_B$ is the rapidity observation window, Δy is the rapidity distance between the centers of observation windows (formula (22) was obtained in the case $\Delta y > \delta y$).

For the calculation of the $\Sigma(n_F,\ n_B)$ formula (20) was used. We used the definitions (19) to calculate ω_n and R_n.

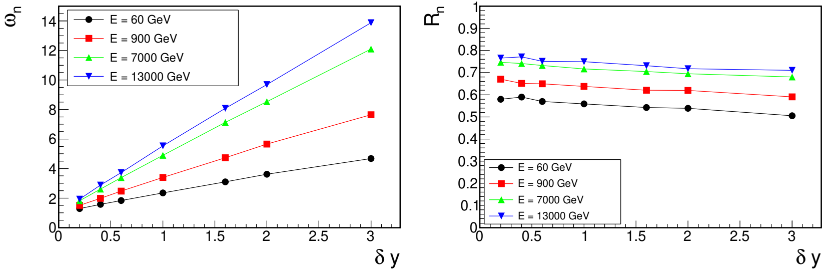

Figure: Results for scaled variance ω_n and robust variance R_n calculated with help of (19) as a function of the rapidity width of the observation window δy for min.bias *pp* interactions at energies 60 - 13000 GeV

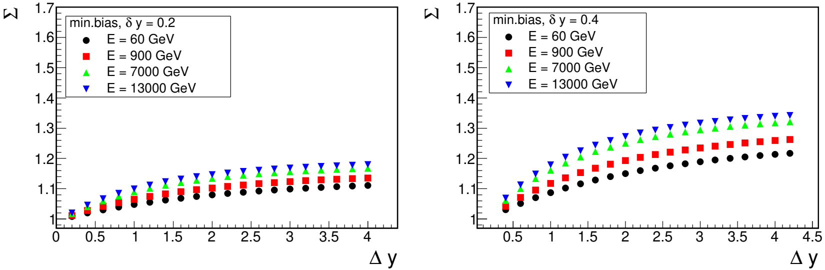

Figure: Results for the strongly intensive variable $\Sigma(n_F,\ n_B)$ calculated with help of (20) as a function of the rapidity distance between the observation windows Δy for min.bias pp interactions at energies 60 - 13000 GeV for rapidity width of the observation windows $\delta y = 0.2$ and $\delta y = 0.4$.

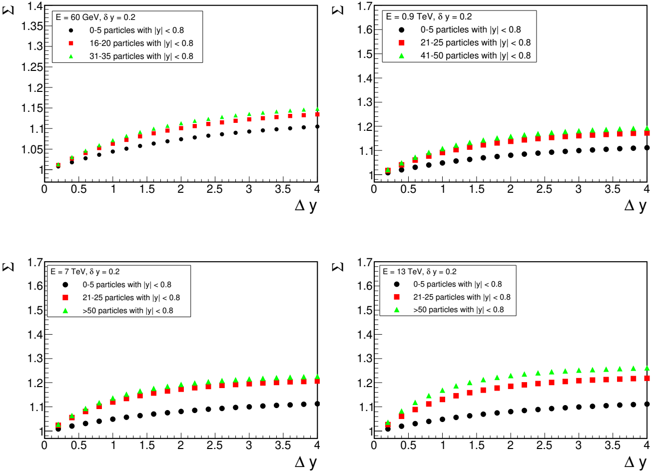

Figure: Results for the strongly intensive variable $\Sigma(n_F,\ n_B)$ calculated with help of (20) as a function of the rapidity distance between the observation windows Δy for *pp* interactions at energies 60 - 13000 GeV for rapidity width of the observation windows $\delta y = 0.2$.

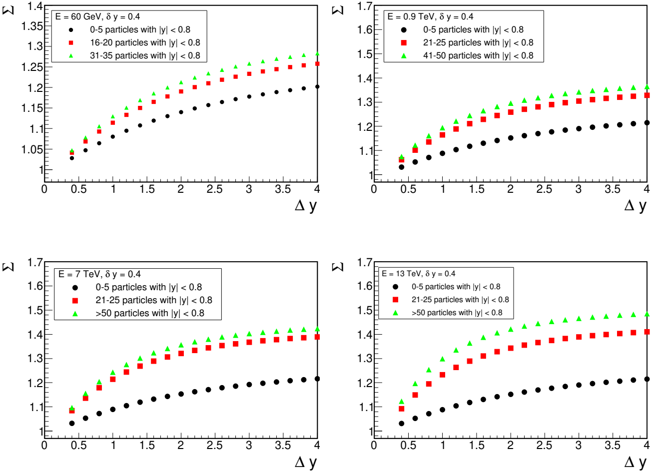

Figure: Results for the strongly intensive variable $\Sigma(n_F,\ n_B)$ calculated with help of (20) as a function of the rapidity distance between the observation windows Δy for *pp* interactions at energies 60 - 13000 GeV for rapidity width of the observation windows $\delta y = 0.4$.

The quark-gluon string model approach and the MC algorithm for the analysis of high energy pp collisions were developed. Strongly internsive variable $\Sigma(n_F,\ n_B)$ was calculated for different energies for two values of the width of the observation rapidity windows as a function of the distance between the centers of this windows. Scaled variance ω_n and robust variance R_n for different energies and for different width of the observation rapidity window was calculated from MC simulation results.

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