SHORT-RANGE NN CORRELATIONS AND QUASI-DEUTERON CLUSTERS IN THE REACTION \( ^{12}\text{C}(p,2pN)^{10}\text{A} \)

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- Motivation: SRC and others…
- \( ^{12}\text{C}(p,pd)^{10}\text{B} \) and \( pd\rightarrow dp \) \( pd\rightarrow \{pp\}_n \) at \( \sim 1 \) GeV
- Elements of formalism for \( p+^{12}\text{C}\rightarrow p+p+N+^{10}\text{B} \) (BM@N)
- Numerical results for \( pp/pn \) ratio and SRC c.m. distribution
- Conclusion

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Dubna, 1957 M.G. Mesheryakov et al. ZHETF, p+^{12}C → d+X at 670 MeV; quasi-elastic knock-out of the fast deuteron clusters
D.I. Blokhintsev (1957) : **fluctons** (6q) in nuclei
Two nucleons being at short distances $r_{NN} < 0.5$ fm have a large relative momentum $q > 1/r_{NN} = 0.4$ GeV/c; Repulsive core in NN-potential → high-momentum part of the w.f. of NN pair

Search for high-momentum components of the nuclear wave functions eA-, pA – elastic and inelastic scattering. A special attention was paid to the lightest nuclei – the deuteron, $^3$He, $^4$He (A.Gilman, F.Gross, J.Phys.G:Nuc.Part.Phys.28 (2002)B13)
A new trend in this study is investigation of short-range correlations (SRC) in nuclei – NN-pair in nucleus with almost zero c.m. momentum but large (equal) internal momenta $q_1=-q_2$, $q > p_F = 250-300$ MeV/c. (M.Strikman, L. Frankfurt, 1978):

* High-momentum part ($q > p_F$) accounts for 20% nucleons.
* pn- SRC pairs dominate by factor of 20 as compared to pp- and nn- due to the tensor forces.
* SRC are connected with neutrino-nucleus interaction, neutron stars structure, modification of the bound nucleon structure (EMC effect).

$E_e = 5$ GeV, $A(e,e'np), A(e,e'pp)$
FIG. 34. The slope of the EMC effect ($R_{EMC}$, ratio of nuclear to deuteron cross section) for $0.35 \leq x_A \leq 0.7$ plotted vs $a_2(A)$, the SRC scale factor (the relative probability that a nucleon belongs to an SRC $NN$ pair) for a variety of nuclei. The fit parameter $a = -0.070 \pm 0.004$ is the intercept of the line constrained to pass through the deuteron (and is therefore also the negative of the slope of that line). From Hen et al., 2013.
Project of BM@N to study SRC in JINR with 4=GeV/c/nucleon beam of $^{12}$C and proton target. Inverse kinematics allows to detect all final particles including the residual nucleus.

SRC@BMN proposal
http://bmnshift.jinr.ru/wiki/doku.pho
Quasi-elastic knockout of fast deuteron clusters $^{12}\text{C}(p,pd)^{10}\text{B}$ and hard $pd\rightarrow dp$


Mechanisms of the breakup reaction $pd\rightarrow (pp)n$. The same mechanisms are used for the reaction $pd\rightarrow dp$. 
Zero-point at $q \approx 180$ MeV/c

$^{12}$C $(p, pd) ^{10}$B
Low excited st.
$\theta_p = 147^\circ$

$E^* = 0 - 5$ MeV

$2S$

J. Erő et al. / Quasi-free scattering

$^{12}$C $(p, pd) ^{10}$B
High excited st.
$\theta_p = 147^\circ$

$E^* \sim 20$ MeV

J. Ero et al. NPA 372 (1981) 371

Suppression of the high $E^*$
REACTION $^{12}\text{C}+\text{p} \rightarrow ^{10}\text{B}+\text{P}+\text{P}+\text{N}$
\[ M_{fi} = M(A \rightarrow B+<NN>) \frac{1}{p_{NN}^2 - M_{NN}^2 + i\varepsilon} M(p<NN> \rightarrow ppN), \]

\[ d\sigma = (2\pi)^4 \delta^4(P_i - P_f) \frac{1}{4I} |M_{fi}|^2 \prod_{j=1}^n \frac{d^3p_j}{2E_j(2\pi)^3} \]

In the rest frame of A:

\[ M(^{12}C \rightarrow ^{10}B+<NN>) = -\left( \frac{A}{2} \right)^{1/2} <\Psi_A|\Psi_B, \Psi_{NN}, \Psi_{\nu_A}> \]

\[ \times \left( \varepsilon_A^{B+<NN>} + \frac{q^2}{2\mu} \right) \Psi_{\nu_\Lambda M_\Lambda}(q) \sqrt{2m_A2m_B2m_{<NN>}}, \]

\[ \varepsilon_A^{B+<NN>} = m_B + m_{<NN>} - m_A, \]

\[ \mu = m_Bm_{<NN>}/(m_B + m_{<NN>}), \]

\[ q = \frac{m_Bp_{<NN>} - m_{<NN>}p_B}{m_B + m_{<NN>}} \]
Spectroscopic factors within the translationally-invariant shell model (TISM)

\[ S^x_A = \left( \frac{A}{x} \right)^{1/2} < \psi_A | \psi_B \psi_\nu \Lambda (R_{A-x} - R_x) \psi_x >. \]

\[ \psi_{TISM}^A = |AN[f](\lambda \mu)\alpha LST JMMT > \]

\[ N_A - N_B = N_x + \nu \]

Mixing shell-model configurations:

\[ \psi_{j,t}^A = \sum_{f;L,S} \alpha_{f;L,S}^{A,J,T} |AN[f](\lambda \mu)\alpha LST JMMT > \]

\[ |AN_A \alpha > = \sum_{\beta \gamma \Lambda M \Lambda N_B N_x \nu} < AN_A \alpha | A - xN_B \beta, \nu \Lambda M, xN_x \gamma > \]

\[ |BN_B \beta > |xN_x \gamma > |\nu \Lambda M \Lambda >. \]
Matrix element for \( p +^{12} C \rightarrow p + p + N +^{10} B \)

\[
M_{fi}(pA \rightarrow ppNB) = \left( \frac{A}{2} \right)^{1/2} \sum_{M_{Jd}, J, \bar{M}, M_A} \sum_{\alpha, \alpha_f, N, \Lambda, \mathcal{L}} \alpha_i^{AJ_iT_i} \alpha_f^{A-2J_fT_f} \cdot \cdot \cdot
\]

\[
< A \alpha_i | A - 2 \alpha_f, N \Lambda; d' > (\Lambda M_{d'}, J_{d'} M_{J_{d'}} | \bar{J} \bar{M})(J_f M_{J_f} | \bar{J} \bar{M} | J_i M_i)
\]

\[
(T_f M_{T_f} T_{d'} M_{T_{d'}} | T_i M_{T_i}) U(\Lambda L_{d'} \bar{J} S_{d'}; \mathcal{L} J_{d'}) \begin{pmatrix} L_f & S_f & J_f \\ \bar{L} & S_{d'} & \bar{J} \\ L_i & S_i & J_i \end{pmatrix}
\]

\[
[(2L_i + 1)(2S_i + 1)(2J_f + 1)(2\bar{J} + 1)]^{1/2} \Psi_{\Lambda \Lambda \Lambda}^{\text{dist}}(k_B)
\]

\[
\times < p_1 \sigma_1, p_2 \sigma_2, \ldots, p_r \sigma_r | \hat{M}(p < NN \rightarrow p_1 p_2 p_r)| p \sigma_p, -k_B \Psi_{NN} >
\]

\( ^{12} C: L_i = S_i = J_i = 0, T_i = 0; |10B >= |s^4 p^6 > \)
Matrix element of the \( p + <NN> \rightarrow p + p + N \)

In the Light front dynamics

\[
M_{fi}^{LFD}(p < NN \rightarrow p_1p_2) = \frac{\Psi_{d}^{LFD}(k_{\perp}, \xi)}{1 - \xi} M_{fi}(pN \rightarrow p_1p_2),
\]

\[
\xi = \frac{p_{r}^+}{p_{r}^+ + p_{N}^+}, \quad q_{\perp} = (1 - \xi)p_{r\perp} - \xi p_{N\perp},
\]

\[
M_{pN}^2 = \frac{m_p^2 + p_{N\perp}^2}{\xi(1 - \xi)}.
\]

\[
\Psi_{d}^{LFD}(q) = \sqrt{\varepsilon(q)}\varphi_{d}^{nonrel}(q)
\]

Factorization of spin averaged \( |M_{fi}|^2 \) for \( S = 0 \) of the \( <NN> \) pair and \( \Lambda = 0 \) in the \( <NN> - B \) relative motion.

\( M_{fi}(pN \rightarrow p_1p_2) \) is connected to on-shell pN-pN scattering, via cross section \( \frac{d\sigma}{dt}(s,t) \).
Momentum distribution in $< NN > -^{10}B_5$

$N\Lambda = 20, 22$ for $|s^4p^6>$

$$R_{20}^2 = \frac{6}{\sqrt{\pi} p_0^3} \left[ 1 - \frac{2}{3} \left( \frac{p}{p_0} \right)^2 \right]^2 \exp\left\{ -\left( \frac{p}{p_0} \right)^2 \right\},$$

(1)

$$R_{22}^2 = \frac{16}{15\sqrt{\pi} p_0^3} \left( \frac{p}{p_0} \right)^4 \exp\left\{ -\left( \frac{p}{p_0} \right)^2 \right\}$$

(2)

$N\Lambda = 00$ for $|s^2p^8>$

$$R_{00}^2 = \frac{4}{\sqrt{\pi} p_0^3} \exp\left\{ -\left( \frac{p}{p_0} \right)^2 \right\}$$

where $p_0 = \sqrt{\mu/r_0} = \sqrt{\mu p_0^{h.o.}}$;

$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{5}{3}$ – the reduced mass of the $d -^{10}B$ system in $m_N$;

$r_0$ is the h.o. shell model parameter;

$p_0^{h.o.} = \hbar/r_0$
Center mass motion of SRC NN pairs in nuclei


Hard breakup of a pp-SRC pair in a hard two-nucleons knockout $A(e, e' pp)$ reactions at recoil proton momentum $p_{rec} \geq 350$ MeV/c assuming factorization

$$d\sigma(e, e' pp) \sim n_{SRC}(\vec{p}_1, \vec{p}_2) \approx n_{c.m.}^A(\vec{p}_{c.m.}) n_{rel}^{NN}(\vec{p}_{rel})$$

$n_{c.m.}^A(\vec{p}_{c.m.})$ is approximated by the 3-D Gaussian $g(x)g(y)g(z)$,

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right]$$

$^{12}C$:

$\sigma_x \approx \sigma_y \approx \sigma_z = (145 \pm 5)$ MeV/c

i.e. $p_0 = \sqrt{2}(145 \pm 5)$ MeV/c = $(205 \pm 7)$ MeV/c
The c.m. distribution in mean-field model (Cioffi degli Atti, Simula, 1996)

\[
<k_{c.m.}^2> = \frac{2(A-2)}{A-1} <k^2>
\]

\[
n_{c.m.}(k_{c.m.}) = C \exp(-\alpha_{c.m.}k_{c.m.}^2) <k_{c.m.}^2> = \frac{3}{2\alpha_{c.m.}}
\]

\[
\alpha_{c.m.} = \frac{3(A-1)}{4(A-2)} \frac{1}{2M<T>}
\]

\[
T_s = \frac{3}{2} \frac{p_0^2}{2M} \text{ and } T_p = \frac{5}{2} \frac{p_0^2}{2M}
\]

\[
<T> = \frac{4T_s + 8T_p}{12} = \frac{13}{6} \frac{p_0^2}{2M}
\]
Fig. 2. Single nucleon momentum distribution in the reaction $^{12}\text{C}(e, ep)^{11}\text{B}$ at electron beam energy 497 MeV for the p-shell (a) and s-shell nucleons (b) corresponding to transitions to the states of the residual nucleus $^{11}\text{B}$ with excitation energy $0 < E^* < 6.5$ MeV and $15 < E^* < 35$ MeV, respectively. The curves show the results of our calculations in the plane wave impulse approximation.
Fig. 3. Distribution over the c.m. momentum of the SCR pair $p_{c.m.}$ in the $^{12}$C for the mean-field model with $\alpha_{c.m.} = 0.95 \text{ fm}^2$ (full line) and the TISM wave function squared with $p_0 = 146.9$ MeV/c for the 0S-type (dashed), and 2S-type (dashed-dotted). All distributions are arbitrary normalized at $p_{c.m.} = 0$ to the same value.

\[ \nu = N_A - N_x - N_{A-x} \]

$N_x = 0, N_A = 8$

$R_{v\Lambda}(k_{c.m.})$

$s^4 p^6, \nu = 2$

$s^2 p^8, \nu = 0$
Fig. 3. Distribution over the c.m. momentum of the SCR pair $p_{c.m.}$ in the $^{12}$C for the mean-field model with $\alpha_{c.m.} = 0.95 \text{fm}^2$ (full line) and the TISM wave function squared with $p_0 = 146.9$ MeV/c for the 0S-type (dashed), and 2S-type (dashed-dotted). All distributions are arbitrary normalized at $p_{c.m.} = 0$ to the same value.
pp and deuteron internal momentum distribution

\[ \phi(q)^2, \text{ a.u.} \]

\[ q, \text{ GeV/c} \]

pp(1S0) scattering
Lensky V. et al.,
EPJ A26 (2005)107
CD – Bonn NN
\[ S^x_A = \binom{A}{2}^{1/2} \frac{\sqrt{2J_f} + 1}{\sqrt{2T + 1}} PC(S, T), \]

\[ |M_{fi}(A(p, 2pN)B)|^2 \propto \frac{(2S + 1)^2}{2T + 1} n_{cm}(k_{c.m.}) n_{pN}(q_{rel}) |M_{pN}|^2 [I_{pN} PC(S, T)]^2, \]

\[ R = \frac{pp}{pn} = \frac{pp}{(pn)_{S=0T=1} + (pn)_{S=1T=0}} = \frac{1}{14} R_{rel} \]

Table 1. The \((ST = 01)/(ST = 10)\) ratio \(R_{rel}\) versus \(q_{min}\) at \(q_{max} = 1.0 \div 2.0\) GeV/c

<table>
<thead>
<tr>
<th>(q_{min}, ) GeV/c</th>
<th>(R_{rel})</th>
<th>(q_{min}, ) GeV/c</th>
<th>(R_{rel})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.15</td>
<td>0.6</td>
<td>0.27-0.3</td>
</tr>
<tr>
<td>0.3</td>
<td>0.06-0.07</td>
<td>0.7</td>
<td>0.39-0.54</td>
</tr>
<tr>
<td>0.4</td>
<td>0.09 -0.10</td>
<td>0.8</td>
<td>0.55-0.88</td>
</tr>
<tr>
<td>0.5</td>
<td>0.17-0.2</td>
<td>0.9</td>
<td>0.78-1.5</td>
</tr>
</tbody>
</table>
\[ R_{rel} = \frac{\int_{q_{min}}^{q_{max}} dq q^2 \psi_{pp;ST=01}^2(q)}{\int_{q_{min}}^{q_{max}} dq q^2 \psi_{d;ST=01}^2(q)}; \]

\[ \psi_{10}^2(q) = u^2(q) + w^2(q); \psi_{01}(q), pp(1S_0) - \text{scattering}; \]

\[ \int_{0}^{\infty} dq q^2 \psi_{ST}^2(q) = 1; \]

\[ R = \frac{pp}{pn} = \frac{pp}{(pn)_{S=0T=1} + (pn)_{S=1T=0}} = \frac{1}{14} R_{rel} \]

If \( R = 0.01 \), then \( R_{obs} \approx 0.03 \) due to charge-exchange in FSI (M.Duer et al. PRL 122 (2019)).

\[ R_{exp} \approx 5\% \]
PARENTAGE COEFFICIENTS of TISM

\[ < A N_i = 8 [ \frac{f_i}{i} (\lambda_{i\mu_i}) \alpha_i L_i S_i T_i | A - 2 N_f [ \frac{f_f}{f_f} (\lambda_{f\mu_f}) \alpha_f L_f S_f T_f, \nu \Lambda; N_x L_x S_x T_x : L_i S_i T_i > ] \]

| \(N_f\) | \(6\) | \(f_f\) | \(442\) | \((\lambda_f \mu_f)\) | \(00\) | \(22\) | \(\nu \Lambda\) | \(22\) | \(N_x L_x\) | \(00\) | \(\frac{2 T_f + 1 S_f + 1}{L_f}\) | \(\frac{13 D_I}{13 D_I}\) | \(-\sqrt{35} / 792\) | \(\frac{35}{792}\) | \(-\sqrt{3} / 550\) | \(\frac{3}{550}\) | \(-\sqrt{7} / 110\) | \(\frac{7}{110}\) | \(\text{PC}\) | \(\sqrt{1 / 264}\) | \(\sqrt{1 / 264}\) | \(\sqrt{35 / 792}\) | \(\sqrt{35 / 792}\) | \(\sqrt{3 / 550}\) | \(\sqrt{3 / 550}\) | \(\sqrt{7 / 110}\) | \(\sqrt{7 / 110}\) |
| \(\frac{2 T_f + 1 S_f + 1}{L_f}\) | \(\frac{13 D_I}{13 D_I}\) | \(\frac{31 D_I}{31 D_I}\) | \(\frac{13 D_{II}}{13 D_{II}}\) | \(\frac{31 D_{II}}{31 D_{II}}\) | \(\frac{13 D_I}{13 D_I}\) | \(\frac{31 D_I}{31 D_I}\) | \(\frac{13 D_{II}}{13 D_{II}}\) | \(\frac{31 D_{II}}{31 D_{II}}\) |
| \(6\) | \(442\) | \(433\) | \(442\) | \(433\) | \(442\) | \(03\) | \(13\) | \(13\) | \(04\) |
| \(7\) | \(11\) | \(11\) | \(00\) | \(00\) |
| \(8\) | \(11\) | \(00\) | \(11\) | \(00\) |
| \(31 S\) | \(31 S\) | \(31 S\) | \(31 S\) | \(31 D_I\) | \(31 D_I\) | \(31 D_{II}\) | \(31 D_{II}\) | \(31 D_I\) | \(31 D_I\) | \(31 D_{II}\) | \(31 D_{II}\) |
| \(\sqrt{2 / 99}\) | \(\sqrt{2 / 99}\) | \(\sqrt{8 / 275}\) | \(\sqrt{8 / 275}\) | \(\sqrt{1 / 55}\) | \(\sqrt{9 / 55}\) | \(\sqrt{21 / 275}\) | \(\sqrt{21 / 275}\) | \(\sqrt{3 / 110}\) | \(\sqrt{3 / 110}\) | \(\sqrt{27 / 110}\) | \(\sqrt{3 / 110}\) | \(-\sqrt{3 / 110}\) |
CONCLUSION

• Translationally-invariant shell model (TISM) applied for $S_A^x$ and $n_{cm}(k_{cm})$ of the deuterons in the $^{12}C$ works reasonable well for the $^{12}C(p, pd)^{10}B$ reaction at 670 MeV with transition to the g.s. of $^{10}B$ ($s^4p^6$) and its excited states $E_B^* > 20$ MeV ($s^2p^8$).

• TISM can be applied to BM@N data on quasi-elastic knock-out of nucleon from SRC NN pairs from the $^{12}C$ in exclusive reaction $^{12}C + p \rightarrow p + p + N + ^{10}B$

• The corresponding formalism is developed in the plane-wave approximation taking into account relativistic effects in the $p+ < NN > \rightarrow p + N + N$ within the LFD approach.

• $pp/pn$ ratio obtained within TISM is in agreement with the data.

• Observed in $^{12}C(e, epp)^{10}B$ S-wave $k_{c.m.}$ momentum distribution is a puzzle for TISM. Corresponding measurements of $^{12}C(p, pd)^{10}B$ at BM@N conditions for $s^4p^6$ and $s^2p^8$ will be very important.
Thank you for attention!
BACKWARD QUASI-ELASTIC pd→(pp)_s n SCATTERING
Mechanisms of the breakup reaction $pd \rightarrow (pp)n$. The same mechanisms are used for the reaction $pd \rightarrow dp$. 
Transition matrix element

\[ T_{fi} = (A_x)^{1/2} \sum_{x', \nu_A} <\psi_A | \psi_B \psi_{x'}, \psi_{\nu_A} > \Phi_{\nu_A}(k_B) T_{px' \rightarrow Nx}. \]

\[ T_{px' \rightarrow Nx} = <k_N k_x \chi_N \psi_x | \tau(px' \rightarrow Nx) | k_p, -k_B \chi_p \psi_{x'} > \]

\[ N_A - N_B = N_x + \nu \]

How to take into account ISI and FSI?
\[ S_A^x = \left( \frac{A}{x} \right)^{1/2} \sum_{\mathcal{L} \mathcal{J} \mathcal{M}} (J_B M_B \mathcal{J} \mathcal{M} | J_A M_A) (\Lambda M_A J_x M_x | \mathcal{J} \mathcal{M}) \]

\[ (T_B M_{T_B} T_x M_{T_x} | T_A M_{T_A}) U(\Lambda L_x J S_x; \mathcal{L} \mathcal{J}_x) \]

\[ [(2L_A + 1)(2S_A + 1)(2J_B + 1)(2\overline{J} + 1)]^{1/2} \begin{pmatrix} L_B & S_B & I_B \\ L & S_x & \overline{J} \\ L_A & S_A & I_A \end{pmatrix} \]

\[ < AN_A [f_A](\lambda_A \mu_A) \alpha_A L_A S_A T_A | A - x N_B [f_B](\lambda_B \mu_B) \alpha_B L_B S_B T_B; v \Lambda, x N_x [f_x](\lambda_x \mu_x) \alpha_x L_x S_x T_x (\mathcal{L}) : L_A S_A T_A > \]

\[ n_{cm}(p) = \left( \frac{\alpha}{\pi} \right) \exp \left[ -\alpha p^2 \right] \]  

(3)

\[ \alpha = 1 \text{ fm}^2 \text{ or } p_0 = \hbar/\sqrt{\alpha} = 197 \text{ MeV/c} \]

From the deuteron knock-out \(^{12}C(p, pd)^{10}B\) from p-shell
(|\(^{10}B > = |s^4p^6 >\) (J. Erö et al., 1981) one has
\[ p_0 = 155 \text{ MeV/c} \]
(not for 1S-wave distribution in Eq.(3), but for 2S Eq.(1)!!)

\(^4He: \alpha = 2.4 \text{ fm}^2 \text{ or } p_0 = \hbar/\sqrt{\alpha} = 127.3 \text{ MeV/c}, \]
that is compatible with \[ p_0 = (144.6 \pm 18.2) \text{ MeV/c} \] from the deuteron knock-out \(^{12}C(p, pd)^{10}B\) from the \(\alpha\)-core, (J. Erö et al., NPA 372, 1981),
|\(^{10}B > = s^2p^8 >\)