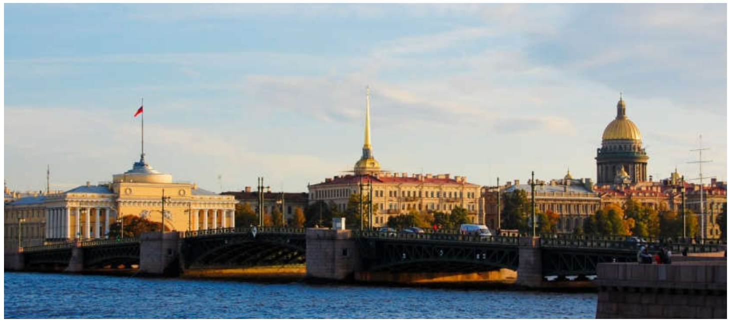

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Bottom-up holographic approach to meson spectroscopy

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The main problem:

QCD is strongly coupled!

Analytical approach – only models

Possible solution: **Dual theory**

Examples of strong-weak duality

(1). In (1+1)-dimensional space-time: The Sine-Gordon model and Thirring model

$$\mathcal{L}_{SG} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha}{\beta^2}\left(\cos\left(\beta\phi\right) - 1\right)$$
$$\mathcal{L}_T = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m_f\bar{\psi}\psi - \frac{\kappa}{2}\left(\bar{\psi}\gamma^\mu\psi\right)^2$$

are the same at quantum level if

$$\boxed{\frac{4\pi}{\beta^2} = 1 + \frac{\kappa}{\pi}}$$

(2). In (3+1)-dimensional space-time: Seiberg duality

It is a nonabelian extension of Montonen-Olive electric-magnetic duality

$$E_i \to B_i \qquad B_i \to -E_i$$

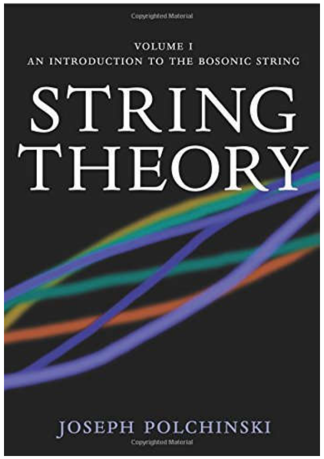
$$\overline{e \to e_m} \Longrightarrow$$ Electrically charged fermion $\longrightarrow$ Magnetic monopole

Charges are quantized: $ee_m \sim 1 \Longrightarrow \boxed{e \sim \frac{1}{e_m}}$

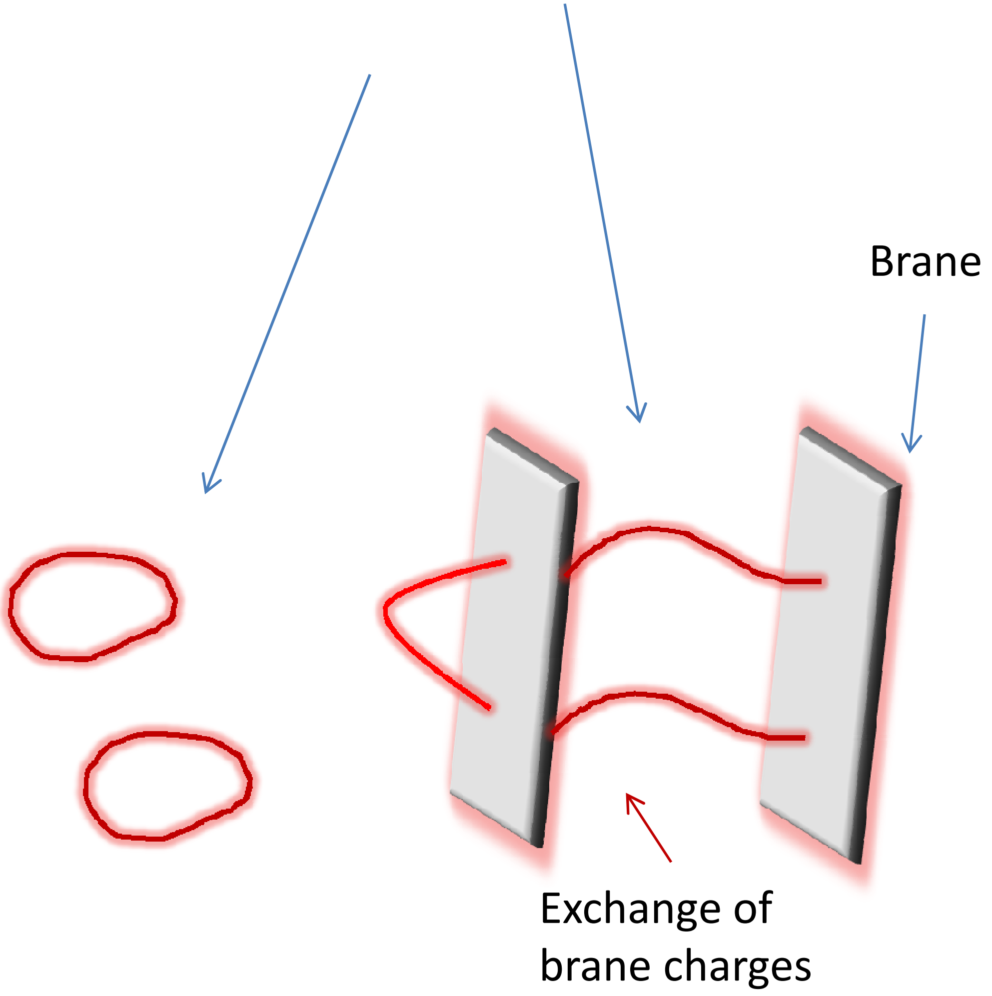
A possible way of the origin of dualities: The quantum theory is unique but one may make a change of variables in the path integral defining this theory,

$$Z[J_k] = \int \mathcal{D}\phi_j\, e^{i\int d^Dx\,(L[\phi_j]+J_k\mathcal{O}_k[\phi_j])}$$

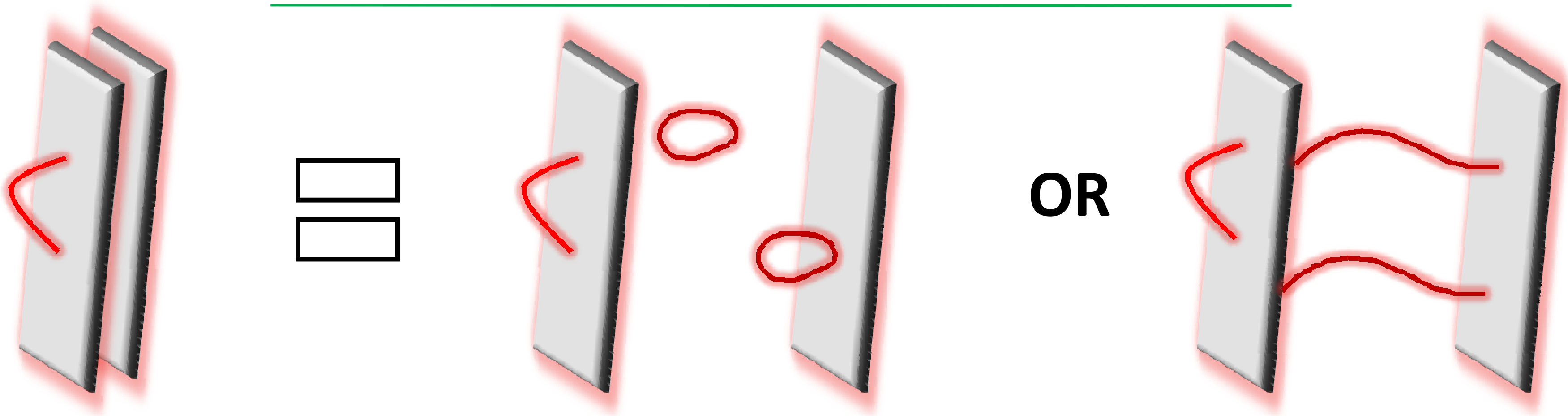
This leads to different Lagrangians in the classical limit.
Likely even the dimension *D* can be changed!



Strings: Closed and Open



THE CONCEPT

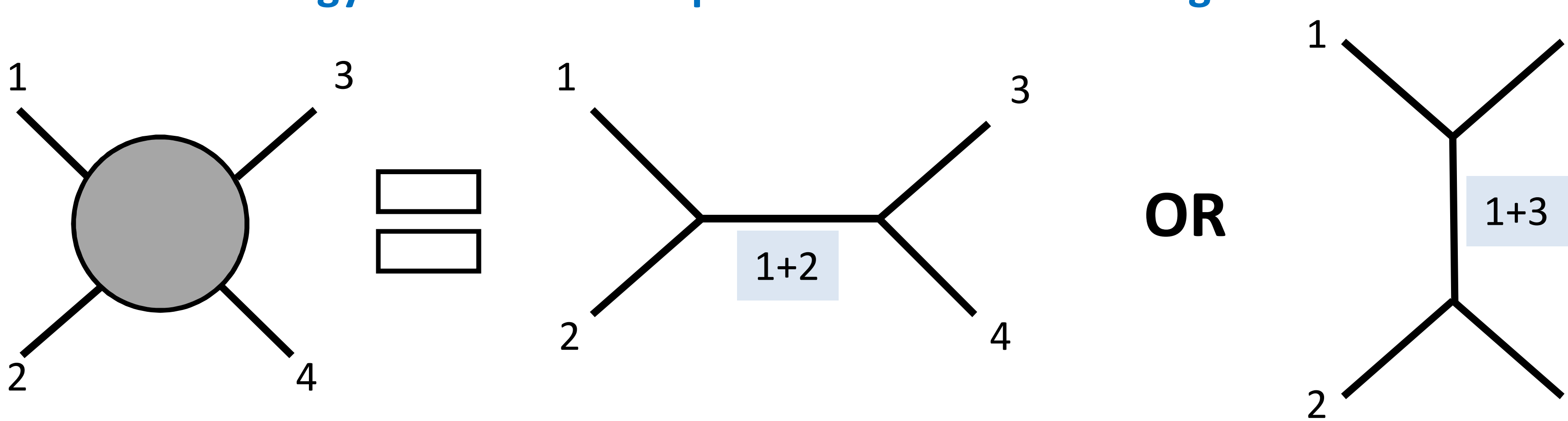


D-brane interaction (at low energies)

via closed strings (= gravitation)

via open strings (= gauge theory)

A remote analogy with dual description of hadron scattering via resonance exchange



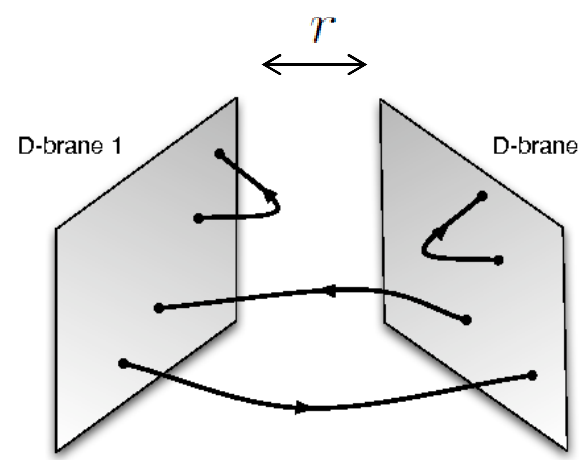
hadron interaction (at low energies)

annihilation (= direct) channel

exchange (= cross) channel

Consider 2 parallel D-branes:

4 vector fields $A_\mu^{11},\ A_\mu^{22},\ A_\mu^{12},\ A_\mu^{21}$



In the limit $\boxed{r \to 0}$ we will arrive at

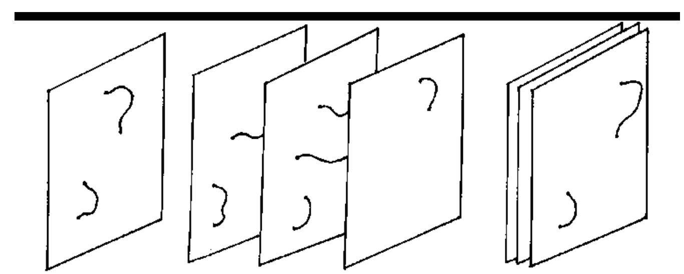
$$U(2) = \underline{U(1)} \times SU(2)$$ gauge theory!

decouples (describes the motion of the whole system)

$$g_{\mathrm{YM}}^2 = 2(2\pi)^{p-2} g_s (\alpha')^{\frac{p-3}{2}}$$

Stack of *N* coincident D_3 branes describes

$\mathcal{N} = 4$ SYM theory with *SU(N)* gauge group!



$$\mathscr{L} = \frac{1}{g_{\mathrm{YM}}^2} \mathrm{tr}\left[-\frac{1}{2}F_{\mu\nu}^2 - (D_\mu \phi_i)^2 - i\bar{\lambda}_I \gamma^\mu D_\mu \lambda^I + O(\phi^4) + O(\lambda\lambda\phi)\right]$$

$$F_{\mu\nu} := F_{\mu\nu}^a t^a$$
$$g_{\mathrm{YM}}^2 = 4\pi g_s$$

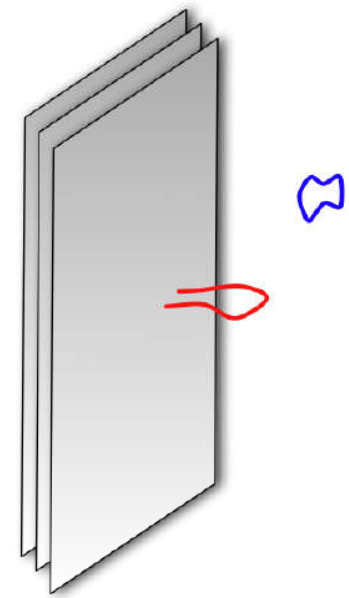
- gauge field A_μ,
- 6 scalar fields ϕ_i, ← scalar fields $X(x^m)$ – 6 brane excitations normal to D_3 !
- 4 Weyl fermions λ_I

(D_3 brane breaks 6 translational symmetries → 6 Goldstone bosons)

Stack of N_c coincident D_3-branes at low energies

(less than the string scale $1/l_s$ when only the massless string states can be excited)

1. Description in terms of gauge fields:



$SU(N_c)$ $\mathcal{N}=4$ SYM ✚ free closed strings

Indeed, schematically we have for the effective action of the massless modes,

$$S = S_{\text{bulk}} + S_{\text{brane}} + S_{\text{int}}$$

$$S_{bulk} \sim \frac{1}{2\kappa^2}\int \sqrt{g}\mathcal{R} \sim \int (\partial h)^2 + \kappa(\partial h)^2 h + \cdots,$$

where we have written the metric as $g = \eta + \kappa h.$

Similarly, $S_{int} \sim \kappa.$

Taking the limit $l_s \to 0$ $(\alpha' \to 0)$ we have $\kappa \sim g_s \alpha'^2 \to 0$

In addition all higher derivative terms in the brane action vanish, leaving just the pure $SU(N_c)$ $\mathcal{N}=4$ SYM

Thus we obtain two decoupled systems!

2. Gravitational description:

$$ds^2 = H^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + H^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right)$$

$$H = 1 + \frac{R^4}{r^4} \qquad R^4 = 4\pi g_s N_c \ell_s^4 \leftarrow (\alpha')^2 \qquad r^2 = y_1^2 + \cdots + y_6^2$$

$r \gg R \quad \Rightarrow \quad H \simeq 1 \quad \Rightarrow$ **<u>flat space</u>**

$r \ll R \quad \Rightarrow \quad \boxed{ds^2 = ds^2_{AdS_5} + R^2 d\Omega_5^2\,,}$

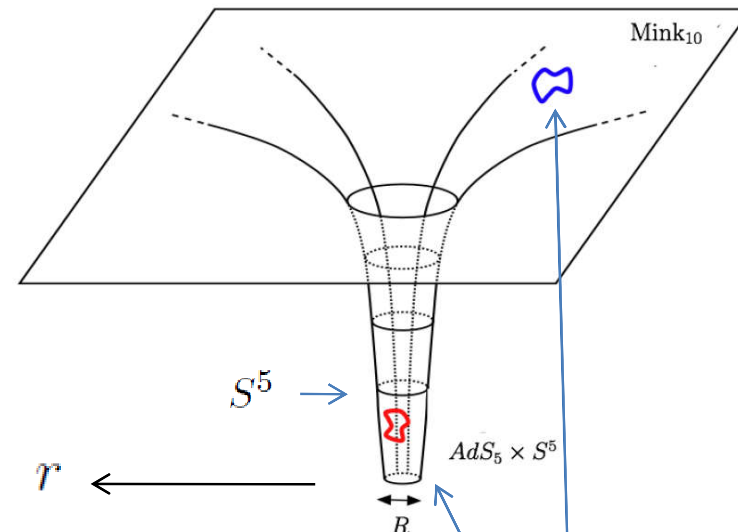
where

$$ds^2_{AdS_5} = \frac{r^2}{R^2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{R^2}{r^2}dr^2$$

$$\|$$

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right) \text{ where } r = R^2/z$$

(Poincaré coordinates; locally M_4 near each z_0!)



$r \longleftarrow$

(distance to the stack of branes)

Do not interact!

Gravitational redshift: $r \to 0$

$$E_\infty = \frac{r}{R}E_h \to 0$$

<u>In the strong gravity region, the 10-dimensional metric factorizes into</u> $\boxed{AdS_5 \times S^5}$

Thus we obtain two decoupled systems again!

The conjecture of holographic duality

J. Maldacena, "The Large N limit of superconformal field theories and supergravity," *Adv. Theor. Math. Phys.* **2** (1998) 231, hep-th/9711200.

(the most cited work in theoretical physics!)

SU(N$_c$)
$\mathcal{N} = 4$ SYM

IIB superstrings on
$AdS_5 \times S^5$

+

+

Free closed strings

Free closed strings

$\lambda \ll 1$

$\lambda \gg 1$

0

1

$$\left(\frac{R}{l_s}\right)^4 = N_c g_{\mathrm{YM}}^2 \equiv \lambda$$

(t'Hooft coupling)

Stack of *N$_c$ D$_3$*-branes

The resulting hypothesis

AdS/CFT correspondence (= gauge/gravity duality = holographic duality)
is a conjectured equivalence between a quantum gravity (in terms of string theory) compactified on anti-de Sitter space (**AdS**) and a Conformal Field Theory (**CFT**) on AdS boundary

The most promoted example (Maldacena, 1997):

Type IIB string theory on $AdS_5 \times S^5$ in the low-energy (i.e. supergravity) approximation	$\Leftrightarrow$	$\mathcal{N} = 4$ SYM theory with *SU(N)* gauge group on AdS_5 boundary (= 4D Minkowski) in the limit $g^2 N \gg 1$

Essential ingredient: a one-to-one mapping of the global symmetries

Isometries of S^5 $\Leftrightarrow$ *SO(6) R*-symmetry of $\mathcal{N} = 4$ Super Yang-Mills theory

Isometries of AdS_5 $\Leftrightarrow$ Conformal group *SO(2,4)* in 4D space

S^5: $X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2 = R^2$ $\qquad$ AdS_5: $X_1^2 + X_2^2 - X_3^2 - X_4^2 - X_5^2 - X_6^2 = R^2$

The term "*Holography*": Realization of the t'Hooft holographic principle (1993)

[Witten; Gubser, Polyakov, Klebanov (1998)]

Essence of the holographic method

$$\langle e^{\int d^d x J(x) \mathcal{O}(x)} \rangle_{\text{CFT}} = \int \mathcal{D}\phi \; e^{-S[\phi,\, g]}\Big|_{\phi(x,\partial AdS)=J(x)}$$

generating functional

action of dual gravitational theory evaluated on classical solutions

AdS boundary

$$\Pi_n \equiv \langle \mathcal{O}_{I_1}(x_1) \ldots \mathcal{O}_{I_n}(x_n) \rangle = \frac{1}{\sqrt{g}} \frac{\delta}{\delta \phi^{I_1}(x_1)} \cdots \frac{1}{\sqrt{g}} \frac{\delta}{\delta \phi^{I_n}(x_n)} S[\phi,\, g]$$

The output of the holographic models: <u>Correlation functions</u>

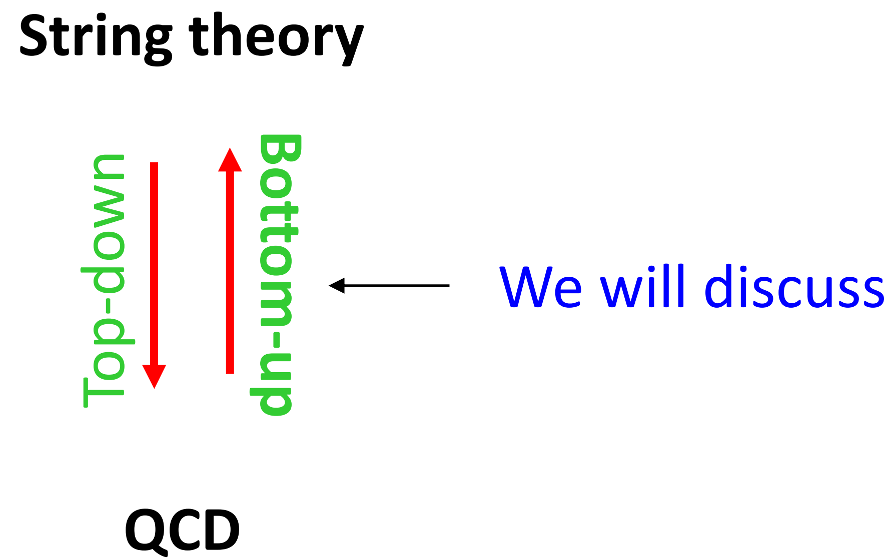
Poles of the 2-point correlator → mass spectrum

Residues of the 2-point correlator → decay constants

Residues of the 3-point correlator → transition amplitudes (--> formfactors)

Alternative way for finding the mass spectrum is to solve e.o.m. $\phi(x_\mu, z) = e^{ixp}\phi(z)$

Holographic QCD – a program for implementation of holographic duality for QCD following some recipies from the AdS/CFT correspondence



5D Anti-de Sitter space

$$\tau^2 + y_0^2 - y_1^2 - y_2^2 - y_3^2 - y_4^2 = R^2$$

$$u = \tau + y_4 \qquad v = \tau - y_4$$

$$uv + y_\mu^2 = R^2$$

$$ds^2 = dudv + dy_\mu dy^\mu$$

Exclude v and introduce

$$z = \frac{R^2}{u}, \qquad x_\mu = \frac{z}{R} y_\mu$$

$$\boxed{ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)}$$

invariant under dilatations

$$x_\mu \to \rho x_\mu, \quad z \to \rho z$$

4D Minkovski space at $z \to 0$

$$p_x = -i\partial_x = \frac{R}{z} p_y$$

Physical meaning of *z*: Inverse energy scale

holographic coordinate

Bottom-up holographic models

Typical ansatz:

$$S = \int d^4x dz \sqrt{g}\, F(z) \mathcal{L} \qquad F(0) = 1$$

5D Anti-de Sitter space

$$ds^2 = \frac{R^2}{z^2}(dx_\mu dx^\mu - dz^2), \qquad z > 0$$

Vector mesons:

$$z = \epsilon \to 0 \qquad V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu q \quad \text{or} \quad V_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \vec{\tau} q$$

$$A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \gamma_5 q \quad \text{or} \quad A_M(x,\epsilon) \leftrightarrow \bar{q}\gamma_\mu \gamma_5 \vec{\tau} q$$

From the AdS/CFT recipes: $m_5^2 R^2 = (\Delta - J)(\Delta + J - 4) \qquad J = 0, 1$

Masses of 5D fields are related to the canonical dimensions of 4D operators!

In the given cases: $\Delta = 3,\ J = 1 \Rightarrow m_5^2 = 0$ gauge 5D theory!

Hard wall model

(Erlich et al., PRL (2005); Da Rold and Pomarol, NPB (2005))

The AdS/CFT dictionary dictates: local symmetries in 5D → global symmetries in 4D

The chiral symmetry: $SU_L(2) \times SU_R(2)$

The typical model describing the chiral symmetry breaking and meson spectrum:

$$\boxed{S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}} \qquad 0 < z \leq z_m$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\; A_{L,R} = A^a_{L,R}t^a,\;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

The pions are introduced via $X = X_0 \exp(i2\pi^a t^a) \qquad t^a = \sigma^a/2$

$$V = (A_L + A_R)/2 \qquad A = (A_L - A_R)/2 \qquad m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$$

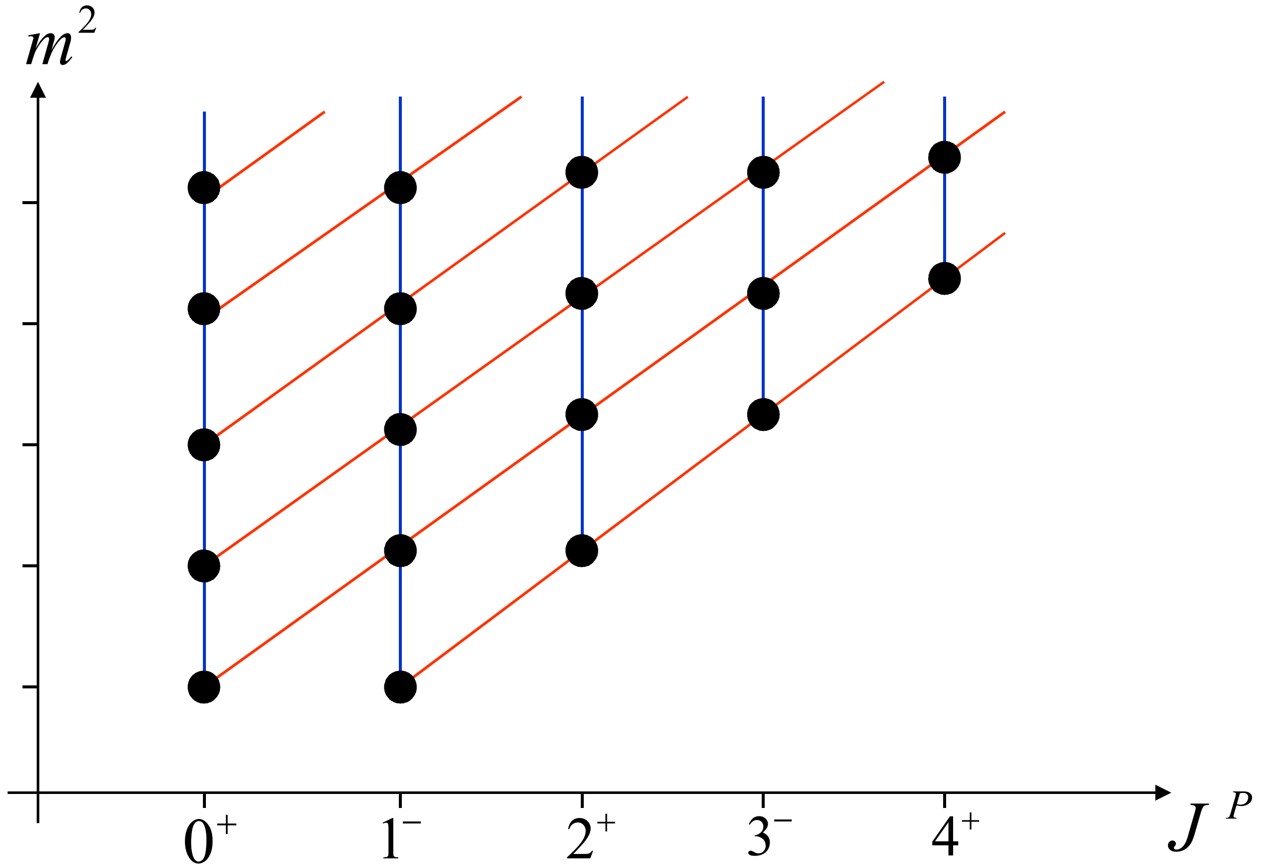
At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	J	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Chiral symmetry breaking – yes

Regge spectrum – no!

Regge and radial Regge <u>linear</u> trajectories



$$m^2(J) = m_0^2 + \alpha' J$$ – Regge trajectories

$$m^2(n) = \mu_0^2 + \alpha n$$ – Radial Regge trajectories

Soft wall model (Karch et al., PRD (2006))

$$\boxed{I = \int d^5x \sqrt{g}\, e^{-\Phi} \mathcal{L}}$$

$$g_{MN}\, dx^M dx^N = e^{2A(z)}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) \qquad \Phi = \Phi(z)$$

The IR boundary condition is that the action is finite at $z = \infty$

Plane wave ansatz: $V_\mu(x,z) = \varepsilon_\mu e^{ipx} v(z)$ $\quad p^2 = m^2$ $\quad$ Axial gauge $V_z = 0$

E.O.M.: $\partial_z \left(e^{-B} \partial_z v_n\right) + m_n^2 e^{-B} v_n = 0$ $\qquad B = \Phi(z) - A(z)$

Substitution $v_n = e^{B/2} \psi_n$

$$\boxed{-\psi_n'' + U(z)\psi_n = m_n^2 \psi_n} \qquad U(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''$$

With the choice $B = \Phi - A = az^2 + \log z$ $\Longrightarrow$ $U = a^2 z^2 + \dfrac{3}{4z^2}$

One has the radial Schroedinger equation for harmonic oscillator with orbital momentum ***L=1***

$$-\psi'' + \left[z^2 + \frac{L^2 - 1/4}{z^2}\right]\psi = E\psi \qquad E = |a|m$$

To have the Regge like spectrum: $\Phi = az^2$

To have the AdS space in UV asymptotics: $A = -\log z$ $\Longrightarrow$ $e^{2A} = \dfrac{1}{z^2}$

The spectrum: $\boxed{m_n^2 = 4|a|(n+1)}$ $\qquad n = 0, 1, 2, \ldots$

The extension to massless higher-spin fields leads to (for ***a > 0***)

$$\boxed{m_{n,J}^2 = 4a(n+J)}$$

Generalization to the arbitrary intercept $m_n^2 = 4|a|(n+1+b)$

$$e^{-az^2} \to \Gamma(1+b)U^2(b,0;az^2)e^{-az^2}$$

(Afonin, PLB (2013))

Tricomi function

Calculation of vector 2-point correlator: $$W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x,\epsilon)]$$

4D Fourier transform

source

$$V^\mu(q,z) = v(q,z)V_0^\mu(q) \qquad v(q,\epsilon) = 1$$

E.O.M.:

$$\partial_z\left(\frac{e^{-az^2}}{z}\partial_z v\right) + \frac{e^{-az^2}}{z}q^2 v = 0$$

Action on the solution

$$\boxed{I = \int d^4x V_0^\mu V_{0\mu}\,\frac{e^{-az^2}}{z}\,v\partial_z v\Big|_{z=\epsilon}^{z=\infty}}$$

$$\int d^4x e^{iqx}\langle J_\mu(x)J_\nu(0)\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(-q^2)$$

$$\Pi_V(-q^2) = c^2\,\frac{\partial_z v}{q^2 z}\Big|_{z=\epsilon}$$

$$v(q,z) = \Gamma\left(1-\frac{q^2}{4|a|}\right)e^{(a-|a|)z^2/2}U\left(\frac{-q^2}{4|a|},0;\,|a|z^2\right)$$

$$\Pi_V(-q^2) = c^2\left[\frac{a-|a|}{q^2} - \frac{1}{2}\psi\left(1-\frac{q^2}{4|a|}\right)\right] + \text{const}$$

$$\boxed{\Pi_V(-q^2) = c^2\left[\frac{a-|a|}{q^2} + \sum_{n=0}^{\infty}\frac{2|a|}{4|a|(n+1)-q^2}\right] + \text{const}}$$

$$\Pi_V(Q^2)_{Q^2\to\infty} = \frac{c^2}{2}\left[\log\left(\frac{4|a|}{Q^2}\right) - \frac{2a}{Q^2} + \frac{4a^2}{3Q^4} + \mathcal{O}\left(\frac{a^4}{Q^8}\right)\right] \qquad Q^2 = -q^2$$

$$\Pi_V(Q^2)_{\text{OPE}} = \frac{N_c}{24\pi^2}\log\left(\frac{\mu^2}{Q^2}\right) + \frac{\alpha_s}{24\pi}\frac{\langle G^2\rangle}{Q^4} + \xi\frac{\langle\bar{q}q\rangle^2}{Q^6} + \mathcal{O}\left(\frac{\mu^8}{Q^8}\right)$$

$$\Rightarrow \qquad c^2 = \frac{N_c}{12\pi^2}$$

Possible extensions

- Various modifications of metrics and of dilaton background
- Alternative descriptions of the chiral symmetry breaking
- Inclusion of additional vertices (Chern-Simon, …)
- Account for backreaction of metrics caused by the condensates (dynamical AdS/QCD models)
- Construction of acceptable AdS/QCD models from a 5D gravitational setup

Some applications

- Meson, baryon and glueball spectra
- Low-energy strong interactions (chiral dynamics)
- Hadronic formfactors
- Description of anomalies in QFT
- Thermodynamic effects (QCD phase diagram)
- Condensed matter (high temperature superconductivity *etc.*)
- ...

Deep relations with other approaches

- Light-front QCD
- Soft wall models: QCD sum rules in the large-N_c limit
- Hard wall models: Chiral perturbation theory supplemented by infinite number of vector and axial-vector mesons
- Renormgroup methods

Thank you
for your attention!