On estimations of the deconfinement temperature in AdS/QCD

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Holography and QCD

The AdS/CFT correspondence between a string theory on $AdS_5 \times S_5$ and SYM theory on ∂AdS_5 can be viewed as a general correspondence between

weakly coupled theories & strongly coupled theories
$\Rightarrow$ a way to find a holographical dual to QCD in the large N_c limit

Thus appeared the AdS/QCD correspondence, which proved to be a fruitful way to study QCD phenomena. In particular, the bottom-up arrroach naturally incorporates some phenomenological features of real QCD once we believe the dual model to be trustworthy.
Of interest for us here are the following issues:

(1) the description of the QCD bound states

- operators $\mathcal{O}(x)$ in $4D$ theory $\Leftrightarrow$ fields $\Phi(x,z)$ in $5D$ dual theory
- canonical dimension Δ of the p-form operator $\mathcal{O}(x) \Leftrightarrow 5D$ mass of $\Phi(x,z)$: $m_5^2R^2 = (\Delta - p)(\Delta + p - 4)$,

  Example 1: vector mesons $\Leftrightarrow \bar{q}\gamma^\mu t^a q$ with $p=1$ and $\Delta = 3 \Leftrightarrow A_\mu^a(x,z)$ with $m_5^2R^2 = 0$

  Example 2: 0^{++} glueballs $\Leftrightarrow G_{\mu\nu}G^{\mu\nu}$ with $p=0$ and $\Delta = 4 \Leftrightarrow \varphi(x,z)$ with $m_5^2R^2 = 0$
- the desired matter content defines $\mathcal{L}_{matter}$ in $5D$

$\longrightarrow$ radial Regge trajectories $\Leftrightarrow$ finding normalizable solutions of $5D$ equations of motion with $q^2 = M^2(n)$, $n = 0, 1, 2, \dots$, that match certain boundary conditions,
$\Phi(x,z) = \sum_{n=0}^{\infty} \phi_n(z)\phi^{(n)}(x)$

(2) the description of thermodynamical properties [Herzog (2007)]

- a universal gravitational part of the $5D$ action $\sim N_c^2$
- $\mathcal{L}_{gravity} = -\frac{1}{2k_g}(\mathcal{R} - 2\Lambda)$, k_g is the coefficient proportional to the $5D$ Newton constant, $\mathcal{R}$ is the Ricci scalar and Λ – the cosmological constant.

$\longrightarrow$ the deconfinement phase transition $\Leftrightarrow$ a Hawking-Page phase transition at T_c between a low temperature thermal AdS space and a high temperature black hole in $5D$

$$S_{5D} = \int d^4x dz \sqrt{-g} f^2(z) \left(\mathcal{L}_{gravity} + \mathcal{L}_{matter}\right)$$

Holographic models are distinguished by the choice of the dilaton background, $f^2(z)$, and the interval the fifth z coordinate spans $z \in [0, z_{max}]$ (it may happen that $z_{max} = \infty$).

Critical temperature in AdS/QCD

Consider evaluating $S_{gravity}$ on different AdS backgrounds:

(1) thermal AdS metric: $ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$, $t \in [0, \beta]$;

(2) metric of AdS with a black hole: $ds^2 = \frac{R^2}{z^2}\left(h(z)dt^2 - d\vec{x}^2 - \frac{dz^2}{h(z)}\right)$,
where $h(z) = 1 - (z/z_h)^4$ and R denotes the AdS radius.

Transition between two backgrounds
$\Longleftrightarrow$ (De)confinement transition

The Hawking temperature is related to the black hole horizon z_h via the relation $T_c = 1/(\pi z_h)$.
The free action densities V identified with the regularized gravitational action are:

$$V_{\rm Th}(\epsilon) = \frac{4R^3}{k_g}\int_0^\beta dt \int_\epsilon^{z_{max}} dz f^2(z) z^{-5}, \quad V_{\rm BH}(\epsilon) = \frac{4R^3}{k_g}\int_0^{\pi z_h} dt \int_\epsilon^{\min(z_{max}, z_h)} dz f^2(z) z^{-5}.$$

The two geometries are compared at a radius $z = \epsilon$ where the periodicity in the time direction is locally the same $\Rightarrow \beta = \pi z_h \sqrt{h(\epsilon)}$.
The order parameter of the phase transition is defined by ΔV:

$$\Delta V = \lim_{\epsilon \to 0}\left(V_{\rm BH}(\epsilon) - V_{\rm Th}(\epsilon)\right)$$

The Hawking-Page phase transition occurs at a point where $\Delta V = 0$.
However, $\Delta V = 0$ yields z_h as a function of model dependent parameters – z_{max} and/or those possibly introduced in $f(z)$. We must appeal to $\mathcal{L}_{matter}$ to give physical meaning to these parameters and to connect T_c to a particular type of a holographic model.

Bottom-up AdS/QCD models

What matter content should we consider?

- vector case: reproducing the masses of non-strange vector mesons
  $J=1 \quad \mathcal{L}_v = -\frac{1}{4g_5^2} g^{MP}g^{NQ}\left(\partial_M A_N - \partial_N A_M\right)\left(\partial_P A_Q - \partial_Q A_P\right)$,
- scalar case: achieving accordance with assumed glueball masses
  $J=0 \quad \mathcal{L}_{sc} = \frac{1}{2k_s}\left(g^{MN}\partial_M\varphi\partial_N\varphi - m_5^2\varphi^2\right)$,

ρ meson
vs
0^{++} glueball

Which holographic model to choose?

- Hard Wall (HW) model [Erlich *et al.* (2005)]: z_{max} is finite and $f^2(z) = 1$

$$m_{HW}^{J=1} = M_{J=1}(0) = \frac{2.405}{z_{max}}, \quad m_{HW}^{J=0} = M_{J=0}(0) = \frac{3.832}{z_{max}}$$
$$T_{HW}^{J=1} = 0.1574 m_{HW}^{J=1}, \quad T_{HW}^{J=0} = 0.0988 m_{HW}^{J=0}.$$

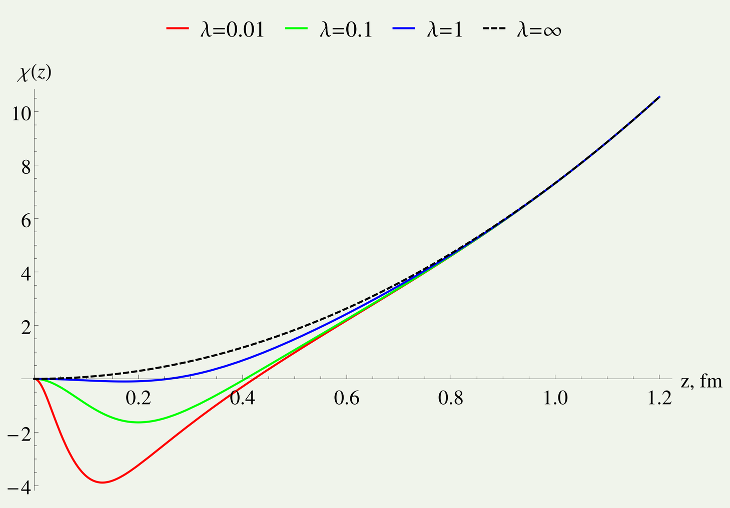
- Soft Wall (SW) model [Karch *et al.* (2006)]: $z_{max} = \infty$ and $f^2(z) = e^{-\kappa^2 z^2}$

$$M_{J=1}^2(n) = 4\kappa^2(n+1), \quad M_{J=0}^2(n) = 4\kappa^2(n+2),$$
$$T_{SW}^{\text{any } J} \simeq 0.4917 \cdot \kappa.$$

- Generalized Soft Wall (GSW) model [Afonin (2013)]:
  $z_{max} = \infty$ and $f^2(z) = e^{-\kappa^2z^2}U^2(b, J-1; \kappa^2z^2)$, where U is the Tricomi hypergeometric function introducing a free intercept parameter b in the spectrum while keeping SW asymptotes in UV and IR, $b = 0$ reduces GSW to SW

$$M_{J=1}^2(n) = 4\kappa^2(n+1+b), \quad M_{J=0}^2(n) = 4\kappa^2(n+2+b)$$
$$T_{GSW}^{J=1}/\kappa \simeq 0.670 \cdot b + 0.496, \quad T_{GSW}^{J=0}/\kappa \simeq 0.123 \cdot b + 0.314.$$

Isospectral models



An idea from SUSY quantum mechanics $\Rightarrow$ a family of strictly isospectral potentials associated with a given potential for Schrödinger-type equations:

$$-\psi_n''(z) + \hat{\mathcal{V}}_\lambda(z)\psi_n(z) = M^2(n)\psi_n(z)$$

The family members are distinguished through a parameter λ, with $\lambda = \infty$ corresponding to the original potential.

Usage: generating a family of the dilaton profiles realted to the original SW $\chi(z) = \kappa^2 z^2$, thus constructing new models while keeping the spectrum fixed [Vega and Cabrera (2016)].

Lattice and experimental data

(I) the deconfinement temperature T_c

- $\rightarrow$ lattice with physical quarks [Borsanyi *et al.* (2010)]: 150 – 170 MeV;
- $\rightarrow$ lattice with non-dynamical quarks and $N_c \to \infty$ [Lucini *et al.* (2012)]: ~ 250 MeV;
- $\rightarrow$ lattice for $SU(3)$ theory [Boyd *et al.* (1996); Iwasaki *et al.* (1997)]: 260 – 270 MeV;
- $\rightarrow$ experimental results favour the range of 150 – 160 MeV.

(II) 0^{++} glueball masses

- $\rightarrow$ quenched lattice: 1.5 or 1.7 GeV;
- $\rightarrow$ unquenched lattice: 1.8 GeV;
- $\rightarrow$ lattice approximation for $N_c \to \infty$;
- $\rightarrow$ radial excitations from lattice - not more than 2 states usually;
- $\rightarrow$ state among $f_0(1370)$, $f_0(1500)$, $f_0(1710)$.

(III) identification of ρ radial excitations; the idea of the universal slope for the light mesons

Holographic estimations

T_c from the vector channel:

Within the isospectral family T_c changes significantly.
For example: for the lightest ρ meson fit we get 142 MeV at $\lambda = 1$ and 112 MeV at $\lambda = 0.01$.

Fit & Model	T_c (MeV)		
	ρ	ω	'universal' traj.†
lightest state & HW	122	123	'168'
lightest state & SW	191	192	263
$n = 0,1,2$ & GSW	118	121	–

'universal' traj.† – the assumption of an equal mean slope for various radial trajectories of light non-strange mesons [Bugg (2004)].

T_c from the scalar glueball channel:

In large N_c limit: $\left.\frac{T_c}{m_{gl}}\right|_{lattice} = 0.1799$, $\left.\frac{T_c}{m_{gl}}\right|_{HW} = 0.0988$, $\left.\frac{T_c}{m_{gl}}\right|_{SW} = 0.1739$.

Fit	m_{gl} (MeV)	T_{HW} (MeV)	T_{SW} (MeV)		
			$\lambda = \infty$	$\lambda = 1$	$\lambda = 0.1$
Morningstar and Peardon (1999)	1730(100)	171(10)	301(17)	253(15)	173(10)
Meyer (2004)	1475(75)	146(7)	256(13)	215(11)	147(8)
Chen *et al.* (2006)	1710(95)	169(9)	297(17)	250(14)	171(10)
large N_c: Lucini *et al.* (2004)	1455(70)	144(7)	253(12)	212(10)	145(7)
unquenched: Gregory *et al.* (2012)	1795(60)	177(6)	312(10)	262(9)	179(6)
$f_0(1500)$ meson Close and Zhao (2005)	1464(47)	145(5)	255(8)	214(7)	146(5)
	1519(41)	150(4)	264(7)	222(6)	152(4)
$f_0(1710)$ meson Cheng *et al.* (2015)	1674(14)	165(1)	291(2)	244(2)	167(1)

$\Leftarrow 0^{++}$ glueball

radial glueball trajectory fit
$\Downarrow$

Fit	$\sqrt{\sigma}$ or r_0^{-1} (MeV)	$m^2 = 4\kappa^2(n+2+b)$		T_{GSW} (MeV)		
		κ (MeV)	b	$\lambda = \infty$	$\lambda = 1$	$\lambda = 0.1$
Morningstar and Peardon (1999)	410	1017(151)	−1.28(0.23)	154(39)	151(36)	150(34)
Meyer (2004)	440	1094(49)	−1.54(0.07)	133(10)	132(10)	132(9)
unquenched: Gregory *et al.* (2012)	420	1652(138)	−1.71(0.05)	175(16)	175(16)	175(16)
large N_c: Lucini *et al.* (2004)	440	1120(88)	−1.58(0.08)	131(14)	131(13)	131(13)
large N_c: Meyer (2004)	440	735(121)	−1.00(0.35)	143(55)	135(44)	130(37)

Conclusions

This work attempts to clarify what input parameters of the bottom-up models should be selected to make a conceptually and phenomenologically sensible prediction of T_c.

- Minimal option: the 'universal' slope value for the radial trajectories of light non-strange mesons. That means a fixation of the dilaton parameter in $f^2(z) = e^{-\kappa^2z^2}$ to $\kappa \simeq 530$ MeV, that coincides with the case of $f_0(1500)$ being predominantly a 0^{++} glueball. This results in $T_c \simeq 260$ MeV corresponding exceedingly well to the lattice results for pure $SU(3)$ and large N_c limit.
- HW and SW reproducing the masses of the ground states. 0^{++} glueball candidates within the range of 1.5 – 1.7 GeV give $T_c \in 260 - 290$ MeV in SW and 150 – 170 MeV in HW. For the ρ or ω mesons we get ~ 190 MeV in SW and ~ 120 MeV in HW. Clearly the results of HW and SW differ, though for the glueballs they appear to coincide with lattice expectations in different regimes.
- GSW allows the consideration of many excited stated. Different vector meson spectra may result in various predictions for T_c, not providing a clear way to select the best fit. Additionally, a lot of variation through the isospectral family. For the scalar glueballs: the isothermality is automatically achieved, and the predicted values are close to the unquenched lattice estimations of T_c.