

Hadron Production in High Energy Particle Collisions

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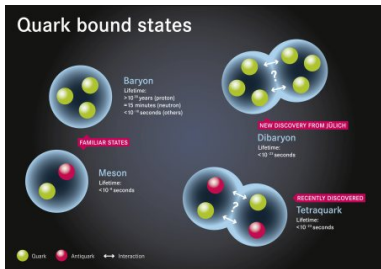
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Contents

- 1 Introduction
- 2 Quark-hadron duality and hadronization
- 3 Flux tube (strings)
- 4 $QCD_4 \rightarrow QCD_{xy} + QCD_{zt}$ compactification
- 5 Hadron production
- 6 Probability
- 7 Hadron rate
- 8 Relation to the experiment
- 9 Conclusion

Introduction



Hadronization models:

1) Local parton-hadron duality approach

(i) String model (1+1) tube, Lund model

(ii) Cluster model (fragmentation from the colorless clusters)

(iii) Glassma (fragmentation from the color condensate)

2) Dynamic approach

(i) Running gluon mass in the pure gluodynamic picture

ii) Renorm. group approach

Quark-hadron duality and hadronization

$$\mathcal{M} = \langle out | in \rangle = \langle \bar{q}_{HAD} | q_{QGP} \rangle = \langle \bar{q}_{QGP} | U | q_{QGP} \rangle \quad (1)$$

$$\langle |\mathcal{M}|^2 \rangle = Tr |(\gamma^0 U) G^{-+}(x_1, x_2)|^2 \quad (2)$$

$$\langle |\mathcal{M}|^2 \rangle = -Tr \{ G_{21}^{-+} \varrho(1, 2) G_{12}^{-+} \};$$

$$\varrho(1, 2) = \gamma^0 U_1 U_2^\dagger \gamma^0 \quad (3)$$

Quark-hadron duality and hadronization

When $G_{21}^{-+}; \varrho(1, 2); G_{12}^{-+} \rightarrow G^{-+}, \varrho(x_1 - x_2)$

In the momentum representation

$$\langle |\mathcal{M}|^2 \rangle = -Tr \left\{ \int \frac{dp_1 dp_2}{(2\pi)^8} \bar{G}^{-+}(p_1) \varrho(p_1, p_2) G^{-+}(p_2) \right\}, \quad (4)$$

$$\frac{d \langle |\mathcal{M}|^2 \rangle}{dp} = -Tr \left\{ \int \frac{dq}{(2\pi)^8} \bar{G}^{-+}(p + q/2) \varrho(p + q/2, p - q/2) G^{-+}(p - q/2) \right\} \quad (5)$$

$$p = \frac{p_1 + p_2}{2}; \quad q = p_1 - p_2, \quad (6)$$

Quark-hadron duality and hadronization

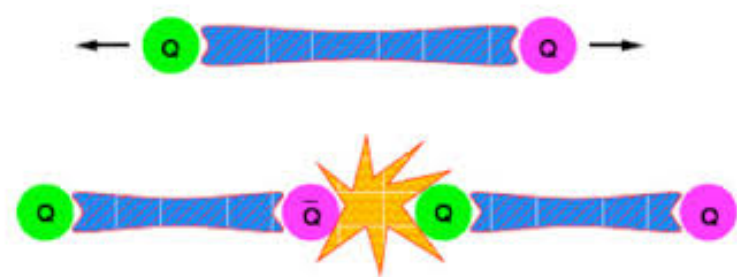
Diagrammally

, (7)

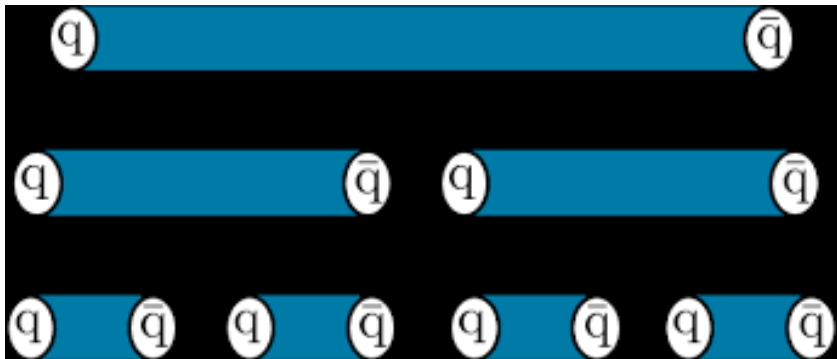
$$\frac{E(\mathbf{p})dN_h}{d^3p} = \int dp^0 (E(\mathbf{p})) \frac{d \langle |\mathcal{M}|^2 \rangle}{dp}, \quad (8)$$

Flux tube picture

$$\sqrt{s} \simeq 8 - 20 \text{ GeV} \quad (9)$$

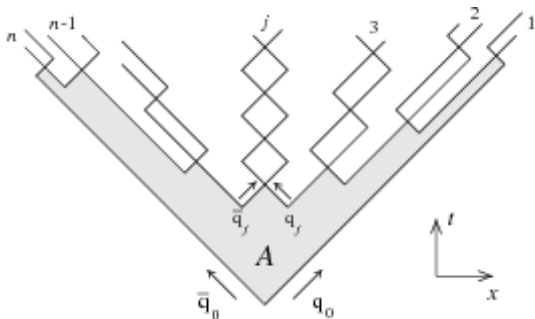


Flux tube pictutre

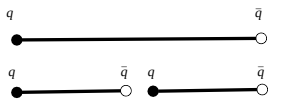
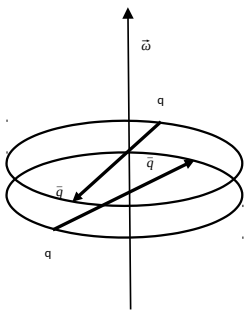


Flux tube pictutre

I,II have taken into account in LUND model.



Flux tube picture



Flux tube picture

1. Supported in experiments on hadronic, e^+e^- and pp collisions
2. Theoretical background - E. Whitten papers

$$j^\mu = -m\varepsilon^{\mu\nu}\partial_\nu\phi, \quad (10)$$

Conclusion

- I. The longitudinal dominance
- II. The transverse confinement

$QCD_4 \rightarrow QCD_{xy} + QCD_{zt}$ compactification

HOW TO GET IT

1. To start from the exact (3+1) QCD
2. To reduce it up to (1+1) QCD

$QCD_4 \rightarrow QCD_{xy} + QCD_{zt}$ compactification

$QCD_4 \rightarrow QCD_{xy} + QCD_{zt}$ compactification
(A.V.Koshelkin, C.-Y.Wong)

$$\Psi(x) = \Psi(\mathbf{r}_\perp)\psi(z, t), \quad (11)$$

Transverse motion

$$\begin{aligned} (p_1 + ip_2)\Psi_+(\mathbf{r}_\perp) &= (m(\mathbf{r}_\perp) + E_\nu^2)\Psi_-(\mathbf{r}_\perp), \\ (p_1 - ip_2)\Psi_-(\mathbf{r}_\perp) &= (E_\nu^2 - m(\mathbf{r}_\perp))\Psi_+(\mathbf{r}_\perp), \end{aligned} \quad (12)$$

Longitudinal motion

$$\partial_z^2 \psi(t, z) - \partial_t^2 \psi(t, z) = m_{qT}^2 \psi(t, z), \quad (13)$$

$QCD_4 \rightarrow QCD_{xy} + QCD_{zt}$ compactification

$$\tau = \sqrt{t^2 - z^2}, \quad \eta = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right) \quad (14)$$

$$\tau^2 \frac{\partial^2 \psi(\tau, \eta)}{\partial \tau^2} + \tau \frac{\partial \psi(\tau, \eta)}{\partial \tau} - \frac{\partial^2 \psi(\tau, \eta)}{\partial \eta^2} = 0. \quad (15)$$

$$\psi_+(\tau, \eta) = f(\eta - \ln(\tau/\tau_0)); \quad \psi_-(\tau, \eta) = g(\eta + \ln(\tau/\tau_0))$$

$$\psi(\tau = \tau_0, \eta) = \frac{1}{\sqrt{\sigma\pi^{1/2}}} \exp \left(-\frac{\eta^2}{2\sigma^2} \right), \quad (16)$$

$$\psi_{\pm}(\tau, \eta) = \frac{1}{\sqrt{\sigma\pi^{1/2}}} \exp \left(-\frac{(\eta \mp \ln(\tau/\tau_0))^2}{2\sigma^2} \right). \quad (17)$$

Hadron production

$$\frac{E(\mathbf{p})dN_h}{d^3p} = -Tr \int dp^0(E(\mathbf{p})) \int \frac{dq}{(2\pi)^8} \times \left\{ \bar{G}^{-+}(p+q/2)\varrho(p+q/2, p-q/2)G^{-+}(p-q/2) \right\}. \quad (18)$$

$$G^{-+}(p) = 2\pi i |\Psi(\mathbf{p})|^2 \delta(p^0 - \mu + \varepsilon(\mathbf{p})). \quad (19)$$

Hadron production

I. The longitudinal dominance

II. The transverse confinement

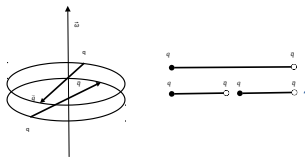
$$\frac{E(\mathbf{p})dN_h}{d^3p} = \sum_{a=1}^N \int \frac{d^2q_{\perp}}{(2\pi)^6} (E(\mathbf{p})) \varrho_a(\mathbf{p}; \mathbf{q}) \times$$

$$|\Psi_{\bar{q}_a}(\mathbf{p}_{\perp} + \mathbf{q}_{\perp}/2)|^2 |\Psi_{q_a}(\mathbf{p}_{\perp} - \mathbf{q}_{\perp}/2)|^2 \times$$

$$\int dq_z |\psi_{\bar{q}_a}(p_z + q_z/2)|^2 |\psi_{q_a}(p_z - q_z/2)|^2, \quad (20)$$

Hadron production

$$\rho_a(\mathbf{p}, q_z) = \int \frac{d^2 q_{\perp}}{(2\pi)^6} E(\mathbf{p}) \varrho_a(\mathbf{p}; \mathbf{q}) |\Psi_{\bar{q}_a}(\mathbf{p}_{\perp} + \frac{\mathbf{q}_{\perp}}{2})|^2 |\Psi_{q_a}(\mathbf{p}_{\perp} - \frac{\mathbf{q}_{\perp}}{2})|^2 \quad (21)$$



$$\rho_a(\mathbf{p}, q_z) = \mathcal{P}_a(\mathbf{p}) \delta(q_z). \quad (22)$$

Hadron production

$$\frac{dN_h}{d^2pdy} = \sum_{a=1}^N \mathcal{P}_a(\mathbf{p}_\perp, y) |\Psi_{\bar{q}_a}(y)|^2 |\Psi_{q_a}(y)|^2. \quad (23)$$

$$y = \frac{1}{2} \ln \left(\frac{E(\mathbf{p}) + p_z}{E(\mathbf{p}) - p_z} \right). \quad (24)$$

Probability

$$\mathcal{P}_a(\mathbf{p}_T) = A \exp\left(-\frac{\Delta G}{T}\right) = A \exp\left(-\frac{E + \alpha \ln\left(\frac{C_{qh}(P, T)}{C_{qq}(P, T)}\right)}{T}\right) \quad (25)$$

$$\Delta G^{(a)}(E_T(p_T = 0)) = m_h \sqrt{g_{00}} = m_h \sqrt{1 - v^2}, \quad (26)$$

where m_h is the mass of a hadron, g_{00} is the zeroth component of the metric tensor $g_{\mu\nu}$ for the rotating frame of reference.

Probability

$$\Delta G^{(a)}(E_T(p_T = 0)) = g_{\mu\nu} U^\mu p^\nu, \quad (27)$$

where $p^\nu = (m_h, 0)$, U^μ is the hydrodynamic velocity

$$U^\mu = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^1}{\sqrt{1-v^2}}, \frac{v^2}{\sqrt{1-v^2}}, 0 \right), \quad v^2 = (v^1)^2 + (v^2)^2. \quad (28)$$

Since the created hadrons are assumed to leave instantly the hadronization area, there is no motion of the hadronic matter as a whole in the laboratory frame of reference. Therefore, we have for the hydrodynamic velocity in this frame of reference

$$U^\mu = (1, 0, 0, 0). \quad (29)$$

Probability

Taking into account that $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ in the laboratory frame of reference we find

$$\Delta G^{(a)} = E_T, \quad (30)$$

where $E_T = \sqrt{\mathbf{p}_T^2 + m_h^2}$ is the transverse energy of a hadron,
 $\mathbf{p}_T = m_h \mathbf{v} / \sqrt{1 - v^2}$.

Probability

When matter moves along the collision axes with a velocity v^3 , we have in the laboratory frame

$$p^\nu = (E_T \cosh y_{v^3}, 0, 0, E_T \sinh y_{v^3}). \quad (31)$$

However, then

$$U^\nu = (\cosh y_{v^3}, 0, 0, \sinh y_{v^3}). \quad (32)$$

Since $g_{\mu\nu}$ still is $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$

$$\Delta G^{(a)} = g_{\mu\nu} U^\mu p^\nu = E_T, \quad (33)$$

where $E_T = \sqrt{\mathbf{p}_T^2 + m_h^2}$ is the transverse energy of a hadron,
 $\mathbf{p}_T = m_h \mathbf{v} / \sqrt{1 - v^2}$.

Hadron rate

$$\frac{dN_h}{dyd^2p_\perp} = \frac{N_f N_c^2 \theta(T_c - T)}{512\pi^8 m_h T K_1(m_h/T)} \exp\left(-\frac{E_T}{T}\right) \sum_{a=1}^N \frac{\Sigma_a^2}{\sigma_a^2} \left\{ \exp\left(-\frac{2(y-y_a)^2}{\sigma_a^2}\right) + \exp\left(-\frac{2(y+y_a)^2}{\sigma_a^2}\right) \right\} \quad (34)$$

Hadron rate

$$\frac{dN_h}{dy} = \frac{N_f N_c^2 \theta(T_c - T)(1 + (T/m_h))}{256\pi^7 K_1(m_h/T) \exp(m_h/T)}$$
$$\sum_{a=1}^N \frac{1}{\sigma_a^2} \left\{ \exp\left(-\frac{2(y - y_a)^2}{\sigma_a^2}\right) + \exp\left(-\frac{2(y + y_a)^2}{\sigma_a^2}\right) \right\} \quad (35)$$

Relation to the experiment

N.Abgrall et al, Eur.Phys.J. **C74**, 2794 (2014).

The yield of the negatively charged pions, arising in the result of inelastic pp interaction of the target protons and the incident protons, having the momenta 20, 31, 40, 80, 158 GeV/c (which are $\sqrt{s} = 6.3, 7.7, 8.8, 12.3, 17.3$ GeV), has been measured at the beam rapidities $y_b = 1.877, 2.094, 2.223, 2.569, 2.909$.

Relation to the experiment

$$\frac{dN_h(m_T, y=0)}{dym_T dm_T} = \frac{2 \langle N_{ch}(s) \rangle \exp\left(m_h/T - \frac{4y_0^2}{y_b^2}\right)}{(2\pi)^{1/2} \sigma m_h T (1 + (T/m_h))} \exp\left(-\frac{m_T}{T}\right) \quad (36)$$

$$\frac{dN_h}{dy} = \frac{\langle N_{ch}(s) \rangle}{\pi^{1/2} y_b} \left\{ \exp\left(-\frac{4(y-y_0)^2}{y_b^2}\right) + \exp\left(-\frac{4(y+y_0)^2}{y_b^2}\right) \right\} \quad (37)$$

Relation to the experiment

$$\zeta\sqrt{s} = \int E_T \cosh y \frac{dN_h}{dyd^2p_\perp} =$$

$$\langle N_{ch}(s) \rangle m_h \cosh(y_0) \exp\left(\frac{\sigma^2}{8}\right) \frac{1 + 2(T/m_h) + 2(T/m_h)^2}{(1 + (T/m_h))} \quad (38)$$

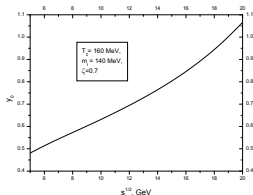


Figure: 1. Dependence of y_0 on the energy of a proton beam.

Relation to the experiment

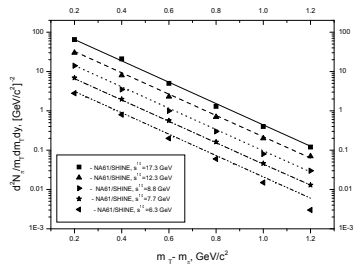


Figure: 2. The lines of various types are the p_T distributions of pions which are given by Eq.(11) at $T_c = 160\text{MeV}$ and $\kappa = 1\text{GeV}/F$, which are normalized by the experimental value of the pion rate at $(m_T - m_\pi) = 0.2\text{GeV}/c$ v.s. the pion rate in $p - p$ collisions (the scattered symbols) at the same projectile energies.

Relation to the experiment

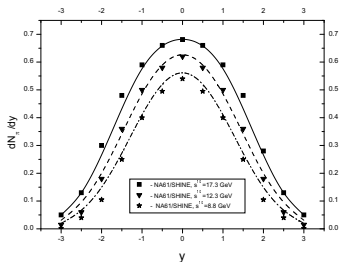


Figure: 3. The rapidity distributions given by Eq.(12) at $\sqrt{s} = 17,3\text{GeV}$ (solid lines), $\sqrt{s} = 12,3\text{GeV}$ (dashed line), $\sqrt{s} = 8,8\text{GeV}$ (dot-dashed line), and at $T_c = 160\text{MeV}$ v.s. the rapidity distributions in $p - p$ collisions

Conclusion

1. Based on the quark-hadron duality concept the new approach to the problem hadronization of the deconfinement is developed.
2. Under the longitudinal dominance and transverse confinement both the rapidity and p_T distributions of the hadron created in collisions of high energy particles are explicitly derived when the hadronization is governed by the first order phase transition.
3. The pion production in high energy proton collisions is studied in detail.
4. The derived hadron rate has been compared with the experimental results on pion production in pp collisions. A good relation to the experimental data is found along the whole range of the energies, $\sqrt{s} = 6.3 - 17.3\text{GeV}$, of the proton projectile used in the experiments.

Acknowledgment

THANK YOU FOR YOUR ATTENTION!!!