QCD phase diagram: baryon density, isospin and chiral imbalance

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Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

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in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
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details can be found in

JHEP 06 (2020) 148 arXiv:2003.10562 [hep-ph]
Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

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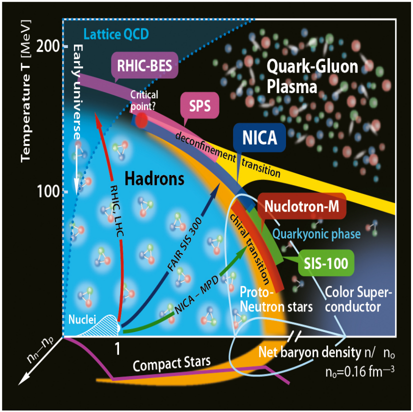
БАЗИС
Фонд развития теоретической физики и математики

QCD Dhase Diagram and Methods

QCD at T and μ
(QCD at extreme conditions)

- neutron stars
- heavy ion collisions
- Early Universe



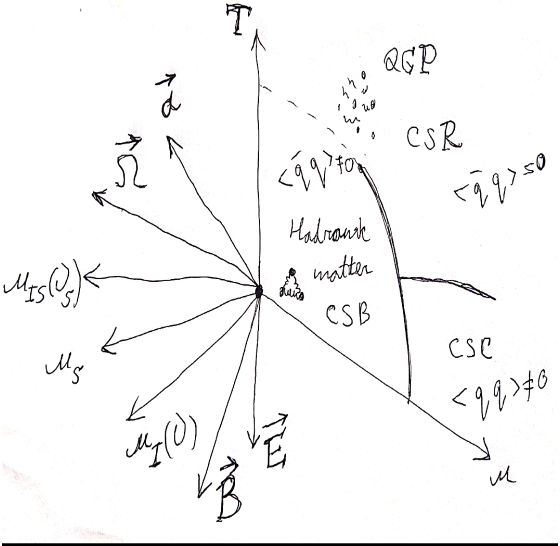
Methods of dealing with QCD

- First principle calcultion – lattice QCD
- Effective models
- DSE, FRG
- .....

More external conditions to QCD

More than just QCD at (μ, T)

- **more chemical potentials** μ_i
- magnetic fields
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More than just QCD at (μ, T)

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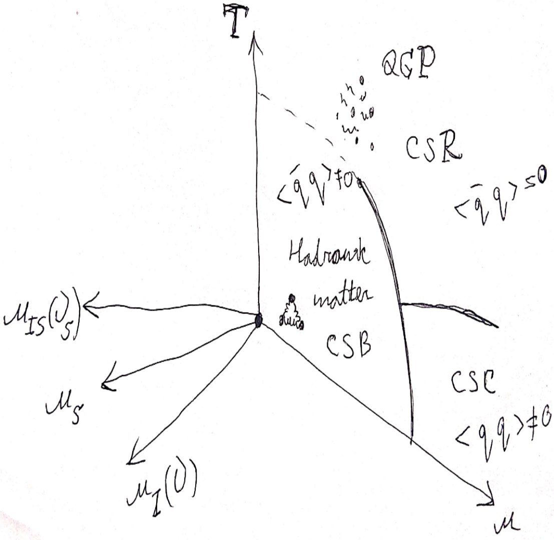

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Baryon chemical potential μ_B

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$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q, \qquad n_B = \frac{1}{3}(n_u + n_d)$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

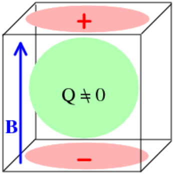
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

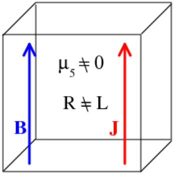
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

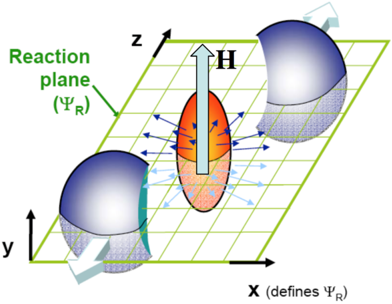
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q}\gamma^0\gamma^5 q$$

Chiral magnetic effect

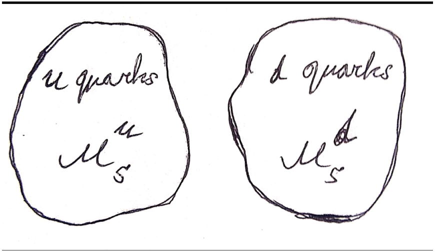






$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

Chiral isospin imbalance n_{I5} and μ_{I5} generation

- Chiral isospin imbalance n_{I5} and hence μ_{I5} can be **generated by** $\vec{E} \,||\, \vec{B}$

  (*for* n_5 *and* μ_5 *M. Ruggieri, M. Chernodub, H. Warringa et al*)

- in **dense quark matter**
  - Chiral separation effect
    (Thanks for the idea to Igor Shovkovy)
  - Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

NJL model can be considered as **effective model for QCD**.

the model is **nonrenormalizable**

Valid up to $E < \Lambda \approx 1$ GeV $\qquad \mu, T < 600\,\mathrm{MeV}$

Parameters G, Λ, m_0 chiral limit $m_0 = 0$

dof– **quarks**, no gluons only **four-fermion interaction**

attractive feature — dynamical CSB

the main drawback – lack of confinement (PNJL)

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets;
τ_k $(k = 1, 2, 3)$ are Pauli matrices.

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\mathrm{i}\partial\!\!\!/ + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi_a^2\Big].$$

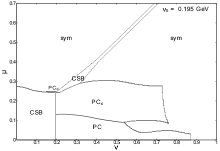
$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

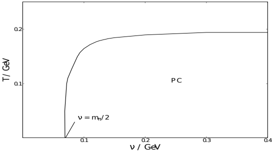
Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates

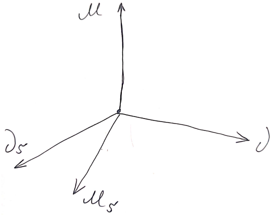
$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0.$$

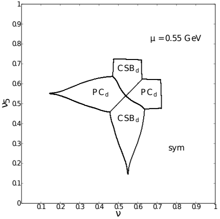
where M and π are already constant quantities.

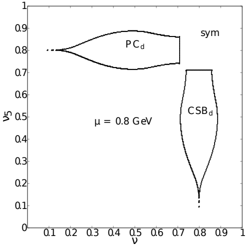
Phase diagram, lots of plots











Chiral imbalance leads to the generation of PC in dense quark matter (PC_d)

Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$

$\Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \nu, \nu_5, ..., M, \pi, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \pi, \nu, \nu_5)$$

$$M \longleftrightarrow \pi, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

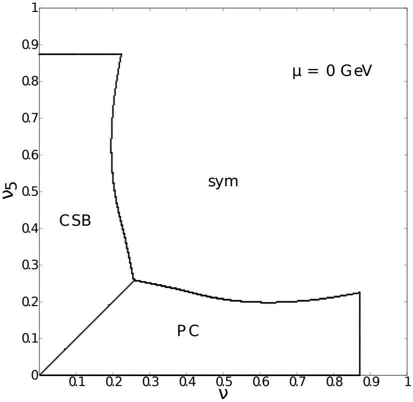


Figure: NJL model results

$$\Omega(M, \pi, \nu, \nu_5) = \Omega(\pi, M, \nu_5, \nu)$$

$$\mathcal{D}: \; M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

Duality was found in

- In the framework of NJL model
- In the leading order of **large** N_c approximation or in **mean field**
- In the chiral limit

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d} \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}$$

$$\mathcal{L}_{\text{NJL}} = \sum_{f=u,d} \bar{q}_f\left[\mathrm{i}\gamma^\nu\partial_\nu - m_f\right]q_f + \frac{G}{N_c}\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\right]$$

m_f is **current quark masses**, m_f : $\frac{m_u+m_d}{2} \approx 3.5\text{MeV}$

typical values of $\mu, \nu, ..., T, .. \sim 10 - 100s$ MeV, for example, 200-400 MeV

One can work in the chiral limit $m_f \to 0$

- $m_f = 0$ $\to$ $m_\pi = 0$
- physical m_f a few MeV $\to$ physical $m_\pi \sim 140$ MeV

In the chiral limit $m_f = 0$ the Duality $\mathcal{D}$ is exact

Duality between CSB and PC is **approximate** in **physical point**

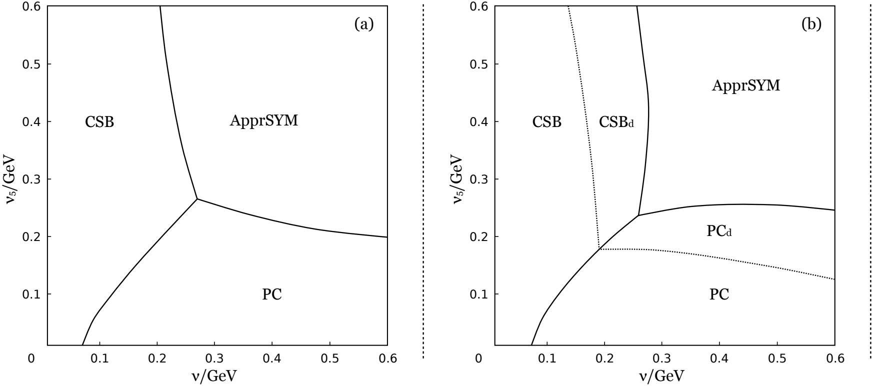


Figure: (ν, ν_5) phase diagram

Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

$\mu_B \neq 0$ impossible on lattice but if

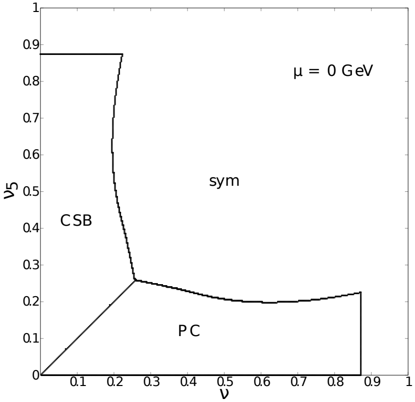
$$\mu_B = 0$$

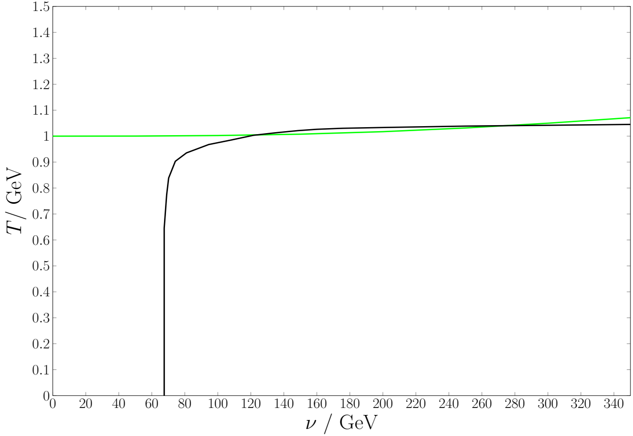
- **QCD at** μ_5 —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()





T_c^M as a function of μ_5 (green line) and $T_c^\pi(\nu)$ (black)

Uses of Dualities

How (if at all) it can be used

discussed in Particles 2020, 3(1), 62-79

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

Circumvent the sign problem

- **Large N_c orbifold equivalences connect gauge theories with different gauge groups and matter content in the large N_c limit.**

M. Hanada and N. Yamamoto, JHEP 1202 (2012) 138, PoS LATTICE **2011** (2011)

Duality

$$\text{QCD at } \mu_1 \longleftrightarrow \text{QCD at } \mu_2$$

- QCD with μ_2 —- sign problem free,
- QCD with μ_1 —- sign problem (no lattice)

$$\text{Investigations of (QCD with } \mu_2)_{\text{on lattice}} \Longrightarrow \text{(QCD with } \mu_1)$$

Inhomogeneous phases (case)

Homogeneous case

In vacuum the quantities $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on space coordinate x.

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_\pm(x)\rangle = \pi, \quad \langle\pi_3(x)\rangle = 0.$$

In a dense medium $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ might depend on spatial coordinates

CDW ansatz for CSB, the single-plane-wave LOFF ansatz for PC

$$\langle\sigma(x)\rangle = M\cos(2kx^1), \quad \langle\pi_3(x)\rangle = M\sin(2kx^1),$$
$$\langle\pi_1(x)\rangle = \pi\cos(2k'x^1), \quad \langle\pi_2(x)\rangle = \pi\sin(2k'x^1)$$

Duality in inhomogeneous case is shown

$$\mathcal{D}_I : \qquad M \longleftrightarrow \pi, \quad \nu \longleftrightarrow \nu_5, \quad k \longleftrightarrow k'$$

schematic (ν_5, μ)-phase diagram

- exchange axis ν to the axis ν_5,
- rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

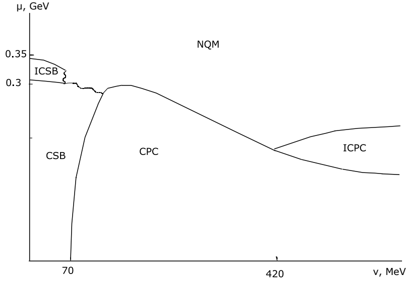


Figure: (ν, μ)-phase diagram.

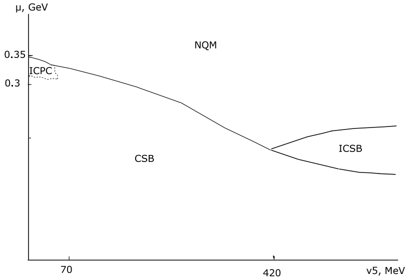


Figure: (ν_5, μ)-phase diagram

M. Buballa, S. Carignano, J. Wambach, D. Nowakovski, Lianyi He et al.

Two colour QCD case

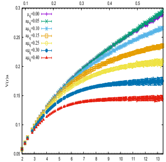
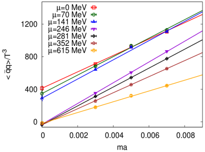
QC_2D

There are similar transitions:

- confinement/deconfinement
- chiral symmetry breaking/restoration

At large μ

- deconfinement
- chiral symmetry is restored

At large T

- deconfinement
- chiral symmetry is restored

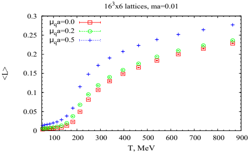


Figure 1 Polyakov loop as a function of T for three values of the baryon chemical potential.

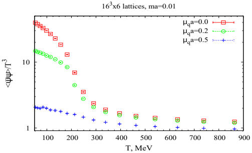


Figure 2: Chiral condensate as a function of T for three values of the baryon chemical potential. The ordinate axis is given on a logarithmic scale.

A lot of quantities coincide up to few dozens percent

SU(2)	SU(3)

Critical temperature

Phys. Lett. B712 (2012) 279-283, JHEP 02 (2005) 033

$T_c/\sqrt{\sigma}$=0.7092(36) $T_c/\sqrt{\sigma}$=0.6462(30)

Topological susceptibility

Nucl.Phys.B 715 (2005) 461-482

$\chi^{\frac{1}{4}}/\sqrt{\sigma}$=0.3928(40) $\chi^{\frac{1}{4}}/\sqrt{\sigma}$=0.4001(35)

Shear viscosity

JHEP 1509 (2015) 082, Phys.Rev. D 76 (2007) 101701

η/s=0.134(57) η/s= 0.102(56)

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

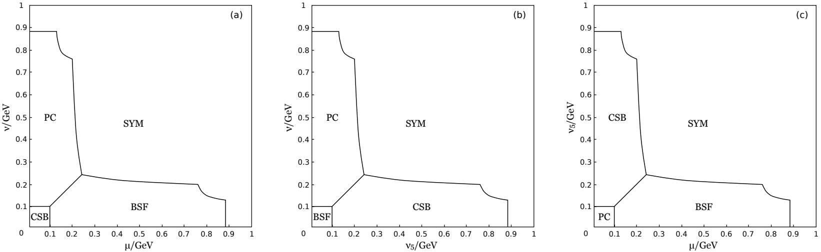
If you use Habbard-Stratanovich technique and auxiliary fileds

$$\sigma(x) = -2H(\bar{q}q),\ \vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q)$$
$$\Delta(x) = -2H\Big[\overline{q^c}i\gamma^5\sigma_2\tau_2 q\Big] = -2H\Big[q^T C i\gamma^5\sigma_2\tau_2 q\Big]$$
$$\Delta^*(x) = -2H\Big[\bar{q}i\gamma^5\sigma_2\tau_2 q^c\Big] = -2H\Big[\bar{q}i\gamma^5\sigma_2\tau_2 C\bar{q}^T\Big]$$

Condensates are

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi_1, \quad \langle\Delta(x)\rangle = \Delta, \quad \langle\Delta^*(x)\rangle = \Delta^*.$$

Dualities in QC_2D



$$(a) \qquad \mathcal{D}_1: \qquad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|$$

$$(b) \qquad \mathcal{D}_3: \qquad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1$$

$$(c) \qquad \mathcal{D}_2: \qquad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|$$

- Based on the **duality** one can show that there is **no mixed phase**, i.e. two non-zero condensates simultaneously.

  This greatly simplifies the numeric calculations.

- Phase diagram is **highly symmetric** due to **dualities**

  The **whole phase diagram**, including diquark condensation, **in two color case** can be obtained from the results of **three color case** without any diquark condensation.

Dualities $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$ were found in

- In the framework of NJL model

- In the large N_c approximation (or mean field)

$$\mathcal{D}_3: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad\qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \qquad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$
$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \qquad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \qquad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

There is also $\mathcal{D}_1$ and $\mathcal{D}_2$

Dualities were found in

- In the framework of NJL model non-pertubartively (or beyond mean field)

- In QC_2D non-pertubartively (at the level of Lagrangian)

Duality in $SU(3)$ QCD

QCD Lagrangian is

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + \bar{\psi}\Big[\mu\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]\psi$$

$$\mathcal{D}: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad\qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \quad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \quad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \quad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

Duality was found in

- In the framework of NJL model non-pertubartively

  (beyond mean field or at all orders of N_c approximation)

- In QCD non-pertubartively (at the level of Lagrangian)

$$\mathcal{D} \;\in\; \quad SU_R(2) \quad \in\; SU_L(2) \times SU_R(2)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$M \neq 0$ breaks the chiral symmetry

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Other Dualities

They are not that strong but still...

They could still be usefull

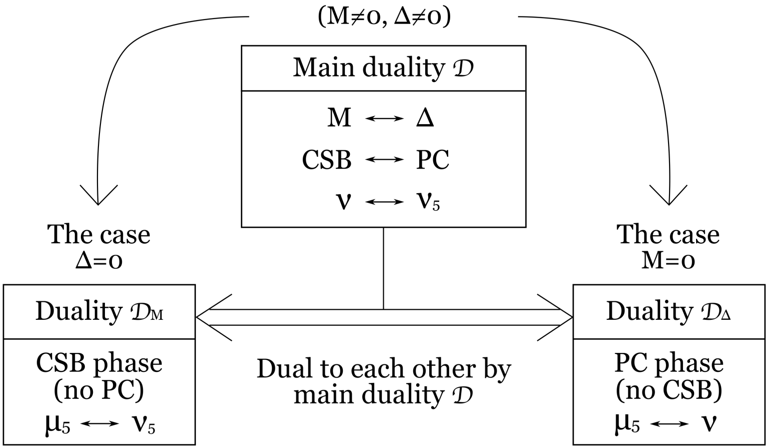


Figure: Dualities

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied
  PC *in dense matter with chiral imbalance*
  also in dense electic neutral matter in β-equilibrium
- It was shown that there exist dualities in QCD and QC_2D

  Richer structure of Dualities in the two colour case
- There have been shown ideas how dualities can be used

  Duality is not just entertaining mathematical property but an instrument with very high predictivity power
- Dualities have been shown non-perturbetively in the two colour case
- Duality have been shown non-perturbarively in QCD