Singular background in a model of material plane interacting with Dirac particles

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Introduction

The proposed by Symanzik approach for modeling of interaction of a macroscopic material body with quantum fields is considered. Its application in quantum electrodynamics enables one to establish the most general form of the action functional describing the interaction of 2-dimensional material objects with photon and fermion fields. The models making it possible to calculate in these systems the Casimir energy, characteristics of bound states and scattering processes for material with different shape and properties are presented. Applications of the models to descriptions of other physical phenomena are considered. The specific of regularization and renormalization procedures used by calculations and the physical meaning of obtained results are discussed.

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Formulation of model

The main idea of the proposed by Symanzik (K.Symanzik, Nucl. Phys. **B 190** , 1 (1981)) approach for modeling the interacting of quantum field with space-time inhomogeneities (defects) is to describe this system with the action functional of the form:

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_V(\varphi) = \int L(\varphi(x))d^D x, \ \ S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x))d^{D'} x,$$

and Γ is a subspace of dimension $D' \leq D$ in D-dimensional space.

Formulation of model

From the basic principles of QED (gauge invariance, locality, renormalizability) it follows that for thin film without charges and currents, which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ and spinor fields $\bar{\psi}(x), \psi(x)$ reads

$$S_{def}(\varphi) = S_\Phi(A) + S_\Phi(\bar{\psi}, \psi).$$

The action $S_\Phi(A)$ is a surface Chern-Simon action

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$), a is a constant dimensionless parameter.

Formulation of model

The fermion defect action can be written as

$$S_\Phi(\bar\psi,\psi) = \int \bar\psi(x)[\lambda + u^\mu\gamma_\mu + \gamma_5(\tau + v^\mu\gamma_\mu) + \omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu = 0,1,2,3$, are the Dirac matrices, $\gamma_5 = i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ,v_μ, $\omega^{\mu\nu} = -\omega^{\nu\mu}$, $\mu,\nu = 0,1,2,3$ are 16 dimensionless parameters.
It is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

Formulation of model

The full action of the model, which satisfies the requirement of locality, gauge invariance and renormalizability, has the form

$$S(\bar{\psi},\psi,A) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi$$
$$+S_{def}(A) + S_{def}(\bar{\psi},\psi).$$

Due to the requirements of renormalizability the fields interaction is described by standard contribution $e\bar{\psi}\hat{A}\psi$ to the QED action.

Formulation of model

The full action functional for electromagnet field in the space-time with film defect including the usual free action of the photon field is written as

$$S(A,\Phi) = S_0(A) + S_\Phi(A), \;\; S_0 = -\frac{1}{4}\int d^4x F^{\mu\nu}(x)F_{\mu\nu}(x).$$

For stationary defects $\partial_0\Phi(x) = 0$.

For the plane $x_3 = l$ the defect action reads

$$S_\Phi = \frac{a}{2}\int d^4x\epsilon^{3\mu\lambda\kappa}\delta(x_3 - l)A_\mu(x)\partial_\lambda A_\kappa(x).$$

For the sphere with radius r_0 :

$$\Phi(x) = \sqrt{x_1^2 + x_2^2 + x_3^2} - r_0,\ \ \vec{\partial}\Phi(x) = \frac{\vec{x}}{|\vec{x}|} = \vec{n}(\vec{x})$$

For the cylinder of radius R placed along the x_3 axis :

$$\Phi(x) = x_1^2 + x_2^2 - R^2,\ \ \vec{\partial}\Phi(x) = (x_1, x_2, 0) = \vec{n}(x)R$$

The limit $a \to \infty$ corresponds to perfectly conducting surface with conditions $n_\mu \tilde{F}^{\mu\nu}|_S = 0$.

Formulation of model

The quantitative description of all physical phenomena caused by interaction of the the photon field with film and classical charges and currents can be obtained if the generating functional of Greens functions is known. For gauge condition $\phi(A) = 0$ it is of the form

$$G(J) = C \int e^{iS(A,\Phi)+iJA}\delta(\phi(A))DA$$

where the constant C is defined by normalization condition $G(0)|_{a=0} = 1$, i.e. in pure photodynamic without defect ln $G(0)$ vanishes.

Formulation of model

The full action $S(A, \Phi)$ of the system can be written as $S(A, \Phi) = 1/2\ A_\mu K_\Phi^{\mu\nu} A_\nu$. The integral is gaussian and is calculated exactly:

$$G(J) = \exp\left\{\frac{1}{2} Tr \ln(D_\Phi D^{-1}) - \frac{1}{2} J D_\phi J\right\}$$

where D_Φ is the propagator $D_\Phi = iK_\Phi^{-1}$ of photodynamic with defect in gauge $\phi(A) = 0$, and D is the propagator of free photon field in the same gauge. For the static defect, function $\Phi(x)$ is time independent, and $\ln G(0)$ defines the Casimir energy.

Casimir energy

In considered model the Casimir energy E_{Cas} is obtained by calculation of gaussian functional integral describing the interaction of vacuum fluctuation with defect:

$$E_{Cas} = -\frac{i}{T} \ln G(0) = -\frac{i}{T} \ln \left[C \int e^{-S(A,\Phi)} DA \right].$$

It holds

$$E_{Cas} = -\frac{i}{2T} Tr \ln(DD_0^{-1})$$

where D is the propagator in the model with defect, and D_0 is the propagator for the model in homogenous space.

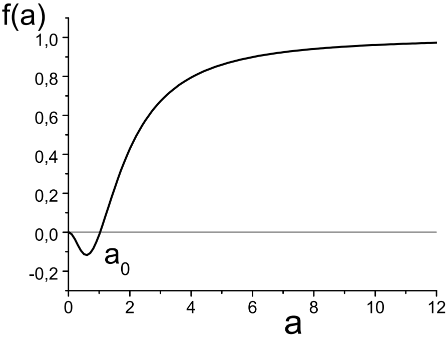
Casimir energy for two parallel planes

For the simplest case of two plane parallel infinite films the Casimir energy was calculated in V. N. Markov and Yu. M. Pis'mak, J. Phys. A 39:21, 6525 (2006); arXive:hep-th/0505218. If the defects are concentrated on planes $x_3 = 0$ and $x_3 = r$, the defect action has the form:

$$S_\Phi = S_{2P} = \frac{1}{2}\int (a_1\delta(x_3) + a_2\delta(x_3 - \mathrm{r}))\varepsilon^{3\mu\nu\rho} A_\mu(x) F_{\nu\rho}(x) dx.$$

For this geometry, it is convenient to use notations like
$x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$, $\vec{x}^2 = x_0^2 - x_1^2 - x_2^2$, $|\vec{x}| = \sqrt{\vec{x}^2}$.

Casimir energy for two parallel planes



Function $f(a)$ determining the Casimir forces between two parallel planes. It is even (f(a)=f(-a)), has the minimum $f(a_m) = -0,11723$ at $a_m = \pm 0,5892$, and $f(a_0) = 0$ at $a_0 = 0$ and $a_0 = \pm 1,03246$.

Interaction of Dirac field with plane $x_3 = 0$

We will consider the material plane $x_3 = 0$ as a defect. In this case, in the Dirac part of the action

$$S(\overline{\psi},\psi) = \int \overline{\psi}(x)(i\hat{\partial} - m + \Omega(x_3))\psi(x)dx,$$

the interaction of the spinor field with the plane is described with matrix $\Omega(x_3) = Q\delta(x_3)$. Since $\Omega(x_3)$ and $\delta(x_3)$ have the dimension of mass, the matrix Q is dimensionless. For homogeneous isotropic material plane in more general case, the matrix Q could be presented in the form:

$$Q = r_1 I + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 +$$
$$+r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2$$

with I - identity 4x4 matrix, γ_3, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ are the Dirac matrices.

Modified Dirac equations

The movement of spinor particle in the field of defect $\Omega(x_3)$ is described by the Dirac equation

$$(i\hat{\partial} - m + \Omega(x_3))\psi(x) = 0.$$

It is one of the Euler-Lagrange equations, which is obtained by variational differentiating of the action over $\overline{\psi}(x)$. Taking the derivative over $\psi(x)$ we obtain the second equation

$$(\partial_\mu \overline{\psi}(x))\gamma^\mu + \overline{\psi}(x)(m - \Omega(x_3)) = 0.$$

The condition $\bar{\psi}(x) = \psi^*(x)\gamma_0$ fulfils if $\gamma_0\Omega^+(x) = \Omega(x)\gamma_0$. It is the case for real values of parameters $r_j,\ j = 1, ..., 8$.

Modified Dirac equations

We denote $\psi(x)$ the solution of the modified Dirac equation, and $\psi_-(x) = \psi(x)$ for $x_3 < 0$, $\psi_+(x) = \psi(x)$ for $x_3 > 0$. The spinors $\psi(x)_\pm$ for $x_3 \neq 0$ satisfy the free Dirac equation and boundary condition

$$\lim_{x^3 \to +0} \psi_+(x) = S \lim_{x^3 \to -0} \psi_-(x),$$

One can choose the regularization procedure for $\delta(x_3)$ in such a way that the matrix S is expressed in terms of Q as

$$S = \exp\{-i\gamma^3 Q\}.$$

Modified Dirac equations

We introduce the following notation. If M is 2×2 matrix

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \ \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

then $M^{(\pm)}$ are the 4×4 matrixes

$$M^{(+)} = \begin{pmatrix} M_{11} & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 \\ M_{21} & 0 & M_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \ M^{(-)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_{11} & 0 & M_{12} \\ 0 & 0 & 0 & 0 \\ 0 & M_{21} & 0 & M_{22} \end{pmatrix}. \tag{1}$$

We denote also by τ_0 the unit 2×2 - matrix, and τ_1, τ_2, τ_3 the Pauli-matrices:

$$\tau_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Modified Dirac equations

The matrix S is presented in the form

$$S = S^{(+)} + S^{(-)}$$

with

$$S^{(\pm)} = e^{i\chi_\pm}\left(\varsigma_{0\pm}\tau_0^{(\pm)} + i\varsigma_{1\pm}\tau_1^{(\pm)} + \varsigma_{2\pm}\tau_2^{(\pm)} + \varsigma_{3\pm}\tau_3^{(\pm)}\right).$$

Here, $\chi_\pm$, $\varsigma_{j\pm}$, $0 \le j \le 3$ are real number which can be expressed in terms of parameters $r_1, \dots, r_8$ of the model and

$$\varsigma_{0\pm}^2 + \varsigma_{1\pm}^2 - \varsigma_{2\pm}^2 - \varsigma_{3\pm}^2 = 1.$$

Free Dirac equations

The free Dirac equation in coordinate space reads

$$(i\hat{\partial} - m)\psi(x) = 0.$$

By substitution $\psi(x)$ in the form

$$\psi(x) = \frac{1}{(2\pi)^4}\int e^{-ipx}\psi(\bar{p})d\bar{p},\ \ \bar{p} = (p^0, p^1, p^2)$$

one obtains

$$(\hat{p} - m)\psi(\bar{p}) = 0.$$

For real $p_3 = \sqrt{p_0^2 - p_1^2 - p_2^2 - m^2}$ the considered spinor $\psi(x)$ describes the scattering state and by imaginary p_3 - the bound state.

Free Dirac equations

The general solution $\psi(\bar{p})$ of the Dirac equation can be presented as an arbitrary linear combination of linear independent spinors

$$\psi_1(\bar{p}) = \left\{ \begin{array}{c} 1 \\ 0 \\ \frac{-p_3}{m+p_0} \\ \frac{-p_1-ip_2}{m+p_0} \end{array} \right\}, \ \psi_2(\bar{p}) = \left\{ \begin{array}{c} 0 \\ 1 \\ \frac{-p_1+ip_2}{m+p_0} \\ \frac{p_3}{m+p_0} \end{array} \right\}, \ p_3 = \pm\sqrt{\bar{p}^2 - m^2}$$

for $p_0 > 0$ and

$$\psi'_1(\bar{p}) = \left\{ \begin{array}{c} \frac{p_1-ip_2}{m-p_0} \\ \frac{p_3}{p_0-m} \\ 0 \\ 1 \end{array} \right\}, \psi'_2(\bar{p}) = \left\{ \begin{array}{c} \frac{p_3}{m-p_0} \\ \frac{p_1+ip_2}{m-p_0} \\ 1 \\ 0 \end{array} \right\}.$$

for $p_0 < 0$.

Scattering of Dirac particle on material plane

The characteristics of the scattering process depend essentially on the choice of parameters of the model, on polarization, energy and incidence angle of particles. If the incidence wave function of particle is defined by spinor $\psi(\bar{p}) = a_1\psi(\bar{p}) + a_2\psi_2(\bar{p})$, then for the particles moving orthogonal to the x_3-axes the transmission coefficient has the form

$$T = \frac{4\xi(|a_1|^2 T_- + |a_2|^2 T_+)}{(|a_1|^2 + |a_2|^2)d_+ d_-}$$

with

$$\xi = \frac{p_0 - m}{p_0 + m}, \;\; d_\pm = 4\xi\varsigma_{0\pm}^2 + (\varsigma_{1\pm} - \varsigma_{2\pm} + \xi(\varsigma_{1\pm} + \varsigma_{2\pm}))^2 \,,$$
$$T_\pm = (\varsigma_{1\pm} - \varsigma_{2\pm})^2 + 2\xi(\varsigma_{0\pm} + \varsigma_{3\pm} + 1) + \xi^2(\varsigma_{1\pm} + \varsigma_{2\pm})^2$$

The features of scattering processes

The parameters of the model can be chosen so that the transmission coefficient is almost equal to unity at low particle energy and is almost zero for particles with high energy. One can choose the parameters and so that at high energies the particles almost completely pass through the plane, and at low energies they are almost completely reflected.

Substituting $p_3 \to \pm i\kappa$ with $\kappa = |\kappa| = \sqrt{m^2 + p_1^2 + p_2^2 - p_0^2}$ we obtain the spinors describing the bound states

$$\psi_{\pm}(\bar{p}) = \psi(p)|_{p_3 \to \mp i\kappa}.$$

They can be presented as follows

$$\psi_+(\bar{p}) = a_1\psi_{1+}(\bar{p}) + a_2 e^{i\phi}\psi_{1+}(\bar{p}),\ \psi_-(\bar{p}) = d_1\psi_{1-}(\bar{p}) + d_2 e^{i\phi}\psi_{1-}(\bar{p}),$$
$$\psi'_+(\bar{p}) = a'_1\psi'_{1+}(\bar{p}) + a'_2 e^{i\phi}\psi'_{1+}(\bar{p}),\ \psi'_-(\bar{p}) = d'_1\psi'_{1-}(\bar{p}) + d'_2 e^{i\phi}\psi'_{1-}(\bar{p}).$$

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Bound state of Dirac particle on the plane

Here

$$\psi_{1\pm}(\bar{p}) = \begin{Bmatrix} 1 \\ 0 \\ \pm ik \\ fe^{i\phi} \end{Bmatrix}, \quad \psi_{\bar{2}}(\bar{p}) = \begin{Bmatrix} 0 \\ 1 \\ fe^{-i\phi} \\ \mp ik \end{Bmatrix},$$

$$\psi'_{1\pm}(\bar{p}) = \begin{Bmatrix} -fe^{-i\phi} \\ \pm ik \\ 0 \\ 1 \end{Bmatrix}, \quad \psi'_{2\pm}(\bar{p}) = \begin{Bmatrix} \mp ik \\ -fe^{i\phi} \\ 1 \\ 0 \end{Bmatrix},$$

$$k = \frac{\kappa}{m + |p_0|}, \quad \frac{p^1 + ip^2}{m + |p_0|} = -\frac{p_1 + ip_2}{m + |p_0|} = fe^{i\phi}, \ f = |f|.$$

The spinors $\psi_\pm, \psi'_\pm$ fulfill the relations

$$\psi_+(\bar{p}) = S\psi_-(\bar{p}), \ \psi'_+(\bar{p}) = S\psi'_-(\bar{p}), \ S = e^{-i\gamma^3 Q}.$$

which can be presented as systems of linear equations for coefficients $a_1, a_2, d_1, d_2, \ a'_1, a'_2, d'_1, d'_2$ contained in $\psi_\pm, \psi'_\pm$.

Dispersion relation

The solvability condition (dispersion relation) can be presented as

$$(p_1^2+p_2^2)(\cos(\chi_+-\chi_-)-\varsigma_{0-}\varsigma_{0+}+\varsigma_{1-}\varsigma_{1+}-\varsigma_{2-}\varsigma_{2+}-\varsigma_{3-}\varsigma_{3+})-$$
$$-2(p_0\,\varsigma_{1-}+m\,\varsigma_{2-}+\kappa\,\varsigma_{0-})(p_0\,\varsigma_{1+}+m\,\varsigma_{2+}-\kappa\,\varsigma_{0+})=0.$$

In virtue of $p_0^2+\kappa^2-p_1^2-p_2^2-m^2=0$,it follows from dispersion relation that p_0,κ,m satisfy the equation

$$(p_0^2+\kappa^2-m^2)(\cos(\chi_+-\chi_-)-\varsigma_{0-}\varsigma_{0+}+\varsigma_{1-}\varsigma_{1+}-\varsigma_{2-}\varsigma_{2+}-\varsigma_{3-}\varsigma_{3+})-$$
$$-2(p_0\,\varsigma_{1-}+m\,\varsigma_{2-}+\kappa\,\varsigma_{0-})(p_0\,\varsigma_{1+}+m\,\varsigma_{2+}-\kappa\,\varsigma_{0+})=0.$$

describing the relation between the dimensionless magnitudes $p_o/m,\kappa/m$ characterizing the bound state. Its solution is presented on the $(p_0-\kappa)$-plane by hyperbola or by two straight lines.

Dispersion relation

Since for Dirac particle the physical value of p_0, κ are positive, the bound state can be realized if there are points of the $(p_0 - \kappa)$-plot presenting the dispersion relation in the region $p_0 > 0, \kappa > 0$. This part of plot can be connected or disconnected, and the possible values of p_0, κ for bound state can be both restricted and non-restricted from above.

It is possible to chose the parameters r_i in such a way that the dispersion law has the form

$$p_0^2 - v_F^2(p_1^2 + p_2^2) = 0,\ 0 \leq v_F \leq 1,$$

and

$$\kappa^2 = m^2 + (p_1^2 + p_2^2)(1 - v_F^2).$$

It describe the propagation of massless particle in the defect plane with the Fermi-velocity v_F. The motion of such particles explains numerous effects in graphene.

O(2,2)-symmetry

Using the notations $t_\pm = f^2 - k^2 \pm 1$

$$\varsigma_\pm = \begin{pmatrix} \varsigma_{0\pm} \\ \varsigma_{1\pm} \\ \varsigma_{2\pm} \\ \varsigma_{3\pm} \end{pmatrix}, \ \mathbf{p} = \begin{pmatrix} p^0 \\ \kappa \\ p^1 \\ p^2 \end{pmatrix}, \ \xi_\pm = \begin{pmatrix} \xi_{0\pm} \\ \xi_{1\pm} \\ \xi_{2\pm} \\ \xi_{3\pm} \end{pmatrix} = \Omega_\pm \varsigma_\pm \qquad (2)$$

$$\Omega_\pm = \frac{1}{2f} \begin{pmatrix} 2k & \mp t_+ & \pm t_- & 0 \\ \pm t_+ & 2k & 0 & t_- \\ \pm t_- & 0 & 2k & t_+ \\ 0 & t_- & -t_+ & 2k \end{pmatrix}, \qquad (3)$$

O(2,2)-symmetry

we can present the dispersion relation

$$p_0^2 + \kappa^2 - p_1^2 - p_2^2 - m^2 = 0$$

and solvability condition (second dispersion relation) as

$$\mathbf{p}G\mathbf{p} = m^2, \ \xi G\xi = 0, \ \xi = e^{i\chi_+}\xi_+ + e^{i\chi_-}\xi_-.$$

Here $G = \mathrm{diag}\{1, 1, -1, -1\}$ is the metric tensor of the group $O(2, 2)$. The matrixes $\Omega_\pm$ fulfill the equalities $\Omega_\pm^T G\Omega = G$. Hence they are elements of the group $O(2, 2)$ and $\xi_\pm G\xi_\pm = \varsigma_\pm G\varsigma_\pm = 1$

O(2,2)-symmetry

The dispersion relations are invariant in respect to transformations

$$\begin{pmatrix} y_1 & -\frac{t_+}{\rho} z_1 & \frac{t_-}{\rho} z_1 & 0 \\ \frac{t_+}{\rho} z_1 & \frac{t_+^2}{\rho^2} y_1 - \frac{t_-^2}{\rho^2} y_2 & \frac{t_+ t_-}{\rho^2}(y_2 - y_1) & -\frac{t_-}{\rho} z_2 \\ \frac{t_-}{\rho} z_1 & \frac{t_+ t_-}{\rho^2}(y_1 - y_2) & \frac{t_+^2}{\rho^2} y_2 - \frac{t_-^2}{\rho^2} y_1 & -\frac{t_+}{\rho} z_2 \\ 0 & -\frac{t_-}{\rho} z_2 & \frac{t_+}{\rho} z_2 & y_2 \end{pmatrix}.$$

of vectors $\varsigma_\pm$.
It is element of $O(2,2)$, depends on 2 real parameters a_1, a_2 and $y_1 = \cos(\rho a_1/t_+)$, $y_2 = \cos(\rho a_2/t_-)$, $z_1 = \sin(\rho a_1), z_2 = \sin(\rho a_2)$ $\rho = \sqrt{t_+^2 - t_-^2}$.

Properties of bound states

We consider the quantities of the form

$$\bar{\psi}_{\pm}(\bar{p}, x_3)\Gamma\,\psi_{\pm}(\bar{p}, x_3) = \bar{\psi}_{\pm}(\bar{p})\Gamma\psi_{\pm}(\bar{p})e^{-2\kappa|x_3|}$$

with 4×4 basic Dirac matrices Γ using convenient notations

$$N_{0+} = a_1^* a_1 + a_2^* a_2,\ \ N_{1+} = a_1^* a_2 + a_2^* a_1,$$
$$N_{2+} = a_1^* a_1 - a_2^* a_2,\ \ N_{3+} = i(a_1^* a_2 - a_2^* a_1),$$
$$N_{0-} = d_1^* d_1 + d_2^* d_2,\ \ N_{1-} = d_1^* d_2 + d_2^* d_1,$$
$$N_{2-} = d_1^* d_1 - d_2^* d_2,\ \ N_{3-} = i(d_1^* d_2 - d_2^* d_1),$$
$$\vec{\gamma} = \{\gamma^1, \gamma^2, \gamma^3\},\ \vec{\sigma} = \{\sigma^1, \sigma^2, \sigma^3\},\ \sigma^j = \sum_{k,l=1}^{3} \varepsilon^{jkl}\gamma^k\gamma^l,\ j = 1, 2, 3$$

with totally antisymmetric ε^{jkl}, $\varepsilon^{123} = 1$.

Properties of bound states

For scalar and pseudoscalar invariant densities

$$\bar{\psi}_{\pm}(x)\psi_{\pm}(x) = e^{-2\kappa|x^3|}d_{\pm}(\bar{p}),\ \bar{\psi}_{\pm}(x)\gamma_5\psi_{\pm}(x) = ie^{-2\kappa|x^3|}d_{5\pm}(\bar{p}),$$

components of electric and axial 4-currents

$$\bar{\psi}(x)\gamma^{\mu}\psi(x) = e^{-2\kappa|x^3|}j^{\mu}_{\pm}(\bar{p}),\ \bar{\psi}(x)\gamma^{\mu}\gamma_5\psi(x) = e^{-2\kappa|x^3|}j^{\mu}_{5\pm}(\bar{p}),$$

for anomalous electric and magnetic dipole moments

$$\bar{\psi}_{\pm}(x)\vec{\gamma}\gamma^0\psi_{\pm}(x) = ie^{-2\kappa|x^3|}\vec{e}_{\pm}(\bar{p}),\ \bar{\psi}_{\pm}(x)\vec{\sigma}\psi_{\pm}(x) = ie^{-2\kappa|x^3|}\vec{m}_{\pm}(\bar{p}),$$

we obtain the following results

Properties of bound states

$$d_\pm = N_{0\pm}\left(1 - f^2 - k^2\right) \pm 2N_{3\pm}fk, \;\; d_{5\pm} = \pm 2N_{2\pm},$$

$$j^0_\pm = N_{0\pm}\left(1 + f^2 + k^2\right) \mp 2N_{3\pm}fk, \;\; j^0_{5\pm} = 2N_{1\pm}f,$$
$$j^1_\pm = j^{\|}_\pm \cos(\phi) + j^{\perp}_\pm \sin(\phi), \;\; j^1_{5\pm} = j^{\|}_{5\pm}\cos(\phi) - j^{\perp}_{5\pm}\sin(\phi),$$
$$j^2_\pm = j^{\|}_\pm \sin(\phi) - j^{\perp}_\pm \cos(\phi), \;\; j^2_{5\pm} = j^{\|}_{5\pm}\sin(\phi) + j^{\perp}_{5\pm}\cos(\phi),$$
$$j^3_\pm = 0, \; j^3_{5\pm} = N_{2\pm}\left(1 - f^2 + k^2\right)$$
$$j^{\|}_\pm = 2(N_{0\pm}f \mp N_{3\pm}k), \; j^{\perp}_\pm = \pm 2N_{1\pm}k,$$
$$j^{\|}_{5\pm} = N_{1\pm}\left(1 + f^2 - k^2\right), \; j^{\perp}_{5\pm} = N_{3\pm}\left(f^2 + k^2 - 1\right) \mp 2N_{0\pm}fk,$$

Properties of bound states

$$e^1_\pm = e^\perp_\pm \sin(\phi),\ \ e^2_\pm = -e^\perp_\pm \cos(\phi),\ \ e^3 = -2(N_{3\pm} f \mp N_{0\pm} k),$$
$$m^1_\pm = m^\parallel_\pm \cos(\phi) - m^\perp_\pm \sin(\phi),\ \ m^2_\pm = m^\parallel_\pm \sin(\phi) + m^\perp_\pm \cos(\phi),$$
$$m^3_\pm = N_{2\pm}\left(k^2 - f^2 - 1\right),\ \ e^\perp = 2fN_{2\pm},$$
$$m^\parallel_\pm = iN_{1\pm}\left(f^2 - k^2 - 1\right),\ \ m^\perp_\pm = \left(N_{3\pm}\left(1 + f^2 + k^2\right) \mp 2N_{0\pm} fk\right).$$

Properties of bound states

We see also that

$$j^0_\pm p^0 - j^1_\pm p^1 - j^2_\pm p^2 = m d_\pm,\ j^0_{5\pm} p^0 - j^1_{5\pm} p^1 - j^2_{5\pm} p^2 = 0,$$
$$\kappa j^3_{5\pm} = m d_{5\pm} = 0,\ m\,\vec{e}_\pm \vec{m}_\pm = -\kappa j^3_{5\pm} d_\pm,\ \vec{e}_\pm \vec{m}_\pm = d_\pm d_{5\pm},$$
$$\vec{e}^{\,2}_\pm - \vec{m}^2_\pm = d^2_{5\pm} - d^2_\pm,\ d_\pm p^0 - j^0_\pm m = \kappa e^3_\pm$$

$$N_{0\pm} = \frac{(d_\pm + j^0_\pm)}{2},\ N_{1\pm} = \frac{j^0_{5\pm}}{2f},\ N_{2\pm} = \frac{j^3_{5\pm}(m + p^0)}{2m},$$
$$N_{3\pm} = \frac{d_\pm(1 + f^2 + k^2) + j^0_\pm(f^2 + k^2 - 1)}{4fk}.$$

Conclusions

- In the framework of the Symanzik approach, we build the model of QED field interaction with 2D material. The action of the model consist of the usual QED action and extra defect contribution which contains parameters that characterize the material properties.
- In one universal model, it is possible to describe many physical phenomena and processes resulting form the interaction of QED fields with 2D macro-object.
- The characteristics of photons and Dirac particles scattering on the defect plane can be calculated in the model, also the properties of states localized near the defect plane can be investigated.

Conclusions

- The model and obtained on its basis results could be used for the theoretical description of the interaction of photons and Dirac particles with 2D materials (graphene, thin films, sputters, sharp boundaries of a solid body). Simple modifications of the model allows to take into account the effects of external electromagnetic fields.
- An experimental verification of presented results can be useful to a deeper understanding of the principles on which one can develop theoretical approaches to studying the effects of the interaction of condensed matter with quantum fields.

Thank you for your attention!