Low and high energy constraints in AdS/QCD models

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LXX International conference "NUCLEUS – 2020. Nuclear physics and elementary particle physics. Nuclear physics technologies"
11–17 October 2020

Introduction

- AdS/QCD models — bottom-up holographic models for QCD.
- Inspired by the ideas of gauge/gravity duality, and AdS/CFT-correspondence.
- A 5D theory over the AdS space is constructed, that is dual to QCD in large-N_c limit (in bottom-up approach).
- Example: a correlator (partition function) can be calculated using the on-shell 5D action.

5D Vector Model

The simplest action:

$$S_{5\mathrm{D}} = \frac{1}{2g_5^2} \int d^4x\, dz \sqrt{g}\, e^{\varphi} \left(-\partial^M V^N \partial_M V_N + m_5^2 V^N V_N \right),$$

with the conditions $\partial^\mu V_\mu = 0$ and $V_z = 0$ (axial gauge).

g_5^2 — normalization constant for the field.
$\varphi = cz^2$ — dilaton; standard Soft-Wall model.
Hard-Wall: cutoff with respect to z instead of dilaton.

Quadratic field structure ensures disappearance of three and higher point correlation functions in the large-N.

5D Vector Model

The 5D mass of the vector field:

$$m_5^2 R^2 = (\Delta - 1)(\Delta - 3).$$

Δ — canonical dimension of the dual operator.
For quark current $\Delta = 3 \Rightarrow m_5^2 = 0$.
Can be generalized for higher-spin operators/p-form fields.

Regge-like linear spectrum: $m_n^2 = 4|c|(n+1)$.
Correct analytical structure of OPE.

From the Soft- to the No-Wall Model

The origin of the dilaton is unclear, but it can be absorbed into m_5^2 via substitution:

$$V_N = e^{-cz^2/2} v_N.$$

We can reformulate the SW model into the so-called No-Wall model:

$$S_{5\mathrm{D}} = \frac{1}{2g_5^2} \int d^4x\, dz \sqrt{g} \left(-\partial^M v^N \partial_M v_N + m_5^2(z) v^N v_N \right),$$

$$m_5^2(z) R^2 = a + bz^2 + c^2 z^4.$$

(Today — only the $a = 0$ case.)

Equations and Solutions

4D Fourier-transformed equations:

$$\left[-q^2 - z\partial_z\left(\frac{1}{z}\partial_z\right) + \frac{m_5^2(z)R^2}{z^2}\right] v_\mu(q,z) = 0.$$

A solution is sought in the form $v_\mu(q,z) = v_\mu(q)v(q,z)$, where $v_\mu(q)$ is interpreted as the source for the operators in 4D.

Solution ($Q^2 = -q^2$, to make comparison with the OPE in QCD):

$$v(\mathcal{Q},\zeta) = \Gamma\left(1 + \mathcal{Q}^2 + \beta\right) e^{-\zeta^2/2} U\left(\mathcal{Q}^2 + \beta, 0; \zeta^2\right),$$

where $\mathcal{Q}^2 = \frac{Q^2}{4|c|}$, $\beta = \frac{b}{4|c|}$, and $\zeta^2 = |c|z^2$.

Vector Correlator

A correlator can be expressed using the e.o.m solution:

$$\Pi_V(\mathcal{Q}^2) = \lim_{\zeta \to 0} \left(-\frac{R}{g_5^2} \frac{\partial_\zeta v(\mathcal{Q}, \zeta)}{\zeta} \right) .$$

Taking the limit of $\zeta \to 0$:

$$v(\mathcal{Q}, \zeta) = 1 +$$
$$+ \left\{ \left(\mathcal{Q}^2 + \beta\right) \left[\ln \zeta^2 + \psi \left(1 + \mathcal{Q}^2 + \beta\right) + 2\gamma - 1\right] - \frac{1}{2} \right\} \zeta^2 .$$

Usually: substitute solution into the correlator, and then omit the divergent logarithmic terms.
We will discard the logarithm before the substitution.

Final Answer

$$\Pi(\mathcal{Q}^2) = -\frac{2R}{g_5^2}|c| \left\{ \left(\mathcal{Q}^2 + \beta\right) \left[\psi\left(1 + \mathcal{Q}^2 + \beta\right) + 2\gamma - 1\right] - \frac{1}{2} \right\}.$$

The digamma function's poles define the mass spectrum:

$$-\mathcal{Q}_n^2 = \frac{m_n^2}{4|c|} = n + 1 + \beta, \qquad n = 0, 1, 2, \ldots.$$

This correlator has the expected analytical structure:

$$\Pi(q^2) \sim \sum_{n=0}^{\infty} \frac{F_n^2}{q^2 - m_n^2}.$$

Can be shown explicitly by using the representation:

$$\psi(1 + x) = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n + x} - \gamma, .$$

Large-Q^2 Expansion

Using $\psi(1+x) \underset{x\to\infty}{=} \ln x + \frac{1}{2x} - \frac{1}{12x^2} + \mathcal{O}(x^{-3})$ we can expand correlator:

$$\Pi(\mathcal{Q}^2) \simeq -\frac{2R}{g_5^2}|c| \left[\left(\mathcal{Q}^2 + \beta\right) \ln \mathcal{Q}^2 + \frac{\beta^2 - 1/6}{2\mathcal{Q}^2} + \right.$$
$$\left. + \frac{\beta(1-2\beta^2)}{12\mathcal{Q}^4} + \dots \right].$$

OPE in QCD:

$$\Pi_{\mathrm{OPE}}(\mathcal{Q}^2) \simeq -\frac{N_c}{6\pi^2}|c|\mathcal{Q}^2 \ln \mathcal{Q}^2 + \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{96|c|\mathcal{Q}^2} + \frac{\xi}{9}\frac{\pi\alpha_s\langle \bar{q}q\rangle^2}{8c^2\mathcal{Q}^4} + \dots .$$

OPE Comparison

First, we get the normalization factor:

$$\frac{R}{g_5^2} = \frac{N_c}{12\pi^2}.$$

Then, we get a relation between gluon condensate and the spectral parameters:

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = \frac{N_c}{2\pi^2} \left(\frac{1}{6} - \beta^2 \right) (4c)^2.$$

For the phenomenological values of $4|c| \approx 1.2\text{GeV}^2$ and $|\beta| \approx 0.3$ we get a sensible gluon condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle \approx (360\,\text{MeV})^4$.

Experiment Comparison (Vectors)

These values of $4|c| \approx 1.2\mathrm{GeV}^2$ and $|\beta| \approx 0.3$ describe well (within the large-N_c limit accuracy) spectra of radially-excited mesons $m_n^2 = 4|c|(n+1+\beta)$.

n	0	1	2	3
m_V^{th}	917	1430	1800	2110
m_ω	782.65 ± 0.12 $\omega(782)$	1410 ± 60 $\omega(1420)$	1670 ± 30 $\omega(1650)$	1960 ± 25 $\omega(1960)$
m_ρ	775.26 ± 0.25 $\rho(770)$	1465 ± 25 $\rho(1450)$	1720 ± 20 $\rho(1700)$	$\sim$ $\rho(2150)$

All masses in MeV.

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

Experiment Comparison (Axials)

Note: The description of excited isovector ρ and a_1 mesons is even better.

n	0	1	2	3
m_A^{th}	1249	1660	1990	2270
m_{f_1}	1281.9 ± 0.5 $f_1(1285)$	1426.3 ± 0.9 $f_1(1420)$	1971 ± 15 $f_1(1970)$	2310 ± 60 $f_1(2310)$
m_{a_1}	1230 ± 40 $a_1(1260)$	1655 ± 16 $a_1(1640)$	1930^{+30}_{-70} $a_1(1930)$	2270^{+55}_{-40} $a_1(2270)$

All masses in MeV.

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Correlators at Zero Momentum

The same prediction for the intercept parameter can be acquired at zero momentum.
In our regularization $\Pi(0)$ is the finite quantity with all constants fixed.
Usually there are subtracted infinite constants which make it ambiguous.

$$\Pi(0) = -\frac{2R}{g_5^2}|c| \left\{ -\frac{1}{2} + \beta \left[\psi\left(1+\beta\right) + 2\gamma - 1 \right] \right\}.$$

Correlators at Zero Momentum

PCAC $\Rightarrow \Pi_A(0) = f_\pi^2$.

Axial vector resonances should not contribute to $\Pi_A(0)$ because of a large mass gap (f_π^2 absorbs all contributions to $\Pi_A(0)$).
This means that $\beta_a\left[\psi(1+\beta_a)+2\gamma-1\right]=0 \Rightarrow$

- either $\beta_a = 0$,
- or $\beta_a = 0.31$ — the same value as at the large Q^2.

Correlators at Zero Momentum

If $\beta_a\left[\psi(1+\beta_a)+2\gamma-1\right]=0$, then:

$$\boxed{4|c|=\frac{48\pi^2}{N_c}f_\pi^2.}$$

- Agrees well with the phenomenology (for $N_c=3$, $f_\pi=87\text{MeV}$, we get empirical value of $4|c|$)
- Usually derived in QCD sum rules
- We've acquired it in the limit $Q^2\to 0$

Correlators at Zero Momentum

$\Pi_V(0)$ is also non-zero, but for different reasons. From the

$$-4L_{10} = \frac{d}{dQ^2}\left(\Pi_V - \Pi_A\right)\big|_{Q^2=0}\,,$$

we get an equation for the vector intercept parameter:

$$\psi\left(1+\beta_v\right)+2\gamma-1+\beta_v\psi\left(1,1+\beta_v\right)-\beta_a\psi\left(1,1+\beta_a\right) = \frac{96\pi^2 L_{10}}{N_c}.$$

Correlators at Zero Momentum

Using the value $L_{10}(0) = (-6.36 \pm 0.09|_{\text{expt}} \pm 0.16|_{\text{theor}}) \cdot 10^{-3}$, we get $\beta_v \approx -0.26$.

- Close to the estimated value $\beta_v \approx -0.3$
- In agreement with $\beta_v = -\beta_a$ for $L_{10} = -7.5 \cdot 10^{-3}$

Alternatively, solve the equation for $\beta_a = -\beta_v$ and $N_c = 3$:

$$\psi(1+\beta_v) + \beta_v \psi(1, 1+\beta_v) - \\ - \psi(1+\beta_a) - \beta_a \psi(1, 1+\beta_a) = 32\pi^2 L_{10},$$

which results in $\beta_a = -\beta_v \approx 0.27$.

Summary

- We demonstrate phenomenologically how the constant contributions to correlators, which are interpreted as part of "contact" terms and neglected at high momentum expansions, play a role at low momentum.
- AdS/QCD predictions of radial Regge spectrum for spin-1 mesons can be reproduced both at high and low momentum.
- Supports the idea that bottom-up holographic models interpolate between high and low energy sectors.
- These calculations can be generalized with the non-zero constant a in the ansatz for $m_5^2(z)R^2 = a + bz^2 + c^2z^4$ and extended to the scalar and tensor cases.

Based on: arXiv:2006.14439

Acknowledgments: The reported study was funded by RFBR, project number 19-32-90053

Thank you for your attention!