Glauber Monte Carlo model at partonic level for pp collisions in a wide energy range

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Introduction

The behavior of the cross sections for hadronic collisions at different energies performed at the LHC and SPS (CERN) colliders is of great scientific interest. But unfortunately, due to the peculiarities of QCD, it is problematic to construct a detailed model derived from the first principles of the theory.

Therefore, various empirical models are created and tested, which are, in a sense, a generalization of the available experimental statistics. One of the fundamental models for describing interactions involving hadrons and nuclei is the Glauber model.

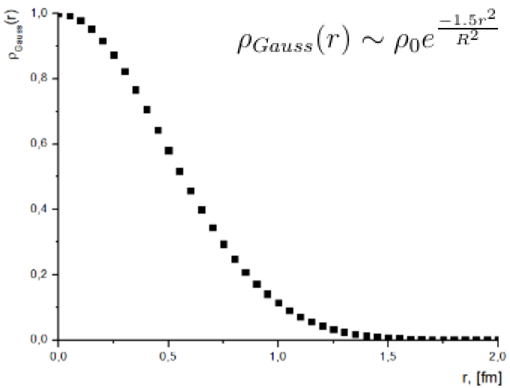
This model has recently been increasingly used to describe nucleus-nucleus collisions at the parton level. However, before moving on to nuclear collisions, it would be worthwhile to study in detail whether this approach works adequately for elementary nucleon collisions.

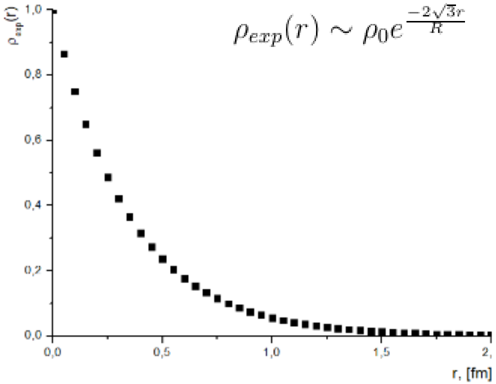
That is why the purpose of our work was to systematically study the Glauber Monte Carlo model at partonic level for pp collisions in a wide energy range.

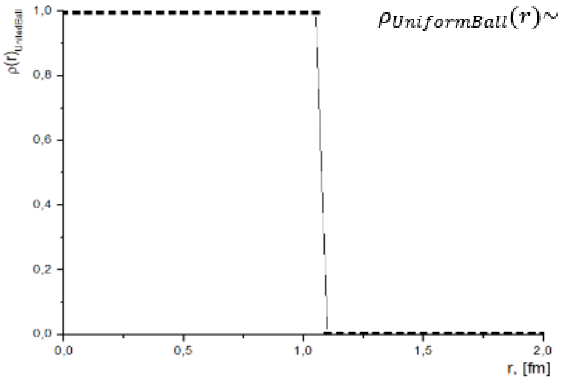
Theory

- Analysis of proton collisions is performed using the Monte Carlo generator of the Glauber model. In this model, not nuclei are considered, but protons, instead of nucleons in nuclei, respectively, partons.
- It were considered three variants of the spatial distribution of partons in protons (see Fig.).
- The root-mean-square radius of the proton is chosen equal to R = 0.831fm [1], and the parton cross section is σ_{parton} = 3.3 mb [2].

$$\rho_{UniformBall}(r) \sim \begin{cases} 0, & \text{if } r > \sqrt{\frac{5}{3}}R \\ \rho_0, & \text{if } r < \sqrt{\frac{5}{3}}R \end{cases}$$







[1] J.-P. Karr, D. Marchand, // Nature 575, 61-62 (2019)
[2] N.S.Amelin, N. Armesto, C. Pajares and D. Sousa // Eur. Phys. J. C 22

Investigated quantities

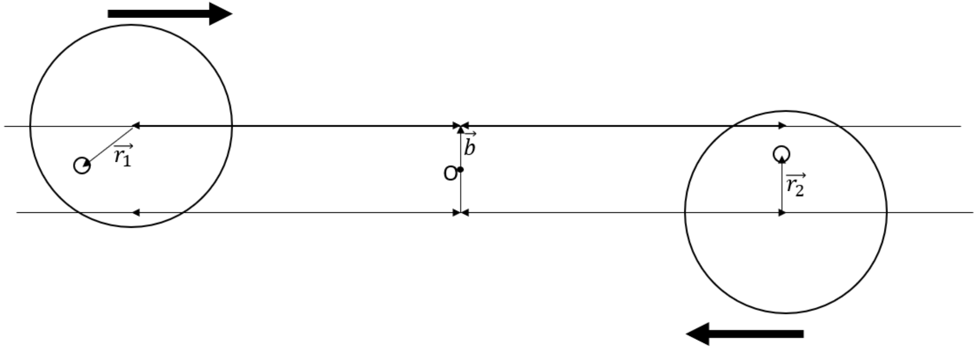
- The profile function σ (b) of pp collision is the probability of interaction of protons at a given impact parameter b
- The number of partons in protons A_1, A_2 and the average number of participating partons <N_{part}>
- Elastic σ_{el}, inelastic σ_{inel} and total σ_{tot} scattering cross sections
- The slope of the diffraction cone B is a characteristic of elastic pp-interaction $\left(\frac{d\sigma_{elastic}}{dt} \sim Ce^{Bt}\right.$, for

$$\sigma_{inel} = \int_0^\infty 2\pi b\sigma(b)\, db \qquad \sigma_{el} = \sigma_{total} - \sigma_{inel}$$

$$\sigma_{total} = \int_0^\infty 4\pi b(1 - \sqrt{1 - \sigma(b)})\, db \qquad B = \frac{1}{2}\frac{\int_0^\infty b^3(1 - \sqrt{1 - \sigma(b)})\, db}{\int_0^\infty b(1 - \sqrt{1 - \sigma(b)})\, db}$$

[3] V.N.Kovalenko, Dipole Description of pp-Interaction // ТМФ, 184:3 (2015), 465–474; Theoret. and Math. Phys., 184:3 (2015), 1295–1303

Simulations in code



-The impact parameter b and the number of partons A_1 and A_2 in each proton are fixed to simulate a collision

-At the first stage, the spatial coordinates of the partons are generated according to the given distributions.

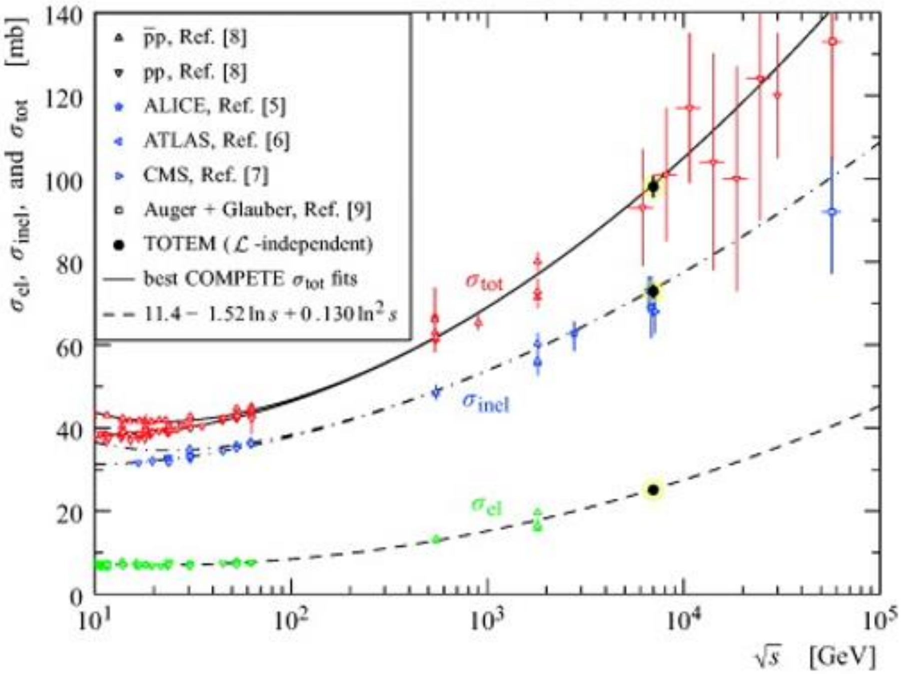
- At the second stage, the participating partons are determined, i.e. the partons of the projectile and target, which are at a distance of interaction from each other. If there is one pair of interacting partons, it is considered that there is an inelastic event.

- Next, the statistics of the probability of inelastic interaction are accumulated and then σ (b) is calculated.

-Next, the cross sections and the slope of the diffraction cone are calculated using σ (b)

Fixing the number of partons by energy

Let's find the dependence of our parameters on energy from the experimental data:



Experimental energy dependence of cross sections from article [4]

Formulas corresponding to the equations for the curves of fitting experimental data in the article [4]:

$$\sigma_{el} = 11.4 - 1.52 \ln s + 0.130 \ln^2 s$$
$$\sigma_{inel} = 30.915 - 0.937 \ln s + 0.188 \ln^2 s$$
$$\sigma_{tot} = \sigma_{inel} + \sigma_{el}$$

Relation between the number of partons $A_1 = A_2$ and energy $\sqrt{s}$ on the example of exponential distribution:

$\sqrt{s}$, $[GeV]$	14,97	165,4	583,3	1468	3067	5713	9495
$A_1 = A_2$	4	5	6	7	8	9	10

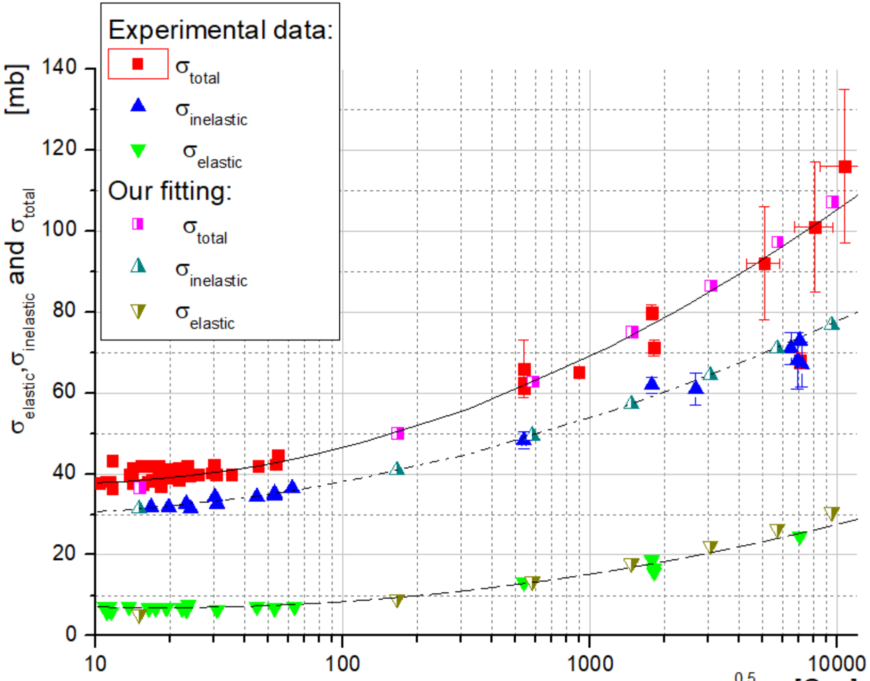
[4] G. Antchev, P. Aspell et al.//Phys. Rev. D 101, 014004

Fixing the number of partons by energy

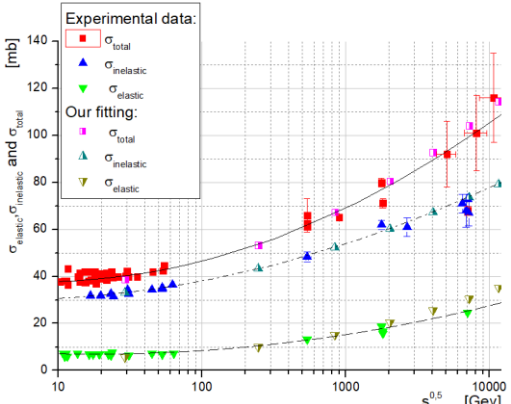
Let's show the energy dependence of the scattering cross sections for various distributions of partons in protons.

It can be seen that all distributions agree well with the experimental data, but the exponential distribution manifests itself best of all.

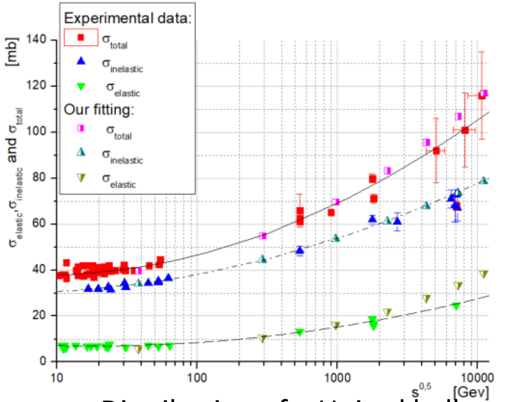
Calculated and experimental dependences of cross sections on energy:



Exponential distribution



Gaussian distribution



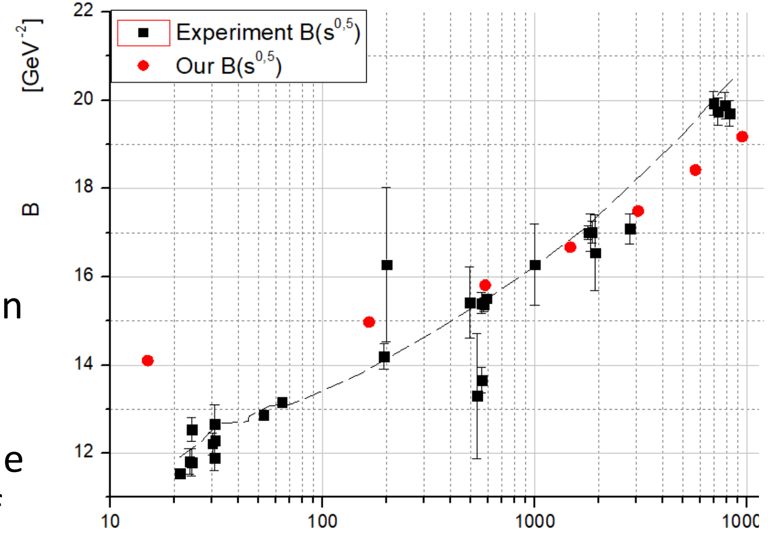
Distribution of a United ball

Fixing the number of partons by energy

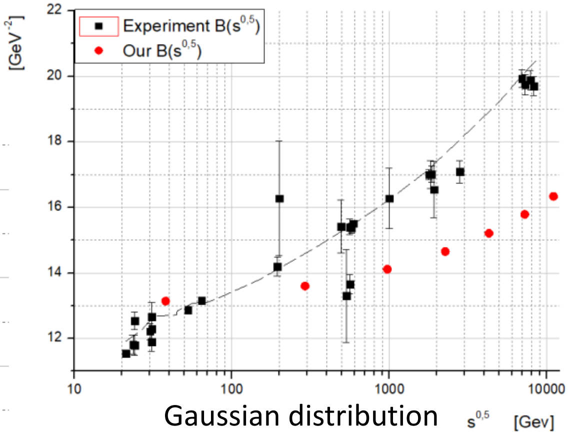
Calculated and experimental ([5],[6]) dependences of slope of the diffraction cone on energy:

For the slope of the diffraction cone, the situation is similar to.

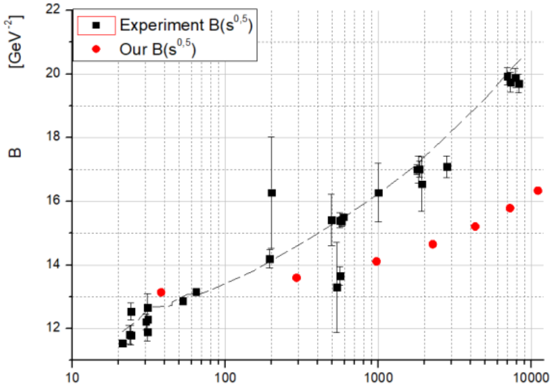
We tried to improve the distributions by varying the parton cross section σ_{parton}, but calculations have shown that the scattering cross sections are stable to such changes, and the slope of the diffraction cone slightly increases with an increase of the σ_{parton}.



Exponential distribution



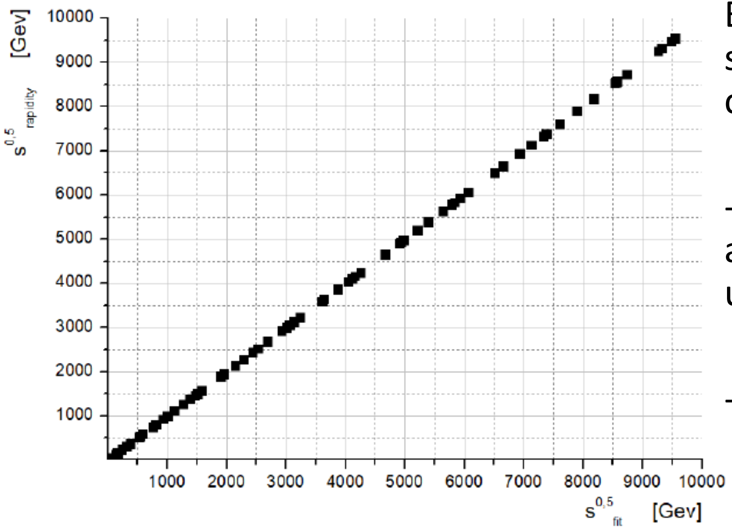
Gaussian distribution



Distribution of a United ball

[5] V. A. Schegelsky, M. G. Ryskin// Phys. Rev. D, 85:9, 094024 (2013)8
[6] V.A. Khoze , M. Ryskin and M. Taševský (2020)

Asymmetric case

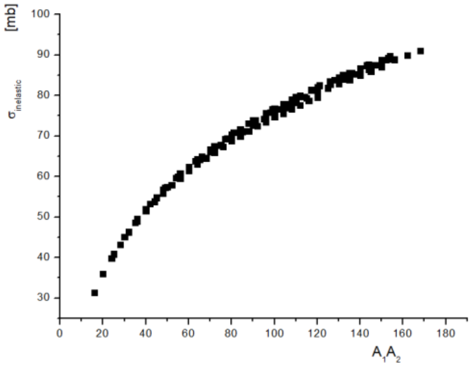


Plot of energy dependences for asymmetric collisions of protons obtained by different methods of approximation

Before that, we considered symmetric cases, because we were not sure if the fitting formulas could be applied to the asymmetric case. But using the concept of rapidity, we showed that:

-from symmetric collisions, you can get the value of the energy of an asymmetric collision and it will be exactly the same as if we used the fitting formula

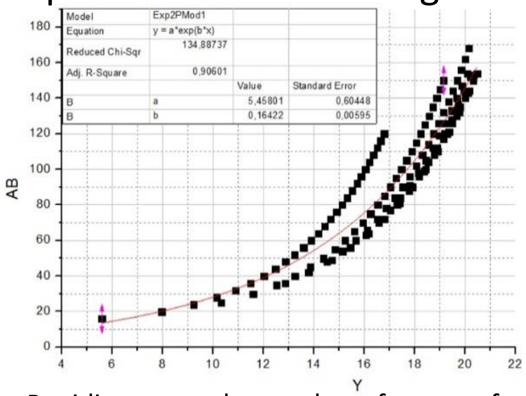
-energy depends on the number of particles as on the product



The number of partons from energy

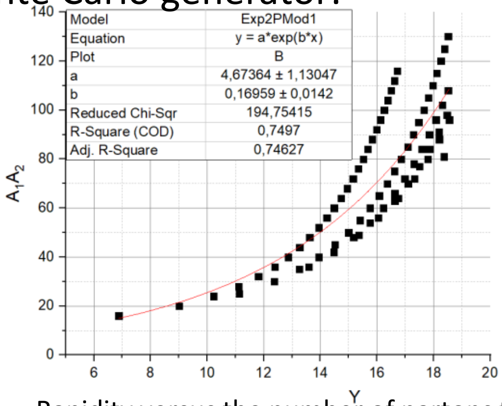
Subsequently, we derived a formula for the dependence of energy on the number of particles for the asymmetric case. Now, knowing the coefficients a and b for each specific distribution, it became possible to analytically calculate the energy from the number of particles without using a Monte Carlo generator:

$$\begin{cases} A_1A_2 = ae^{bY}; \\ Y = 2\ln\left(\dfrac{\sqrt{s}}{m_p}\right); \end{cases} \Rightarrow \sqrt{S} = \left(\frac{A_1A_2}{a}\right)^{\frac{1}{2b}} m_p;$$



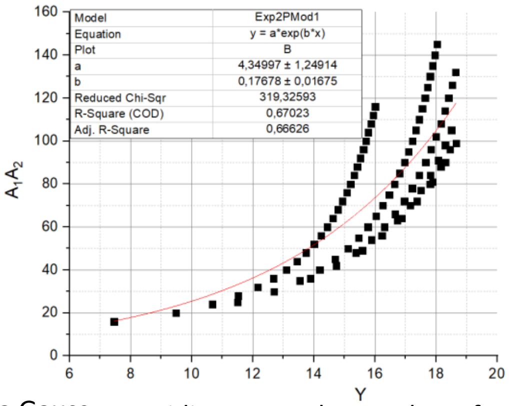
Rapidity versus the number of partons for an exponential distribution

$$\sqrt{S} = \left(\frac{A_1A_2}{5{,}5\pm0{,}6}\right)^{\frac{1}{0{,}32\pm0{,}01}} 0{,}938$$



Rapidity versus the number of partons for a Gauss distribution

$$\sqrt{S} = \left(\frac{A_1A_2}{4{,}7\pm1{,}1}\right)^{\frac{1}{0{,}34\pm0{,}02}} 0{,}938$$



Rapidity versus the number of partons for a Uniform Ball distribution

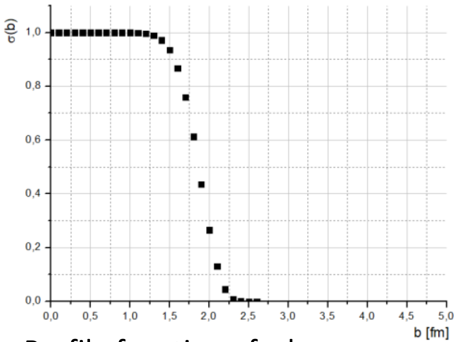
$$\sqrt{S} = \left(\frac{A_1A_2}{4{,}3\pm1{,}2}\right)^{\frac{1}{0{,}36\pm0{,}02}} 0{,}938$$

Froissard's theorem

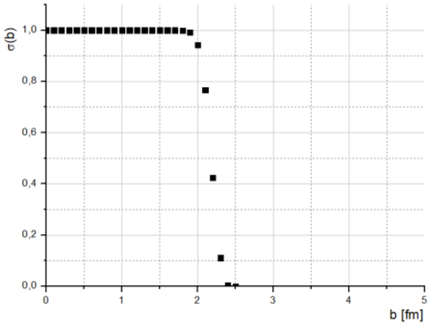
By Froissard's theorem, the total and inelastic cross sections can grow no faster than the square of the logarithm of the energy. This corresponds to the fitting formula in this work :

$$\sigma_{inel} = 30.915 - 0.937 \ln s + 0.188 \ln^2 s$$

For the case when protons are uniform balls, it can be proved that all quantities tend to certain limits.



Profile function of a homogeneous ball at energy $\sqrt{s}$=100TeV



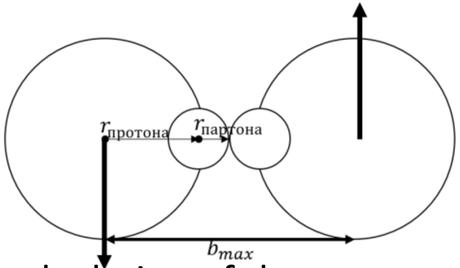
Profile function of a homogeneous ball at energy $\sqrt{s}$=1000TeV

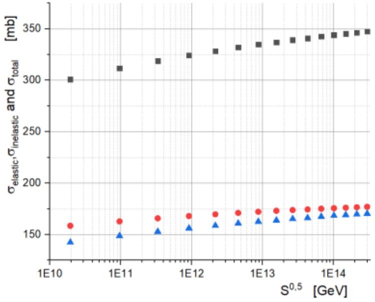
The graphs above show that σ (b) of uniform balls take the form of a step function of height 1 as the energies tend to infinity.

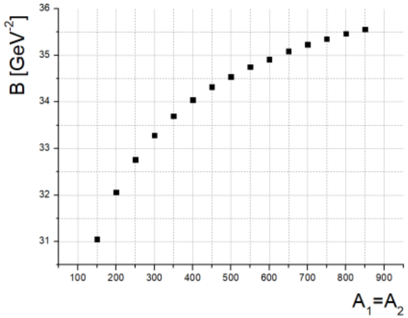
Froissard's theorem

$b_{max} = 2(0.831\sqrt{5/3} + 0.162)$ fm=2.4697 fm, where $0.831\sqrt{5/3}$ fm is the radius of the proton boundary, and 0.162 fm is the parton radius obtained from the parton cross section 3.3 mb.

Scheme of collisions of protons with the largest impact parameter :







Analytical calculation of the cross sections and slope of the diffraction cone:

$$\sigma_{inel} = \int_0^\infty 2\pi b\sigma(b)\, db = \int_0^{b_{max}} 2\pi b\, db = \pi b_{max}^2 = 191,624mb$$

$$\sigma_{total} = \int_0^\infty 4\pi b(1-\sqrt{1-\sigma(b)})\, db = \int_0^{b_{max}} 4\pi b\, db = 2\pi b_{max}^2 = 383,248m$$

$$\sigma_{el} = \sigma_{total} - \sigma_{inel} = \pi b_{max}^2 = 191,624mb$$

$$B = \frac{1}{2}\frac{\int_0^\infty b^3(1-\sqrt{1-\sigma(b)})\, db}{\int_0^\infty b(1-\sqrt{1-\sigma(b)})\, db} = \frac{1}{2}\frac{\int_0^{b_{max}} b^3\, db}{\int_0^{b_{max}} b\, db} = \frac{b_{max}^2}{4} = 39,16GeV^{-2}$$

Cross sections and slope of the diffraction cone as an approximation of the graphs:

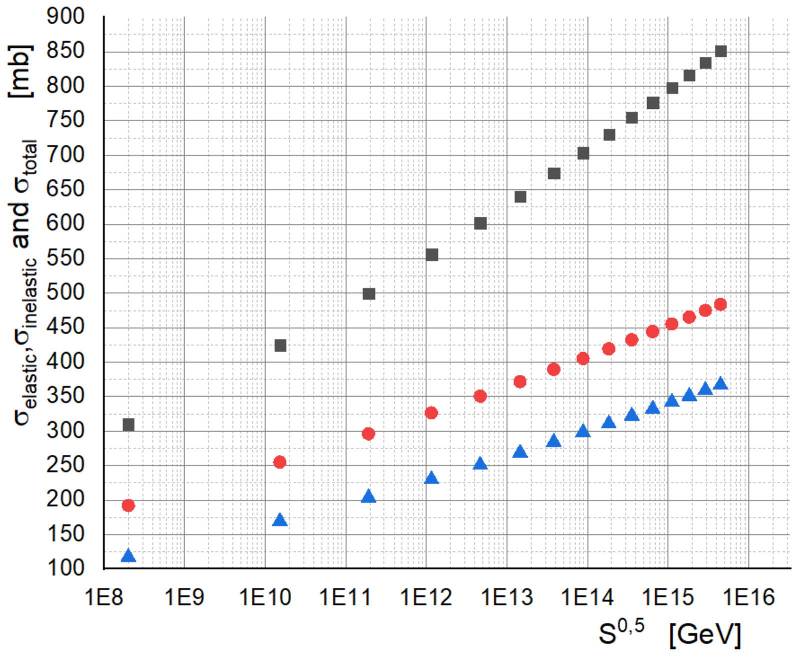
$$\sigma_{inel} = 181.0 \pm 3mb$$

$$\sigma_{total} = 358.0 \pm 0.7mb$$

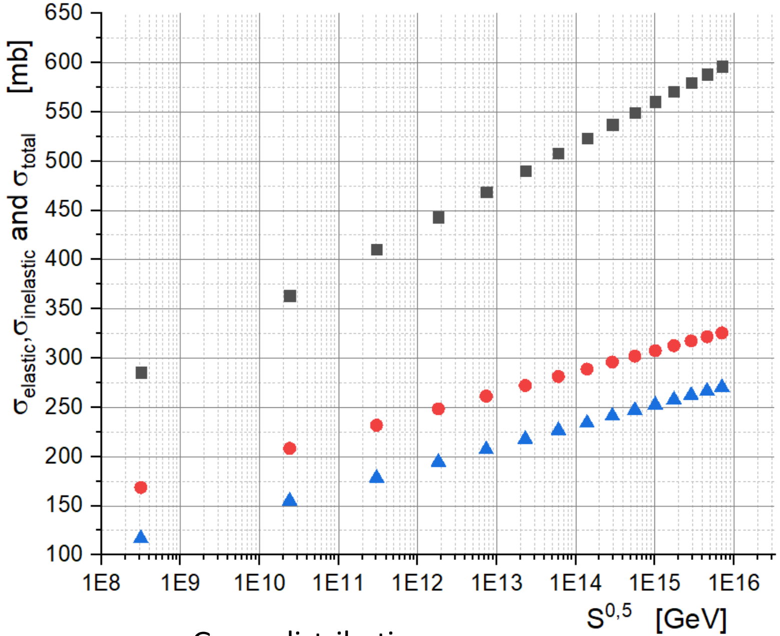
$$\sigma_{el} = 177.2 \pm 0.5mb$$

$$B = 36.57 \pm 0.07GeV^{-2}$$

Conclusion: the values differ by no more than 7%



Exponential distribution



Gauss distribution

To check Froissard's theorem, we approximate the graphs above with the formula: $\sigma_{inel} = A - B\ln(s) + C\ln^2(s)$

Distribution	A, [mb]	B, [mb]	C, [mb]
UnitedBall	41+-1	7,14+-0,008	-0,009+-0,001
Exp	7,6+-0,8	5,63+-0,06	0,210+-0,001
Gauss	-8,2692+-0,77908	8,91+-0,05	0,0067+-0,0009

This means that our calculations satisfy Froissard's theorem; moreover, they satisfy the stronger statement that the coefficient in front of $\ln^2(s)$ does not exceed $\frac{\pi}{m_\pi^2} \approx 172{,}43\mathrm{Gev}^{-2}$=67,14mb [7]

[7] Introduction to the physics of the total cross section at LHC, Giulia Pancheri, Yogendra N. Srivastava

Conclusion

Output:

-The MC Glauber model of pp collisions at the parton level is developed, which satisfactorily describes the cross sections and slope of the diffraction cone. The best agreement with experiment is obtained for the exponential form of the spatial parton distribution.

-The self-consistency of the model has been verified in the transition to a moving frame of reference.

-An explicit form is obtained for the dependence of the number of initial partons on the beam energy.

-High energy limit considered