

# PROPERTIES OF ISOSCALAR GIANT MULTIPOLE RESONANCES IN MEDIUM-HEAVY CLOSED-SHELL NUCLEI: A SEMIMICROSCOPIC DESCRIPTION

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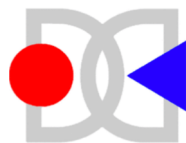
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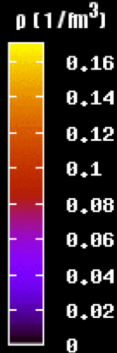
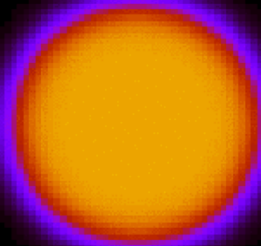
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The particle-hole dispersive optical model (PHDOM), having a few unique abilities, is used to describe main properties of isoscalar giant multipole resonances ( $L \leq 3$ ) in medium-heavy spherical nuclei. Besides the energy, main properties include the projected transition density, strength function and probabilities of direct one-nucleon decay. Overtones of the isoscalar monopole and quadrupole giant resonances are also considered. Calculation results obtained for the  $^{208}\text{Pb}$  nucleus are compared with available experimental data.



# Isoscalar Multipole Giant Resonances

**L=0**



**GMR**

Breathing mode

$$\sum r_i^2$$

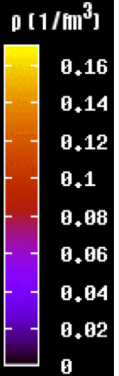
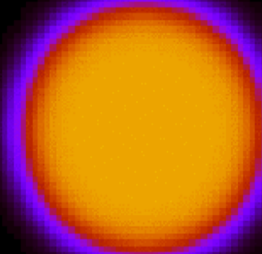
$$2\hbar\omega$$

Squeezing mode

$$\sum r_i^3 Y_1$$

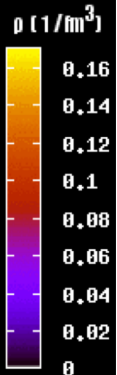
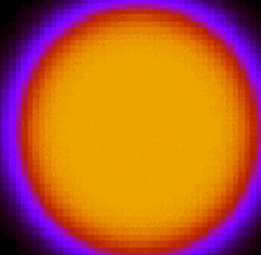
$$3\hbar\omega$$

**L=1**



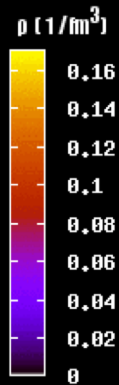
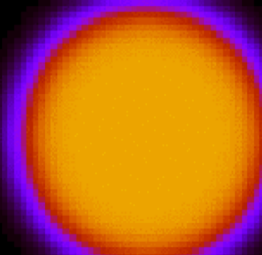
**ISGDR**

**L=2**



**GQR**

**L=3**



**GOR**

By M. Itoh of Tohoku Univ.

## Brief description of the model

The particle-hole dispersive optical model (PHDOM) was formulated recently (*Urin M.H., Phys. At. Nucl., 2011; Phys. Rev. C, 2013*) to describe in a semi-microscopic way the main properties of high-energy particle-hole-type excitations in medium-heavy-mass spherical nuclei.

Within PHDOM the main relaxation modes of the mentioned excitations are commonly taken into account. These modes are:

- (i) Landau damping (that is due to the shell-structure of nuclei);
- (ii) coupling to single-particle continuum (that is due to the fact, that nuclei are the open Fermi-systems);
- (iii) coupling to many-quasiparticle configurations, or to chaotic states (the spreading effect).

The model is an extension of the standard and non-standard versions of the continuum RPA on taking (phenomenologically and in average over the energy) the spreading effect into account.

## The unique feature of the model is its abilities to describe:

- ❑ the energy-averaged strength functions, corresponding to a single-particle long-range external field (or to a probing operator) in a wide interval of the excitation energy, including the given giant resonance and its low- and high-energy “tails”;
- ❑ direct-nucleon-decay properties of high-energy p-h-type nuclear excitations (partial probabilities of direct nucleon decay from a given energy interval);
- ❑ the energy-averaged double transition density, which determines the energy-averaged cross sections of hadron-nucleus scattering, accompanying by excitation of the particle-hole-type states in a wide excitation energy interval.

## Ingredients of the model:

- ✓ the Landau-Migdal p-h interaction;
- ✓ a phenomenological nuclear mean field partially consistent with interaction;
- ✓ the imaginary part of the effective optical model potential (this part determines the corresponding real part via a proper dispersive relationship).

Most of model parameters are taken from independent data, except of the intensity of the imaginary part, which is adjusted to reproduce in calculations the observable total width of the given giant resonance.

► the isospin symmetry of a model Hamiltonian

$$H_0 = \sum_a H_0(a) \qquad F = \frac{1}{2} \sum_{a \neq b} F(a, b)$$

$$F(x_a, x_b) \rightarrow (F + F' \tau_a \tau_b) \delta(r_a - r_b)$$

The mean Coulomb  $U_C = \sum_a \frac{1}{2} (1 - \tau_a^{(3)}) U_C(r_a)$

– the main source of the weak violation of the isospin symmetry.

$$[H, T^{(-)}] = U_C^{(-)}$$



RPA



$$U_1 = \frac{1}{2} v(r) \tau^{(3)} \quad v(r) = 2F' n^{(-)}(r) - \text{self-consistency condition}$$

$$n^{(-)}(r) = n^n(r) - n^p(r)$$

## ► the translation symmetry of a model

$$1^- \text{ spurious state} \quad \omega \rightarrow 0 \quad EWSR \rightarrow 1$$

$$F(x, x') = C(F(r) + F' \tau \tau') \delta(r - r')$$

$$F(r) = f^{in} f_{WS}(r) + f^{ex} (1 - f_{WS}(r, R, a))$$

$$(EWSR)_{V_L} = \frac{\hbar^2}{2m} \int \left[ \left( \frac{dV_L}{dr} \right)^2 + L(L+1) \left( \frac{V_L}{r} \right)^2 \right] n^{(+)}(r) r^2 dr$$

## The nuclear mean field

$$U(x) = U_0(x) + U_{so}(x) + U_1(x) + U_c(x)$$

$$U_0(x) = -U_0 f_{WS}(r, R, a)$$

$$U_{SO}(x) = -U_{SO} \frac{1}{r} \frac{df_{WS}}{dr} l_s$$

$$U_1(x) = \frac{1}{2} v(r) \tau^{(3)}$$

$$U_c(x) = \frac{1 - \tau^{(3)}}{2} U_c(r)$$

## The Landau-Migdal particle-hole interaction

$$F(x, x') = C(F(r) + F' \tau \tau') \delta(r - r')$$

$$F(r) = f^{in} f_{WS}(r) + f^{ex} (1 - f_{WS}(r, R, a))$$



## Spreading effect – PHDOM

$[-iW(\omega) + P(\omega)]f_{WS}(r)$  - the "optical-model like" addition to the nuclear mean field

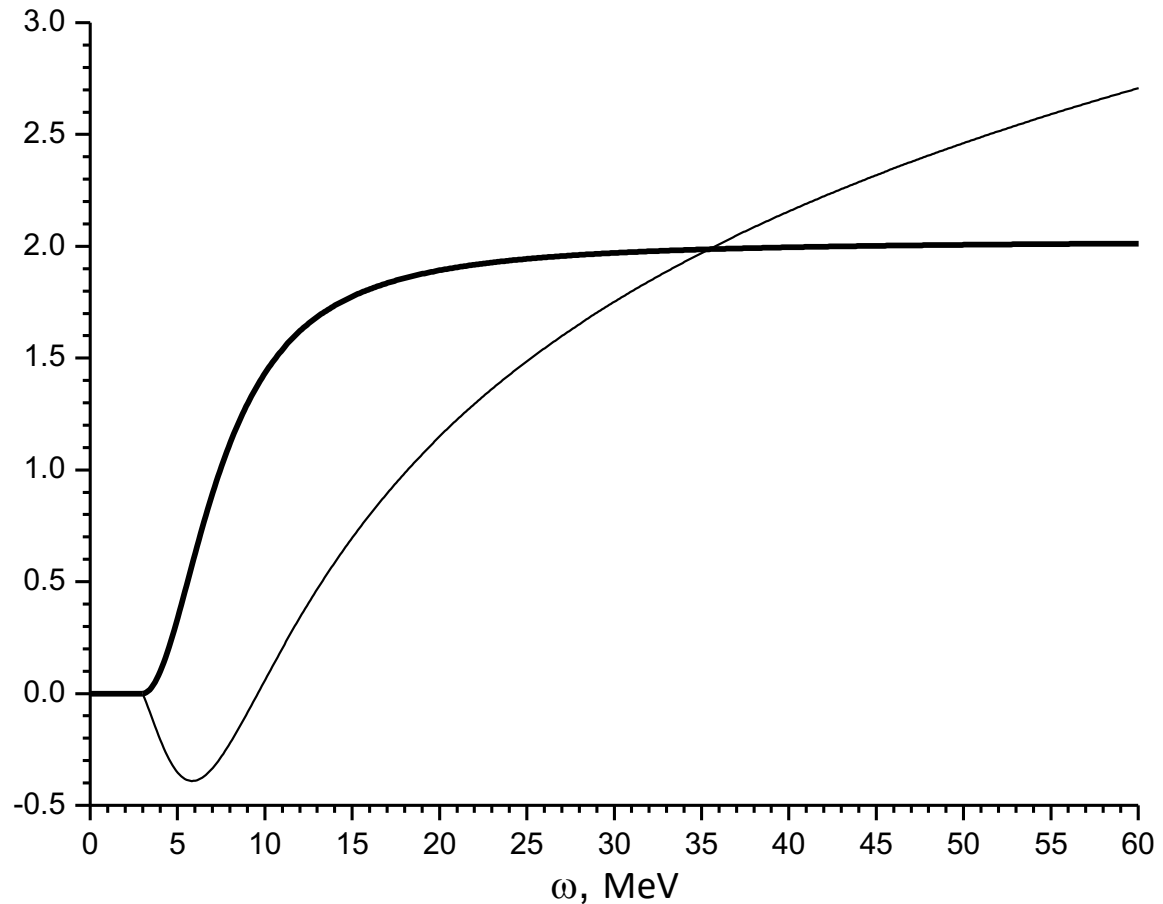
$$2W(\omega \geq \Delta) = \alpha \frac{(\omega - \Delta)^2}{1 + (\omega - \Delta)^2 / B^2}, \quad W(\omega \leq \Delta) = 0$$

$W(\omega), P(\omega), \text{MeV}$

$$\alpha = 0.20 \text{ MeV}^{-1}$$

$$B = 4.5 \text{ MeV}$$

$$\Delta = 3 \text{ MeV}$$



# Basic equations for describing high-energy isoscalar multipole excitations within the PHDOM

Equation for the effective p-h propagator:

$$\widetilde{A}_L(r, r', \omega) = A_L(r, r', \omega) + \int A_L(r, r_1, \omega) \frac{F(r_1)}{r_1^2} \widetilde{A}_L(r_1, r', \omega) dr_1$$

$$\rho_L(r, r', \omega) = -\frac{1}{\pi} \text{Im} \widetilde{A}_L(r, r', \omega) \text{ - double transition density}$$

$$S_L(\omega) = -\frac{1}{\pi} \text{Im} \int V_L(r) \widetilde{A}_L(r, r', \omega) V_L(r') dr dr' \text{ - strength function}$$

$$S_L(\omega) = -\frac{1}{\pi} \text{Im} \int V_L(r) A_L(r, r', \omega) \widetilde{V}_L(r', \omega) dr dr'$$

$$\int \widetilde{A}_L(r, r', \omega) V_L(r') dr' = \int A_L(r, r', \omega) \widetilde{V}_L(r', \omega) dr'$$

Equation for the effective field:

$$\widetilde{V}_L(r, \omega) = V_L(r) + \frac{F(r)}{r^2} \int A_L(r, r', \omega) \widetilde{V}_L(r', \omega) dr'$$

Projected transition density:

$$\rho_{V_L}(r, \omega) = \int \rho_L(r, r', \omega) V_L(r') dr' / S_L^{1/2}(\omega)$$

$$S_L(\omega) = \left( \int \rho_{V_L}(r, \omega) V_L(r) dr \right)^2$$

$$\frac{1}{r^2} \rho_{V_L}(r, \omega) = -\frac{1}{\pi} \text{Im} \frac{\widetilde{V}_L(r, \omega)}{F(r) S_L^{1/2}(\omega)}$$

# The IS radial component of the energy-averaged “free” p-h propagator:

$$A_L(r, r', \omega) = A_L^i + A_L^{ii} + A_L^{iii}$$

$$A_L^i(r, r', \omega) = \sum_{(\lambda), \mu} n_\mu (t_{(\lambda)(\mu)}^L)^2 \chi_\mu(r) \chi_\mu(r') g_{(\lambda)}(r, r', \varepsilon_\mu + \omega)$$

$$A_L^{ii}(r, r', \omega) = \sum_{\lambda, (\mu)} n_\lambda (t_{(\lambda)(\mu)}^L)^2 \chi_\lambda(r) \chi_\lambda(r') g_{(\mu)}(r, r', \varepsilon_\mu - \omega)$$

$$A_L^{iii}(r, r', \omega) = \sum_{\lambda, \mu} n_\lambda n_\mu (t_{(\lambda)(\mu)}^L)^2 \chi_\lambda(r) \chi_\lambda(r') \chi_\mu(r) \chi_\mu(r') \times$$

$$\times \frac{2(iW(\omega) - P(\omega)) f_\lambda f_\mu}{(\varepsilon_\lambda - \varepsilon_\mu - \omega)^2 + (iW(\omega) - P(\omega))^2 f_\lambda^2 f_\mu^2}$$

$$t_{(\lambda)(\mu)}^L = \frac{1}{\sqrt{2L+1}} \langle (\lambda) \| Y_L \| (\mu) \rangle \quad f_\lambda = \int f_{ws}(r) \chi_\lambda^2(r) dr$$

## The optical-model radial Green functions satisfy to the equations:

$$\left[ h_{0,(\lambda)} - (\varepsilon_{\mu} + \omega + (iW(\omega) - P(\omega))f_{\mu}f_{WS}(r)) \right] \times \\ \times g_{(\lambda)}(r, r', \varepsilon_{\mu} + \omega) = -\delta(r - r')$$

$$\left[ h_{0,(\mu)} - (\varepsilon_{\lambda} - \omega + (iW(\omega) - P(\omega))f_{\lambda}f_{WS}(r')) \right] \times \\ \times g_{(\mu)}(r, r', \varepsilon_{\lambda} - \omega) = -\delta(r' - r)$$

$h_{0,(\lambda)}$  - are the radial parts of a s-p Hamiltonian (including the spin-orbit and centrifugal terms)  
 $h_{0,(\mu)}$

# Direct-one-nucleon-decay strength functions and branching ratios

$$S_{L,\mu}^{\uparrow}(\omega) = \sum_{(\lambda)} n_{\mu} (t_{(\lambda)(\mu)}^L)^2 \left| \int \chi_{(\lambda)}^{(-)*}(r, \varepsilon_{\mu} + \omega) \widetilde{V}_L(r, \omega) \chi_{\mu}(r) dr \times \right. \\ \left. \times \int \chi_{\mu}(r') \widetilde{V}_L^*(r, \omega) \chi_{(\lambda)}^{(+)}(r', \varepsilon_{\mu} + \omega) dr' \right|$$

$$S_L^{\uparrow}(\omega) = \sum_{\mu} S_{L,\mu}^{\uparrow}(\omega) \quad b_{L,\mu}^{\uparrow}(\delta) = \frac{\int_{(\delta)} S_{L,\mu}^{\uparrow}(\omega) d\omega}{\int_{(\delta)} S_L^{\uparrow}(\omega) d\omega}$$

$$b_{L,tot}^{\uparrow}(\delta) = \sum_{\mu} b_{L,\mu}^{\uparrow}(\delta)$$

## Probing operators

$$\mathbf{ISGMR} \quad V_{L=0}(r) = r^2 - \eta_0 \langle r^2 \rangle$$

$$\mathbf{ISGQR} \quad V_{L=2}(r) = r^2$$

$$\mathbf{ISGDR} \quad V_{L=1}(r) = r(r^2 - \eta_1 \langle r^2 \rangle)$$

$$\mathbf{ISGOR} \quad V_{L=3}(r) = r^3$$

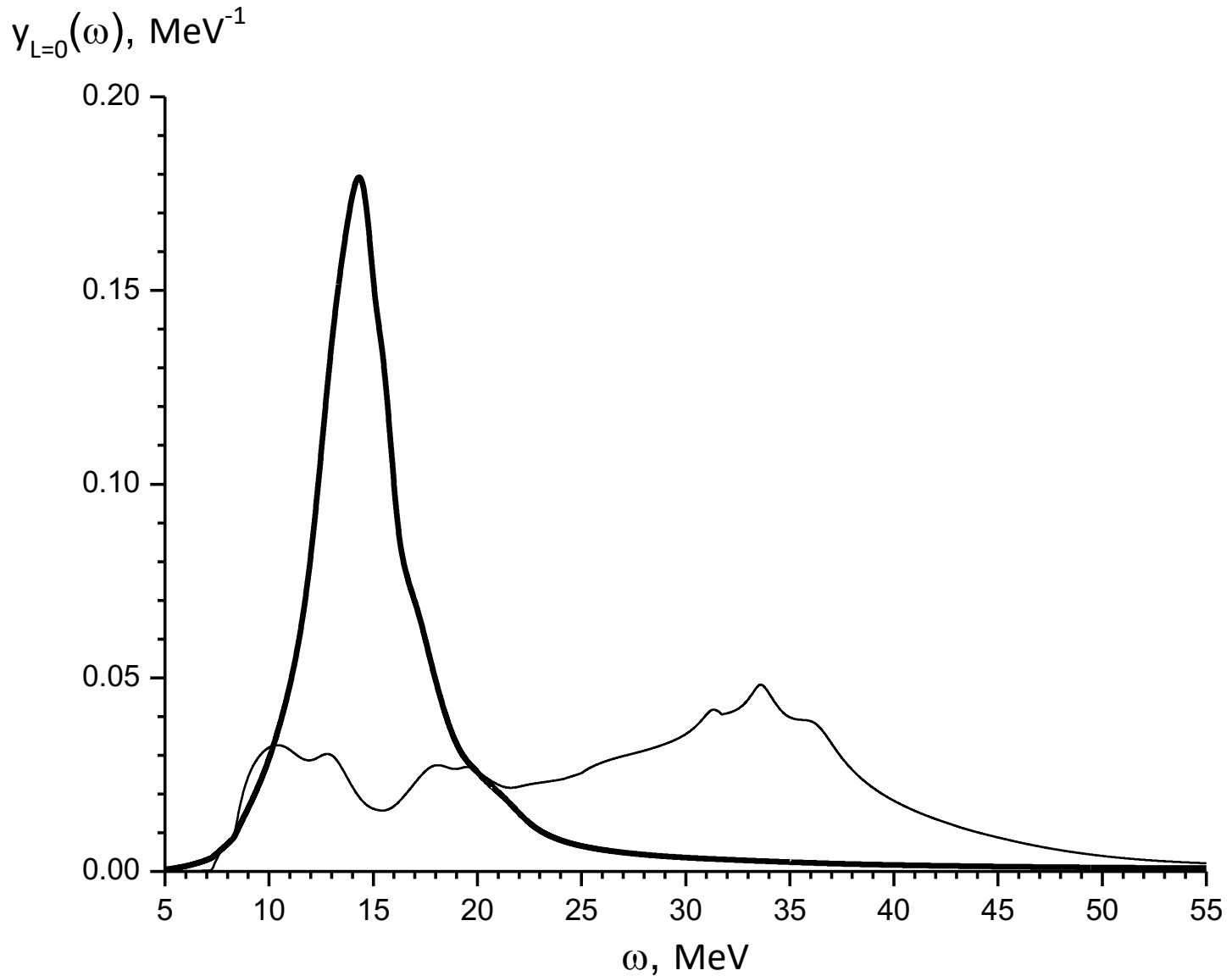
$$y_L(\omega) = \frac{\omega S_L(\omega)}{(EWSR)_{V_L}}$$

$$(EWSR)_{V_L} = \frac{\hbar^2}{2m} \int \left( \left( \frac{dV_L}{dr} \right)^2 + L(L+1) \left( \frac{V_L}{r} \right)^2 \right) n^{(+)}(r) r^2 dr$$

$$V_L^{ov}(r) = r^2(r^2 - \eta_L^{ov} \langle r^2 \rangle)$$

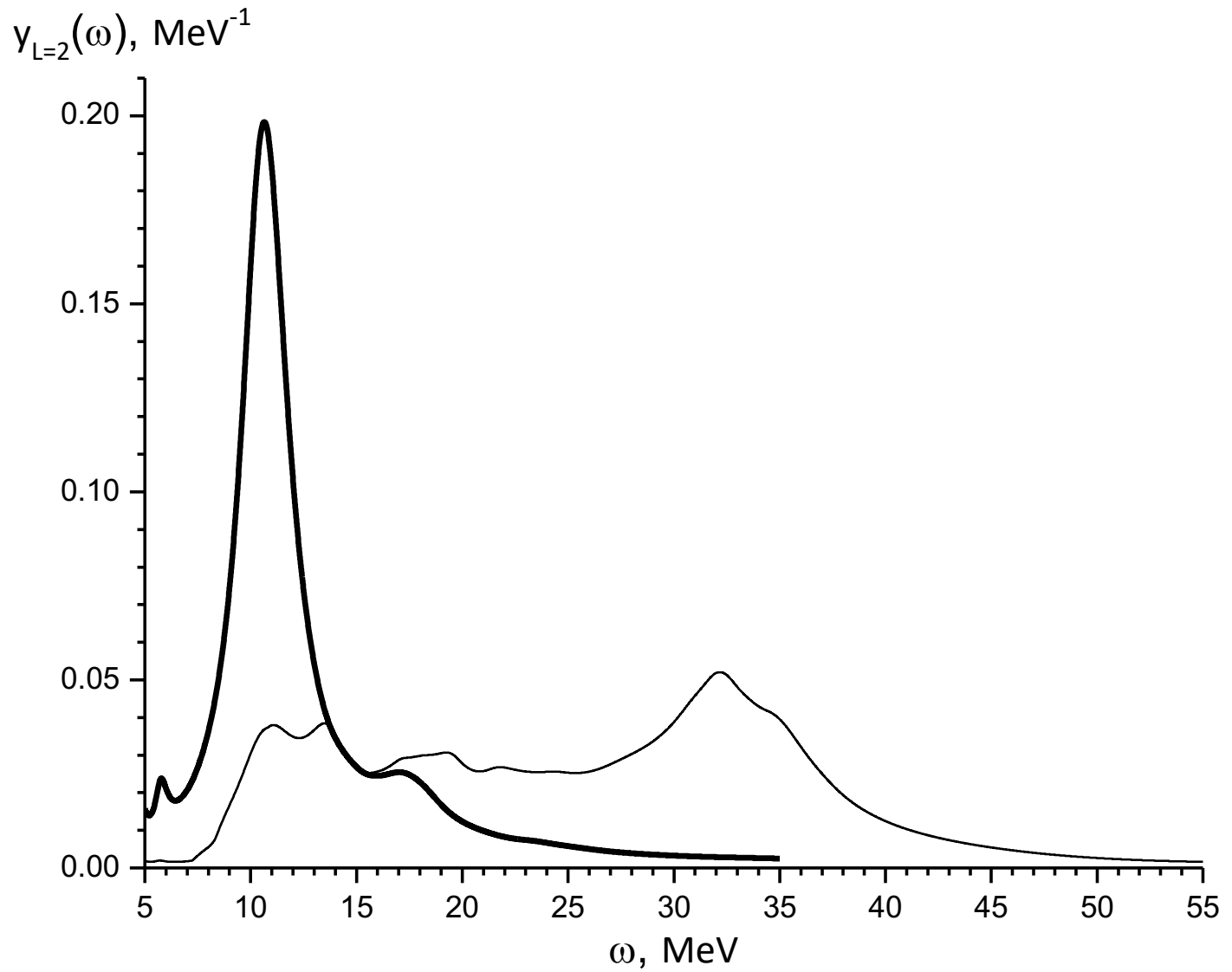
$$\int \rho_{V_L}(r, \omega_{L(peak)}) V_L^{ov}(r) dr = 0$$

**Fig. 1. The relative energy-weighted strength functions calculated within PHDOM for ISGMR(solid line) and ISGMR2 (thin line) in  $^{208}\text{Pb}$ .**

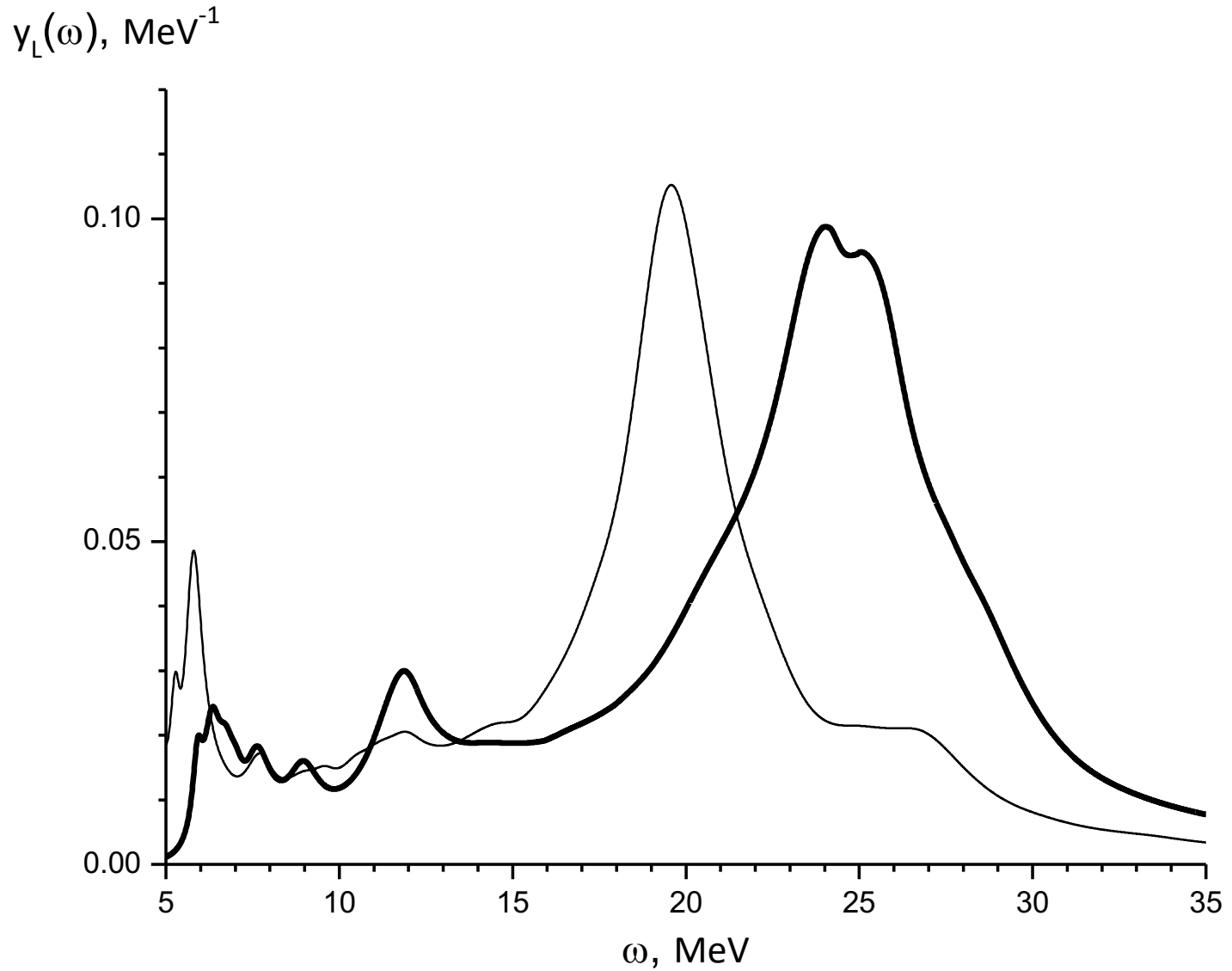




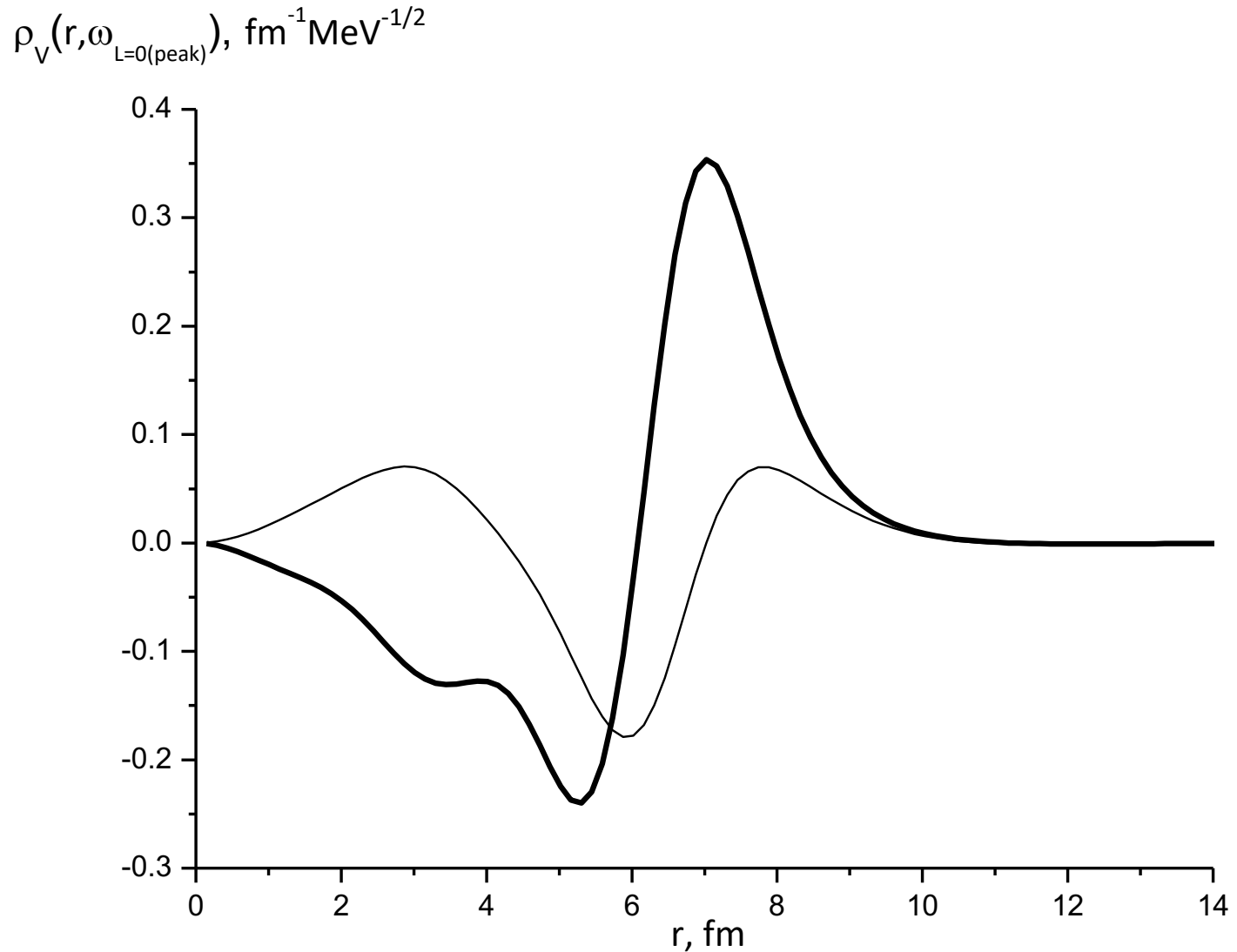
**Fig. 2. The same as in Fig. 1, but for ISGQR and ISGQR2.**



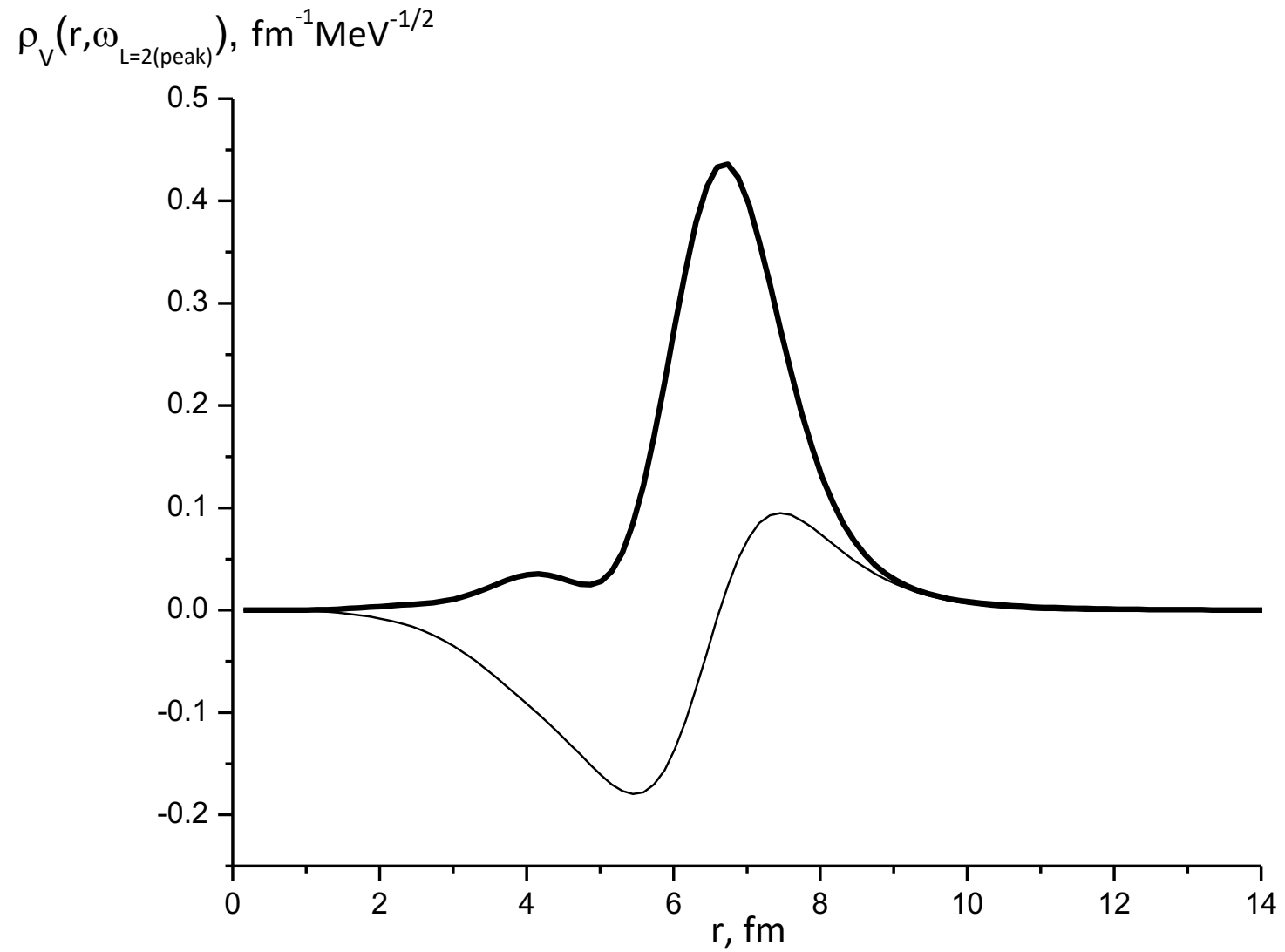
**Fig. 3. The same as in Figs. 1,2, but for ISGDR and ISGOR.**



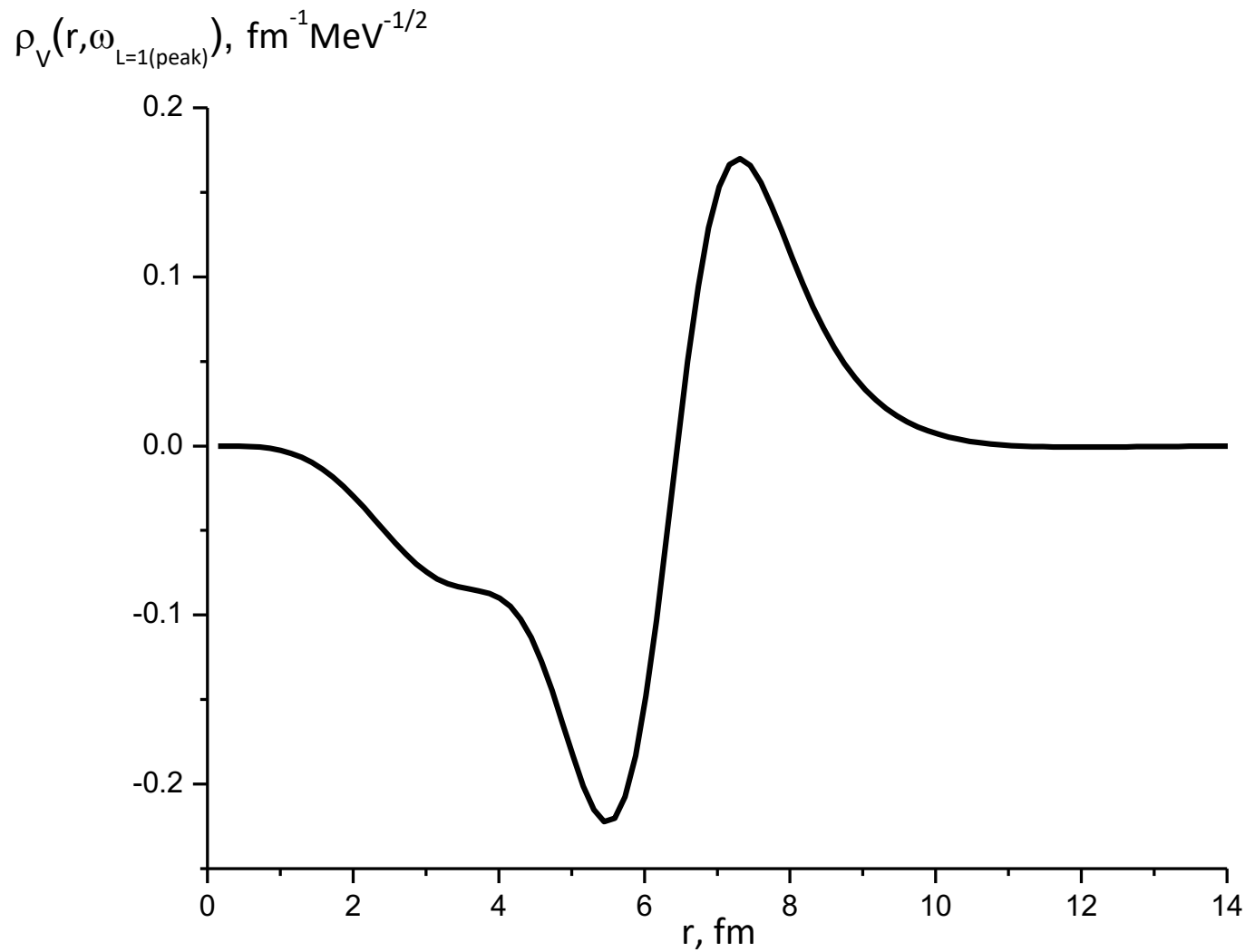
**Fig. 4. The projected transition densities taken at the resonance peak-energy and calculated within PHDOM for ISGMR (solid line) and ISGMR2 (thin line) in  $^{208}\text{Pb}$ .**



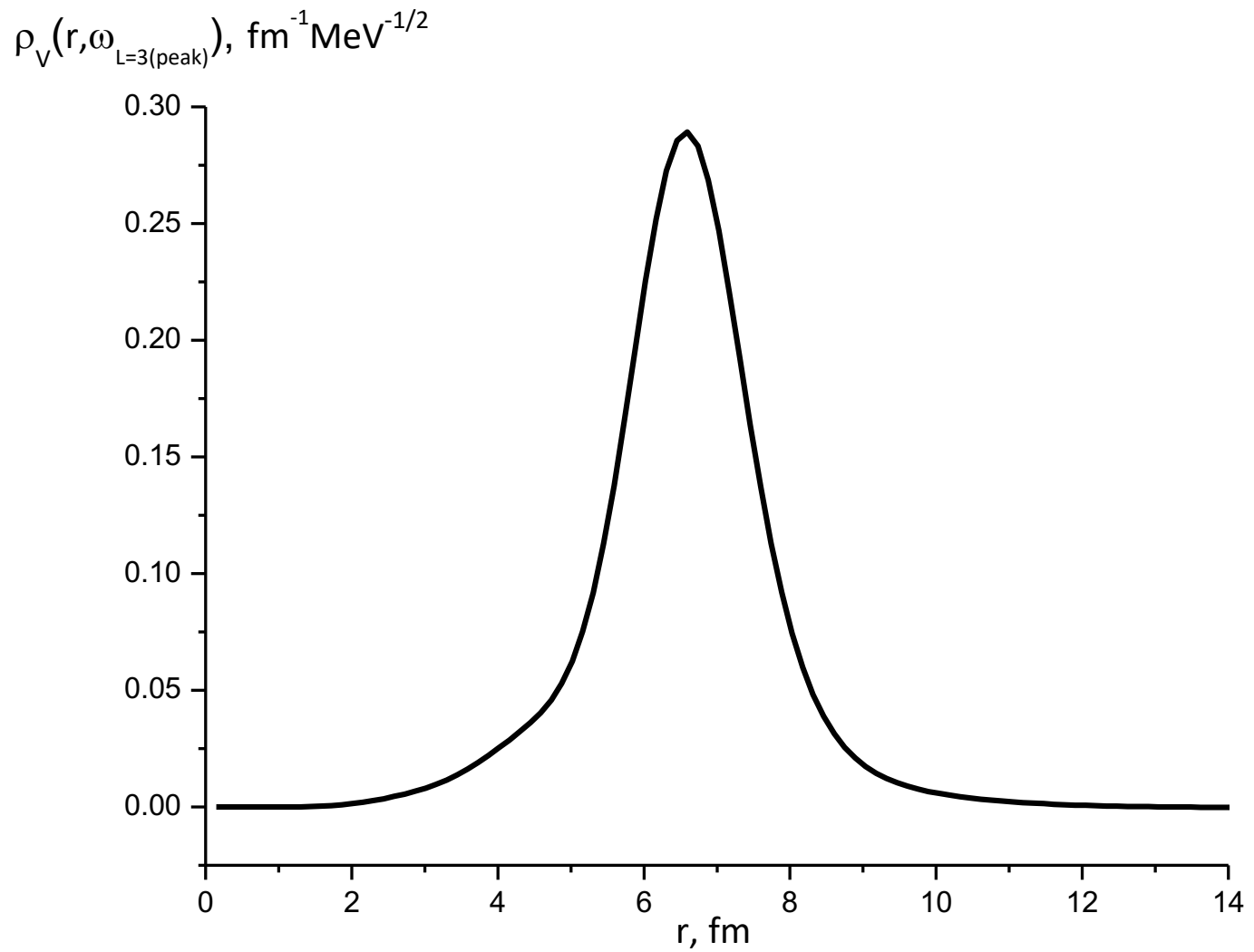
**Fig. 5. The same as in Fig. 4, but for ISGQR and ISGQR2.**



**Fig. 6. The projected transition densities taken at the resonance peak-energy and calculated within PHDOM for ISGDR.**



**Fig. 7. The same as in Fig. 6, but for ISGOR.**



**Table 1. The calculated parameters for the ISGRs in  $^{208}\text{Pb}$ .**

$L, \eta_L$	$\omega_1 - \omega_2$	$x_L$ (%)	$\overline{\omega}_L$	$\omega_{L(\text{peak})}$	$\Gamma_{L(\text{FWHM})}$	
0 $\eta_0 = 1$	5 – 35	100	13.9	14.2	1.4	cRPA (2W=0.1 MeV)
	5 – 35	103	14.5	<b>14.2</b>	<b>4.2</b>	PHDOM
	8 – 20	99±15		13.96±0.20 13.5±0.2	2.88±0.20 3.6±0.4	Expt. [1] Expt. [2]
	7 – 60		13.92			SkT1 ( $m^*/m = 1.0$ )
1 $\eta_1 = 1.72$	15 – 35	81	22.9	-	-	cRPA (2W=0.1 MeV)
	15 – 35	83	23.5	<b>23.9</b>	<b>7.0</b>	PHDOM
	8 – 35	88±15		22.20±0.30 22.5±0.3 22.1±0.3	9.39±0.35 10.9±0.9 3.8±0.8	Expt. [1] Expt. [2] Expt. [3]
	16 – 60		23.40			SkT1 ( $m^*/m = 1.0$ )
2	5 – 35	85	11.0	10.7	0.2	cRPA (2W=0.1 MeV)
	5 – 35	90	11.3	<b>10.6</b>	<b>2.7</b>	PHDOM
	8 – 35	100±13		10.89±0.30 10.9±0.3	3.0±0.3 3.1±0.3	Expt. [1] Expt. [4]
	7 – 60		10.55			SkT1 ( $m^*/m = 1.0$ )
3	5 – 35	77	15.3	18.9	0.4	cRPA (2W=0.1 MeV)
	5 – 35	80	15.5	<b>19.5</b>	<b>3.7</b>	PHDOM
	8 – 35	70±14		19.6±0.5 19.1±1.1	7.4±0.6 5.3±0.8	Expt. [1] Expt. [5]
	15 – 60		19.34			SkT1 ( $m^*/m = 1.0$ )

[1] D.H. Youngblood, et al., Phys. Rev. C 69, 034315 (2004).

[2] M. Uchida et al., Phys. Lett. B549, 58 (2002).

[3] M. Hunaydi, et al., Phys. Rev. C 75, 014606 (2007).

[4] D. H. Youngblood, et al., Phys. Rev. C 23, 1997 (1981).

[5] T. A. Carey, et al., Phys. Rev. Lett. 45, 239 (1980).

**Table 2. Calculated partial branching ratios (%) for direct neutron decay of the ISGRs in  $^{208}\text{Pb}$  ( $S_\mu=1$ ).**

	$b_{L=0,\mu}^\uparrow$	$b_{L=1,\mu}^\uparrow$	$b_{L=2,\mu}^\uparrow$	$b_{L=3,\mu}^\uparrow$
$\mu^{-1} \setminus \omega_2$	12.5 - 15.5 [6]	20 - 25 [3]	9 - 12	16 - 23
$3p_{1/2}$	3.6	1.1	2.8	1.6
$2f_{5/2}$	18.0	5.4	1.5	5.9
$3p_{3/2}$	7.5	2.6	5.8	3.8
$1i_{13/2}$	0.8	11.4	0.2	5.9
$2f_{7/2}$	26.6	9.3	0.2	13.3
$\sum_\mu b_{L,\mu}^\uparrow$	56.5	29.8	10.5	30.5
$\left(\sum_\mu b_{L,\mu}^\uparrow\right)_{expt}$	$22 \pm 6$ [6] $14.3 \pm 3$ [7]	$23 \pm 5$ [3] $10.5$ [8]	-	-
$b_L^{\uparrow,n}$	56.7	66.8	10.6	37.5

[3] M. Hunaydi et al., Nucl. Phys. A 731, 49 (2004).

[6] S. Brandenburg et al., Phys. Rev. C 39, 24448 (1989).

[7] A. Bracco et al., Phys. Rev. Lett. 60, 2603 (1988).

[8] M. Hunaydi et al., Phys. Rev. C 75, 014606 (2007).



**Table 3. Calculated partial branching ratios (%) for direct proton decay of the ISGDRs in  $^{208}\text{Pb}$ .**

$\mu^{-1}$	$b_{L=1,\mu}^{\uparrow}$	$S_{\mu}$ [9]	$S_{\mu} \cdot b_{L=1,\mu}^{\uparrow}$	$(b_{L=1,\mu}^{\uparrow})_{expt}$ [3]
$3s_{1/2}$	3.4	0.55	1.9	$2.3 \pm 1.1$
$2d_{3/2}$	3.0	0.57	1.7	
$1h_{11/2}$	0.2	0.58	0.1	$1.2 \pm 0.7$
$2d_{5/2}$	4.1	0.54	2.2	
$\sum_{\mu} b_{L,\mu}^{\uparrow}$	10.7	-	5.9	$3.5 \pm 1.8$

[3] M. Hunaydi et al., Nucl. Phys. A 731, 49 (2004).

[9] I. Bobeldijk et al., Phys. Rev. Lett. 73, 2648 (1994).

# Conclusion

The particle-hole (p-h) dispersive optical model (PHDOM), was implemented for describing main properties of Isoscalar Giant Multipole Resonances up to  $L=3$  in medium-heavy closed-shell nuclei. The overtones of the monopole and quadrupole isoscalar giant resonances were also studied. The main properties, considered in a large excitation-energy interval, include the following energy-averaged quantities: (i) the strength function related to an appropriate probing operator; (ii) the projected one-body transition density (related to the corresponding operator), and; (iii) partial probabilities of direct one-nucleon decay. Unique abilities of PHDOM were conditioned by a joint description of the main relaxation processes of high-energy p-h configurations associated with a given giant resonance (GR). Two processes (Landau damping and coupling the mentioned configurations to the single-particle continuum) were described microscopically in terms of Landau-Migdal p-h interaction and a phenomenological mean field, partially consistent with this interaction. Another mode, the coupling to many quasiparticle states (the spreading effect) was described phenomenologically in terms of the imaginary part of the properly parameterized energy-averaged p-h self-energy term. The imaginary part determines the real one via a microscopically-based dispersive relationship. The model parameters related to a mean field and p-h interaction were taken from independent data with the isospin symmetry, and translation invariance of the model Hamiltonian also taken into account. Parameters of the imaginary part of the strength of self-energy term were adjusted to reproduce in PHDOM-based calculations of total width of ISGMR for the considered closed-shell nucleus  $^{208}\text{Pb}$  taken as an example. The calculation results were compared with available experimental data. Some of the results were compared with those obtained in microscopic Hartree-Fock based RPA calculations. These comparisons indicate that PHDOM is a powerful tool for describing ISGMPR in medium-heavy closed-shell nuclei. Extension of the model by taking nucleon-nucleon pairing interaction into account in open-shell nuclei is in order.

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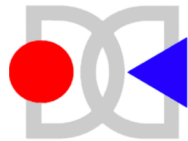
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**THANK YOU FOR ATTENTION!**