MICROSCOPIC DESCRIPTION OF ISOSCALAR GIANT MONOPOLE RESONANCE IN $^{118-132}$Sn

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LXX$^{\mathrm{th}}$ International conference «NUCLEUS-2020»:
Nuclear physics and elementary particle physics. Nuclear physics technologies
Saint Petersburg (online)
11–17 October 2020

Outline

Introduction

1. Isoscalar giant monopole resonances (ISGMR) centroid $\bar{E}$ is connected with the nuclear matter incompressibility. The incompressibility or the compression modulus is important in several physical contexts such as:

a) prompt supernova explosions;

– ***H. A. Bethe***, **Rev. Mod. Phys. 62, 801 (1990)**

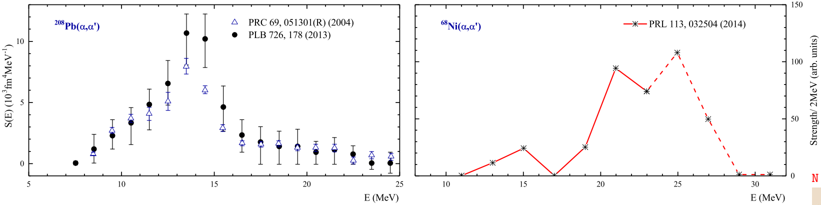
b) the interiors of neutron stars;

– ***J. M. Lattimer and M. Prakash***, **Astrophys. J. 550, 426 (2001)**

c) heavy-ion collisions at intermediate and high energies;

– ***H. Stöcker and W. Greiner***, **Phys. Rep. 137, 277 (1986).**

2. The properties of the GMR in stable and radioactive atomic nuclei have been extensively investigated in many experiments.



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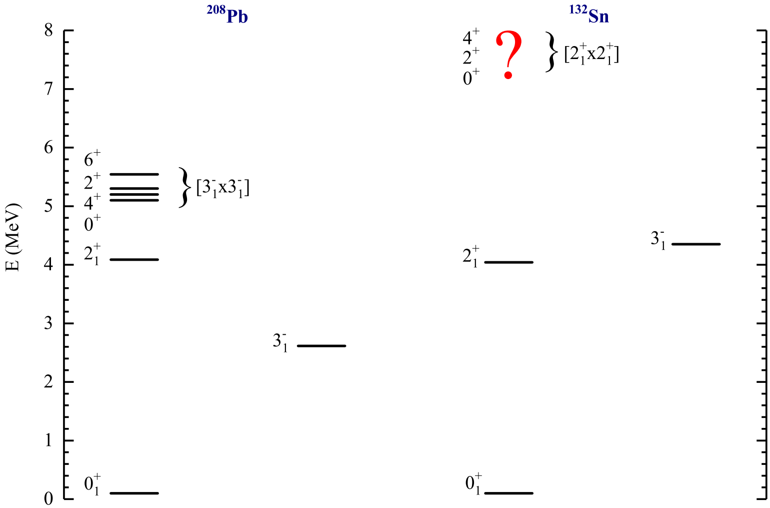
– ***U. Garg and G. Colò***, **Prog. Part. Nucl. Phys. 101, 55 (2018)**

Introduction



J. Enders, P. von Neumann-Cosel, V. Yu. Ponomarev, A. Richter, Nucl. Phys. A612, 239 (1997).

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MAIN INGREDIENTS OF THE MODEL

Realization of QRPA

We employ the effective Skyrme interaction in the particle-hole channel

$$V(\vec{r}_1,\vec{r}_2)=t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)+\frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right]$$
$$+t_2\left(1+x_2\hat{P}_\sigma\right)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$
$$+\,iW_0\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}\right].$$

T. H. R. Skyrme, **Nucl. Phys. 9, 615 (1959).**

D. Vautherin and D. M. Brink, **Phys. Rev. C5, 626 (1972).**

The Hamiltonian includes the pairing correlations are generated by the density-dependent zero-range force in the particle-particle channel

$$V_{pair}(\vec{r}_1,\vec{r}_2)=V_0\left(1-\eta\frac{\rho(r_1)}{\rho_c}\right)\delta(\vec{r}_1-\vec{r}_2),$$

where ρ_c is the nuclear saturation density; η and V_0 are model parameters. For example, $\eta=0$ and $\eta=1$ are the case of a volume interaction and a surface-peaked interaction, respectively.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, **Phys. Rev. C77, 024322 (2008).**

Realization of QRPA

The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

J. P. Blaizot and D. Gogny, **Nucl. Phys.** A284, **429 (1977)**.

The residual interaction in the particle-hole channel V_{res}^{ph} and in the particle-particle channel V_{res}^{pp} can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively.

$$V_{res}^{\mathrm{ph}} \sim \frac{\delta^2\mathcal{H}}{\delta\rho_1\delta\rho_2} \quad V_{res}^{\mathrm{pp}} \sim \frac{\delta^2\mathcal{H}}{\delta\tilde{\rho}_1\delta\tilde{\rho}_2}.$$

G. T. Bertsch and S. F. Tsai, **Phys. Rep.** 18, **125 (1975)**.

Realization of QRPA

We simplify V_{res} by approximating it by its Landau-Migdal form

$$V_{res}(\vec{k}_1, \vec{k}_2) = N_0^{-1} \sum_{l=0} \left[F_l + G_l \sigma_1 \cdot \sigma_2 + (F_l' + G_l' \sigma_1 \cdot \sigma_2) \tau_1 \cdot \tau_2 \right] P_l \left(\frac{\vec{k}_1, \vec{k}_2}{k_F^2} \right) ,$$

where $\tau_{\mathbf{i}}$ is the isospin operator, and $N_0 = 2k_F m^* / \pi^2 \hbar^2$ with k_F and m^* standing for the Fermi momentum and nucleon effective mass.

A. B. Migdal, **Theory of Finite Fermi Systems and Applications to Atomic Nuclei** (Wiley, New York, 1967).

Moreover we keep only Landau parameters F_0 and F_0'. Thus, we can write the residual interaction in the following form:

$$V_{res}^{(\mathrm{a})}(\vec{r}_1, \vec{r}_2) = N_0^{-1} \left[F_0^{(\mathrm{a})}(r_1) + F_0^{\prime(\mathrm{a})}(r_1)(\tau_1 \cdot \tau_2) \right] \delta(\vec{r}_1 - \vec{r}_2) \, ,$$

where $\mathrm{a} = \{\mathrm{ph}, \mathrm{pp}\}$ is the channel index.
The corresponding Landau parameters can be expressed via the Skyrme force parameters.

N. V. Giai and H. Sagawa, Phys. Lett. B106, 379 (1981).

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C77, 024322 (2008).

Realization of QRPA

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\left(X^{\lambda i}_{jj'}\,A^{+}(jj';\lambda\mu)-(-1)^{\lambda-\mu}Y^{\lambda i}_{jj'}\,A(jj';\lambda-\mu)\right),$$

$$A^{+}(jj';\lambda\mu)=\sum_{mm'}C^{\lambda\mu}_{jm\,j'm'}\alpha^{+}_{jm}\alpha^{+}_{j'm'}.$$

The index λ denotes total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and one-phonon excited states are $Q^{+}_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i},Q^{+}_{\lambda\mu i'}]|0\rangle=\delta_{ii'}.$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ -\mathcal{B} & -\mathcal{A}\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}=\omega\begin{pmatrix}X\\ Y\end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies ω and the amplitudes X, Y of the excited states.

P. Ring and P. Schuck, **The Nuclear Many Body Problem (Springer, Berlin 1980).**

Phonon-phonon coupling (**PPC**)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as a linear combination of one- and two-phonon configurations

$$\Psi_\nu(JM) = \left[\sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2} \right]_{JM} \right] |0\rangle$$

with the normalization condition

$$\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \right]^2 = 1.$$

V. G. Soloviev, **Theory of Atomic Nuclei: Quasiparticles and Phonons (Inst. of Phys., Bristol 1992).**

Phonon-phonon coupling (PPC)

Using the variational principle in the form

$$\delta\Big(\langle\Psi_\nu(JM)|\mathcal{H}|\Psi_\nu(JM)\rangle - E_\nu\left[\langle\Psi_\nu(JM)|\Psi_\nu(JM)\rangle - 1\right]\Big) = 0,$$

one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$:

$$(\omega_{Ji} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0;$$

$$\sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)R_i(J\nu) + 2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0.$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is the matrix element coupling one- and two-phonon configurations:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0|Q_{Ji}\mathcal{H}\left[Q^+_{\lambda_1 i_1}Q^+_{\lambda_2 i_2}\right]_J|0\rangle.$$

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. A22, 397 (2004).

A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, V. Werner, Phys. Rev. C90, 011306(R) (2014).

RESULTS AND DISCUSSION

Details of calculations

We use the parametrization SLy4 of the Skyrme interaction. This parametrization have been adjusted to reproduce nuclear matter properties, as well as nuclear charge radii, binding energies of doubly magic nuclei.

E. Chabanat et al., **Nucl. Phys.** A635, **231 (1998)**.

The pairing strength is fixed to be $V_0 = -870$ MeV fm^3 in order to fit the experimental neutron pairing gaps of ^{126,128}Cd, ^{130}Sn, and ^{132}Te obtained by the three-point formula.

A. P. Severyukhin et al., **Phys. Rev.** C101, **054309 (2020)**.

The integral characteristics of $E0$ strength function are the centroid energy E_c and the spreading width Γ

$$E_c = \frac{m_1}{m_0} \quad \text{and} \quad \Gamma = 2.35\sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2},$$

where $m_k = \sum_i B_i E0)\, E_i^k$ are the energy-weighted moments.

G. F. Bertsch, P. F. Bortignon, R. A. Broglia, **Rev. Mod. Phys.** 55, **287 (1983)**.

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Details of calculations

To construct the wave functions of the 0^+ states, in the present study we take into account all two-phonon configurations below 30 MeV that are built from the phonons with different multipoles $\lambda^\pi = 0^+, 1^-, 2^+, 3^-, 4^+$ and 5^- coupled to 0^+. We have checked that extending the configuration space plays a minor role in our calculations.

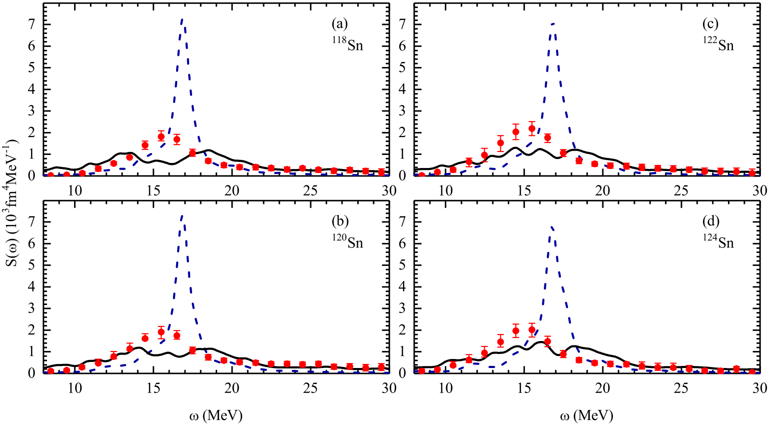
RPA low-lying states: 2_1^+, 3_1^-, 4_1^+ and 5_1^- in ^{132}Sn

λ^π_ν	Energy (MeV) Expt.	Energy (MeV) Theory	$B(E\lambda; \lambda^\pi_1 \to 0^+_{gs})$ (W.u.) Expt.	$B(E\lambda; \lambda^\pi_1 \to 0^+_{gs})$ (W.u.) Theory	Structure $\{n_1 l_1 j_1 n_2 l_2 j_2\}_\tau$
2_1^+	4.04	4.5	5.5 ± 1.5 4.4 ± 1.0	6.9	61% $\{2f_{\frac{7}{2}} 1h_{\frac{11}{2}}\}_\nu$ 33% $\{2d_{\frac{5}{2}} 1g_{\frac{9}{2}}\}_\pi$
3_1^-	4.35	5.5	> 7.1	28.0	12% $\{1i_{\frac{13}{2}} 1h_{\frac{11}{2}}\}_\nu$ 12% $\{2f_{\frac{7}{2}} 3s_{\frac{1}{2}}\}_\nu$ 16% $\{1h_{\frac{11}{2}} 1g_{\frac{9}{2}}\}_\pi$ 11% $\{1g_{\frac{7}{2}} 2p_{\frac{1}{2}}\}_\pi$
4_1^+	4.42	5.1	7.7 ± 0.4	11.3	66% $\{2f_{\frac{7}{2}} 1h_{\frac{11}{2}}\}_\nu$ 19% $\{2d_{\frac{5}{2}} 1g_{\frac{9}{2}}\}_\pi$
5_1^-	4.94	6.9	–	8.5	85% $\{2f_{\frac{7}{2}} 2d_{\frac{3}{2}}\}_\nu$

N. N. Arsenyev, A. P. Severyukhin, in preparation.

D. Rosiak et al., Phys. Rev. Lett. 121, 252501 (2018).

ISGMR in ^{118,120,122,124}Sn SLy4



***N. N. Arsenyev, A. P. Severyukhin*, in preparation.**

***T. Li et al.*, Phys. Rev. Lett. 99, 162503 (2007).**

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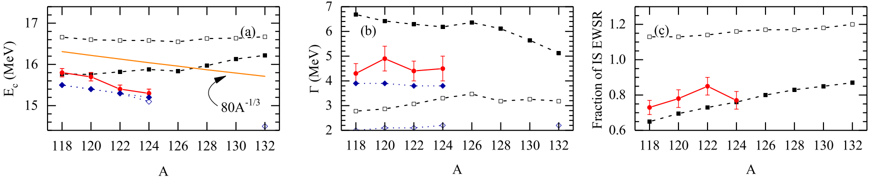
Characteristics of the ISGMR in $^{118-132}$Sn SLy4



N. N. Arsenyev, A. P. Severyukhin, **in preparation.**

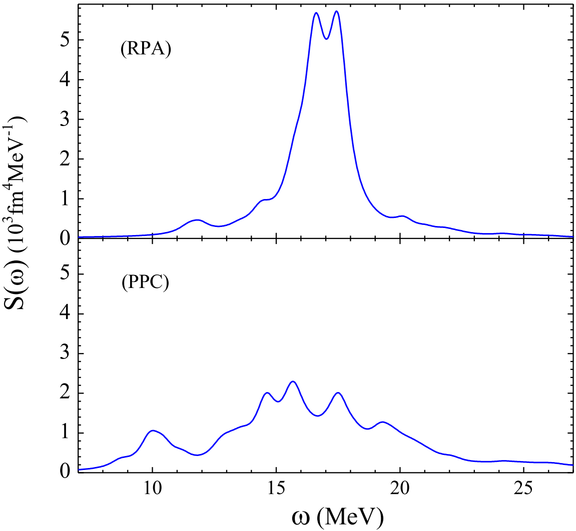
T. Li et al., **Phys. Rev. Lett. 99, 162503 (2007).**

The inclusion of the coupling with the complex configurations plays the key role to explain the peculiarities of the ISGMR strength distributions in all considered nuclei. This fact agrees with the results of calculations within the framework of the self-consistent approach including correlations beyond the QRPA based on the Skyrme T5 EDF (opened and filled triangles).

V. Tselyaev et al., **Phys. Rev. C79, 034309 (2009).**

ISGMR in ^{132}Sn

SLy4



N. N. Arsenyev, A. P. Severyukhin, in preparation.

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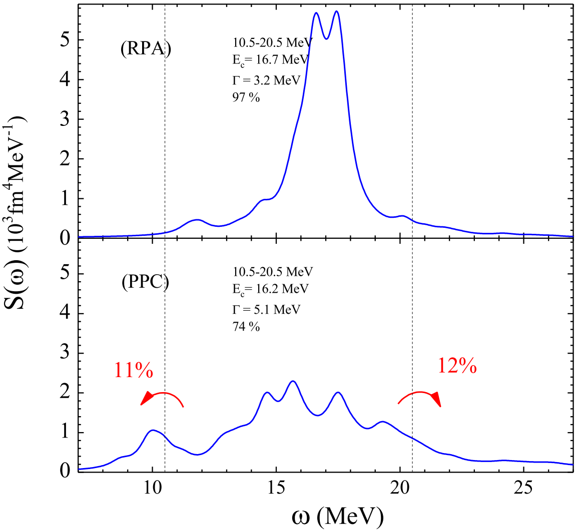
ISGMR in ^{132}Sn

SLy4



N. N. Arsenyev, A. P. Severyukhin, in preparation.

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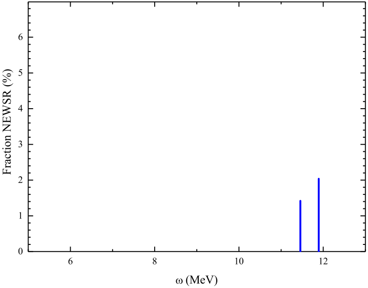
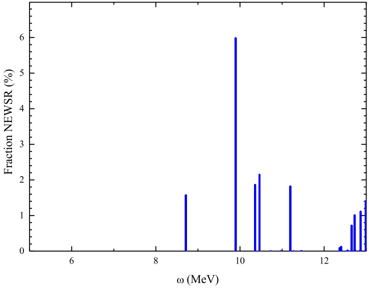
Low-lying 0^+ states in ^{132}Sn

SLy4





State	Energy (MeV)	Structure	Fraction NEWSR (%)
$[0^+_2]_{RPA}$	11.4	$99\%\{4s_{\frac{1}{2}}3s_{\frac{1}{2}}\}_\nu$	1.4
$[0^+_3]_{RPA}$	11.9	$99\%\{3d_{\frac{3}{2}}2d_{\frac{3}{2}}\}_\nu$	2.0

$\lambda^\pi_i = 0^+_i$	Energy (MeV)	Main configurations	Fraction NEWSR (%)
0^+_2	8.7	$93\%[2^+_1 \otimes 2^+_1]_{RPA}$	1.6
0^+_3	9.9	$49\%[4^+_1 \otimes 4^+_1]_{RPA}$ $+34\%[3^-_1 \otimes 3^-_1]_{RPA}$	6.0
0^+_4	10.4	$47\%[4^+_1 \otimes 4^+_1]_{RPA}$ $+43\%[3^-_1 \otimes 3^-_1]_{RPA}$	1.9

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Conclusions

Starting from the Skyrme mean-field calculations with the parameter set SLy4, the properties of the spectrum of 0^+ excitations of ^{132}Sn has been studied within the FRSA model including both the two-phonon configurations effects. The suggested approach enables one to perform the calculations in very large configuration spaces.

We applied this model firstly to the $E0$ strength distributions in ^{118,120,122,124}Sn, for which experimental data are available. Our results taking into account the two-phonon configurations indicate the fragmentation of the monopole strength to the lower energy than the main peak at E=17 MeV and also a high-energy tail above E=20.5 MeV. Our results are in reasonable agreement with experimental data.

Secondly, in the case of ^{132}Sn our results taking into account the two-phonon configurations indicate the shifts 11% and 12% of the total strength to the lower energy region (E_x<10.5 MeV) and higher energy region (E_x>20.5 MeV) respectively. The crucial contribution in the wave function structure of the low-lying 0^+ states (E_x<10.5 MeV), comes from the two-phonon configurations and, in particular, the $[2_1^+ \otimes 2_1^+]_{RPA}$ configuration.

THE END