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# NUCLEAR MATTER DENSITY DISTRIBUTIONS OF THE LIGHT WEAKLY BOUND NUCLEI



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JOINT INSTITUTE FOR  
NUCLEAR RESEARCH

# CONTENT

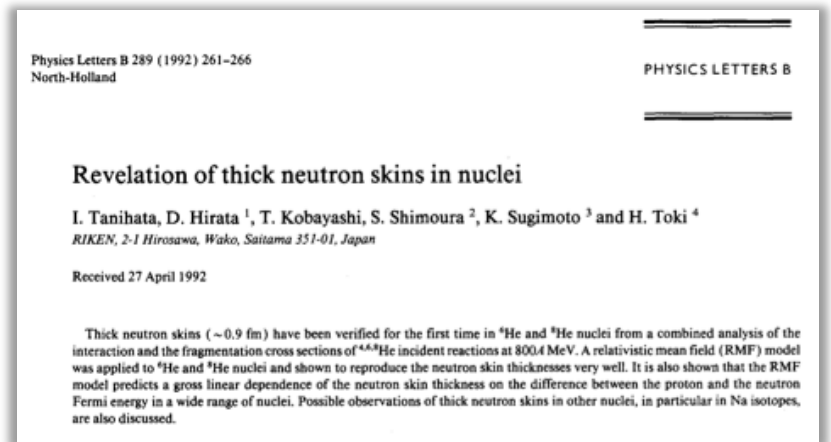
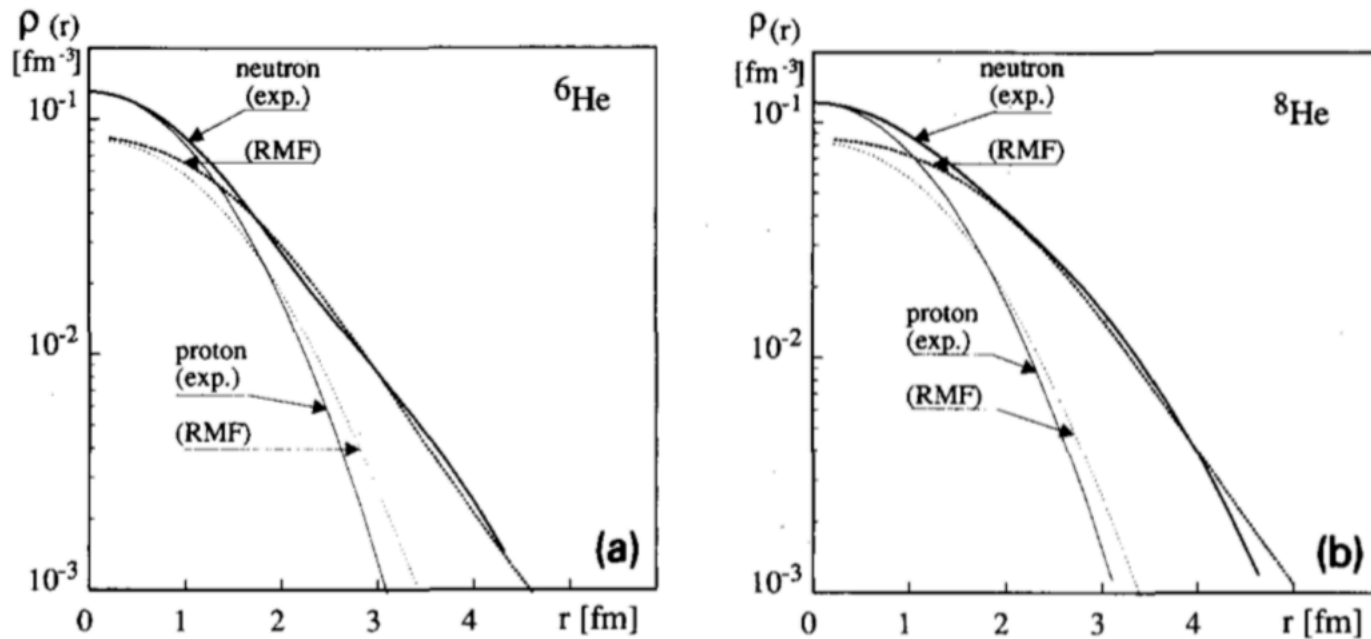
- Why the Density Distribution nuclear matter is important?
- Objects of studying:  ${}^6\text{He}$ ,  ${}^6\text{Li}$  and  ${}^9\text{Be}$
- The three body wave function
- The Density Distribution of nuclear matter within the three body model
- Results and discussions
  - ${}^6\text{He}$ ,  ${}^6\text{Li}$
  - ${}^9\text{Be}$
- Conclusion



# WHY THE DD NUCLEAR MATTER IS IMPORTANT?

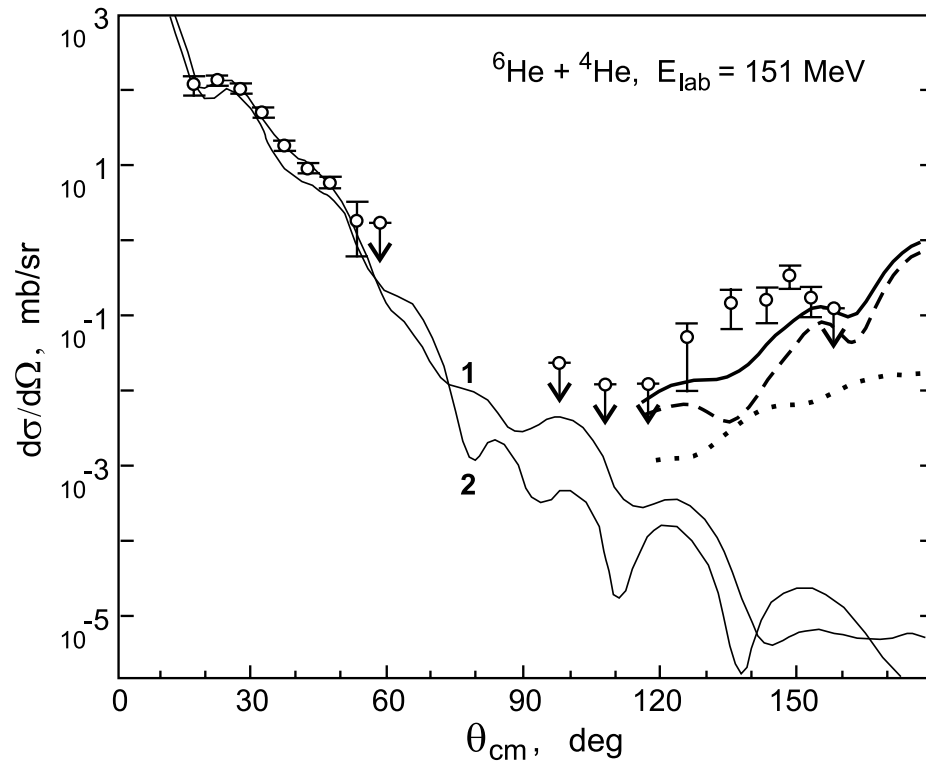
# NEUTRON SKINS IN ${}^6\text{He}$ AND ${}^8\text{He}$ NUCLEI

the Relativistic Mean Field model



# A TWO-NEUTRON TRANSFER REACTION

$\alpha$  ( ${}^6\text{He}$ ,  $\alpha$ )  ${}^6\text{He}$



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## "Di-Neutron" Configuration of ${}^6\text{He}$

Yu. Ts. Oganessian and V. I. Zagrebaev

*Flerov Laboratory of Nuclear Reaction, JINR, Dubna, Moscow Region, Russia*

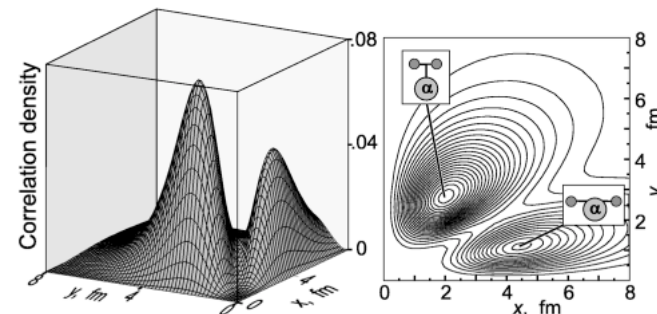
J. S. Vaagen

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(Received 18 December 1998)

Two-neutron transfer reactions induced by the Borromean nucleus  ${}^6\text{He}$  on  ${}^4\text{He}$  and  ${}^1\text{H}$  targets are analyzed within a realistic four-body model. Available experimental data are well described, and it is the "di-neutron" configuration of the  ${}^6\text{He}$  nucleus that is found to make the dominant contribution to the cross sections of the two-neutron transfer reactions. [S0031-9007(99)09437-5]

PACS numbers: 25.60.Je, 24.10.Eq, 25.10.+s, 27.20.+n

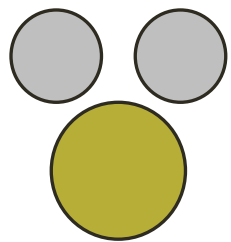




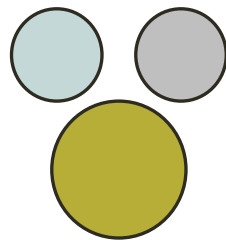
**OBJECTS OF STUDYING:**  
 ${}^6\text{He}$ ,  ${}^6\text{Li}$  AND  ${}^9\text{Be}$  |

# THE THREE BODY NUCLEI

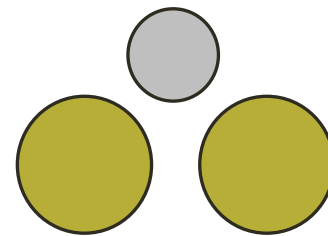
- ✓ cluster structure
- ✓ plenty nuclear reactions
- ✓ interaction potentials of internal clusters



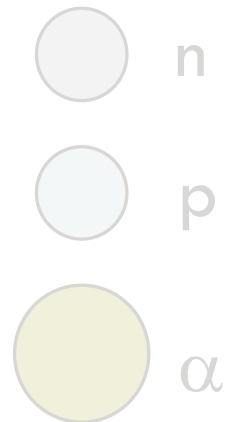
${}^6\text{He}$



${}^6\text{Li}$



${}^9\text{Be}$





# THE THREE BODY MODEL



# THE SCHRÖDINGER EQUATION

$$H\Psi(\mathbf{r}_1, \dots, \mathbf{r}_N) = E\Psi(\mathbf{r}_1, \dots, \mathbf{r}_N).$$

Hamiltonian of the system

$$H = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m_i} - T_{cm} + \sum_{j>i=1}^N V_{ij}.$$

$$|\Psi\rangle = \sum_{i=1}^N C_i |\varphi\rangle \quad \sum_{i=1}^N |C_i|^2 = 1$$

expansion of the wave function into basis

multiplying bra from the left side

$$\sum_{i=1}^N C_i H |\varphi\rangle = E \sum_{i=1}^N C_i |\varphi\rangle, \quad \text{one gets } \sum_{i=1}^N \sum_{k=1}^N C_i H_{ik} = E \sum_{i=1}^N C_i.$$

Matrix form of the Schrödinger equation for non-orthogonal basis:

$$\begin{pmatrix} H_{11} & H_{12} & \dots & H_{1N} \\ H_{21} & H_{22} & \dots & H_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ H_{N1} & H_{2N} & \dots & H_{NN} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix} = E \begin{pmatrix} W_{11} & W_{12} & \dots & W_{1N} \\ W_{21} & W_{22} & \dots & W_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ W_{N1} & W_{2N} & \dots & W_{NN} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix} \quad \text{where } W_{ik} = \langle \varphi_i | \varphi_k \rangle$$

# THE HAMILTONIAN

Kinetic energy

Sack-Biedenharn-Breit  
 $N\alpha$ -potential

Ali-Bodmer  
 $\alpha\alpha$ -potential

$$\tilde{H} = H_0 + \tilde{V}_{\alpha_1 N} + \tilde{V}_{\alpha_2 N} + \tilde{V}_{\alpha_1 \alpha_2},$$

$$\tilde{V} = V + \Delta = V + \lambda\Gamma$$

Pauli Projector

$$\Gamma = \Gamma(f) = \sum_{m_f} |\varphi_{fm_f}(\mathbf{x})\rangle \langle \varphi_{fm_f}(\mathbf{x}')| \delta(\mathbf{y} - \mathbf{y}')$$

# THE BASIS FUNCTION

$$\Psi_{JM_J}^{tot} = \varphi_{J_\alpha=T_\alpha=0} (1,2,3,4) \varphi_{J_\alpha=T_\alpha=0} (5,6,7,8) \Psi^{JM_J} (\mathbf{x}, \mathbf{y})$$

- full WF

$$\Psi^{JM_J} (\mathbf{x}, \mathbf{y}) = \sum_{\lambda L} \Psi_{\lambda L}^{JM_J} (\mathbf{x}, \mathbf{y}),$$

- sum of components

$$\Psi_{\lambda L}^{JM_J} (\mathbf{x}, \mathbf{y}) = \Phi_{\lambda L} (x, y) F_{\lambda L S}^{JM_J} (\hat{x}, \hat{y})$$

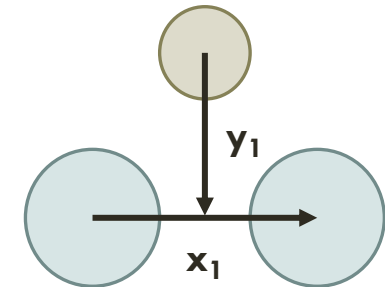
- spin angular part

$$F_{\lambda L S}^{JM_J} (x, y) = \sum_{M_L M_S} \langle LM_L SM_S | JM_J \rangle \Upsilon_{\lambda L}^{LM_L} (\hat{x}, \hat{y}) \chi^{SM_S}.$$

$$\Upsilon_{\lambda L}^{LM_L} (\hat{x}, \hat{y}) = \sum_{\mu m} \langle \lambda \mu l m | LM_L \rangle Y_{\lambda \mu} (\hat{x}) Y_{l m} (\hat{y}),$$

$$\Phi_{\lambda L} (x, y) = \sum_{i=1}^N C_i^{(\lambda)} x^\lambda y^l \exp \left( -\alpha_i^{(\lambda)} x^2 - \beta_i^{(l)} y^2 \right)$$

- radial part



$\mathbf{x}, \mathbf{y}$  – Jacobi coordinates

 -neutron

 -alpha particle



# THE DENSITY DISTRIBUTION FUNCTION OF NUCLEAR MATTER

The function for the 3BS:

$$\rho(\mathbf{R}) = \sum_{cluster=i,j,k} \rho_{cluster}(\mathbf{R}).$$

referring to the (i,jk) Jacobi coordinates set:

$$\rho_{N_i}(\mathbf{R}) = \langle \varphi^\gamma(i, jk) | \delta(\mathbf{R} - \mathbf{y}_i) | \varphi^\gamma(i, jk) \rangle$$

$$\rho_{\alpha_i}(\mathbf{R}) = \langle \varphi^\gamma(i, jk) | \rho_\alpha(\mathbf{R} - \mathbf{y}_i) \delta(\mathbf{y}_i - \mathbf{r}_\alpha) | \varphi^\gamma(i, jk) \rangle$$

referring to the (j,ki) Jacobi coordinates set:

$$\rho_{N_j}(\mathbf{R}) = \sum_{\tilde{\gamma}} A_\Omega^{j \leftarrow i} A_\Omega^{k \leftarrow i} \langle \varphi^{\tilde{\gamma}}(j, ki) | \delta(\mathbf{R} - \mathbf{y}_j) | \varphi^{\tilde{\gamma}}(j, ki) \rangle$$

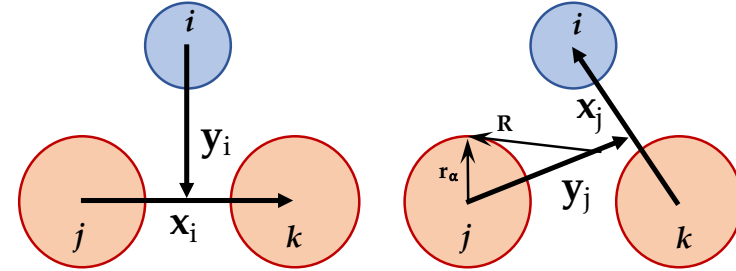
$$\rho_{\alpha_j}(\mathbf{R}) = \sum_{\tilde{\gamma}} A_\Omega^{j \leftarrow i} A_\Omega^{k \leftarrow i} \langle \varphi^{\tilde{\gamma}}(j, ki) | \rho_\alpha(\mathbf{R} - \mathbf{y}_j) \delta(\mathbf{y}_j - \mathbf{r}_\alpha) | \varphi^{\tilde{\gamma}}(j, ki) \rangle.$$

using certain expression

$$\exp(-\rho_0 \mathbf{R} \cdot \mathbf{y}) = 4\pi \sum_k \sqrt{2k+1} i_k(\rho_0 R y) Y_{00}^{kk}(\hat{R}, \hat{y})$$

$$\int_0^\infty y^{2l+k+2} \exp(-\beta y^2) i_k(\mu y) dy = \sqrt{\frac{\pi}{2}} \frac{(2l)!! (\mu)^k}{\left(\frac{1}{2}\beta\right)^{l+k+3/2}} \exp\left(\frac{\mu^2}{\beta}\right) L_l^{k+1/2}\left(-\frac{\mu^2}{\beta}\right)$$

<sup>9</sup>Be on relative Jacobi coordinate sets with cyclically particle permutations taking into account a size of sub-alpha particles:



Final analytical expression depending on only R:

$$\rho_{N_i}(R) = \frac{1}{2} \left(\frac{R}{y_0}\right)^{2l+2} \Gamma\left(\frac{3}{2} + \frac{\lambda}{2}\right) \sum_{ij} C_i C_j \frac{\exp\left(-\frac{(\beta_i + \beta_j)}{y_0^2} R^2\right)}{(\alpha_i + \alpha_j)^{\frac{3}{2} + \frac{\lambda}{2}}}$$

$$\rho_{\alpha_i}(R) = (2\pi)^{\frac{3}{2}} \rho_0 \sum_{ij} C_i C_j \frac{\Gamma\left(\frac{3}{2} + \lambda\right) (2l)!! L_l^{1/2}\left(-\frac{(\gamma_0)R^2}{\beta_i + \beta_j + \gamma_0}\right)}{(\alpha_i + \alpha_j)^{\frac{3}{2} + \lambda} (\beta_i + \beta_j + \gamma_0)^{\frac{3}{2} + l}} \times \exp\left(\left(-\gamma_0 + \frac{\gamma_0^2}{\beta_i + \beta_j + \gamma_0}\right) \left(\frac{R}{y_0}\right)^2\right)$$

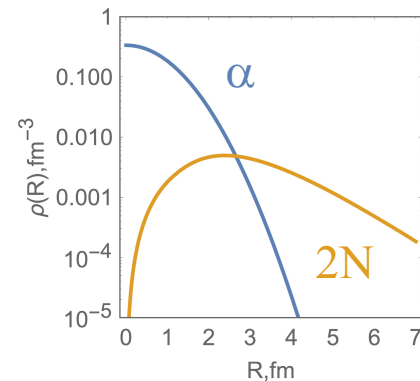
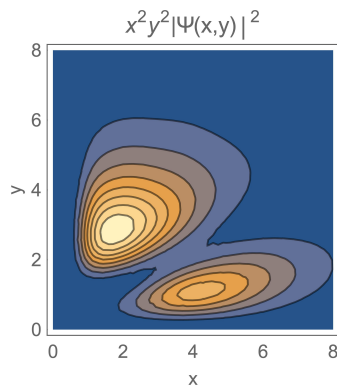
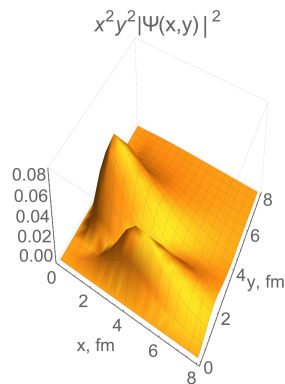


**RESULTS** |

${}^6\text{He } 0^+$

$E_{\text{exp}} = -0.975 \text{ MeV}$

$E_{\text{thr}} = -0.228 \text{ MeV}$

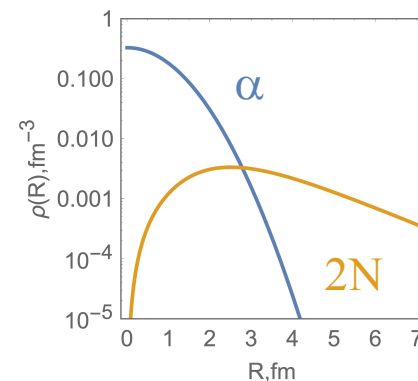
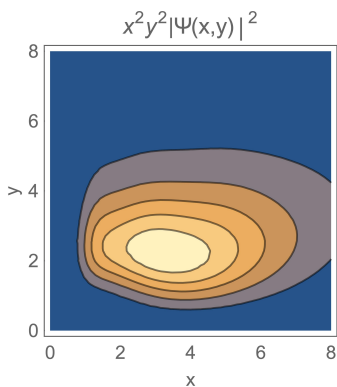
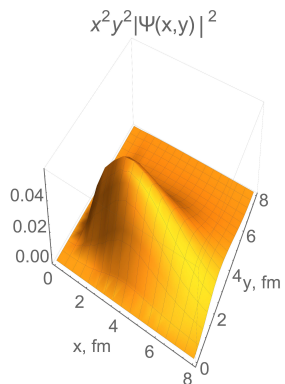


$$\langle R_m^2 \rangle = 2.83$$

${}^6\text{He } 2^+$

$E_{\text{exp}} = 0.825 \text{ MeV}$

$E_{\text{thr}} = 1.842 \text{ MeV}$

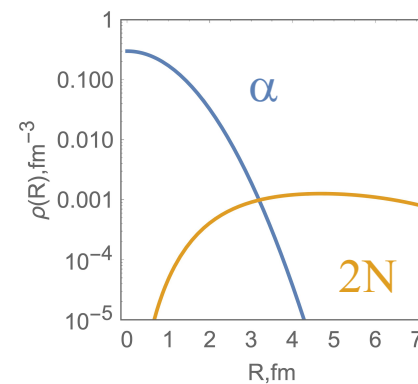
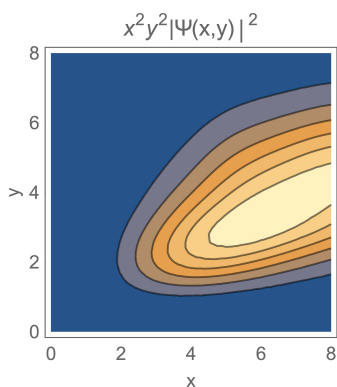
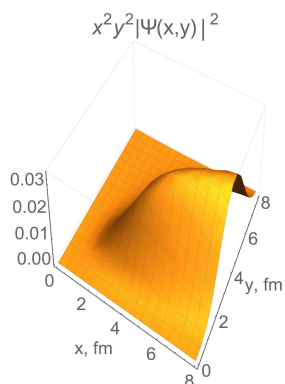


$$\langle R_m^2 \rangle = 3.13$$

${}^6\text{He } 1^+$

$E_{\text{exp}} = 4.0 \text{ MeV}$

$E_{\text{thr}} = 3.947 \text{ MeV}$

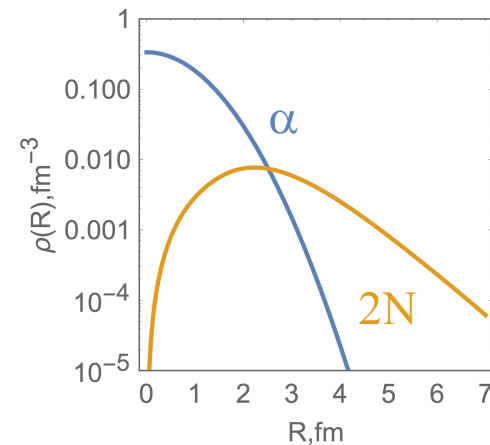
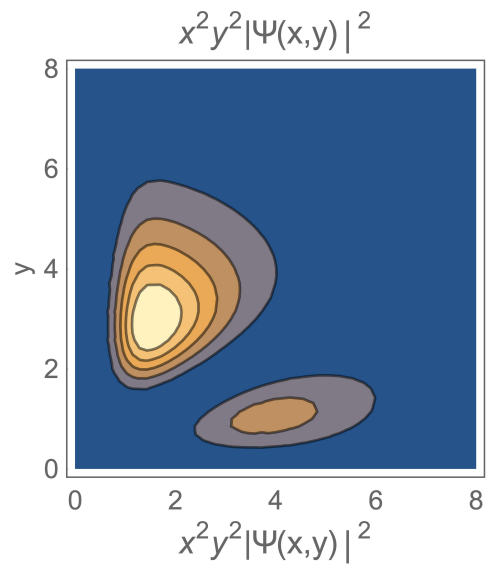
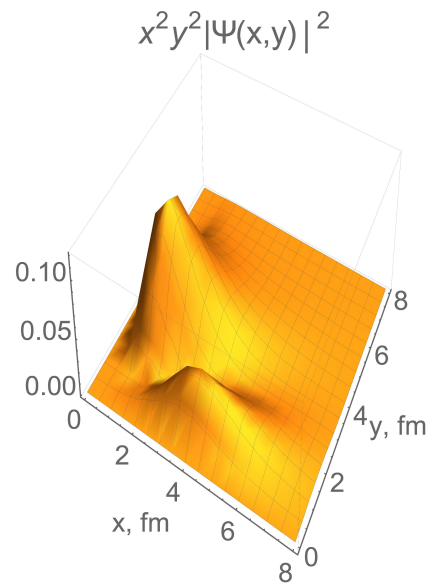


$$\langle R_m^2 \rangle = 3.70$$

${}^6\text{Li } 1^+$

$E_{\text{exp}} = -3.700 \text{ MeV}$

$E_{\text{thr}} = -3.258 \text{ MeV}$

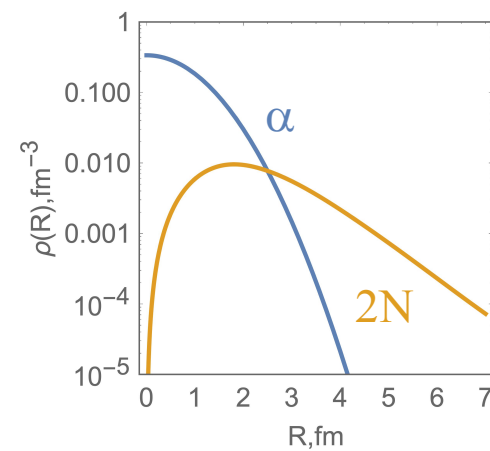
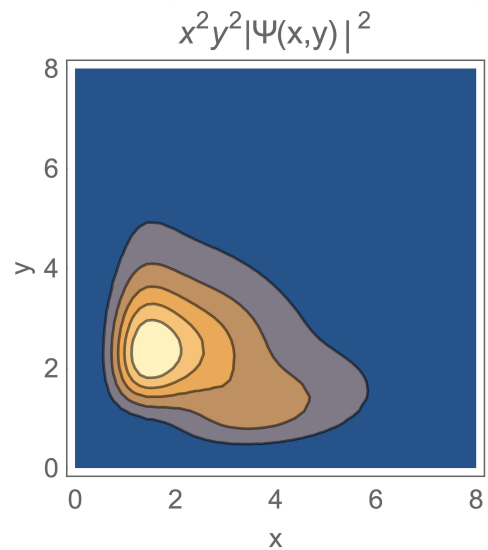
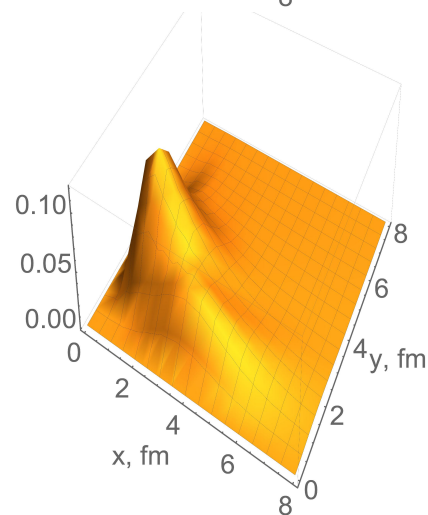


$$\langle R_m^2 \rangle = 2.52$$

${}^6\text{Li } 3^+$

$E_{\text{exp}} = -1.514 \text{ MeV}$

$E_{\text{thr}} = -1.714 \text{ MeV}$



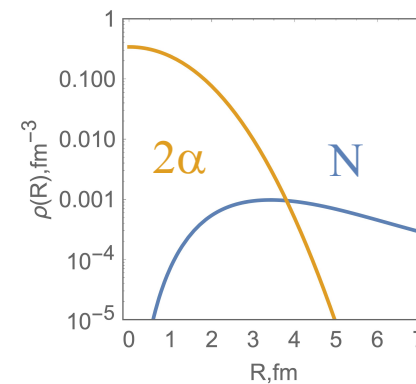
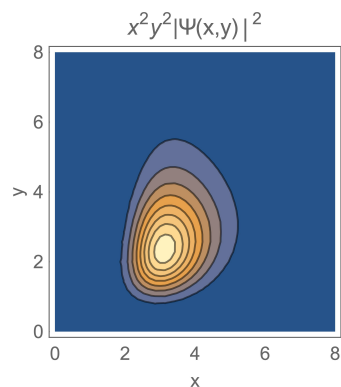
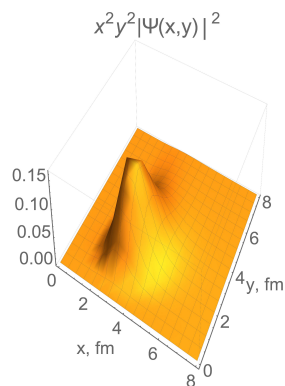
$$\langle R_m^2 \rangle = 2.48$$



${}^9\text{Be } 3/2^-$

$E_{\text{exp}} = -1.57 \text{ MeV}$

$E_{\text{thr}} = -1.417 \text{ MeV}$

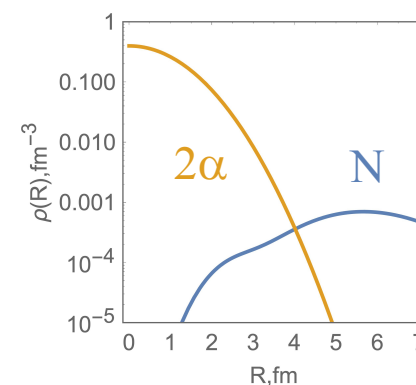
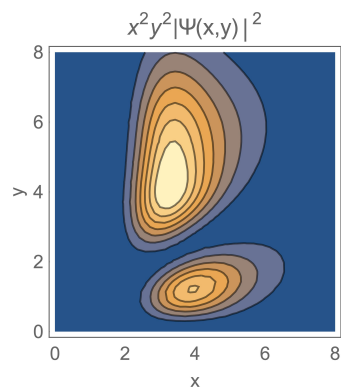
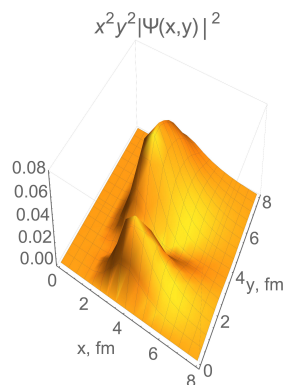


$$\langle R_m^2 \rangle = 2.59$$

${}^9\text{Be } 1/2^+$

$E_{\text{exp}} = 0.11 \text{ MeV}$

$E_{\text{thr}} = 0.874 \text{ MeV}$

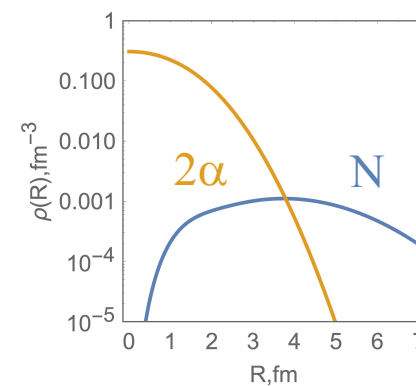
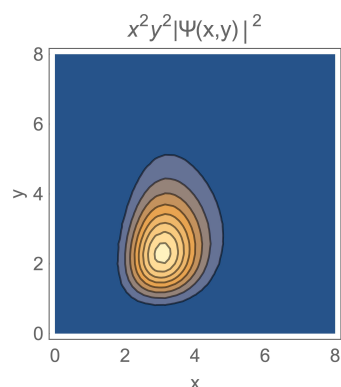
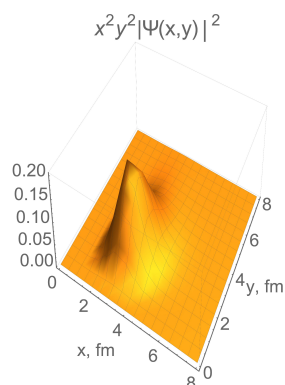


$$\langle R_m^2 \rangle = 2.71$$

${}^9\text{Be } 5/2^-$

$E_{\text{exp}} = 0.86 \text{ MeV}$

$E_{\text{thr}} = 0.696 \text{ MeV}$



$$\langle R_m^2 \rangle = 2.53$$



**CONCLUSIONS** |

# CONCLUSIONS

- The three body model for the  ${}^6\text{He}$ ,  ${}^6\text{Li}$  and  ${}^9\text{Be}$  nuclei has been introduced
- Analytical expressions of the density distribution function of nuclear matter were deduced for the three body system
- Obtained expressions were applied for the  ${}^6\text{He}$ ,  ${}^6\text{Li}$  and  ${}^9\text{Be}$  nuclei including ground and low lying excited states
- Demonstrated the structures of nuclei in terms of nuclear matter
- Taken results may be used for the coupling potentials in the Coupled Reaction Channels calculations



**THANK YOU FOR YOUR ATTENTION** |