# PARTICLE-HOLE DISPERSIVE OPTICAL MODEL FOR OPEN-SHELL NUCLEI. IMPLEMENTATION FOR DESCRIBING $0^+$ GIANT RESONANCES IN TIN ISOTOPES

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The particle-hole dispersive optical model proposed early to describe main properties of various giant resonances in closed-shell nuclei is extended to take into account nucleon pairing in medium-heavy open-shell spherical nuclei. Being formulated in a "high-energy limit", the extended model is implemented for describing main properties of Isoscalar Giant Monopole and Isobaric Analog Resonances in a number of tin isotopes. Calculation results are compared with available experimental data.

## The isospin symmetry of a model Hamiltonian

$$H_0 = \sum_a H_0(a) \qquad F = \frac{1}{2}\sum_{a\neq b} F(a,b)$$

$$F(x_a, x_b) \rightarrow (F + F'\tau_a\tau_b)\delta(r_a - r_b)$$

The mean Coulomb $U_C = \sum_a \frac{1}{2}\left(1 - \tau_a^{(3)}\right) U_C(r_a)$

– the main source of the weak violation of the isospin symmetry.

$$[H, T^{(-)}] = U_C^{(-)} \qquad U_C^{(-)} = \sum_a U_C(a)\tau_a^{(-)}$$

$$\downarrow$$

$$RPA$$

$$\downarrow$$

$$U_1 = \frac{1}{2} v(r)\tau^{(3)} \qquad v(r) = 2F' n^{(-)}(r)$$ - self-consistency condition

$$n^{(-)}(r) = n^n(r) - n^p(r)$$

- **The translation symmetry of a model**

$1^-$ spurious state $\quad \omega \to 0 \qquad EWSR \to 1$

$$F(x, x') = C(F(r) + F'\tau\tau')\delta(r - r')$$

$$F(r) = f^{in} f_{WS}(r) + f^{ex}(1 - f_{WS}(r, R, a))$$

$$(EWSR)_{V_L} = \frac{\hbar^2}{2m} \int \left( \left( \frac{dV_L}{dr} \right)^2 + L(L+1) \left( \frac{V_L}{r} \right)^2 \right) n^{(+)}(r) r^2 dr$$

## The nuclear mean field

$$U(x) = U_0(x) + U_{so}(x) + U_1(x) + U_C(x)$$

$$U_0(x) = -U_0 f_{WS}(r, R, a)$$

$$U_{SO}(x) = -U_{SO} \frac{1}{r} \frac{df_{WS}}{dr} ls$$

$$U_1(x) = \frac{1}{2} v(r) \tau^{(3)}$$

$$U_C(x) = \frac{1 - \tau^{(3)}}{2} U_C(r)$$

## The Landau-Migdal particle-hole interaction

$$F(x, x') = C(F(r) + F' \tau\tau') \delta(r - r')$$

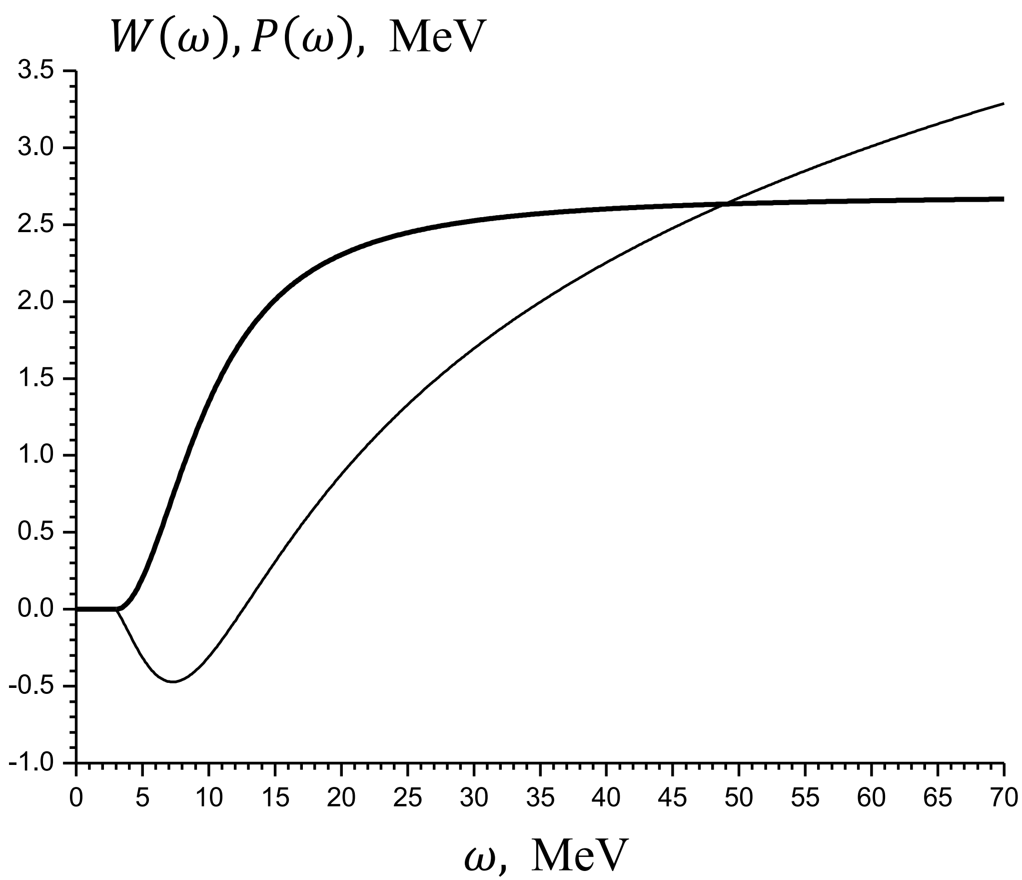
$$F(r) = f^{in} f_{WS}(r) + f^{ex}(1 - f_{WS}(r, R, a))$$

# Spreading effect – PHDOM

$\big(-iW(\omega) + P(\omega)\big) f_{WS}(r)$ - the "optical-model like" addition to the nuclear mean field

$$2W(\omega) = \begin{cases} 0, & \omega < \Delta\,; \\ \alpha\,(\omega-\Delta)^2/[1+(\omega-\Delta)^2/B^2]\,, & \omega \geq \Delta\,. \end{cases}$$

$\alpha = 0.11$ MeV$^{-1}$
$B = 7$ MeV
$\Delta = 3$ MeV



## Basic equations

Equation for the effective p-h propagator:

$$\tilde{A}_{JLS}(r,r',\omega) = A_{JLS}(r,r',\omega) + \int A_{JLS}(r,r_1,\omega)\frac{F_S(r_1)}{r_1^2}\tilde{A}_{JLS}(r_1,r',\omega)dr_1$$

$$\rho_{JLS}(r,r',\omega) = -\frac{1}{\pi}Im\tilde{A}_{JLS}(r,r',\omega)$$ - double transition density

$$S_{JLS}(\omega) = -\frac{1}{\pi}Im\int V^*_{JLS}(r)\tilde{A}_{JLS}(r,r',\omega)V_{JLS}(r')drdr'$$ - strength function

$$\int \tilde{A}_{JLS}(r,r',\omega)V_{JLS}(r')dr' = \int A_{JLS}(r,r',\omega)\tilde{V}_{JLS}(r',\omega)dr'$$

$$S_{JLS}(\omega) = -\frac{1}{\pi}Im\int V^*_{JLS}(r)A_{JLS}(r,r',\omega)\tilde{V}_{JLS}(r',\omega)drdr'$$

Equation for the effective field:

$$\tilde{V}_{JLS}(r,\omega) = V_{JLS}(r) + \frac{F_S(r)}{r^2}\int A_{JLS}(r,r',\omega)\tilde{V}_{JLS}(r',\omega)dr'$$

Projected transition density:

$$\rho_{V_{JLS}}(r,\omega) = \int \rho_{JLS}(r,r',\omega)V_{JLS}(r')dr' \Big/ S_{JLS}^{1/2}(\omega)$$

$$S_{JLS}(\omega) = \left(\int \rho_{V_{JLS}}(r,\omega)V_{JLS}^{*}(r)dr\right)^2$$

$$\frac{1}{r^2}\rho_{V_{JLS}}(r,\omega) = -\frac{1}{\pi} Im \frac{\tilde{V}_{JLS}(r,\omega)}{F_S(r)S_{JLS}^{1/2}(\omega)}$$

## The radial component of the energy-averaged "free" p-h propagator:

$$A_{JLS}(r,r',\omega) = A^{i}_{JLS} + A^{ii}_{JLS} + A^{iii}_{JLS}$$

$$A^{i}_{JLS}(r,r',\omega) = \sum_{(\lambda),\mu} v_\mu^2 \left(t^{JLS}_{(\lambda)(\mu)}\right)^2 \chi_\mu(r)\chi_\mu(r')g_{(\lambda)}(r,r',\varepsilon=\varepsilon_\mu+\omega)$$

$$A^{ii}_{JLS}(r,r',\omega) = \sum_{\lambda,(\mu)} v_\lambda^2 \left(t^{JLS}_{(\lambda)(\mu)}\right)^2 \chi_\lambda(r)\chi_\lambda(r')g_{(\mu)}(r,r',\varepsilon=\varepsilon_\lambda-\omega)$$

$$A^{iii}_{JLS}(r,r',\omega) = \sum_{\varepsilon_\mu,\varepsilon_\lambda<0} \left(t^{JLS}_{(\lambda)(\mu)}\right)^2 \chi_\lambda(r)\chi_\lambda(r')\chi_\mu(r)\chi_\mu(r')a_{\pi\nu}(\omega)$$

$$a_{\pi\nu}(\omega) = \frac{v_\mu^2}{\varepsilon_\lambda-\varepsilon_\mu-\omega-v_\mu^2(iW(\omega)-P(\omega))f_\lambda f_\mu} - \frac{v_\lambda^2}{\varepsilon_\lambda-\varepsilon_\mu-\omega+v_\lambda^2\left(iW(\omega)-P(\omega)\right)f_\lambda f_\mu} + \frac{v_\lambda^2-v_\mu^2}{\varepsilon_\lambda-\varepsilon_\mu+\omega-(v_\lambda^2-v_\mu^2)\left(iW(\omega)-P(\omega)\right)f_\lambda f_\mu}$$

$$t^{JLS}_{(\lambda)(\mu)} = \frac{1}{\sqrt{2J+1}}\langle(\lambda)\|T_{JLS}\|(\mu)\rangle \qquad f_\lambda = \int f_{WS}(r)\chi_\lambda^2(r)dr$$

**The optical-model radial Green functions satisfy to the equations:**

$$\{h_{(\lambda)}(r) - (\varepsilon_\mu + \omega) - (iW(\omega) - P(\omega)) f_\mu v_\mu^2 f_{WS}(r)\} g_{(\lambda)}(r, r', \varepsilon = \varepsilon_\mu + \omega) = \\ = -\delta(r - r')$$

$$\{h_{(\mu)}(r) - (\varepsilon_\lambda - \omega) - (iW(\omega) - P(\omega)) f_\lambda v_\lambda^2 f_{WS}(r)\} g_{(\mu)}(r, r', \varepsilon = \varepsilon_\lambda - \omega) = \\ = -\delta(r - r')$$

$h_{(\lambda)}(r)$ - are the radial parts of a s-p Hamiltonian (including the spin-orbit and centrifugal terms)

In the absence of pairing, when $v_{\lambda,\mu}^2 \to n_{\lambda,\mu}$, the equations for p-h propagator and Green functions go to the respective equations of the PHDOM version formulated for closed-shell nuclei.

## Direct-one-nucleon-decay strength functions and branching ratios

$$e^{2i\xi_{(\lambda)}}\left|M_{JLS,(\lambda),\mu}(\omega)\right|^2 = v_\mu^2\left(t_{(\lambda)(\mu)}^{JLS}\right)^2\int\chi_{(\lambda)}^{(-)*}(r,\varepsilon_\mu+\omega)\tilde{V}_{JLS}(r,\omega)\chi_\mu(r)dr\times$$

$$\times\int\chi_\mu(r')\tilde{V}_{JLS}^{*}(r,\omega)\chi_{(\lambda)}^{(+)}(r',\varepsilon_\mu+\omega)dr'$$

$$S_{JLS,\mu}^{\uparrow}(\omega)=\sum_{(\lambda)}\left|M_{JLS,(\lambda),\mu}(\omega)\right|^2$$

$$b_{JLS,\mu}^{\uparrow}=\frac{\int_{(\delta)}S_{JLS,\mu}^{\uparrow}(\omega)d\omega}{\int_{(\delta)}S_{JLS}^{\uparrow}(\omega)d\omega}$$

$$b_{JLS,tot}^{\uparrow}=\sum_{\mu}b_{JLS,\mu}^{\uparrow}$$

$$b_{JLS}^{\downarrow}=1-b_{JLS,tot}^{\uparrow}$$

In the absence of the spreading effect (within cQRPA): $b_{JLS,tot}^{\uparrow}=1$.

# BCS-model relations

$$N = \sum_{\mu} (2j_\mu + 1) v_\mu^2 \qquad \sum_{\mu} (2j_\mu + 1) \frac{\mathcal{G}}{2E_\mu} = 1$$

$$v_\mu^2 = \frac{1}{2}\left(1 - \xi_\mu / E_\mu\right) \qquad \xi_\mu = \varepsilon_\mu - C \qquad E_\mu = \left(\xi_\mu^2 + \Delta^2\right)^{1/2}$$

$$N = \sum_{\mu \neq \mu_0} (2j_\mu + 1) v_\mu^2 + (2j_{\mu_0} - 1) v_{\mu_0}^2 + 1$$

$$\sum_{\mu \neq \mu_0} (2j_\mu + 1) \frac{\mathcal{G}}{2E_\mu} + (2j_{\mu_0} - 1) \frac{\mathcal{G}}{2E_{\mu_0}} = 1$$

# ”Coulomb” description of IAR and IVGMR

$$V_F = Y_{00}\tau^{(-)}$$

$$\Gamma_A = \frac{2\pi}{S_A} S_C^{(-)}(\omega = \omega_A)$$

$$V_C^{(-)}(\vec{r}) = V_C^{(-)}(r) Y_{00}(\vec{n})\tau^{(-)}$$

$$V_C^{(-)}(r) = \sqrt{4\pi}\left(U_C(r) - \omega_A + \frac{i}{2}\Gamma_A\right)$$

$$\Gamma_A^{\uparrow} = \sum_{\nu} \Gamma_{A,\nu}^{\uparrow} \qquad \Gamma_{A,\nu}^{\uparrow} = \frac{2\pi}{S_A} S_C^{(-),\uparrow}(\omega = \omega_A),$$

$$e^{2i\xi_{(\lambda)}}\Gamma_{A,\nu}^{\uparrow} = \frac{1}{4\pi}\left((2j_\nu + 1)v_\nu^2 + \left(u_{\nu_0}^2 - v_{\nu_0}^2\right)\delta_{\nu,\nu_0}\right)\delta_{(\lambda)(\mu)} \text{ x}$$

$$\text{x } \int \chi^{(-)*}_{\varepsilon=\varepsilon_\nu+\omega,(\pi)}(r)\tilde{V}_{JLS}(r,\omega)\chi_\nu(r)dr \int \chi_\nu(r)\tilde{V}^{*}_{JLS}(r,\omega)\chi^{(+)}_{\varepsilon=\varepsilon_\nu+\omega,(\pi)}(r)dr.$$

$$\Gamma_A^{\downarrow} = \Gamma_A - \Gamma_A^{\uparrow}.$$

**Fig. 1. The strength functions calculated within cQRPA (red line) and PHDOM (blue line) for ISGMR in $^{112}Sn$.**

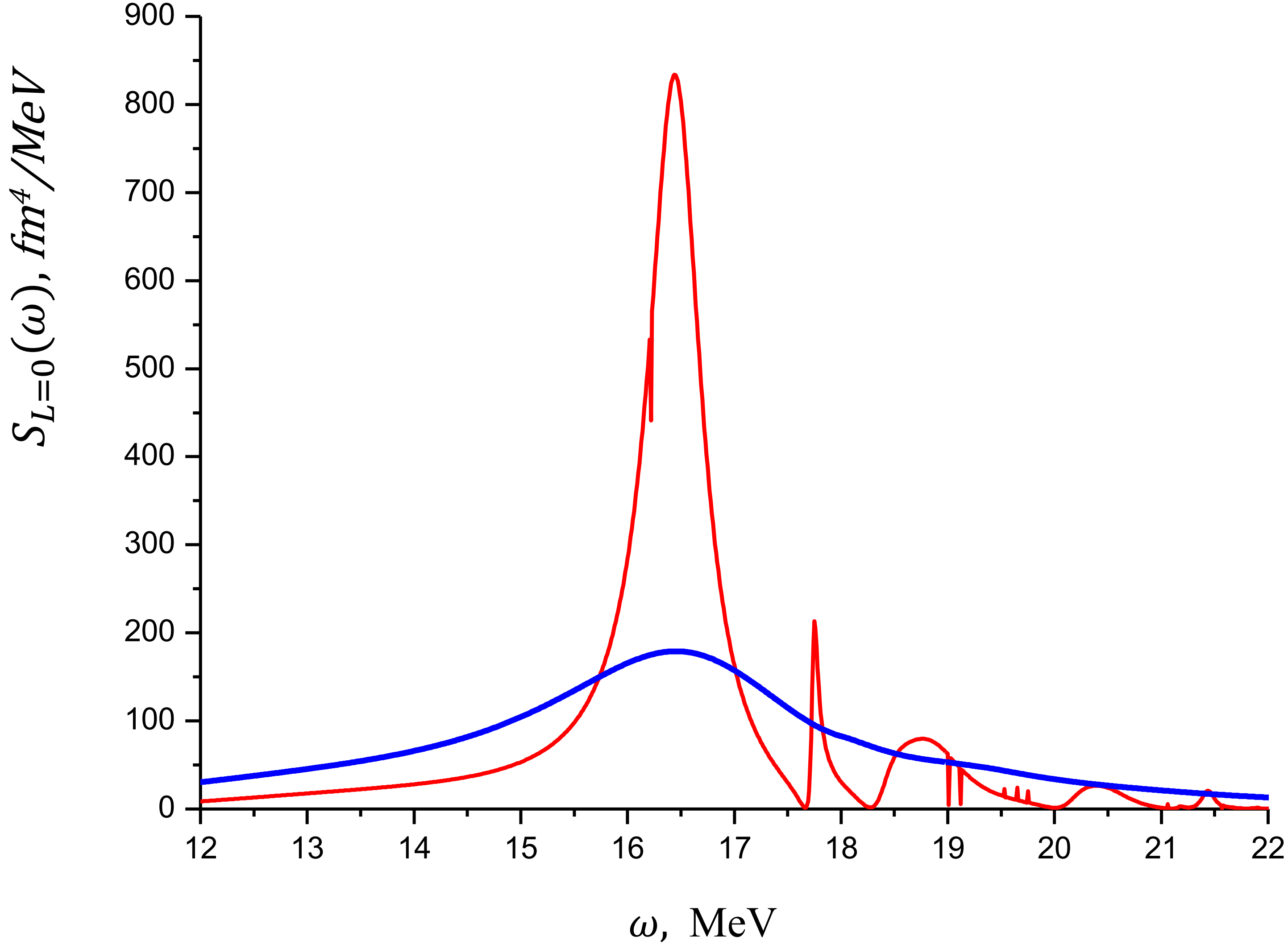
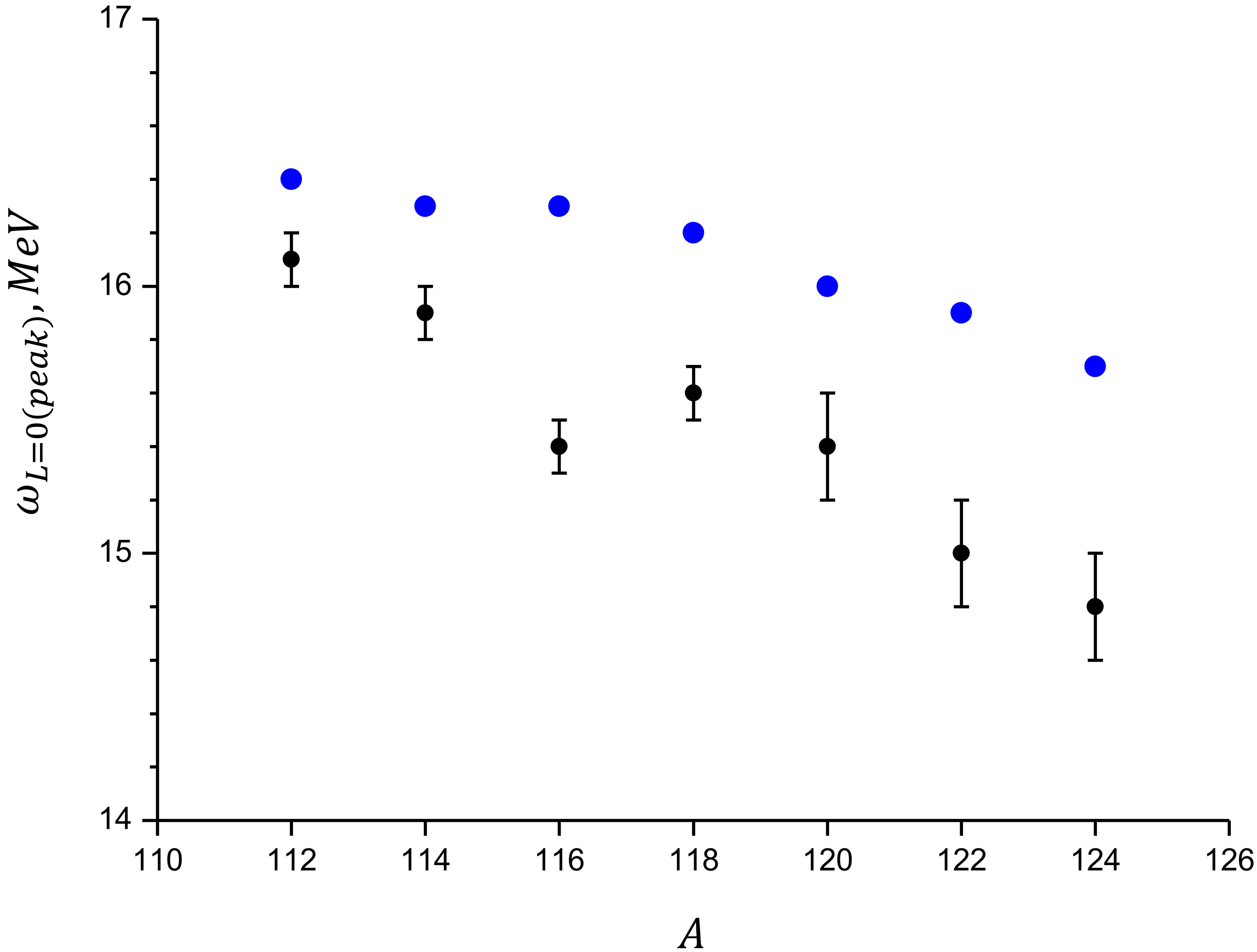
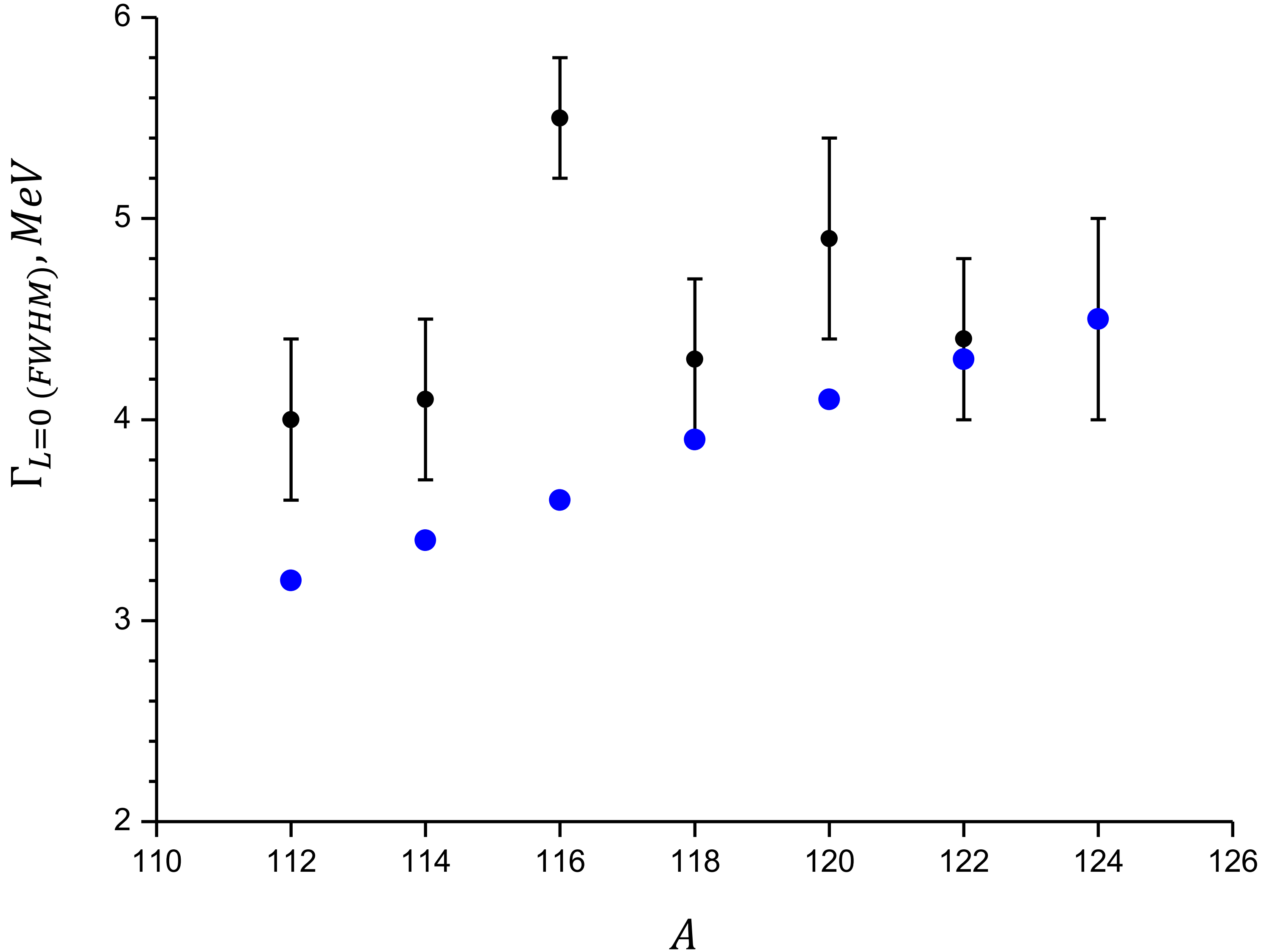


**Fig. 2. Experimental [1] and calculated within PHDOM peak energy for ISGMR in Sn isotopes.**




[1] T. Li, U. Garg, Y. Liu, et al., Phys. Rev. C 81, 034309 (2010).

**Fig. 3. Experimental [1] and calculated within PHDOM total width (FWHM) for ISGMR in Sn isotopes.**




[1] T. Li, U. Garg, Y. Liu, et al., Phys. Rev. C 81, 034309 (2010).

**Fig. 4. The Fermi strength function calculated for IAR in $^{120}$Sn within the cQRPA.**

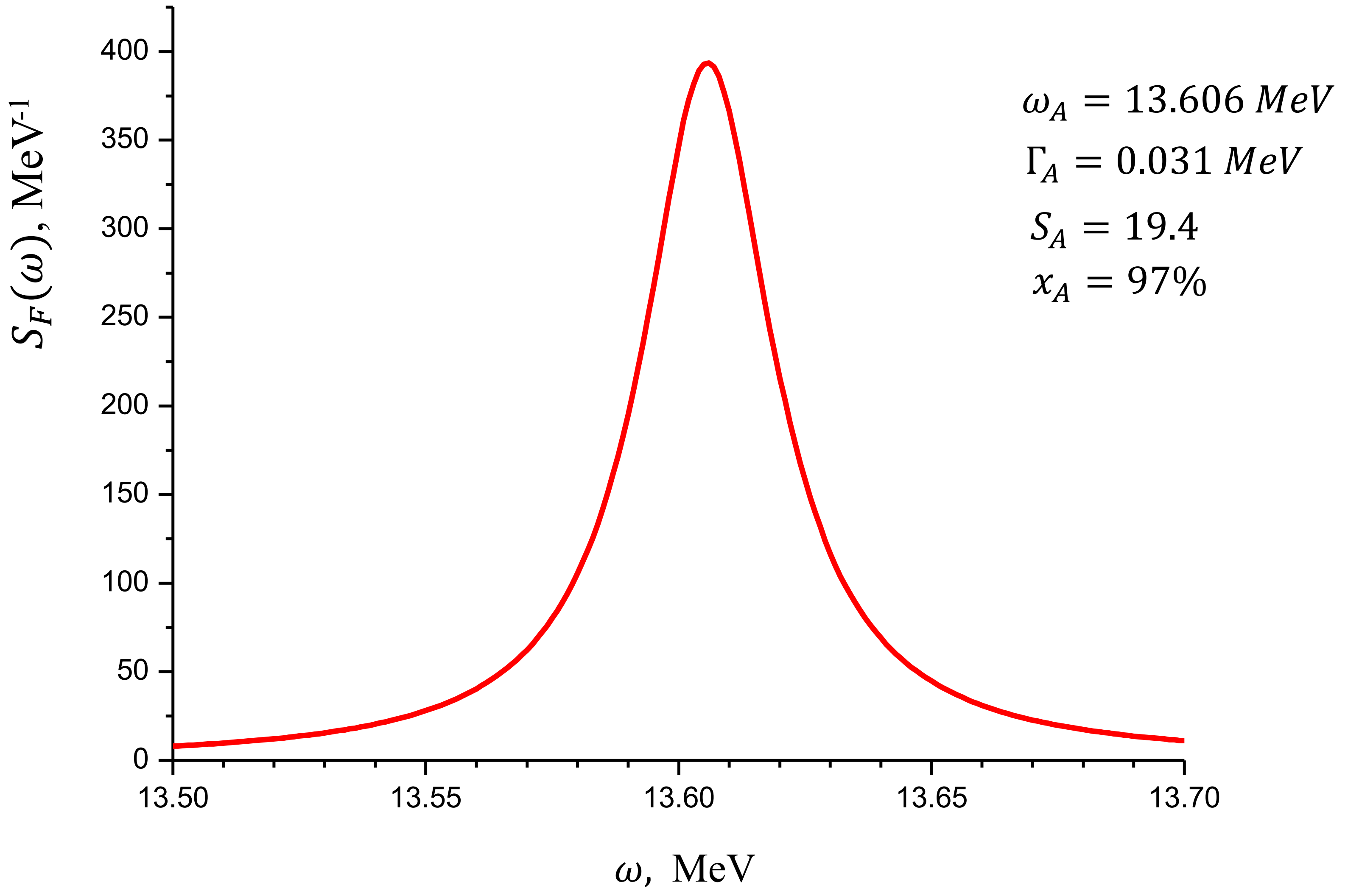
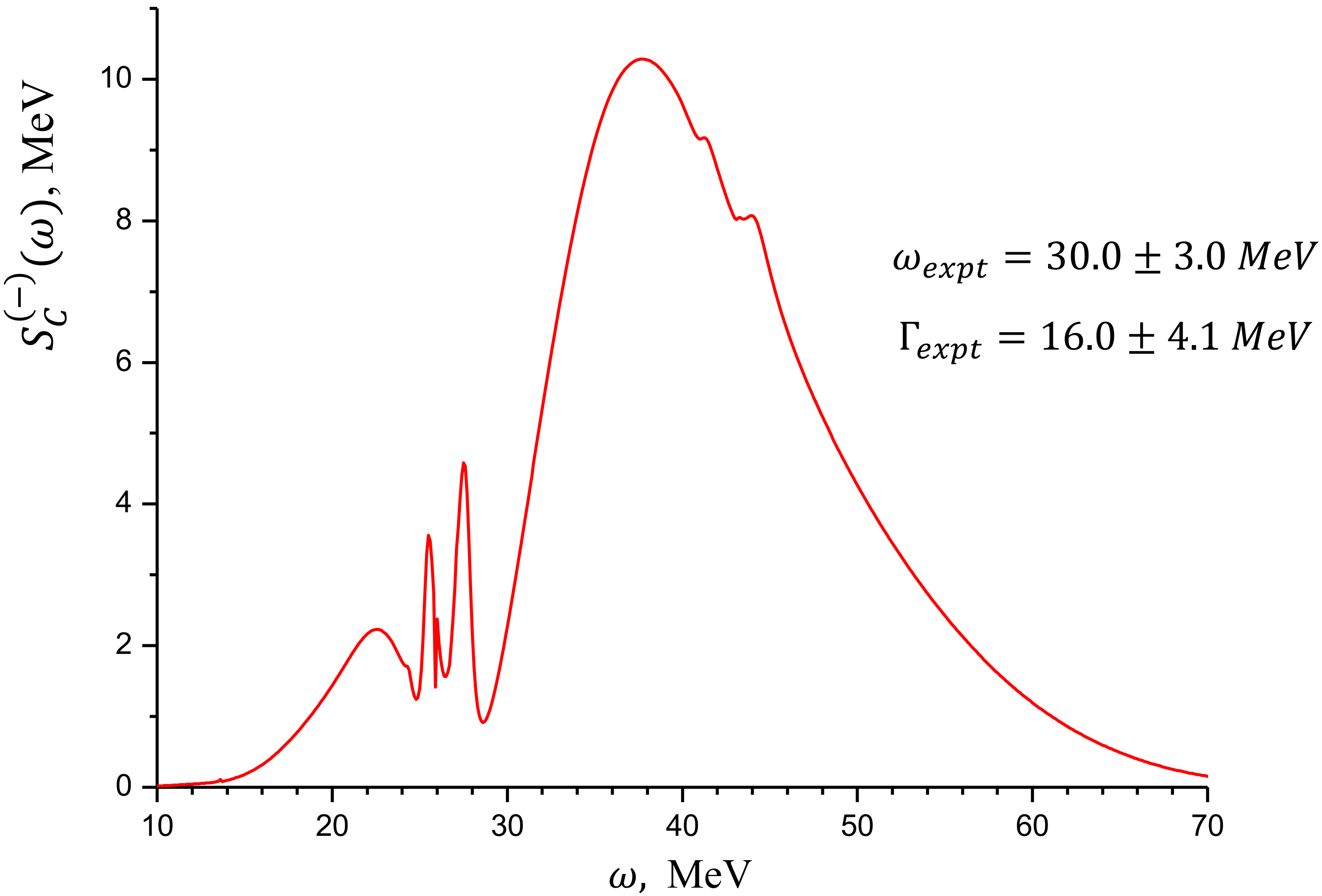


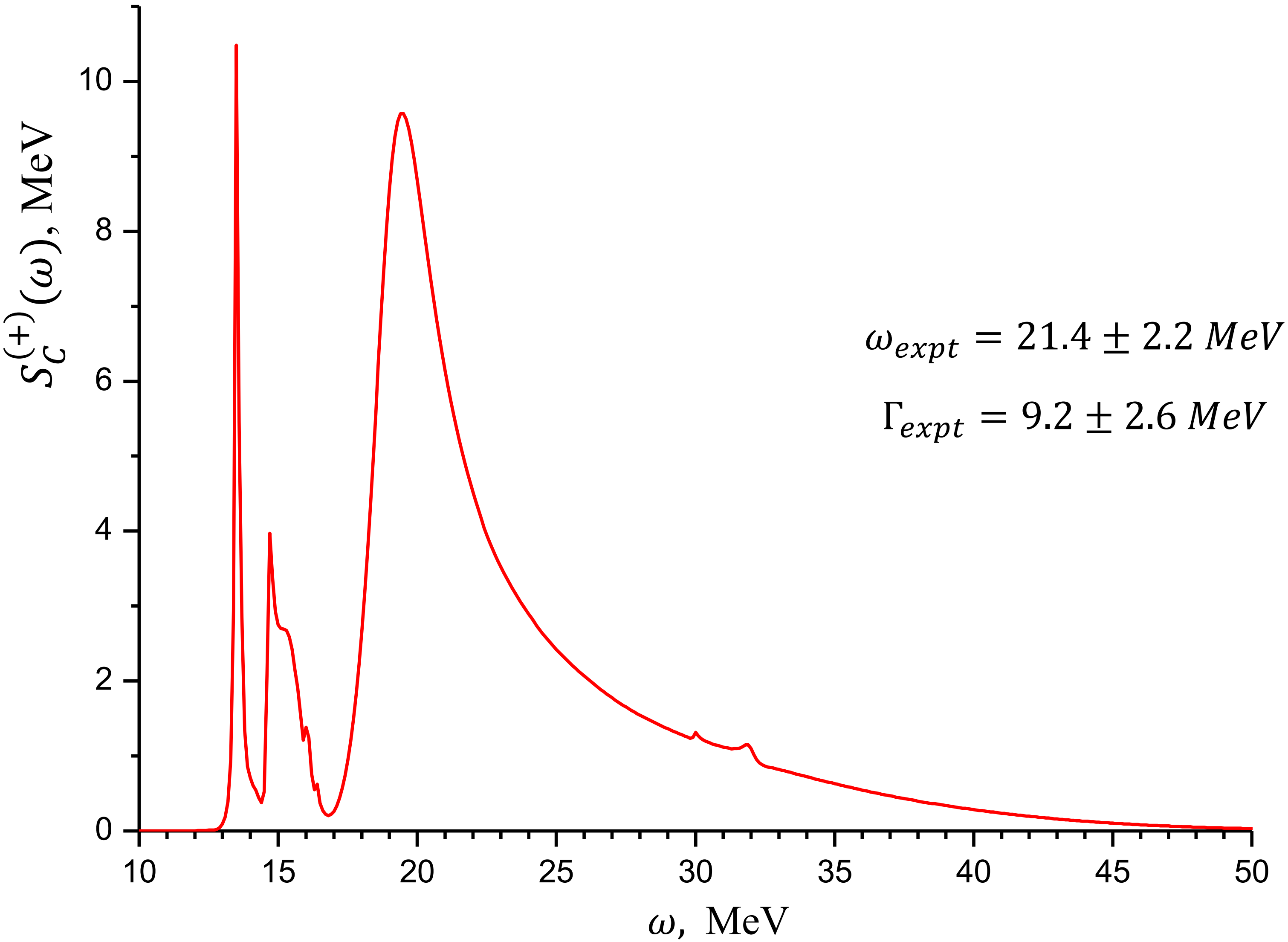
**Fig. 5. The “Coulomb” strength function calculated for IVGMR(-) in $^{120}Sn$ within the cQRPA.**





[2] A. Erell, J. Alster, J. Lichtenstadt, et al., Phys. Rev. C 34, 1822 (1986).

**Fig. 6. The “Coulomb” strength function calculated for IVGMR$^{(+)}$ in $^{120}$Sn within the cQRPA.**





[2] A. Erell, J. Alster, J. Lichtenstadt, et al., Phys. Rev. C 34, 1822 (1986).

## Conclusion

The semi-microscopic particle-hole dispersive optical model (PHDOM), in which main relaxation modes of high-energy particle-hole-type nuclear excitations are together taken into account, has been implemented for describing various giant resonances in medium-heavy closed-shell nuclei. A lot of experimental data concerned with giant resonances in medium-heavy open-shell spherical nuclei makes reasonable an extension of PHDOM for taking nucleon pairing into account. In the present work, an extended PHDOM version is developed in a “high-energy limit” employing the simplest BCS-model. The proposed version is implemented for describing main properties of Isoscalar Giant Monopole Resonance (ISGMR) and Isobaric Analog Resonance (IAR) in a number of tin isotopes. From studies of ISGMR in a chain of tin isotopes one gets information about isotopic dependence of nuclear-matter incompressibility coefficient. Existence and properties of IAR are closely related to the isospin and symmetry in nuclei. Using previous studies of ISGMR, IAR and its overtone as a base, we employ the extended PHDOM version for describing strength function, projected transition density, probabilities of direct one-nucleon decay of ISGMR, and main relaxation parameters of IAR (partial proton and spreading widths, resonance-mixing phase). The obtained results are compared with respective experimental data.

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## THANK YOU FOR ATTENTION!

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