Description of the M1 resonance in ^{208}Pb within the self-consistent phonon-coupling model

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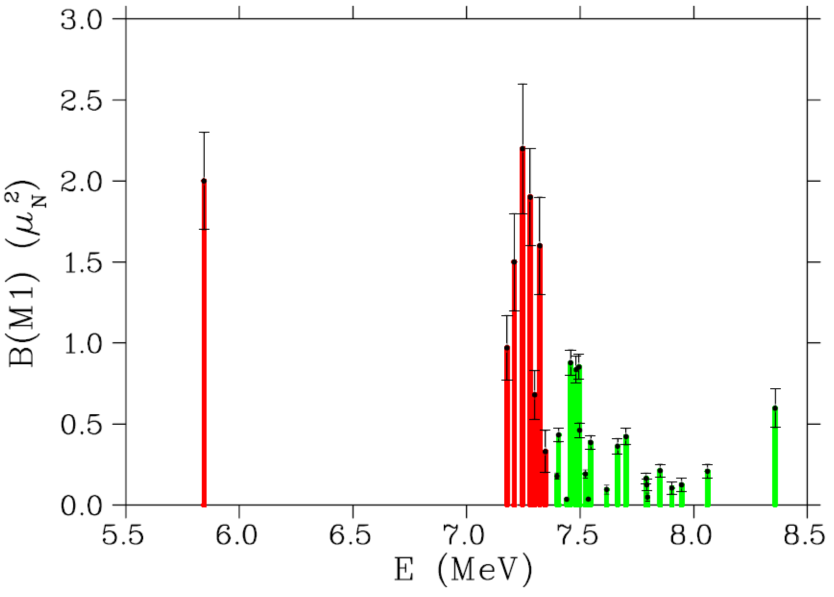
Outline

- History of the problem: experiment and theory.
- Microscopic models of the description of nuclear excitations: Random-Phase Approximation (RPA) and Renormalized Time-Blocking Approximation (RenTBA).
- Self-consistent approach based on the Skyrme energy-density functional (EDF). Spin-related parameters of the Skyrme EDF.
- Results of the calculations of the low-energy M1 excitations in ^{208}Pb within the self-consistent RPA and RenTBA.
- Problem of the fragmentation of the isovector M1 resonance in ^{208}Pb .
- Results for the low-energy electric excitations in ^{208}Pb.
- Conclusions.

Experimental data: M1 excitations in ^{208}Pb



Characteristics of the M1 excitations in ^{208}Pb:

- **Isoscalar state:**
  $E(1_1^+) = 5.844$ MeV
  $B(M1; 1_1^+) = 2.0(3)\ \mu_N^2$
- **Isovector M1 resonance** (interval 6.6–8.1 MeV):
  $\bar{E} = 7.39$ MeV
  $\sum B(M1) = 15.3(7)\ \mu_N^2$
  $B(M1)_{\max} = 2.2\ \mu_N^2$

T. Shizuma et al., Phys. Rev. C **78**, 061303(R) (2008). [^{208}Pb(γ,γ'), M1 strength at $E < S_n = 7.37$ MeV]

R. Köhler et al., Phys. Rev. C **35**, 1646 (1987). [^{207}Pb(n,γ), M1 strength at $E > S_n$]

M1 excitations in ^{208}Pb (historical overview)

- **Experiment:**

  1. W. Knüpfer et al., Phys. Lett. B **77**, 367 (1978). [(e,e'), M1 strength in ^{208}Pb was not detected]
  2. K. Wienhard et al., Phys. Rev. Lett. **49**, 18 (1982). [(γ,γ'), $E(1_1^+) = 5.846$ MeV]
  3. R. Köhler et al., Phys. Rev. C **35**, 1646 (1987). [(n,γ), $E > S_n = 7.37$ MeV]
  4. R. M. Laszewski et al., Phys. Rev. Lett. **61**, 1710 (1988). [(γ,γ'), $E < S_n$]

- **Theory** (*1p1h configuration space*):

  1. J. D. Vergados, Phys. Lett. B **36**, 12 (1971).
  2. P. Ring and J. Speth, Phys. Lett. B **44**, 477 (1973).
  3. V. N. Tkachev et al., Sov. J. Nucl. Phys. **24**, 373 (1976).

- **Theory** (*1p1h + 2p2h configuration space*):

  1. T.-S. H. Lee and S. Pittel, Phys. Rev. C **11**, 607 (1975).
  2. J. S. Dehesa et al., Phys. Rev. Lett. **38**, 208 (1977).
  3. S. P. Kamerdzhiev and V. N. Tkachev, Phys. Lett. B **142**, 225 (1984).
  4. D. Cha et al., Nucl. Phys. A **430**, 321 (1984).

Microscopic models of the description of nuclear excitations: Response function formalism

Q – external-field operator

ω_n – excitation energy

$B_n(Q)$ – transition probability

$R(\omega)$ – response function

Dynamic polarizability: $\Pi(\omega) = -\langle Q|R(\omega)|Q\rangle = \sum_n \frac{\mathrm{sgn}(\omega_n)\, B_n(Q)}{\omega - \omega_n}$

Strength function: $S(E) = -\frac{1}{\pi}\mathrm{Im}\,\Pi(E + i\Delta)$

Microscopic models of the description of nuclear excitations

Random-Phase Approximation (RPA): $1p1h$ configuration space

$R^{\rm RPA} = R^{\rm IPM} - R^{\rm IPM} V R^{\rm RPA}$, Bethe-Salpeter equation for the response function

$R^{\rm IPM} = -G^{\rm MF} G^{\rm MF}$, the response function in the independent-particle model (IPM)

$G^{\rm MF}(\varepsilon) = (\varepsilon - h)^{-1}$, the single-particle mean-field Green function in the energy representation

h – the single-particle Hamiltonian

V – the residual interaction

Self-consistent RPA: $h = \dfrac{\delta E[\rho]}{\delta \rho}$, $V = \dfrac{\delta^2 E[\rho]}{\delta\rho\, \delta\rho}$, $E[\rho]$ – the energy-density functional

Time-Blocking Approximation (TBA): $1p1h + 1p1h \otimes$phonon configuration space

[V. I. Tselyaev, Sov. J. Nucl. Phys. **50**, 780 (1989); Phys. Rev. C **75**, 024306 (2007)]

$R^{\rm TBA} = \tilde{R}^{\rm IPM} - \tilde{R}^{\rm IPM}(V + \bar{W}) R^{\rm TBA}$, Bethe-Salpeter equation for the response function

$\tilde{R}^{\rm IPM}$ – the IPM response function including the time-projection operator

$\bar{W}$ – the induced interaction including the coupling to the intermediate $1p1h \otimes$phonon configurations

Renormalized TBA (RenTBA): the non-linear version of the TBA

[V. Tselyaev, N. Lyutorovich, J. Speth, and P.-G. Reinhard, Phys. Rev. C **97**, 044308 (2018)]

Self-consistent approach and spin-related parameters of the Skyrme EDF

$$h = \frac{\delta E[\rho]}{\delta \rho}, \qquad V = \frac{\delta^2 E[\rho]}{\delta\rho\,\delta\rho}$$

$$E_{\rm Skyrme} = E^{\rm fixed}_{\rm Skyrme} + E^{\rm fit}_{\rm Skyrme}(W_0,\, x_W,\, g,\, g')$$

$$V = V^{\rm fixed} + V^{\rm s.o.} + V^{{\rm LM}\,(\sigma\sigma)}$$

$V^{{\rm LM}\,(\sigma\sigma)}$ – Landau-Migdal ansatz:

$$V^{{\rm LM}\,(\sigma\sigma)} = C_{\rm N}\big(\, g\, \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}' + g'\, \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'\ \boldsymbol{\tau}\cdot\boldsymbol{\tau}'\,\big)$$

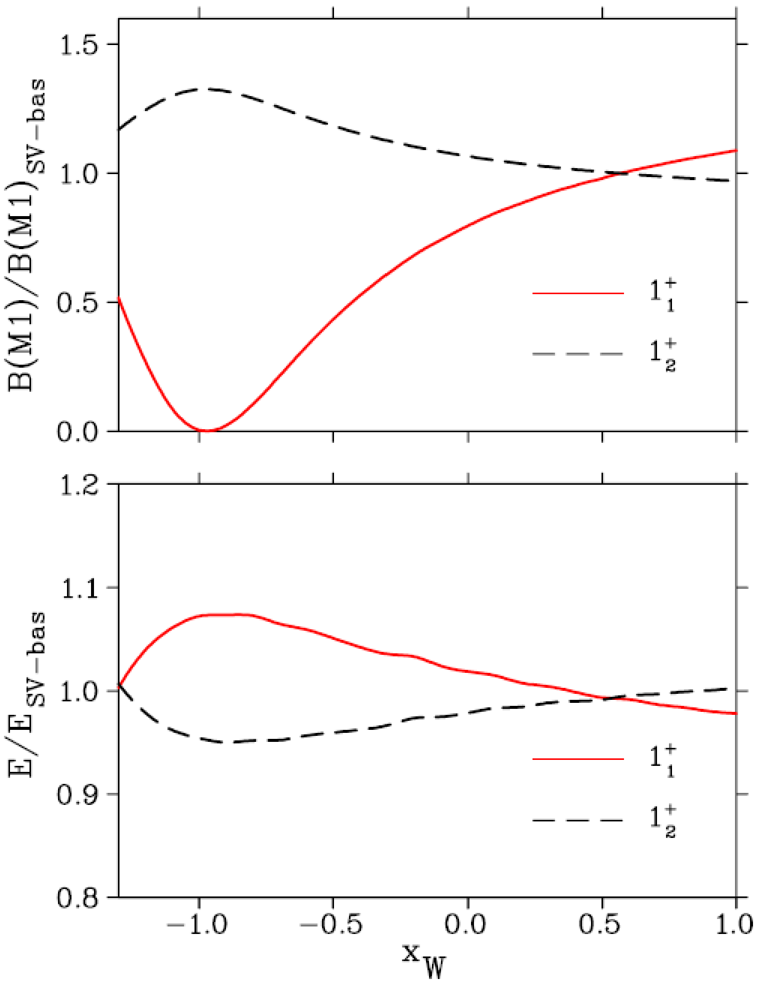
W_0 – spin-orbit strength

x_W – proton-neutron balance of the spin-orbit term

g – Landau parameter for isoscalar spin mode

g' – Landau parameter for isovector spin mode

$$C_0^{\nabla J} = -\tfrac{1}{4}(2 + x_W)W_0 \approx \text{ constant}$$



Determination of the parameters of the modified Skyrme EDF

The field operator of the M1 excitations:

$$Q = \mu_N \sqrt{\frac{3}{16\pi}} \left\{ (\gamma_n + \gamma_p)\,\boldsymbol{\sigma} + \boldsymbol{l} + \left[(1 - 2\xi_s)(\gamma_n - \gamma_p)\,\boldsymbol{\sigma} - (1 - 2\xi_l)\,\boldsymbol{l} \right] \tau_3 \right\}$$

$\gamma_p = 2.793$, $\gamma_n = -1.913$ – the spin gyromagnetic ratios

ξ_s, ξ_l – the renormalization constants ($\xi_l = 0$)

5 parameters W_0, x_W, g, g', ξ_s were determined from 5 conditions:

$B(^{208}\mathrm{Pb}) = 1636.4$ MeV – binding energy of ^{208}Pb

$E(1^+_1) = 5.84$ MeV – energy of the 1^+_1 state of ^{208}Pb

$B(M1; 1^+_1) = 2.0\ \mu_N^2$ – excitation probability of the 1^+_1 state

$\bar{E} = 7.39$ MeV – mean energy of the M1 excitations in the interval 6.6–8.1 MeV

$\sum B(M1) = 15.3\ \mu_N^2$ – summed M1 strength in this interval

V. Tselyaev, N. Lyutorovich, J. Speth, P.-G. Reinhard, and D. Smirnov, Phys. Rev. C **99**, 064329 (2019)

V. Tselyaev, N. Lyutorovich, J. Speth, and P.-G. Reinhard, arXiv:2010.03149v1

Parameters of the modified Skyrme EDFs

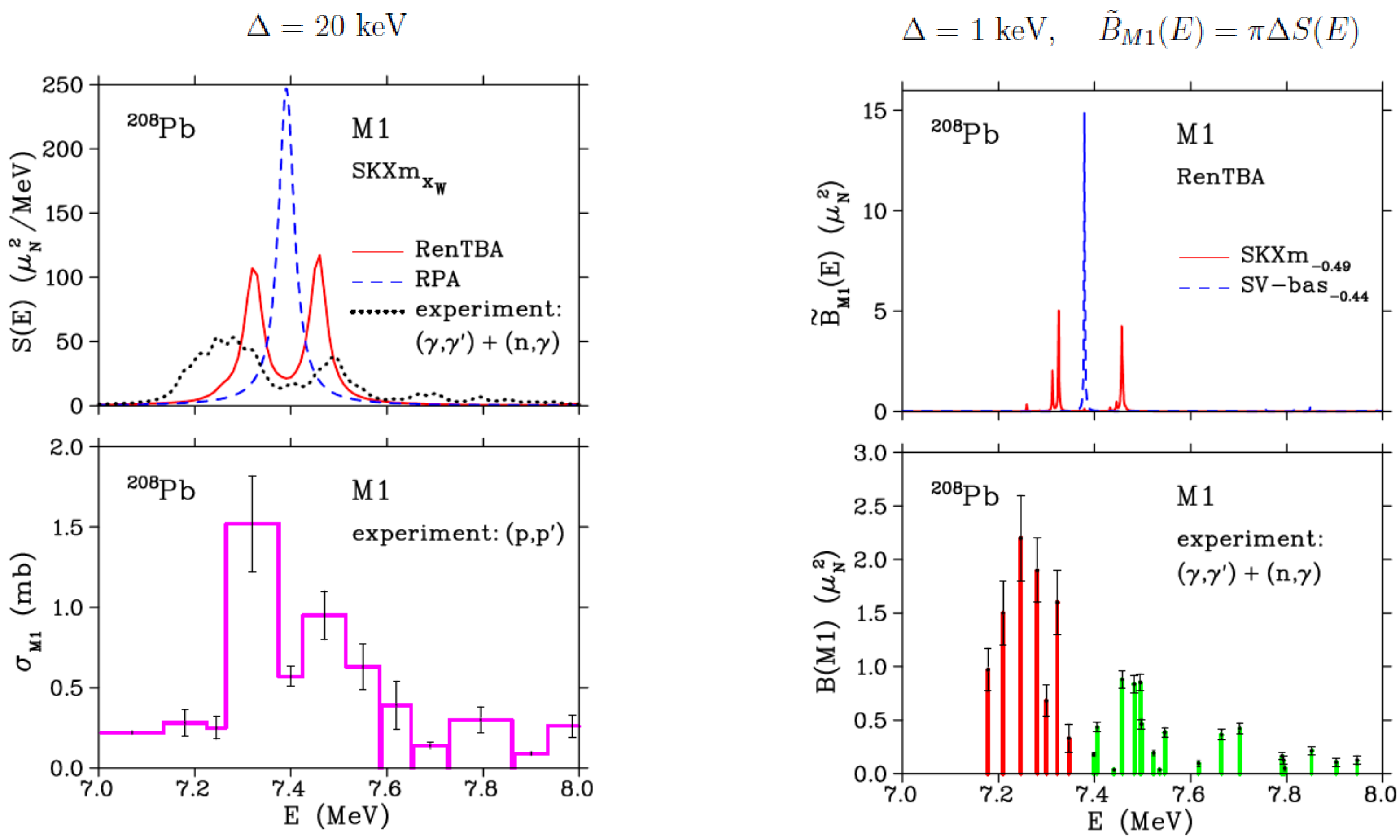
model	EDF	x_W	W_0 (MeV·fm^5)	g	g'	ξ_s
RPA	SKXm$_{-0.54}$	-0.54	226.0	-0.078	0.430	0.156
RPA	SV-bas$_{-0.50}$	-0.50	213.0	-0.028	0.516	0.156
RenTBA	SKXm$_{-0.49}$	-0.49	218.5	0.108	0.930	0.085
RenTBA	SV-bas$_{-0.44}$	-0.44	204.7	0.177	1.030	0.085
	SKXm	0	155.9	0	0	
	SV-bas	0.55	124.6	0	0	

$C_{\mathrm{N}} = 300$ MeV·fm^3

SKXm: B. A. Brown, Phys. Rev. C **58**, 220 (1998)

SV-bas: P. Klüpfel et al., Phys. Rev. C **79**, 034310 (2009)

Results of the calculations and comparison with the experiment



^{208}Pb(γ,γ'): T. Shizuma et al., Phys. Rev. C **78**, 061303(R) (2008)

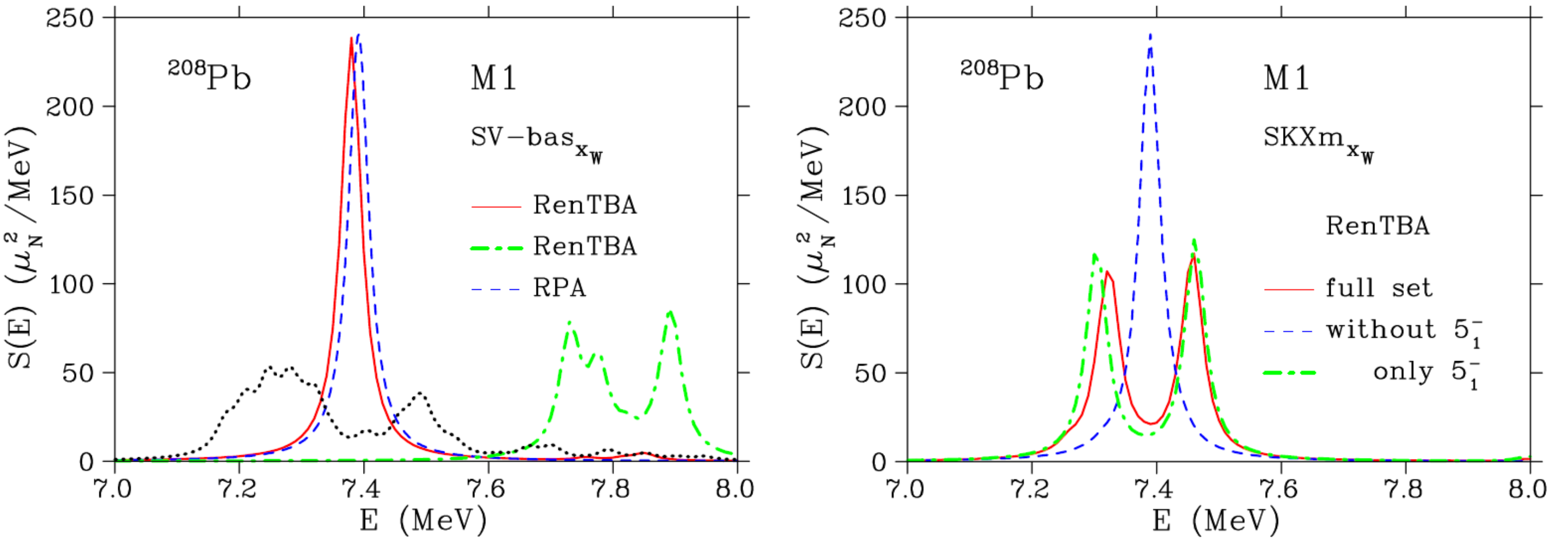
^{207}Pb(n,γ): R. Köhler et al., Phys. Rev. C **35**, 1646 (1987)

^{208}Pb(p,p'): I. Poltoratska et al., Phys. Rev. C **85**, 041304(R) (2012)

Problem of the fragmentation

Problem of the fragmentation of the isovector M1 resonance in ^{208}Pb

and the role of the intermediate configuration $\pi(1h_{9/2} \otimes 3s_{1/2}^{-1}) \otimes 5_1^-$



Ω_c^{min} = the minimum energy

of the $1p1h\otimes$phonon configurations:

$\Omega_c^{min} = \varepsilon_{ph}^{\pi} + \omega(5_1^-)$

$\varepsilon_{ph}^{\pi} = \varepsilon_p^{\pi}(1h_{9/2}) - \varepsilon_h^{\pi}(3s_{1/2})$

EDF	ε_{ph}^{π} (MeV)	$\omega(5_1^-)$ (MeV)	Ω_c^{min} (MeV)
SKXm$_{-0.49}$	4.14	3.24	7.38
SV-bas$_{-0.44}$	4.27	3.55	7.82
experiment	4.21	3.20	7.41

Results for the low-energy electric excitations in ^{208}Pb

The energies *E* and excitation probabilities *B(M1)* of the first excited states of the natural parity in ^{208}Pb

	E (MeV)					$B(M1)$ ($e^2\mathrm{fm}^{2L}$)				
	SKXm$_{-0.49}$		SV-bas$_{-0.44}$			SKXm$_{-0.49}$		SV-bas$_{-0.44}$		
L^π	RenTBA	RPA	RenTBA	RPA	experiment	RenTBA	RPA	RenTBA	RPA	experiment
2_1^+	4.01	4.45	4.00	4.42	4.09	2.6×10^3	3.2×10^3	2.5×10^3	3.0×10^3	3.2×10^3
3_1^-	2.69	2.91	2.88	3.10	2.61	5.6×10^5	6.4×10^5	5.8×10^5	6.4×10^5	6.1×10^5
4_1^+	4.29	4.81	4.30	4.80	4.32	1.1×10^7	1.5×10^7	9.6×10^6	1.3×10^7	1.6×10^7
5_1^-	3.19	3.64	3.49	3.93	3.20	1.9×10^8	2.9×10^8	2.3×10^8	3.6×10^8	4.5×10^8
6_1^+	4.43	5.02	4.53	5.13	4.42	2.6×10^{10}	3.6×10^{10}	1.4×10^{10}	2.2×10^{10}	6.7×10^{10}

Conclusions

- The description of the basic experimental characteristics of the M1 excitations in ^{208}Pb requires refitting some of the spin-related parameters of the Skyrme EDF, both in the RPA and in the RenTBA.
- The observed fragmentation of the isovector M1 resonance in ^{208}Pb which is absent in all the RPA calculations can be to a certain extent described within the self-consistent RenTBA.
- The necessary condition to obtain this fragmentation in the RenTBA is the proximity of the energy of the intermediate $1p1h\otimes$phonon configuration $\pi(1h_{9/2}\otimes 3s^{-1}_{1/2})\otimes 5^-_1$ to the mean energy of the isovector M1 resonance in ^{208}Pb (7.4 MeV) that implies the proximity of the energy of 5^-_1 phonon to the experimental excitation energy of the 5^-_1 state in ^{208}Pb.
- The modified parametrizations of the Skyrme EDF can be used in the description of the low-energy electric excitations within the RenTBA.

Thank you!