SYNTHESIS OF MAGNETIZED HEAVY NUCLEI

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Properties and mass distribution of ultramagnetized atomic nuclei, which arise in heavy-ion collisions, magnetar crusts, during Type II supernova explosions and neutron star mergers are analyzed. For magnetic field strength range 0.1 — 10 *teratesla* the Zeeman effect leads to linear nuclear magnetic response that can be described in terms of magnetic susceptibility. Respectively, binding energies increase for open shell and decrease for closed shell nuclei. Noticeable enhancement in a yield of corresponding explosive nucleosynthesis products with antimagic numbers corroborate with observational results..

1. Introduction

Radioactive nuclides synthesized during nuclear processes make it possible to probe active regions of nuclear reactions in respective sites, cf., e.g., [1-4]. For example, radioactive decay of the iron group isotopes (^{44}Ti, ^{56}Co, ^{57}Co) gives the most plausible source of energy [5], which feeds infrared, optical and ultraviolet radiation of supernova (SN) remnants. The contribution from ^{44}Ti dominates for SNe older than three or four years until an interaction of ejecta with the surrounding matter increases and represents the dominant source. Accordingly, light curves and spectra of infrared and ultraviolet radiation were analyzed by using complex and model-dependent computer simulations [5]. For the initial mass of ^{44}Ti in SNR 1987A was obtained an estimate $(1-2)\cdot 10^{-4}\cdot M_{\text{solar}}$ (in the masses of the Sun). This value significantly exceeds model predictions (see [9] and below). Neutron star mergers give another plausible source [11] of nucleosynthetic components of r-process nuclide enrichment in galactic chemical evolution.

Radioisotopes synthesized at SN explosions can be observed directly by recording the characteristic gamma lines accompanying their decay [1,9]. The radioactive decay chain $^{44}\text{Ti} \rightarrow ^{44}\text{Sc} \rightarrow ^{44}\text{Ca}$ leads to emission of lines with energies of 67.9 keV and 78.4 keV (from ^{44}Sc) and 1157 keV (from ^{44}Ca) of approximately the same intensity. The half-life of ^{44}Ti, which is about 60 years under Earth conditions, allows us to estimate the mass of this isotope in the remnant. The obtained observational values for the total mass of ^{44}Ti nuclides synthesized in SN explosions significantly exceed model predictions; which give the mass of initially synthesized nuclides ^{44}Ti $M_{\text{Ti}} \square 10^{-5}\cdot M_{\text{solar}}$ in the absence of magnetic effects. These predictions are consistent with observational data of SN-type I, see [6] and refs. therein. Consideration of specific SN explosion scenarios leads in some cases, c.f., e.g. [12, 13], to mass values approaching observational data.

Superstrong magnetic fields exceeding *teratesla* (TT) arise at SN explosions [1,2], neutron star mergers [3], in heavy ion collisions [4] and magnetar crusts [7] in conjunction with observations of soft-gamma repeaters and abnormal X-ray pulsars. The nuclides formed in such processes contain information about the structure of matter and mechanisms of explosive processes. In this contribution we analyze an effect of corresponding relatively weak magnetic field on nuclear structure and discuss some possibilities of using radionuclides to probe the internal regions of these processes. The next section briefly describes methods of nuclear statistical equilibrium for description and analysis of nucleosynthesis. Section 3 considers the changes in the structure and properties of atomic nuclei due to Zeeman splitting of energy levels of nucleons. It is shown that such a mechanism dominates for magnetic field strength range 0.1 — 10 *teratesla* and results in linear nuclear magnetic response that can be described in terms of magnetic susceptibility. The magnetic field influence on the composition of nuclei is considered section 3.2. Conclusions are presented in section 4.

2. Abundance of atomic nuclei at statistical equilibrium

Nuclear statistical equilibrium (NSE) approximation is used very successfully to describe abundance of iron group nuclei and nearby nuclides for more than half a century. Under NSE conditions the yield of nuclides is mainly determined by the binding energy of resulting atomic nuclei. Magnetic effects in NSE were considered [1,2,8] and refs. therein. Recall that at temperatures ($T \leq 10^{9.5}$ K) and field strengths ($H \geq 0.1$ TT) the dependence on the magnetic field of the relative yield $y = Y(H) / Y(0)$ is determined mainly by a change of nuclear binding energy ΔB in a field and can be written in the following form

$$y = \exp\{\Delta B / kT\}, \tag{1}$$

The binding energy of a nucleus is given in a form of the energy difference between non-interacting *free* nucleons E_{N} and the nucleus consisting of them E_A, $B = E_{\mathrm{N}} - E_A$. Under conditions of thermodynamic equilibrium at a temperature T, the corresponding energy is expressed as follows.

$$E = \frac{kT^2}{\Sigma}\frac{\partial \Sigma}{\partial T} \tag{2}$$

in terms of a partition function $\Sigma = \sum_i \exp\{-e_i / kT\}$, where e_i denotes an energy of nuclear particles in an i-state, k is the Boltzmann constant. Using Eq. (2) for free nucleons the energy component due to an interaction with a magnetic field can be written in the following form $E_\alpha = -\frac{g_\alpha}{2}\omega_{\mathrm{L}}\mathrm{th}(g_\alpha \omega_{\mathrm{L}} / 2kT)$, where $\mathrm{th}(x)$ is the hyperbolic tangent. Here, the well-known [10] spin g − factors $g_{\mathrm{p}} \approx 5.586$ and $g_{\mathrm{n}} \approx -3.826$ for protons $\alpha = \mathrm{p}$ and neutrons $\alpha = \mathrm{n}$. For values of temperature ($T \square 10^{9.5}$ K) and field strengths ($H \square 1$TT) considered here one gets $E_\alpha \square -10^{0.5}$ keV.

3. Synthesis of ultramagnetized atomic nuclei

The Zeeman – Paschen – Back effect is associated with a shift of nucleon energy levels due to an interaction of nucleon magnetic moments with a field. Dramatic change in nuclear structure occurs under conditions of nuclear level crossing [2,8]. The characteristic energy interval $\Delta\varepsilon \square 1$MeV determines a scale of a field strength $\Delta H_{\mathrm{cross}} \square \Delta\varepsilon / \mu_{\mathrm{N}} \square 10^{1.5}$TT, at which nonlinear effects dominate. Here μ_{N} stands for nuclear magneton. In a case of a small field strength $H \square 10^{1.5}$TT a linear approximation can be used. In following, we also neglect the residual interaction. The most important corresponding effect is a pairing of nucleons caused by the spin-spin interaction and contributing to the binding energy [10] $E_{\mathrm{pair}} \square 10^{0.5}$MeV. Obviously, for a nucleus ${}^{A}_{N}Z$ at a field strength $H \geq 2E_{\mathrm{pair}} / \mu_{\mathrm{N}}(g_{\mathrm{p}}Z - g_{\mathrm{n}}N)$ an interaction with magnetic field dominates over a residual interaction indicating sufficient accuracy of a model of independent nucleons in an average self-consistent mean field. Consequently, for nuclei of average mass numbers $A \sim 50$ in fields of an intensity $H \geq 0.1$TT the residual interaction is not significant. It is worthy to notice that under such conditions the total value of the spin quantum number of nucleons of a subshell (and a nucleus) is maximum possible. This property is similar to the Hund rule, well known for electrons of atoms.

3.1. Zeeman energy in atomic nuclei

The method of self-consistent mean field gives useful and widely used approach for realistic description and analysis of properties of atomic nuclei. The single-particle (sp) Hamiltonian $\hat{H}_{\alpha}$ for nuclei in a relatively weak magnetic field $\mathbf{H}$ within the linear approximation can be written as

$$H_{\alpha} = H_{\alpha}^{0} - (g_{\alpha}^{o}\,\mathbf{l}\ -\ g_{\alpha}\,\mathbf{s})\ \omega_{L} \tag{3}$$

for protons $\alpha = \mathrm{p}$ and neutrons $\alpha = \mathrm{n}$. Here $\hat{H}_{\alpha}^{0}$ represents the sp Hamiltonian for isolated nuclei, the orbital moment and spin operators are denoted by $\hat{\mathbf{l}}$ and $\hat{\mathbf{s}}$, respectively. The interaction of dipole nucleon magnetic moments with a field is represented by terms containing the vector $\omega_{\mathrm{L}} = \mu_{\mathrm{N}}\mathbf{H}$, and values g_{α}^{o} denote orbital $g-$factors $g_{\mathrm{p}}^{\mathrm{o}} = 1$ and $g_{\mathrm{n}}^{\mathrm{o}} = 0$, provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

Thus, the binding energy decreases for magic nuclei with a closed shell, zero magnetic moment and, therefore, zero interaction energy with a magnetic field. In cases of antimagic nuclei with open shells, significant (maximum possible under these conditions, see above) magnetic moment leads to an additional increase in the binding energy B in a field. In this case, the leading component of such a magnetic contribution is represented by the sum over the filled i sp energy levels ε_{i}, $B_{m} = \sum_{i-occ} \varepsilon_{i}$, see [8] and refs. therein. In the representation of the angular momentum for spherical nuclei, the sp states $|\,i>$ are conveniently characterized by quantum numbers (see [10]): n - radial quantum number, angular momentum l, total spin j, and spin projection on the direction of the magnetic field m_{j}. Then, using sp energies $\varepsilon_{nljm_{j}}$ and wave functions $|\,nljm_{j}>$, the magnetic energy change $\Delta B^{m} = B^{m}(H) - B^{m}(0)$ in a field H can be written as

$$\Delta B_{\alpha}^{m} = \kappa_{\alpha}\omega_{\mathrm{L}}, \kappa_{\alpha} = \sum_{i-\mathrm{occ}} \kappa_{\alpha}^{i},$$

$$\kappa_{\alpha}^{i} = \sum_{m,s} |< lm, \frac{1}{2}s\ |\ jm_{j} >|^{2}\ (g_{\alpha}^{o}m + g_{\alpha}s)$$

$$= \begin{cases} (g_{\alpha}^{o}l + g_{\alpha}/2)m_{j}/j, & \text{for } \mathrm{j} = \mathrm{l} + 1/2, \\ (g_{\alpha}^{o}(l+1) - g_{\alpha}/2)m_{j}/(j+1), & \text{for } \mathrm{j} = \mathrm{l} - 1/2, \end{cases} \tag{4}$$

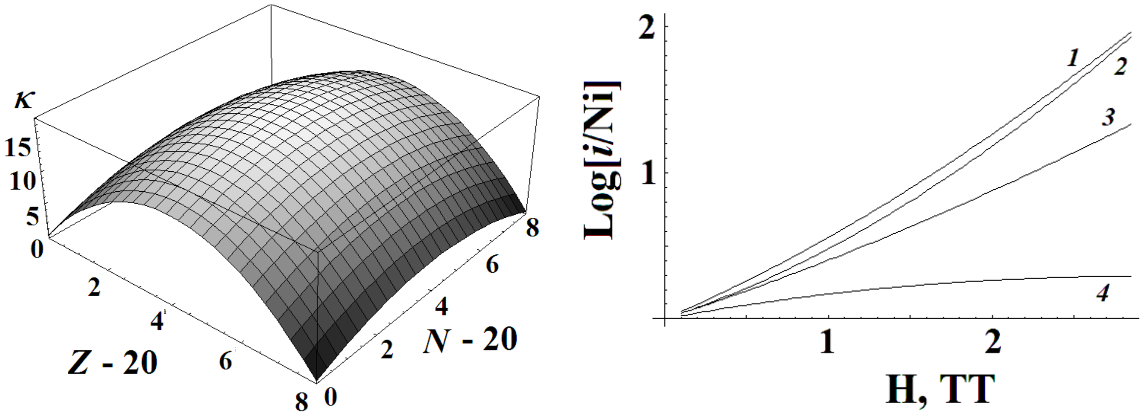
where $(\alpha = \mathrm{n,p})$, $< lm, \frac{1}{2}s\ |\ jm_{j} >$ is the Clebsh-Gordan coefficient. The result Eq. (4) is similar to the one obtained in the Schmidt model [10]. We stress here that in this case parameters κ_{α} are given by combined susceptibility of all independent nucleons spatially confined due a mean field. Thus, the value κ_{α} differs from nuclear g-factor corresponding to an interaction of magnetic moment of a nucleus in the ground state with a field. Within the shell model the magnetic moment is determined by valence unpaired nucleons [10], and respective g-factor of a nucleus is given by maximum spin projection m_{j} with a positive and negative sign for protons and neutrons, respectively. As is mentioned above, Eq. (4) provides more reliable nuclear energy in magnetic fields $H > 0.1$ TT.

3.2. Iron region

In a case of magic numbers $\kappa = 0$, see Fig. 1a, and the dependence on the magnetic field in the synthesis of nuclei is due to a change in an energy of an interaction of free nucleons with a field. The magnetization of a nondegenerate nucleon gas and the arising component of magnetic pressure lead to an effective decrease in binding energy of magic nuclei and, as a result, to the suppression of the yield of corresponding chemical elements. However, we notice that the suppression factor is less significant in the case of realistic magnetic field geometry, see [2]. Significant magnetic moment and parameter κ contribute to an increase in binding of nucleons for ultramagnetized antimagic nuclei in a field. The increase in nucleosynthesis products caused by such an enhancement is weakly sensitive to a structure of magnetic field [2].

Let us consider normalized yield coefficient of antimagic even-even symmetric nuclei of the 1 $f_{7/2}$ and 2 $p_{3/2}$ shells and the double magic nucleus ^{56}Ni, i.e.$[i / \mathrm{Ni}] \equiv y_i / y_{\mathrm{Ni}}$. As seen in Fig. 2, the volume of synthesis of ^{44}Ti and ^{48}Cr increases sharply with increasing magnetic induction, whereas the output of ^{52}Fe varies relatively insignificantly, and the total mass of ^{60}Zn is almost constant. It is worthy to recall in this connection the mysteriously large abundance of titanium obtained in direct observations of SN-type II remnants, see refs. [2,5,9]. Observational data suggest a Ti nucleus yield for type II SNe far exceeding model predictions and similar results for type I SNe. As one can see from Eqs. (3), (4) and Fig. 1b the magnetic increase in the synthesis of nuclides by an order of magnitude corresponds to a field strength of several TT. Such magnetic induction is consistent with simulation predictions and an explosion energy of SNe [1,2].

Accounting for Eqs. (3), (4) and Fig. 1b we notice that such conditions suggest even stronger enrichment by ^{48}Cr isotope, since the maximum magnetic susceptibility corresponds to a half-filled shell. In the case of filling of shell 1 $f_{7/2}$ (iron group nuclei) this condition is satisfied at $Z = N = 24$ (see Section 3.1). Then a significant value of parameter $\kappa_{\mathrm{Cr}} = 17.51$ leads to a noticeable magnetic amplification of synthesis of ^{48}Cr nuclide. The chain of radioactive decay $^{48}\mathrm{Cr} \to ^{48}\mathrm{V} \to ^{48}\mathrm{Ti}$ generates an excess of the dominant titanium isotope.



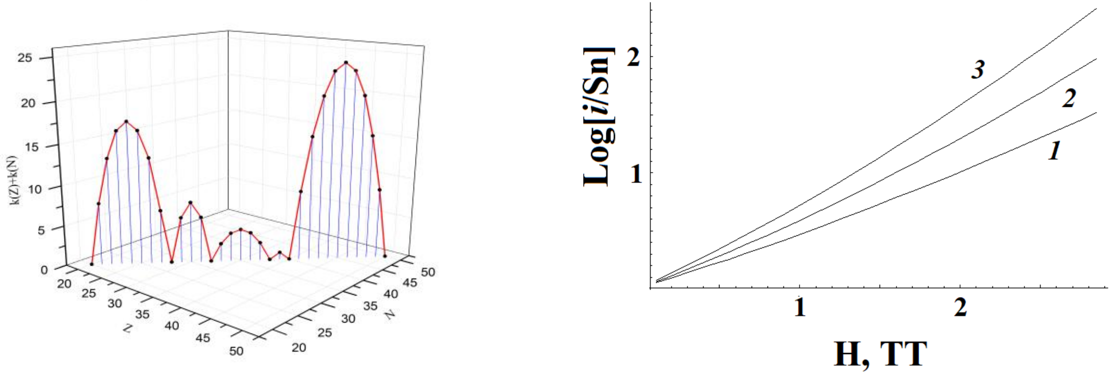
(**a**) (**b**)

Figure 1. Magnetic effects for nuclei in the iron region: (**a**) Dependence on the number of protons and neutrons of the magnetic susceptibility for nuclei with filled 1 $f_{7/2}$ shell. The minimum values $\kappa_{\mathrm{magic}} = 0$ correspond to double magic nuclei at Z(&N) = 20 or 28, the maximum value $\kappa_{\max} \approx 17.51$ for the antimagic nucleus ^{48}Cr at $Z = N = 24$; (**b**) Magnetic field dependence of the yield ratio $[i / \mathrm{Ni}]$ (see text) for ^{56}Ni and ^{44}Ti — 2, ^{48}Cr — 1, ^{52}Fe — 3, ^{60}Zn — 4, at kT=0.5 MeV.

3.3. The r-process nuclides

The r-process nuclides can plausibly originate from neutron star mergers. In a single event such sites produce 100 times larger nuclide volume as compared to Type II SN event. In the first stage of the production of r-process nuclei, matter experiences explosive burning at high temperatures and is heated to conditions of NSE equilibrium [12] and abundance is given by Eq. (1). Significantly amplified magnetic induction can affect nucleosynthesis processes in both cases. As is seen from Eqs. (4) noticeable magnetic modification in nuclear properties is expected for mass numbers corresponding to pronounced magic numbers N&Z = 50, 82, and 126.

As is illustrated in Fig. 2a for mass numbers A= = 40 --- 100 considerable values of magnetic susceptibility are displayed for nuclei corresponding to 1$f_{7/2}$ and 1$g_{9/2}$ shells. Neutron number N = 50 gives a magic number or a point for concentration of nuclear material on the way of r-process scenario. Such a mass enhancement originates also from small cross section of (n, γ) reaction on magic nuclei, see [15]. Normalized yield coefficients of some nuclei of the $1g_{9/2}$ shell and the double magic nucleus ^{100}Sn, i.e. $[i/\mathrm{Sn}]=y_i/y_{\mathrm{Sn}}$ are presented in Fig. 2b. As is seen in Fig. 2b magnetic effects give rise to an enrichment of nuclear components with smaller mass numbers. However, $N = 50$ isotone ^{95}Rn displays more pronounced enrichment indicating thereby that large volume of isotones with $N = 50$ remains robust. Such a property is due to larger magnetic susceptibility for protons than for neutrons. Following arguments of waiting point approximation one can expect some slight magnetic effect in r-process peak with an enhanced portion of small mass number nuclides.



(**a**) (**b**)

Figure 2. Nuclear magnetic effects: (**a**) Dependence on the number of protons and neutrons of the magnetic susceptibility for symmetric nuclei in the region $A = 40$ — **100**; (**b**) Magnetic field dependence of the yield ratio **[*i*/Sn]** (see text) for ^{100}Sn and ^{96}Cd — 1, ^{92}Pd — 2, ^{95}Rn — 3 at kT=0.5 MeV.

4. Conclusions

We considered ultramagnetized atomic nuclei arising at explosions of type II supernovae, neutron star mergers, during collisions of heavy ions, and in magnetar crusts. It is shown that for a field strength of 0.1 — 10 TT magnetic response of nucleons is determined by the Zeeman effect. Accordingly, dominant linear magnetic susceptibility is represented as a combined reactivity of valent nucleons and binding energy increases for open-shell nuclei. For magic nuclei with closed shells the binding energy is effectively reduced due to the field-induced additional pressure in a free nucleon gas. As a result the composition of atomic nuclei formed in an ultramagnetized plasma depends on a field strength. Considerable magnetic modification of nuclear properties is predicted for mass numbers corresponding to large valent shell spins and, respectively, pronounced magic numbers N&Z = **28,** 50, 82, 126…

The magnetic structure change for $1f_{7/2}$ shell nuclei (iron group) enhances nucleosynthesis products of smaller mass numbers. In particular, an increase in the volume part of the titanium ^{44}Ti isotope at field induction of several TT is in satisfactory agreement with data of direct observations of SN remnants [2,6,8,9]. Such an induction of the magnetic field is consistent with SN explosion energy [2]. These conditions of nucleosynthesis imply a significant increase in a portion of the main titanium isotope ^{48}Ti in chemical composition of galaxies.

On an example of synthesis of nuclei with open $1g_{9/2}$ shell and magic number $N = 50$ we see that magnetic effects in r-process give rise to an enrichment of nuclear components with smaller mass numbers as well. However, large volume of isotones with $N = 50$ remains robust. Then magnetic effect in r-process peak is expected to result in some enhancement of volume of small mass number nuclides. Magnetic effects considered can also stimulate dynamical deformations in nuclear collisions, which are important in calculations of subbarrier fusion cross sections [16].

We notice, finally, that arising in heavy ion collisions magnetic fields of ~10^2 TT affect quark and gluon dynamics [4] with potential effect in chiral transition and quarkyonic matter [17] important in experiments at energies of the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-Ion Collider Facility (NICA) at JINR.

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