STUDY OF GROUND STATES OF ^{10,11}B, ^{10,11}C NUCLEI BY FEYNMAN'S CONTINUAL INTEGRALS METHOD

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Motivation

It is well known, that the ^{9}Be and ^{10}Be nuclei composed of two α-clusters, and one and two external (valence) weakly bound neutrons respectively [1-3]. They may be represented as simple nuclear molecules. In this work the structure of neighbour nuclei ^{10,11}B, ^{10,11}C are studying. These nuclei are represented as few body systems consisting from two α-clusters, and two and three external (valence) nucleons.

Feynman's Continual Integrals (FCI) method [4-10] provides a mathematically more simple possibility for calculating the energy and the probability density of the ground states of *N*-particle systems compared to other approaches, e.g., expansion into hyperspherical harmonics in Hyperspherical Harmonics Method (HHM) [10,11]. The FCI method allows application of modern parallel computing solutions on GPU using NVIDIA CUDA technology to speed up the calculations [12-14].

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Theory

Energy E_0 and squared absolute value of the wave function $|\Psi_0(q)|^2$ of the ground state depending on coordinate q can be found using the path integrals introduced by Richard Feynman [4, 5]. The path integral (propagator) in imaginary (Euclidean) time $t=-i\tau$ for a particle with mass of m having potential energy $V(q)$ can be presented as [6-10]

$$K_E(q,\tau;q,0)\approx\left(\frac{m}{2\pi\hbar\tau}\right)^{N/2}\left\langle\exp\left[-\frac{\Delta\tau}{\hbar}\sum_{k=1}^{N}V(q_k)\right]\right\rangle_{0,N}. \quad (1)$$

Here $\tau = N\Delta\tau$, angle brackets denote averaging over random (N-1)-dimensional vectors (trajectories) [8], which can be done using the Monte Carlo method. We used the CUDA technique of parallel computing on GPUs to calculate averages over random trajectories [12–14]. The calculations were made on the HybriLIT heterogeneous cluster at the Joint Institute for Nuclear Research's Laboratory of Information Technologies.

- **Heterogeneous Cluster (http://hybrilit.jinr.ru/) (LIT, Joint Institute for Nuclear Research)**

Energies E_0, E_1 and squared absolute values of the wave function of the ground $|\Psi_0(q)|^2$ and first excited states $|\Psi_1(q)|^2$ determine the first terms of the propagator's asymptotics at the limit of $\tau\to\infty$

$$K_E(q,\tau;q,0)\to|\Psi_0(q)|^2\exp\left(-\frac{E_0\tau}{\hbar}\right)+|\Psi_1(q)|^2\exp\left(-\frac{E_1\tau}{\hbar}\right)+\ldots,\ \tau\to\infty. \quad (2)$$

For convenience in calculations, we use dimensionless variables $\tilde{q}=q/x_0$, $\tilde{V}=V(q)/\varepsilon_0$, $\tilde{E}_0=E_0/\varepsilon_0$, $\tilde{m}=m/m_0$, $\tilde{\tau}=\tau/t_0$, $\Delta\tilde{\tau}=\Delta\tau/t_0$, and $\tilde{K}_E=K_E x_0$, where $x_0=1$ fm, $\varepsilon_0=1$ MeV; m_0 is the neutron mass; $t_0=m_0x_0^2/\hbar\approx 1.57\times10^{-23}$ s, and $b_0=t_0\varepsilon_0/\hbar\approx 0.02412$.

$$b_0^{-1}\ln\tilde{K}_E(\tilde{q},\tilde{\tau};\tilde{q},0)\approx b_0^{-1}\ln|\Psi_0(\tilde{q})|^2-\tilde{E}_0\tilde{\tau}, \quad (3)$$

$$|\Psi_0(\tilde{q})|^2=\text{const}\cdot\tilde{K}_E(\tilde{q},\tilde{\tau};\tilde{q},0), \quad (4)$$

Formulas (1)–(5) naturally generalize to cases with greater numbers of degrees of freedom. Since the wave function of the ground state has no nodal points (lines or surfaces) and does not change sign, the unnormalized wave function can be found using the formula

$$\Psi_0(q)=\sqrt{K_E(q,\tau;q,0)}, \quad (5)$$

Potencials

$$V^{(1^+)}_{p-n}(r)=\sum_{k=1}^{3}u_k\exp\left(-r^2/b_k^2\right) \quad (6)$$

$$V^{(0^+)}_{p-n}(r)=\sum_{k=1}^{3}u_k\exp\left(-r^2/d_k^2\right)\quad V^{(0^+)}_{n-n}(r)=\sum_{k=1}^{3}u_k\exp\left(-r^2/c_k^2\right)$$

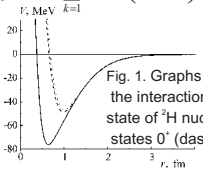


Fig. 1. Graphs of effective potentials: (6) of the interaction between nucleons in the 1^+ state of ^{2}H nucleus (solid line) and in the states 0^+ (dashed and dotted lines).

$$U(r)=-U_1f(r;B_1,a_1)+U_2f(r;B_2,a_2)-U_3f(r;B_3,a_3)f(r;B_4,a_4), \quad (7)$$

$$f(r;B,a)=\left[1+\exp\left(\frac{r-B}{a}\right)\right]^{-1}$$

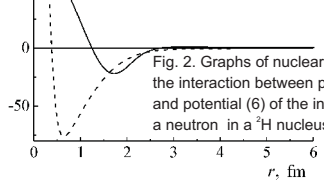


Fig. 2. Graphs of nuclear pseudopotential (7) of the interaction between protons and α-clusters (solid line) and potential (6) of the interaction between a proton and a neutron in a ^{2}H nucleus (dashed line).

$$V^{(N)}_{\alpha-\alpha}(r)=-U_{\alpha1}f(r;B_{\alpha1},a_{\alpha1})+U_{\alpha2}f(r;B_{\alpha2},a_{\alpha2}), \quad (8)$$

$$f(r;B,a)=\left[1+\exp\left(\frac{r-B}{a}\right)\right]^{-1}$$

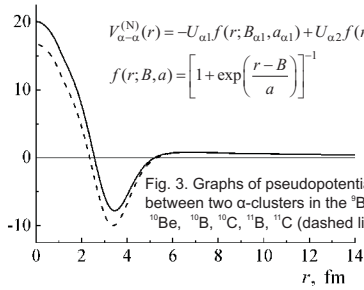


Fig. 3. Graphs of pseudopotential (8) of the interaction between two α-clusters in the ^{8}Be (solid line) and ^{10}Be, ^{10}B, ^{10}C, ^{11}B, ^{11}C (dashed line) nuclei.

Energies of the ground state for 4-body and 5-body systems

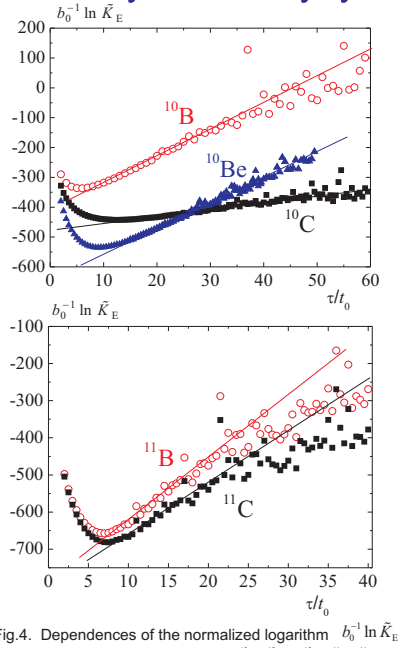


Fig.4. Dependences of the normalized logarithm $b_0^{-1}\ln\tilde{K}_E$ of propagator on imaginary time for ^{10}B, ^{10}Be, ^{10}C, ^{11}B, ^{11}C. Straight lines are the results from a linear regression applied to the graph's linear interval.

The differences between experimental and theoretical values of the separation energy are small:

Nuclei	Separation energy (MeV)	
	Experiment*	Theory
^{10}B (α + *p* + *n* + α)	8.159	8,33±0.27
^{10}Be (α +*n* + *n* + α)	8.38	8,28±0.11
^{10}C (α + *p* + *p* + α)	3.728	2,67±0.15
^{11}B (α + *p* + *n* + *n* + α)	19.613	19,34±0.80
^{11}C (α + *p* + *p* + *n* + α)	16.848	16,51±0.80

* Nuclear reaction video. http://nrv.jinr.ru/.

Probability densities of the ground state for 4-body and 5-body systems

^{10}Be (α+*n*+*n*+α) ^{10}B (α+*p*+*n*+α) ^{10}C (α+*p*+*p*+α) ^{11}B (α+*p*+*n*+*n*+α) ^{11}C (α+*p*+*p*+*n*+α)

Fig. 5. Probability densities $|\Psi|^2$, Jacobi vectors and 3D models

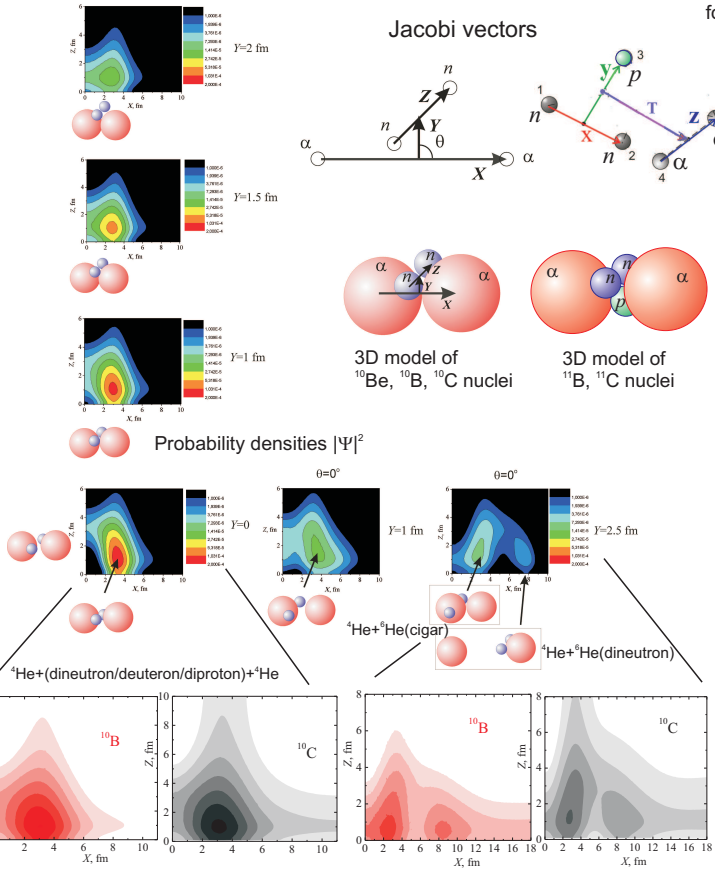
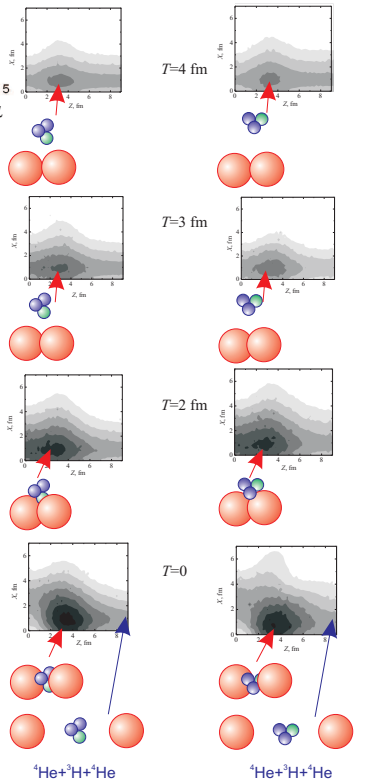


Fig. 6. Probability densities $|\Psi|^2$, Jacobi vectors and 3D models for ^{11}B (α+*p*+*n*+*n*+α)



The structure of ^{11}B nucleus is (α+*t*+α), the structure of ^{11}C nucleus is (α+^{3}He+α).