Structure of low-lying states of ^{9}Be nucleus

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Introduction

The poster presentation is devoted to the memory of our colleague $\boxed{\text{Elena Ibraeva}}$.

Application of Glauber theory of multiple scattering to description of interaction of hadrons with light nuclei played an essential role in her scientific activity.

She was the first to use realistic nuclear wave functions.

In the first similar works for the sake of simplicity the nuclear wave functions were replaced by density functions in form of product of Gauss functions reflecting the general way of nuclear substance densities, but not taking into account the structural peculiarities of nuclei.

Using the realistic wave functions allowed to calculate not only the differential cross sections of both elastic and inelastic processes, but also to obtain the polarization characteristics (polarization and asymmetries), which were started to be measured in experiments.

Realistic wave functions were obtained in nuclear models reproducing as possible as many spectroscopic characteristics of nuclei: position of energy levels and their quantum numbers, moments, radii, spectroscopic factors and etc.

It is a multiparticle shell model [1], three-particle αNN and $2\alpha N$-models for nuclei ^{6}Li and ^{9}Be [2], two-particle model of ^{6}Li nucleus, αtn-model of ^{8}Li nucleus; particle-shell model of nuclei ^{15}N, ^{15}O and ^{15}C [3] and etc.

The successful result is the discovery in works of $\boxed{\text{Ibraeva}}$ of the halo-structure of the excited states of ^{9}Be nucleus with quantum numbers $1/2^+$ and $3/2^+$.

[1] - Бояркина А.Н. Структура ядер 1р-оболочки. – М.: Изд-во МГУ, 1973. – 62 с.
[2] - Kukulin, MSU.
[3] - Zhusupov, KazNU.

Review on previous works

6 low-lying levels of ^{9}Be nucleus are under consideration:

$$J^\pi = 3/2^-(g.s.), 1/2^+, 3/2^+, 1/2^-, 5/2^+, 5/2^-.$$

Inelastic p^{9}Be scattering at E = 180 MeV [4].
Elastic and Inelastic scattering at E = 220 MeV ($3/2^-$ g.s.) [5].
Inelastic p^{9}Be-scattering to the level $J^\pi = 1/2^+$ (E = 180 MeV) [6].
Inelastic p^{9}Be-scattering to the level $J^\pi = 3/2^+$ (E = 180 MeV) [7].

In this work we present results of calculations for the levels $5/2^+, 5/2^-$.

[4] - Dixit S. et al. Phys.Rev.C. 1991. V.43. No. 4. 1758-1776.
[5] - Жусупов М.А., Ибраева Е.Т. Упругое и неупругое рассеяние адронов на легких ядрах в дифракционной теории. Физика ЭЧАЯ. Т. 31, вып. 6. 2000. С.1427-1495.
[6] - Zhusupov M.A., Ibraeva E.T., Krassovitskiy P.M. Phys.At.Nucl. 2015. V.78.No.1.PP.151-158.
[7] - Ibraeva E.T., Zhusupov M.A., Krassovitskiy P.M. Nucl.Phys.A. 933. 2015. 16-33.

Formalism in Brief

The matrix element in Glauber theory [8]:

$$M_{if}(\vec{q}) = \sum_{M_J, M'_J} \frac{ik}{2\pi} \int d^2\vec{\rho} exp(i\vec{q}\vec{\rho})\delta(\vec{R}_A) \left\langle \Psi_f^{J'M'_J} \left| \Omega \right| \Psi_i^{JM_J} \right\rangle \quad (1)$$

where $\vec{\rho}$ - is an impact parameter, which is a two-dimensional vector in the Glauber theory;
$\vec{R}_A$ - is a coordinate of the target nucleus mass center;
$\Psi_i^{JM_J}$ and $\Psi_f^{J'M'_J}$ - initial and final states wave functions of the target nucleus;
$\vec{k}, \vec{k}'$ - are incoming and outgoing momenta of the incident and escape proton;
$\vec{q} = \vec{k} - \vec{k}'$ - is a momentum transfer/transferred in the reaction.

[8] - R.G. Glauber, High - energy collision theory. Lect. Theor. Phys. (New York – London: Interscience, 1959)

Formalism in Brief

The wave function of the ^{9}Be nucleus in $2\alpha n$-model [9,10] with total angular momentum J and its projection M_J:

$$\Psi_{i,f}^{JM_J} = \varphi_1(\varepsilon_{1-4})\varphi_2(\varepsilon_{5-8}) \sum_L \Psi_L^{JM_J}(\vec{r}, \vec{R}), \qquad (2)$$

where $\varphi_1(\varepsilon_{1-4}), \varphi_2(\varepsilon_{5-8})$ - are the wave functions of the α-particles dependent on the internal coordinates of the system of 4 nucleons;
$\Psi_L^{JM_J}(\vec{r}, \vec{R})$ - is a function of relative motion in terms of the Jacobi coordinates, which is expanded by partial waves:

$$\Psi_L^{JM_J} = \sum_{M_L M_S \mu m} \langle L M_L S M_S | J M_J \rangle \langle \lambda \mu \ell m | 1 M_L \rangle r^{\lambda} Y_{\lambda\mu}(\Omega_r) R^{\ell} Y_{\ell m}(\Omega_R) \times$$

$$\times \chi_{SM_S} \sum_{\nu\epsilon} C_{\nu\epsilon}^{\lambda\ell} exp(-\alpha_\nu r^2 - \beta_\epsilon R^2), \qquad (3)$$

where $\langle L M_L S M_S | J M_J \rangle$ - Clebsh-Gordan coefficients defining the scheme of momenta addition;
$Y_{\lambda\mu}(\Omega_r)$, $Y_{\ell m}(\Omega_R)$ - spherical functions;
$\chi_{SM_S} = \chi_{1/2 m_N} \varphi_1(\varepsilon_{1-4})\varphi_2(\varepsilon_{5-8})$ - spin wave function of the valence nucleon and α-particle;
$C_{ij}^{\lambda\ell}, \alpha_i, \beta_j$ - linear and nonlinear variational parameters.

[9] - Voronchev V.T., Kukulin V.I., Pomerantsev V.N.//Few-Body Syst. 1995. V. 18. P. 191;
[10] - Kukulin V.I., Pomerantsev V.N., Rasikov Kh.D. et al.//Nucl. Phys. A. 1995. V. 586. P. 151.

Formalism in Brief

The matrix element (1) after substitution of the wave function (3):

$$M_{if}(q) = \frac{ik}{2\pi} \sum_{M_L M_L' M_S M_S' \mu \mu' m m'} \langle L M_L S M_S | J M_J \rangle \langle L' M_L' S' M_S' | J' M_J' \rangle$$

$$\langle \lambda \mu \ell m | L M_L \rangle \langle \lambda' \mu' \ell' m' | L' M_L' \rangle \sum_{i j i' j'} C_{ij}^{\lambda \ell} C_{i' j'}^{\lambda' \ell'} \times$$

$$\times \int d^2 \rho e^{i \vec{q} \vec{\rho}} \left\langle r^{\lambda} Y_{\lambda \mu}(\vec{r}) R^{\ell} Y_{\ell m}(\vec{R}) e^{-\alpha_i r^2 - \beta_j R^2} | \Omega | r^{\lambda'} Y_{\lambda' \mu'}(\vec{r}) R^{\ell'} Y_{\ell' m'}(\vec{R}) e^{-\alpha_i' r^2 - \beta_j' R^2} \right\rangle \tag{4}$$

The general form of the Glauber multiple scattering operator is written as alternating-sign series of one-, two, . . . , A-fold (where A – is a number of nucleons in the target nucleus) collisions of the incident proton with the nucleons of the nucleus [5]:

$$\Omega = 1 - \prod_{j=1}^{A} (1 - \omega_j(\vec{\rho} - \vec{\rho}_j)) = \sum_{j=1}^{A} \omega_j + \sum_{j<\mu} \omega_j \omega_\mu - \sum_{j\mu<\eta} \omega_j \omega_\mu \omega_\eta + \ldots + (-1)^{(A-1)} \omega_1 \omega_2 \ldots \omega_A, \tag{5}$$

where ω_j - is a profile function, dependent on the elementary $f_{xj}(q)$-amplitude.

Formalism in Brief

Substituting the wave function of the ^{9}Be nucleus in $2\alpha n$-model into the matrix element, it is convenient to transform the Ω-operator to a form conjugated to this model, considering collisions not with separate nucleons, but with α-particle clusters as structureless and the remained nucleon.
In accordance to this approach the series of multiple scattering (5) for the ^{9}Be nucleus is rewritten as follows:

$$\Omega = \sum_{j=1}^{3} \omega_j - \sum_{i<j=1}^{3} \omega_i \omega_j + \omega_{\alpha_1} \omega_{\alpha_2} \omega_n \qquad (6)$$

where $j = 1, 2$ enumerate α_1 and α_2, $j = 3$ enumerates the nucleon.
For further calculations it is necessary to change from single-particle $\{\vec{\rho}_1, \vec{\rho}_2, \vec{\rho}_3\}$ coordinates of nucleons in the Ω operator to the Jacobi coordinates $\left\{\vec{r}, \vec{R}\right\}$ and the coordinate of the ^{9}Be nucleus mass center - $\vec{R}_9$:

$$\vec{r} = \vec{\rho}_1 - \vec{\rho}_2; \qquad \vec{R} = \frac{\vec{\rho}_1 + \vec{\rho}_2}{2} - \vec{\rho}_3; \qquad \vec{R}_9 = \frac{1}{9}\left(4\vec{\rho}_1 + 4\vec{\rho}_2 + \vec{\rho}_3\right) \qquad (7)$$

Differential cross section is matrix element module squared and weighed over total spin:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2J+1} \sum_{M_J M_J'} |M_{if}(q)|^2 \qquad (8)$$

Wave Function of ^{9}Be nucleus in $2\alpha n$-model

The calculation of the wave function in $2\alpha n$-model [9,10] was carried out in variational stochastic method with three couple interactions $V_{\alpha\alpha}$, $V_{\alpha_1,n}$, $V_{\alpha_2,n}$.
Model 1: $V_{\alpha\alpha}$ - is Ali-Bodmer potential (AB) [11], shallow one with repulsive core at small distances, not containing the forbidden states;
Model 2: $V_{\alpha\alpha}$ - is Buck potential (B) [12], deep attractive one with the forbidden states, describing scattering phases with $\lambda = 0, 2, 4$ and 6;
$V_{\alpha n}$ - is the same as in model 1.
In both models $V_{\alpha n}$ was used - a potential with exchange Majorana component which leads to the even-odd splitting of the phase shifts.
Three-dimensional Profiles of WFs:

$$W(r,R) = \sum_{\lambda \ell L} \left| \Psi^{\lambda \ell L} \right|^2 r^2 R^2 \qquad (9)$$

[11] - Ali S., Bodmer A.R. Nucl.Phys. 1966. V.80. P. 99.
[12] - B.Buck, H.Friedrich, C.Wheatley. Nucl.Phys.A. 1977. V.275. P.246.

Wave Function of ^{9}Be nucleus in the excited $J^\pi = 5/2^+$ state with Ali-Bodmer (AB) potential

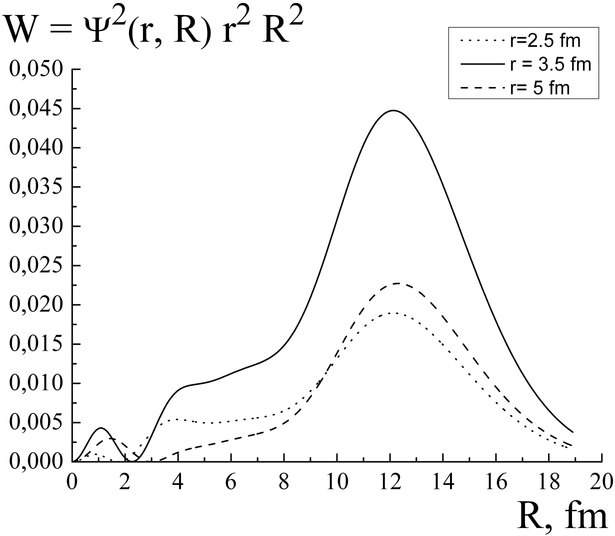


Fig. Three-dimensional profile of ^{9}Be nucleus wave function in the excited $J^{\pi} = 5/2^{+}$ state with Ali-Bodmer (AB) potential

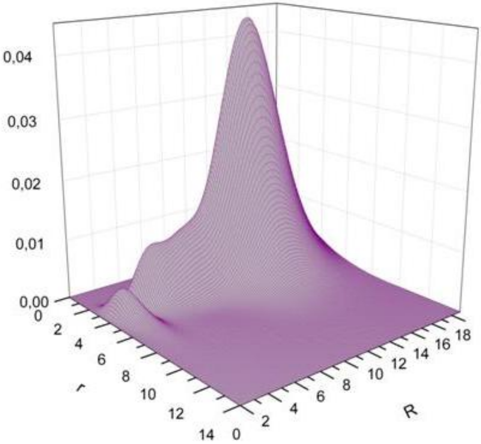


Fig. Three-dimensional profile of ^{9}Be nucleus wave function in the excited $J^\pi = 5/2^-$ state with Ali-Bodmer (AB) potential

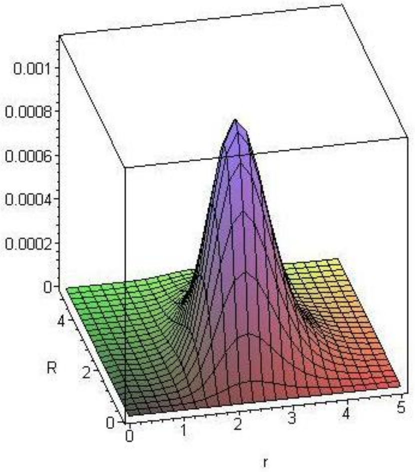


Fig. Three-dimensional profile of ^{9}Be nucleus wave functions in the excited $J^{\pi} = 5/2^{-}$ state with Buck (B) potential

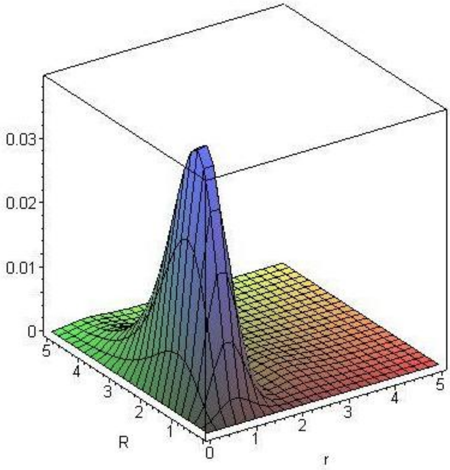
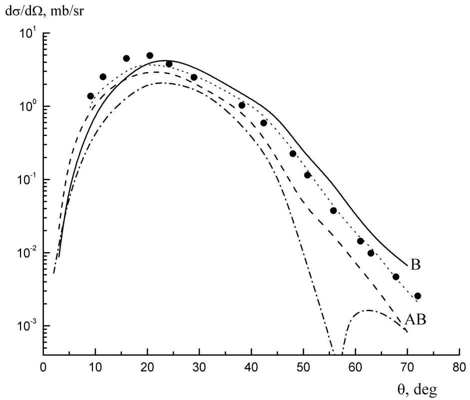


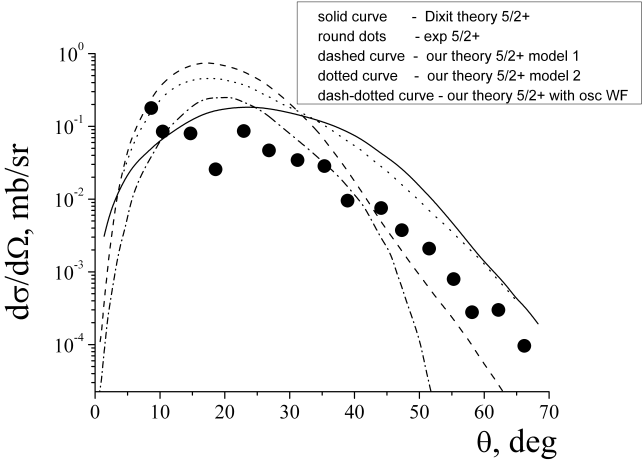
Fig. Differential cross sections of inelastic p^{9}Be-scattering for the level $5/2^-$ with different model wave functions of the ^{9}Be nucleus



The solid and dashed curves – calculation with wave functions in models 1 and 2, dash-dotted curve – with oscillatory wave function, dotted – from paper/work [4]

[4] - Dixit S., Bertozzi W., Buti T.N. et al. Phys.Rev.C. 43, 1758. (1991).

Fig. Differential cross sections of inelastic p^{9}Be-scattering for the level $5/2^+$ with different model wave functions of the ^{9}Be nucleus



[4] - Dixit S., Bertozzi W., Buti T.N. et al. Phys.Rev.C. 43, 1758. (1991).

Conclusion

Thus, the levels of negative parity in ^{9}Be nucleus ($3/2^-$ and $5/2^-$) have three-particle $\alpha\alpha n$-structure with a neutron in $1p$-state.

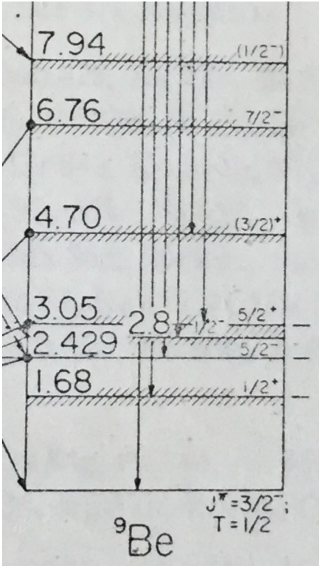
And the levels of positive parity $1/2^+$, $5/2^+$ and $3/2^+$ have $\alpha\alpha n$-nature with the valence neutron in $2s_{1/2}$, $1d_{5/2}$ and $1d_{3/2}$ states respectively.

These levels have clearly manifested halo-structure.

The halo-structure is in the significant distance of the valence neutron from the center of gravity of two α-particles (R $\approx$ 11-12 fermi), while in the states of negative parity this distance does not exceed 3 fermi.

The level $(J^\pi, T) = (1/2^-, 1/2)$ at $E = 2.8 MeV$ was not considered since it has a large width $\Gamma \approx 1$ MeV and it is not well resolved, but its nature is the same as for $3/2^-$ and $5/2^-$, that is the neutron is in the $1p$-shell.

Fig. Energy of low-lying levels of ^{9}Be nucleus [13]



[13] - Ajzenberg-Selove F. Energy Levels of Light Nuclei A=9. Nucl. Phys. A. – 1988. – V. 490. – P. 1.