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A NOVEL ALGORITHM FOR CALCULATING THE PROTON, NEUTRON, AND CHARGE NUCLEAR DENSITIES: COMPARISON WITH THE EXPERIMENTAL DATA

Nucleus-2020

Plan:

- to discuss three types of densities:

  1) proton density
  2) neutron density
  3) charge density

- to suggest a novel method for calculation of proton and neutron densities, short, simple, but accurate enough

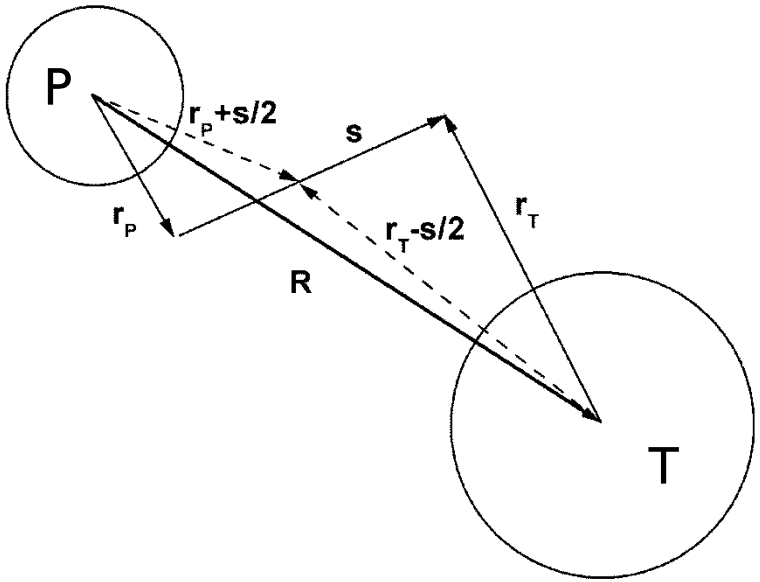
- to check method's accuracy by comparison with independent experimental data

Important part of calculation – energy of strong interaction

$$U_{\mathrm{DF}}(R) = \int d\mathbf{r}_{\mathrm{P}} \int d\mathbf{r}_{\mathrm{T}}\, \rho_{\mathrm{AP}}(r_{\mathrm{P}})\, v_{\mathrm{NN}}(|\mathbf{R} - \mathbf{r}_{\mathrm{T}} + \mathbf{r}_{\mathrm{P}}|)\, \rho_{\mathrm{AT}}(r_{\mathrm{T}}) \qquad (1)$$

double folding

nucleon densities should be known



microscopic methods
are preferable
but computer time
consuming

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^{12}C, ^{16}O, ^{36}S, ^{92}Zr, ^{144}Sm, ^{204}Pb.

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1. As the first step we approximate these densities with the standard Woods-Saxon formula:

$$\rho_{\mathrm{Z\,WS}}(r) = \rho_{\mathrm{ZC\,WS}}\left[1+\exp\left\{\left(r-r_{\mathrm{Z0\,WS}}A^{\frac{1}{3}}\right)/a_{\mathrm{Z\,WS}}\right\}\right]^{-1}$$

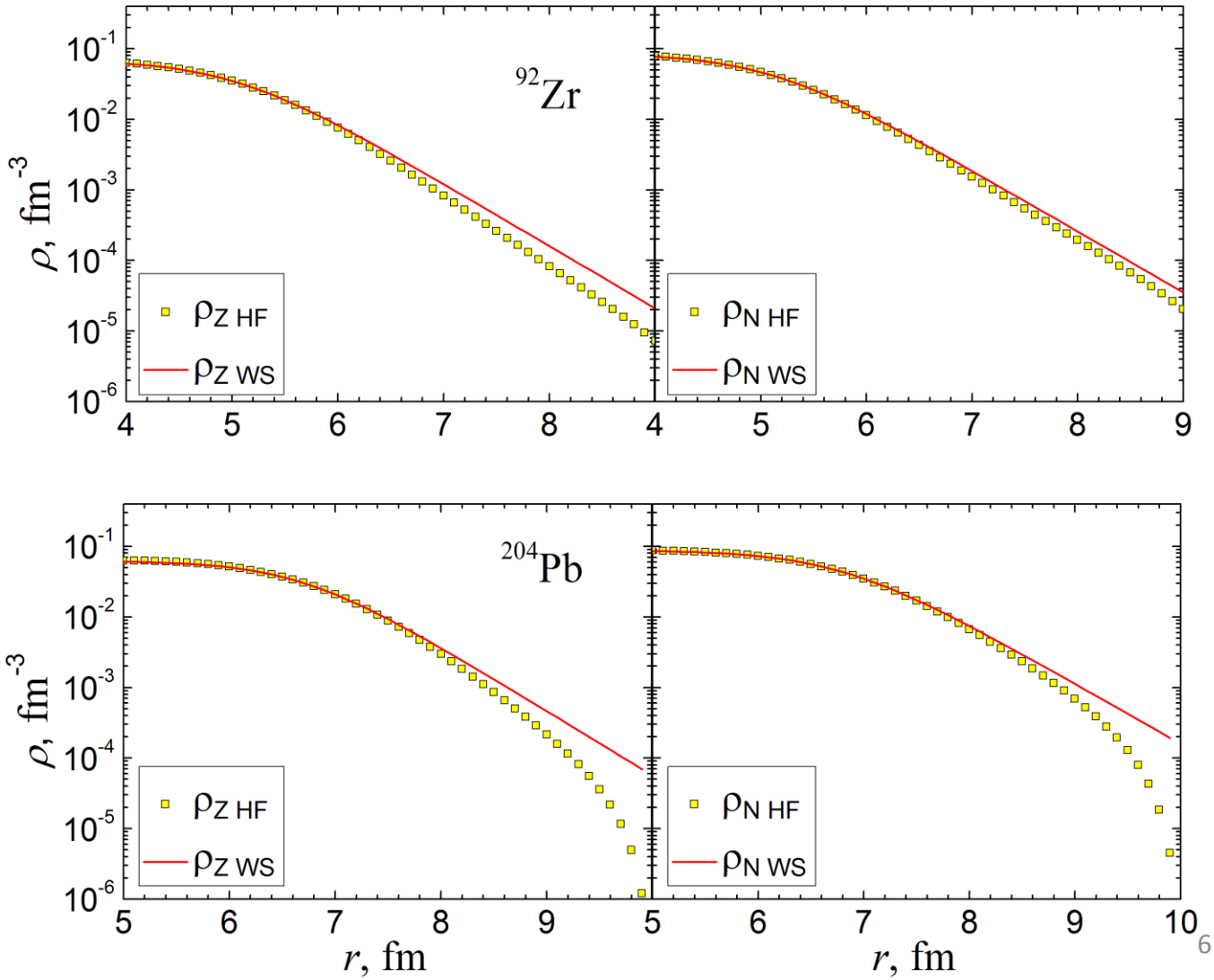
$$\rho_{\mathrm{N\,WS}}(r) = \rho_{\mathrm{NC\,WS}}\left[1+\exp\left\{\left(r-r_{\mathrm{N0\,WS}}A^{\frac{1}{3}}\right)/a_{\mathrm{N\,WS}}\right\}\right]^{-1} \quad (2)$$

normalization

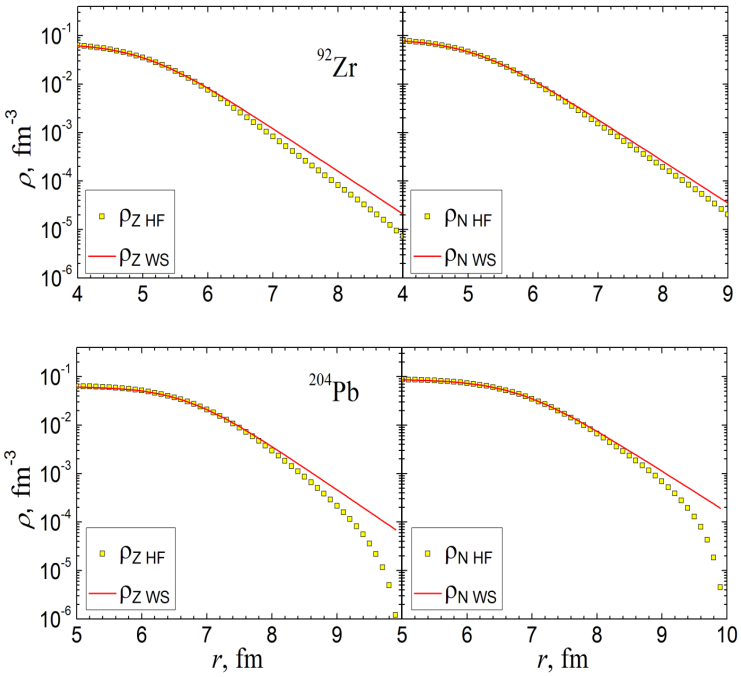
free variable parameters

Nucleon densities:
WS-approximation in comparison with HF-calculations

To improve coincidence, we use correction of nucleon densities in a tail area



2. As the second step we correct the standard Woods-Saxon formula in the tail area:



$$\rho_{\mathrm{WSt}}(r) = \begin{cases} \rho_{\mathrm{WS}} \ , r \le r_{\mathrm{t}}; \\ \rho_{\mathrm{WS}}(r_{\mathrm{t}}) \exp\left[\dfrac{R_{\mathrm{WS}} - r}{a_{\mathrm{t}}(r)}\right], r > r_{\mathrm{t}}. \end{cases} \quad \textbf{(3)}$$

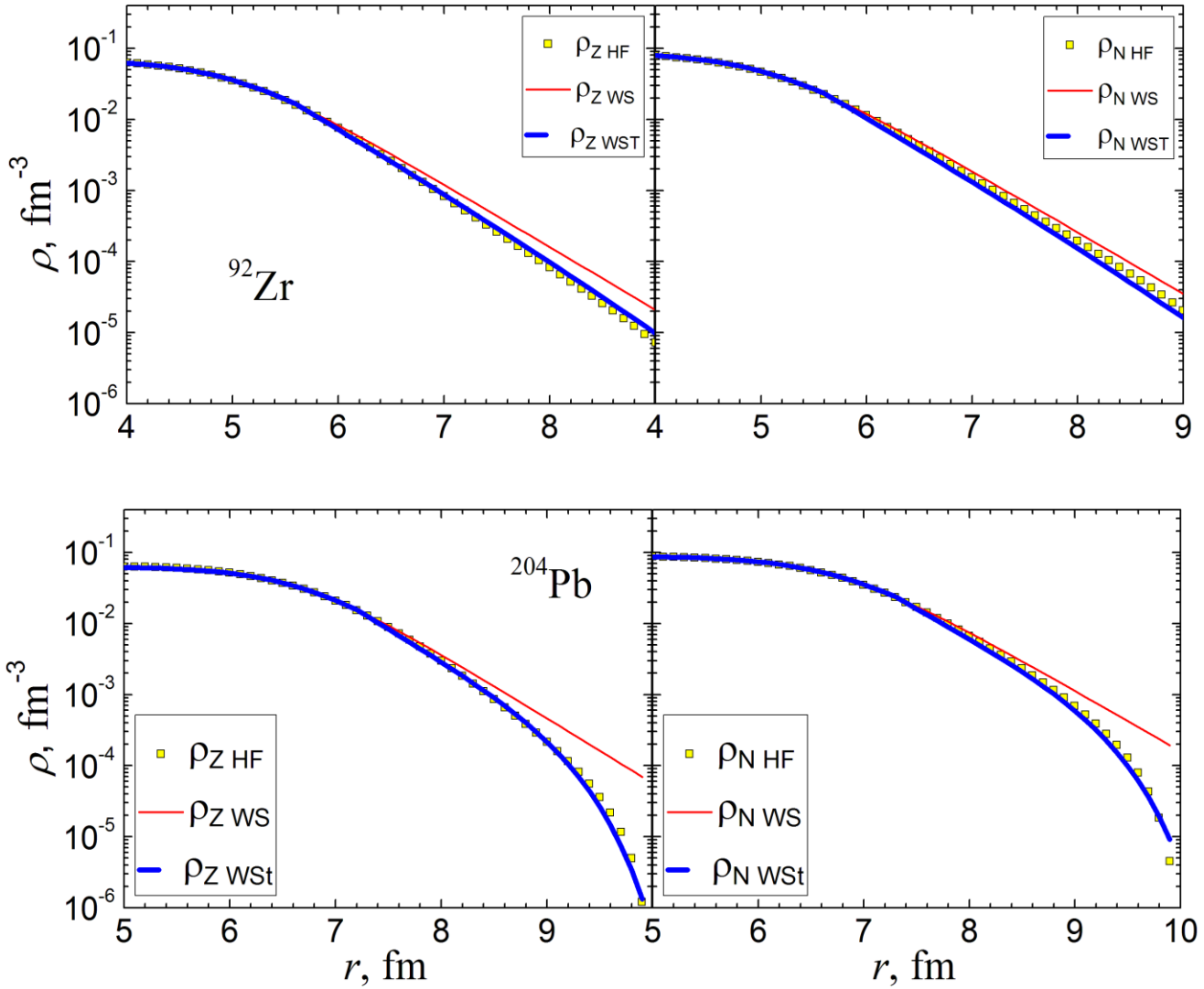
$$a_{\mathrm{t}}(r) = a_{\mathrm{WS}} \left\{ 1 - \delta \cdot \Delta R^{x} \left[\frac{r - r_{\mathrm{t}}}{\Delta R}\right]^{x} \right\} \quad \textbf{(4)}$$

$$r_{\mathrm{t}} = R_{\mathrm{WS}} + \Delta R$$

$$R_{\mathrm{WS}} = r_{0\,\mathrm{WS}} A^{\frac{1}{3}}$$

$$\delta = 0.02 fm^{-x} \Delta R = 0.5 fm$$

Nucleon densities:
Conventional WS-approximation and WSt – approximation with exponential tail – in comparison with HF-calculations

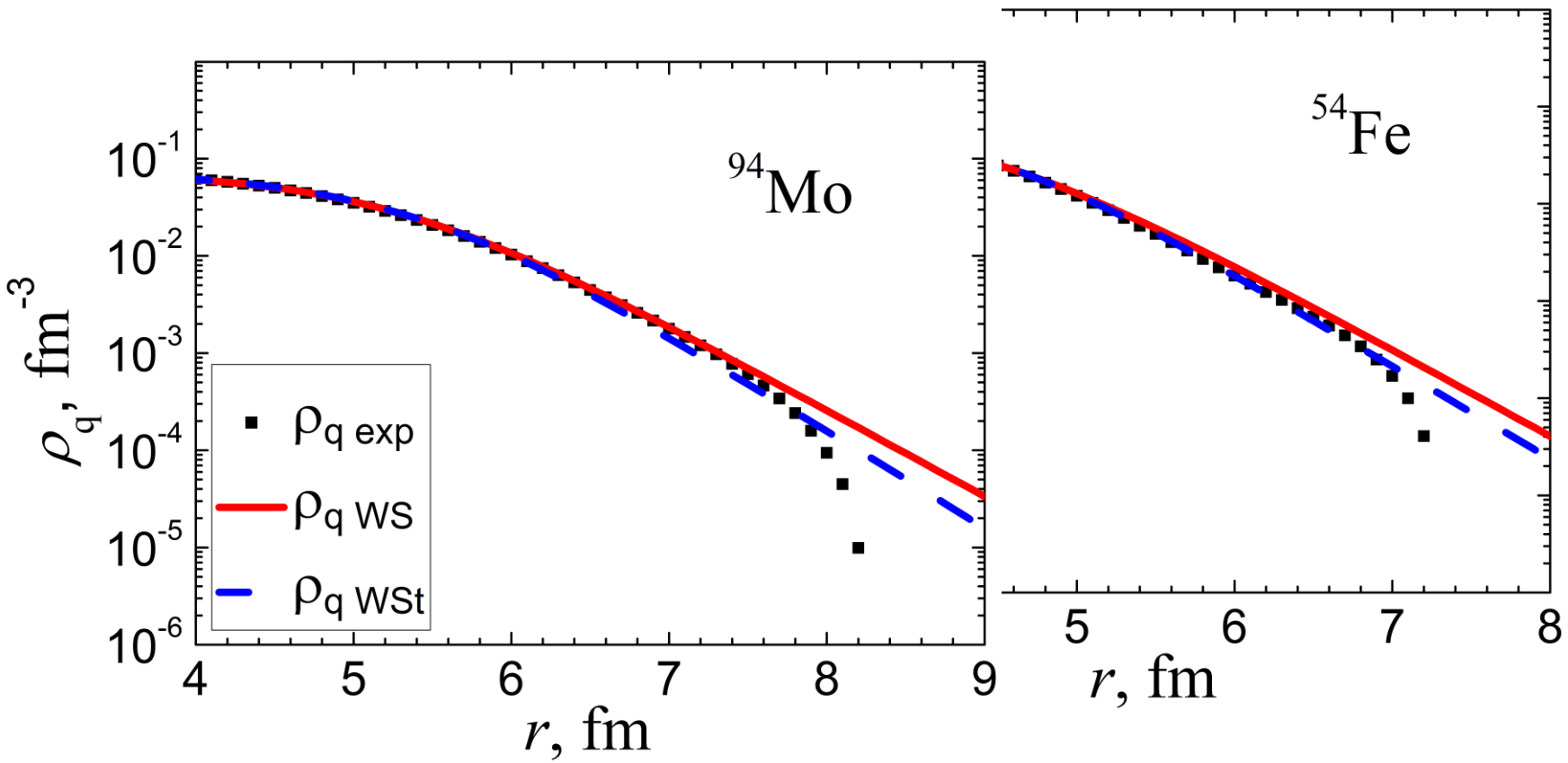


Nucleus	^{12}C	^{16}O	^{36}S	^{92}Zr	^{144}Sm	^{204}Pb
$\rho_{ZC\,WST}$, fm^{-3}	0.0828	0.0764	0.0721	0.0691	0.0668	0.0628
$R_{Z\,WS}$, fm	2.30	2.67	3.55	5.02	5.93	6.67
$a_{Z\,WST}$, fm	0.472	0.471	0.486	0.492	0.477	0.475
ε_Z, %	4.8	3.0	6.1	4.5	3.9	3.0
$x_Z = x_N$	0	0	0.5	1	2	3
$\rho_{NC\,WST}$, fm^{-3}	0.0842	0.0777	0.0846	0.0862	0.0853	0.0884
$R_{N\,WS}$, fm	2.29	2.66	3.62	5.08	5.99	6.78
$a_{N\,WST}$, fm	0.469	0.467	0.501	0.502	0.497	0.510
ε_N, %	5.3	3.5	4.0	2.9	2.3	2.6

3. As the third step we perform a **linear interpolation** for the parameters defining the WSt profile for the nuclei with the charge number between the benchmarking ones.

Charge densities calculated using the WSt nucleon densities in comparison with experimental data

$$\rho_{\mathrm{q}}(r) = \int d\mathbf{r}_{\mathrm{p}} \rho_{\mathrm{Z}}(r_{\mathrm{p}}) f_{\mathrm{p}}(s_{\mathrm{p}}) + \int d\mathbf{r}_{\mathrm{n}} \rho_{\mathrm{N}}(r_{\mathrm{n}}) f_{\mathrm{n}}(s_{\mathrm{n}}) \quad (5)$$



De Vries H., De Jager C.W., De Vries C. // At. Data Nucl. Data Tables. 1987. 36. P. 495-536. https://doi.org/10.1016/0092-640X(87)90013-1.

Comparison with experimental data: Coulomb barrier's heights

$$B_Z = Z_P Z_T / \left(A_P^{1/3} + A_T^{1/3}\right) \text{ MeV}$$

	Reaction	B_Z, MeV	U_{B0WST}, MeV	U_{B0exp}, MeV	ξ_U, %
1	$^{12}C+^{12}C$	7.86	5.74	5.8 ± 0.3	-1.0
2	$^{12}C+^{16}O$	9.98	7.48	7.7 ± 0.4	-2.9
3	$^{12}C+^{92}Zr$	35.27	31.49	32.31	-2.5
4	$^{12}C+^{144}Sm$	49.40	45.96	no	---
5	$^{12}C+^{208}Pb$	59.89	57.36	57.0 ± 0.4	0.63
6	$^{12}C+^{204}Pb$	60.17	57.74	57.55	0.33
7	$^{16}O+^{16}O$	12.70	9.75	11.2 ± 0.6	-13
8	$^{16}O+^{58}Ni$	35.05	30.73	31.67	-3.0
9	$^{16}O+^{92}Zr$	45.49	41.21	41.96	-1.8
10	$^{16}O+^{116}Sn$	54.08	50.02	50.96	-1.8
11	$^{16}O+^{112}Sn$	54.49	50.43	51.02	-1.2
12	$^{16}O+^{144}Sm$	63.91	60.22	61.03	-1.3
13	$^{16}O+^{208}Pb$	77.68	75.23	74.52	0.95
14	$^{16}O+^{204}Pb$	78.03	75.30	74.9	0.53
15	$^{36}S+^{208}Pb$	142.19	143.15	140.4 ± 1.4	2.0
16	$^{36}S+^{204}Pb$	142.78	143.68	143.1 ± 0.2	0.41
17	$^{36}S+^{144}Sm$	116.11	114.59	no	---
18	$^{40}Ca+^{50}Ti$	61.94	57.56	58.21	-1.1
19	$^{36}S+^{90}Zr$	82.23	78.34	77.97	0.47
20	$^{36}S+^{96}Zr$	81.21	77.68	75.61	2.7
21	$^{40}Ca+^{90}Zr$	101.25	97.91	96.88	1.1
22	$^{40}Ca+^{96}Zr$	100.01	97.08	94.59	2.6

$$\xi_U = \frac{U_{B0\,WST}}{U_{B0\,exp}} - 1$$

Conclusions

We have proposed a novel simple method for generating the proton and neutron densities of spherical nuclei with $6<Z<82$. This method is based on

1) the conventional WS-formula corrected in the tail area
2) fitting the parameters for six benchmarking nuclei
3) interpolating the parameters for intermediate nuclei

The resulting calculated charge densities are in good agreement with the experimental charge densities.

The barriers calculated in this approach differ from the experimental ones within 2%.