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Nuclear physics and elementary particle physics. Nuclear physics technologies"

Time-dependent calculation for processes of neutron transfer and nuclear breakup in ^{11}Li+^{28}Si reaction

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Motivation

- Obtaining new information about nuclei located near the nuclear drip line is the important problem in nuclear physics, since for the properties of such nuclei one can expect significant deviations from commonly known laws.
- The aim of this work is the investigation of the reactions with light nuclei ^{11}Li having external weakly bound neutrons, which is formed, so called, exotic “halo” structure of ^{11}Li nucleus.

Time-dependent Schrödinger equation [1] :

- accurate quantum description of interacting particles (nucleons and clusters),
- a small step of the 3D grid (0.3 fm, less than the distance of probability density oscillations),
- classical description of the motion of nuclear centers,
- can be used for light nuclei
- allows you to analyze the dynamics of the processes occurring during the reaction,
- quick calculations.

[1] Sobolev Yu.G., Stukalov S.S., Samarin V.V., NaumenkoM.A., Penionzhkevich Yu.E. INVESTIGATION OF REACTION CROSS SECTIONS FOR BEAM OF ^{8}Li, ^{8}He ON ^{28}Si, ^{59}Co, ^{181}Ta TARGETS // **Plenary report. Conference NUCLEUS-2020**.
[2] Samarin V.V. Time-Dependent Description of Reactions with Weakly Bound Nuclei 8Li, 8B // **Bull. Russ. Acad. Sci. Phys. 84, 990 (2020).**
[3] Azhibekov A.K., Samarin V.V., Kuterbekov K.A., Time-dependent calculations for neutron transfer and nuclear breakup processes in ^{11}Li+^{9}Be and ^{11}Li+^{12}C reactions at low energy // **Chinese Journal of Physics 65, 292 (2020).**
[4] Zagrebaev V.I., Samarin V.V., Greiner W. Sub-barrier fusion of neutron-rich nuclei and its astrophysical consequences // **Phys. Rev. C75, 035809 (2007).**

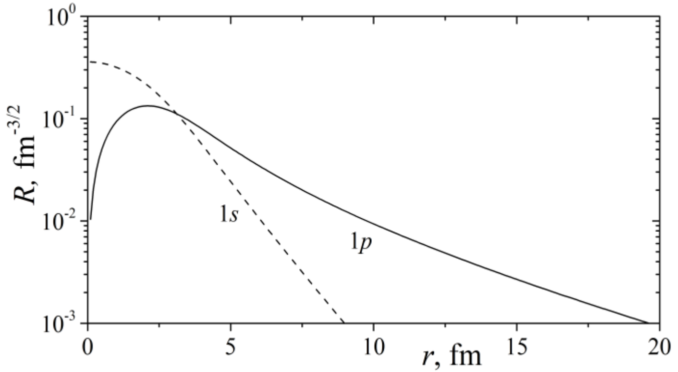
Formalism of the time-dependent microscopic description of nuclear collisions

- Motion of nuclear core centers along classical trajectories.

$$m_1\ddot{\vec r}_1 = -\nabla_{\vec r_1} V_{12}\left(\left|\vec r_1 - \vec r_2\right|\right),\ m_2\ddot{\vec r}_2 = -\nabla_{\vec r_2} V_{12}\left(\left|\vec r_2 - \vec r_1\right|\right).$$

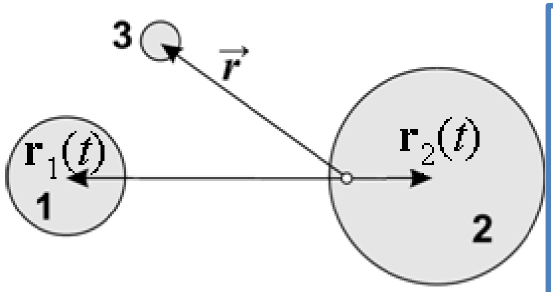
- The nucleon transfer and nuclear breakup processes during the collision of nuclei is described by the **time-dependent** Schrödinger equation [1,2]:

$$i\hbar\frac{\partial}{\partial t}\Psi = \left\{-\frac{\hbar^2}{2m}\Delta + V_1(\vec r - \vec r_1(t)) + V_2(\vec r - \vec r_2(t))\right\}\Psi$$



Radial part $R(r)$ of the wave functions for 1s and 1p levels neutrons in the ^{11}Li nucleus.

The initial wave function is calculated in the shell model. The initial conditions for the wave functions were determined from the shell model with parameters that provide the values of the rms nuclear charge radius and neutron separation energy close to the experimental.



1,2 - two heavy classical particles (nuclear cores),
3 - light quantum particle (nucleon of a projectile or target)

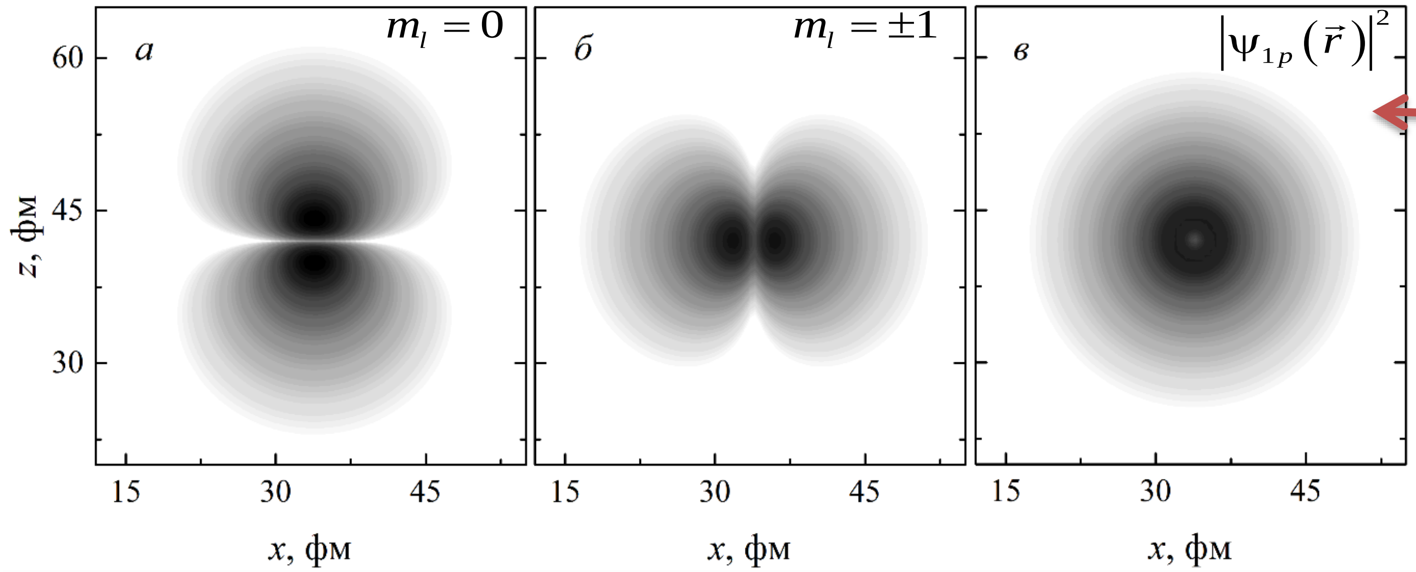
References
[1] V. I. Zagrebaev, V.V. Samarin, W. Greiner. Phys. Rev. C 75, 035809 (2007).
[2] Azhibekov A.K., Samarin V.V., Kuterbekov K.A., Chinese Journal of Physics 65, 292 (2020).

Calculations without taking into account spin-orbital interaction may be used for weakly bound neutron in the light nuclei (as ^{11}Li) because there are no any energy levels in the vicinity of the level of this neutron.

What we study by numerical solutions of the time-dependent Schrödinger equation?

- The use of the time-dependent Schrodinger equation to study rearrangement of neutrons during fusion reactions was proposed by [1,2]. Such approach makes it possible to obtain a microscopic description of dynamics of nuclear fusion [1,2], neutrons stripping and pick-up [3-8], and breakup of nuclei[7,8].
- Determination the evolution of probability density of external weakly bound neutrons of ^{11}Li and the probabilities of transfer and breakup.
- Calculation of the reaction cross sections for nucleon transfer, nuclear breakup, nuclear fusion processes.
- Comparison of results of the TDSE predicted calculations for transfer and nuclear breakup cross sections of ^{11}Li+^{28}Si reactions at low energy, for which experimental measurements are not available.

1. V.I. Zagrebaev, V.V. Samarin, W. Greiner, *Physical Review C***75**, 035809 (2007).
2. V.V. Samarin and K.V. Samarin, *Bull. Russ. Acad. Sci. Phys.***74**, 567 (2010).
3. V.V. Samarin and K.V. Samarin, *Bull. Russ. Acad. Sci. Phys.* **76**, 450 (2012).
4. V.V. Samarin, *EPJ Web of Conference* **66,** 03075 (2014).
5. V.V. Samarin, *EPJ Web of Conference* **86**, 00040 (2015).
6. V.V. Samarin, *EPJ Web of Conference* **117,** 08018 (2016).
7. A.K. Azhibekov, K.A. Kuterbekov, V.V. Samarin, *Euras. Jour. of Phys. and Funct. Mater.* 2(4), 301 (2018).
8. A.K. Azhibekov, V.V. Samarin, K.A. Kuterbekov, *Chinese Journal of Physics* 65, 292 (2020).

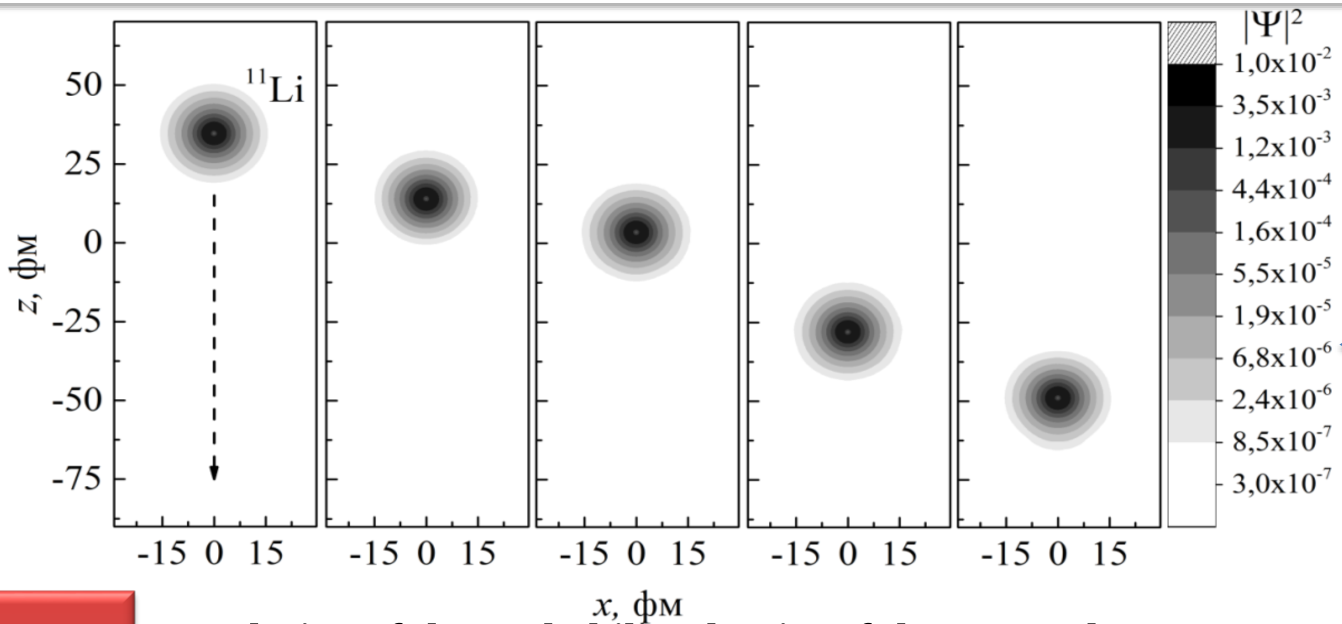


Probability density for external neutrons in the shell model of a spherical nucleus without taking into account the spin-orbit interaction

2. In reality, nuclei move in the center-of-mass system, therefore, for a correct description of the states of nucleons in, it is necessary to take into account the motion of nuclei.

$$\Psi_{nlm_l}(\vec r, t_0) = \psi_{nlm_l}(\vec r)\exp\left(i\frac{m\vec v\vec r}{\hbar}\right)$$

1. The wave functions for external neutrons calculated within the shell model of a spherical nucleus were used to describe the initial state of the nucleon.



Evolution of the probability density of the external neutron for free motion of the ^{11}Li nucleus (E_{lab} = 24.3 MeV)

^{11}Li

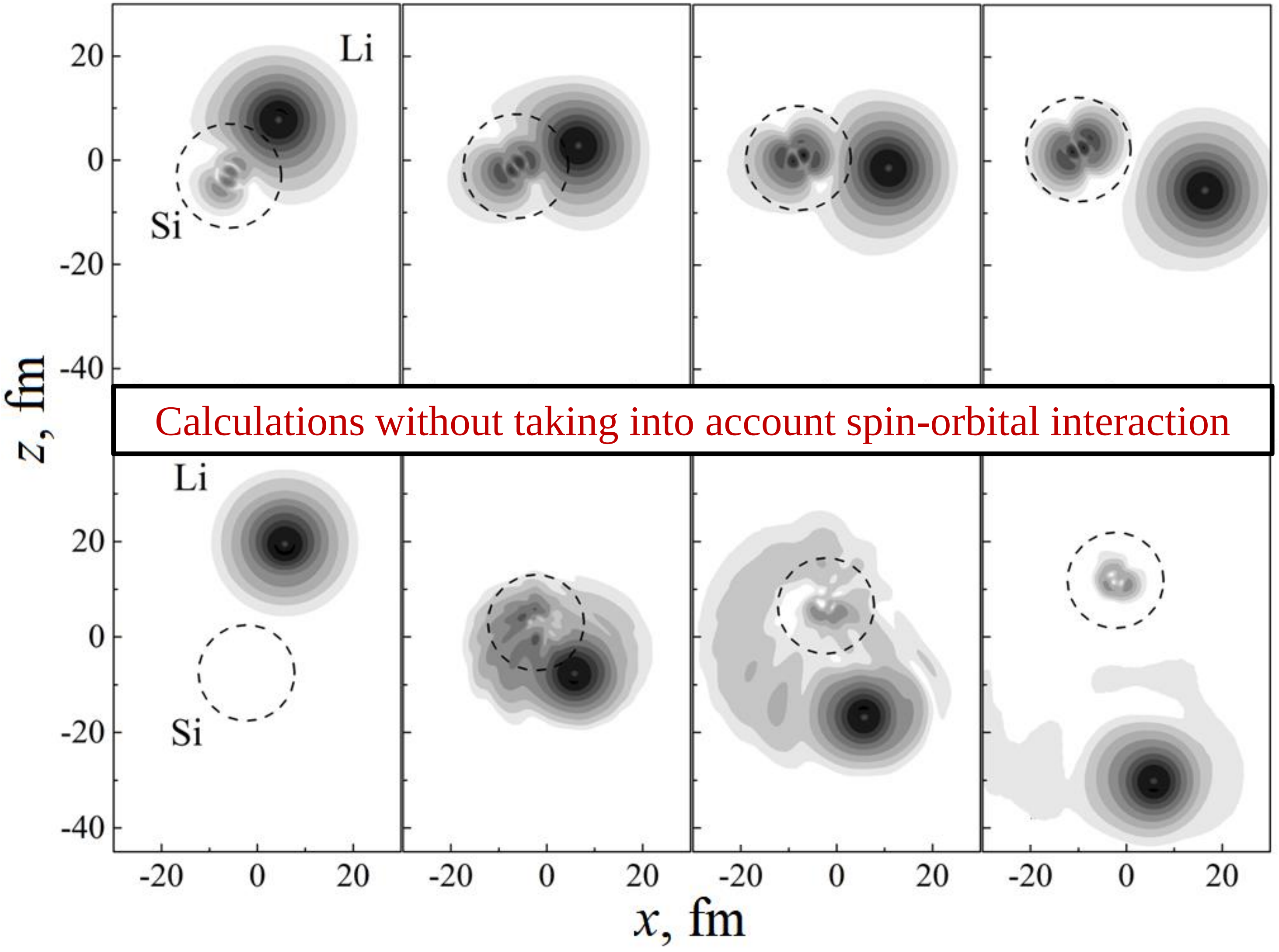
3. Probability of the neutron remaining in the ^{11}Li projectile is calculated as integral over the region S_1 (ball centered at a point $\vec r$)

$$p_r = \int_{S_1} |\Psi|^2 \, dV = 1$$

During the time of free motion of the ^{11}Li nucleus at a distance of 85 fm, the value of the integral decreased to 0.99528.

The lost probability determines the error of the numerical solution of the Schrödinger equation **0.47%**.

Time-dependent microscopic description of external neutrons during the collision ^{11}Li+^{28}Si



Calculations without taking into account spin-orbital interaction

The evolution of the probability density of external neutrons of the ^{11}Li nucleus in the collision with the ^{28}Si nucleus. It can be seen that the external neutrons lost by the ^{11}Li nucleus are transferred to the target nucleus ^{28}Si and leave both nuclei with energy in the continuous spectrum.

The time evolution of the probability density for the outer 1*p* neutrons of **^{11}Li** nucleus in the collision with ^{28}Si for impact parameter $b = 8$ **fm.**

***E* = 1*A* MeV** (**top row**),

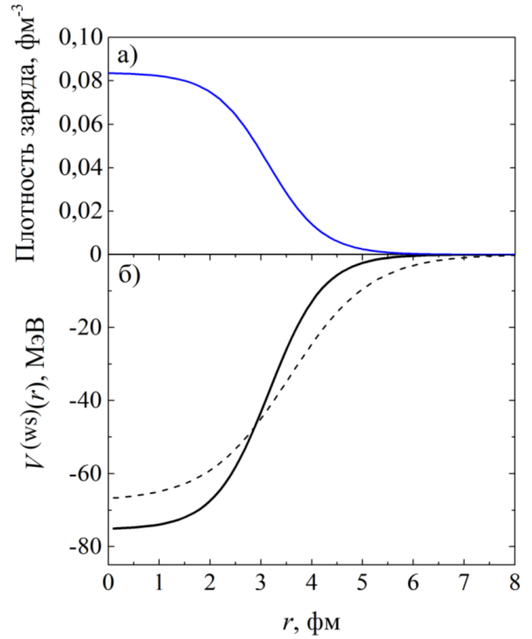
***E* = 30*A* MeV** (**bottom row**).

Dashed circle – ^{28}Si. For top and bottom rows, **time scale** is from left to right.

To describe the effect of the target on the external neutrons of the ^{11}Li projectile at the initial stage of the penetration of projectile neutrons into the ^{28}Si target, we use the shell model of a spherical nucleus, as in [1].

The shape of the potential is close to the experimental data on the charge distribution of the ^{28}Si nucleus.

dashed line – potential with parameters from [1]



Nucleus	$V^{(WS)}$	$r_0^{(WS)}$ / $r_0^{(WS)}A^{1/3}$	$a^{(WS)}$
^{11}Li	–27.679 MeV	1.25 / 2.78 fm	0.650 fm
^{28}Si [1]	–67.576 MeV	1.17 / 3.54 fm	0.800 fm
^{28}Si	–75.265 MeV	1.04 / 3.16 fm	0.537 fm

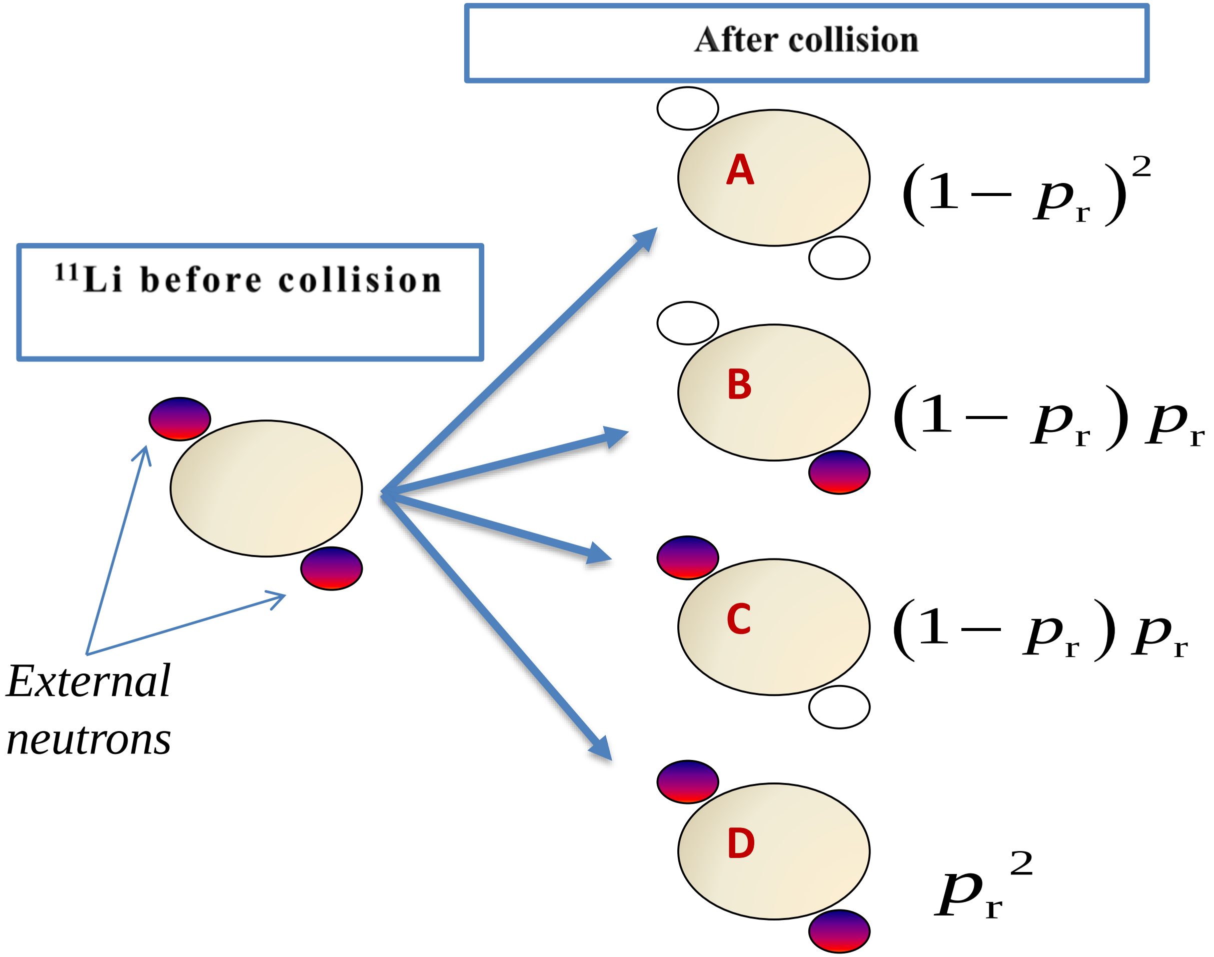
[1] Penionzhkevich Yu.E. et al., Phys.Rev. C. 99, 014609 (2019).

Numerical methods

- We have developed our own computational code for solution **time-dependent** Schrödinger equation (TDSE).
- The time-dependent Schrödinger equation is solved iteratively in time with the fast complex Fourier transform [1, 2].
- We employ a uniform spatial grid in the three-dimensional Cartesian coordinate to represent single-particle orbitals. The lattice spacing in the TDSE method is 0.3 fm, which is substantially smaller than 0.8 fm in a typical time-dependent Hartree-Fock calculation [3].
- We take a box size of 485×300×200 grid points (145.5×90×60 fm^3) for collision calculations, where the reaction plane is taken to be the *xz* plane.
- We have tested the accuracy of the code by solution of TDSE for free particle motion. The total loss of probability density was about 0.001.

[1] G. I. Marchuk, Methods of Computational Mathematics (Springer-Verlag, Berlin, 1981).
[2] M.E. Riley, B. Ritchie, Numerical time-dependent Schrödinger description of charge-exchange collisions, Phys.Rev. A. **59,** P. 3544 (1999).
[3] C. Golabek and C. Simenel, Phys. Rev. Lett. **103**, 042701 (2009).

Probabilities of reaction channels



The sum of the probabilities of events **A, B, C** determines the probability of stripping two neutrons from an ^{11}Li as a result of a collision with a target [1]

$$p_{-2n} = 1 - (p_r)^2$$

Probability of transfer of an external neutron during tangential collisions of nuclei

$$p_{tr} = \int_{S_2} |\Psi|^2 dV$$

Probability of transition of one neutron to continuous spectrum states

$$p_{br} = 1 - p_{tr} - p_r$$

$$p_r = \int_{S_1} |\Psi|^2 dV$$

is probability that the neutron remained in the ^{11}Li projectile

[1] Azhibekov A.K., Samarin V.V., Kuterbekov K.A., Chinese Journal of Physics 65, 292 (2020).

Reaction cross sections (^{11}Li,^{9}Li)

- Due to the low binding energy of external neutron in ^{10}Li (25 keV for ^{9}Li+n), it is highly likely that the neutron transfer from ^{11}Li or nuclear breakup lead to the breakup of the ^{10}Li nucleus.

$$\sigma_{-2n}\left(E\right)=2\pi\int_{b_{\min}}^{\infty} p_{-2n}(R_{\min}(b,E))bdb \qquad (1)$$

The minimum of the integration over the impact parameter is the border dividing fusion and direct reactions (transfer, breakup). In practice, we first examine the maximum impact parameter in which fusion reactions take place for a given incident energy. We call it the minimal impact parameter for direct reactions and denote it as $b_{\min}$. We then repeat reaction calculations at various impact parameters for the region, $b>b_{\min}$, and calculate the cross section by numerical quadrature according to Eq. 1.

Cross section and probabilities of reaction channels for the ^{11}Li + ^{28}Si

$$\sigma_{-2n}(E) = 2\pi \int_{b_{\min}}^{\infty} p_{-2n}(R_{\min}(b,E))bdb$$

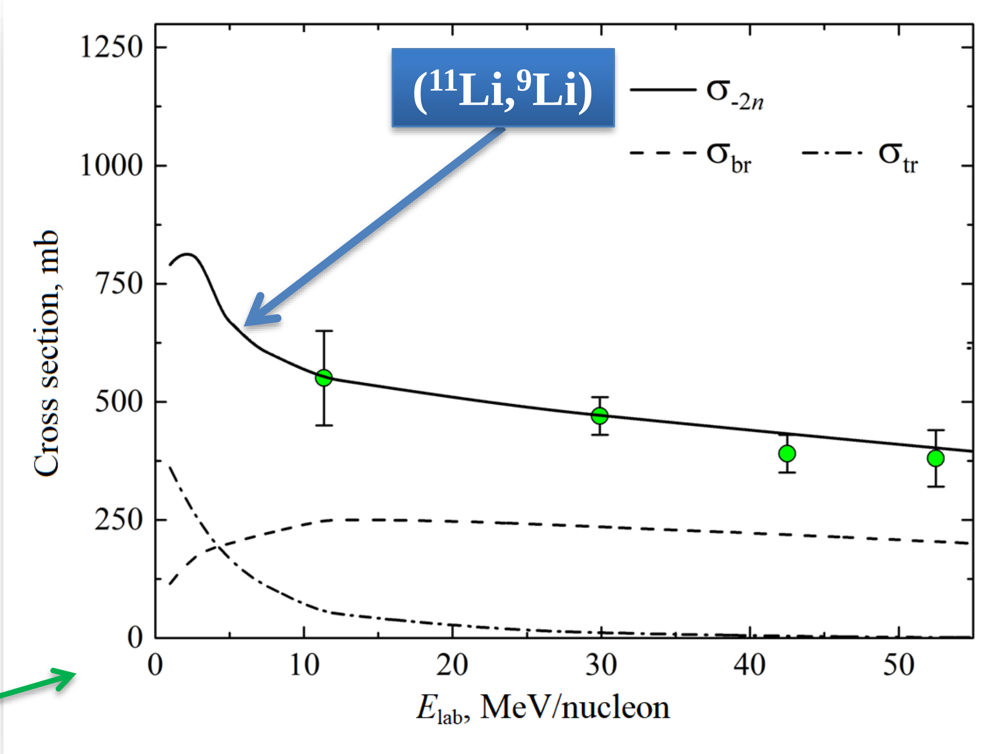
$$\sigma_{\mathrm{tr}}(E) = 2\pi \int_{b_{\min}}^{\infty} p_{\mathrm{tr}}(R_{\min}(b,E))bdb$$

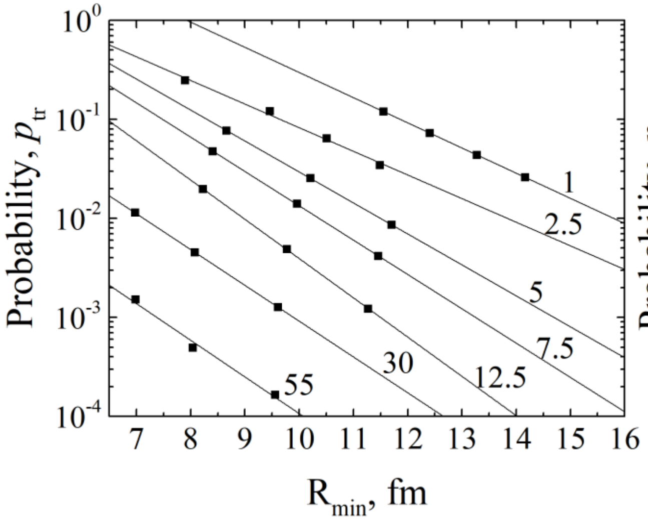
$$\sigma_{\mathrm{br}}(E) = 2\pi \int_{b_{\min}}^{\infty} p_{\mathrm{br}}(R_{\min}(b,E))bdb$$

In the experiment [1], the mechanism of the two-neutron removal reaction was not determined, since the detector system recorded only the final product of the ^{9}Li reaction.

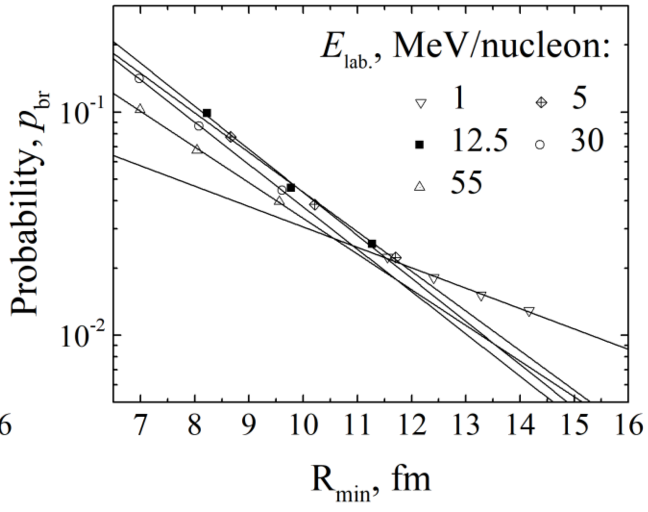
Our calculations show that two neutrons removal cross section for the ^{11}Li+^{28}Si reaction in the low-energy region sharply increases due to the neutron transfer channel with a decrease in the projectile energy.

[1] Warner R.E. et al. Phys.Rev.C 54, 1700 (1996) – **green-filled circles**

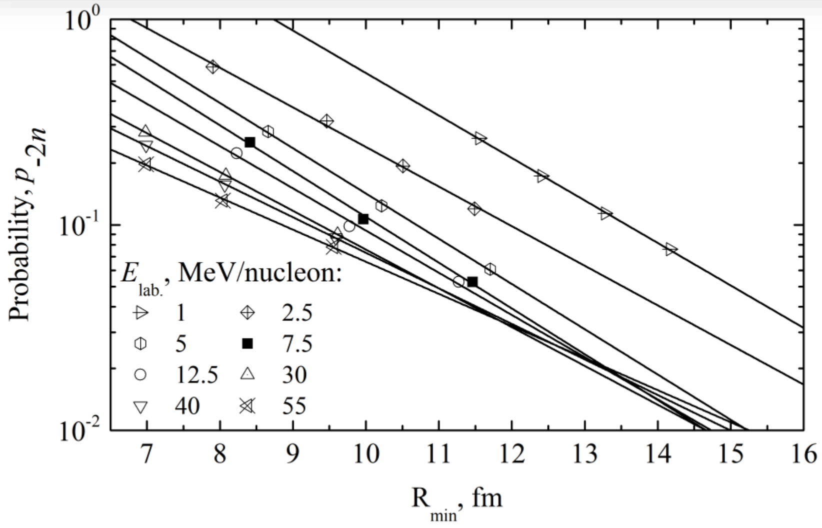




Neutron transfer probabilities



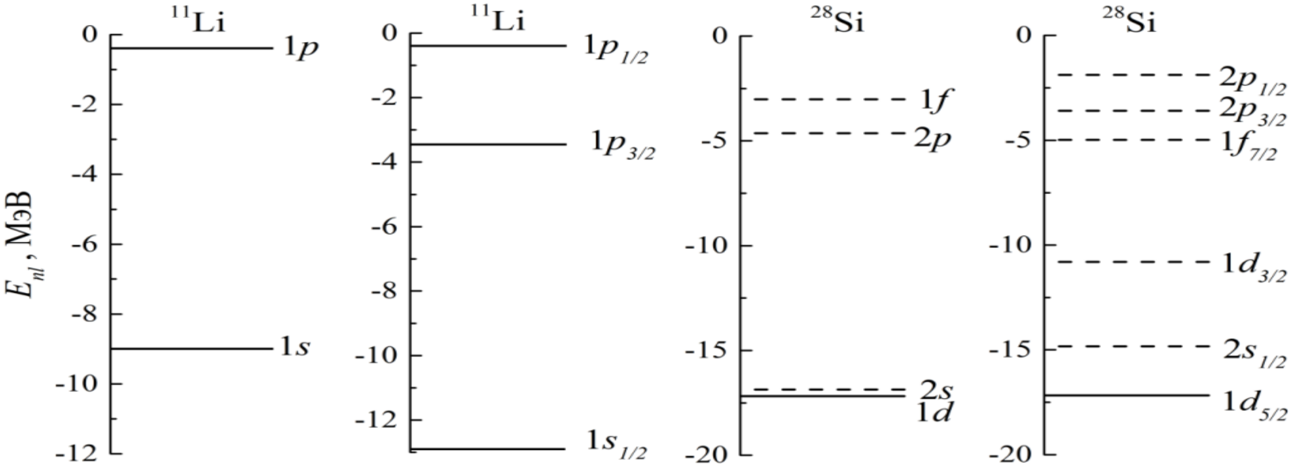
Nuclear breakup probabilities

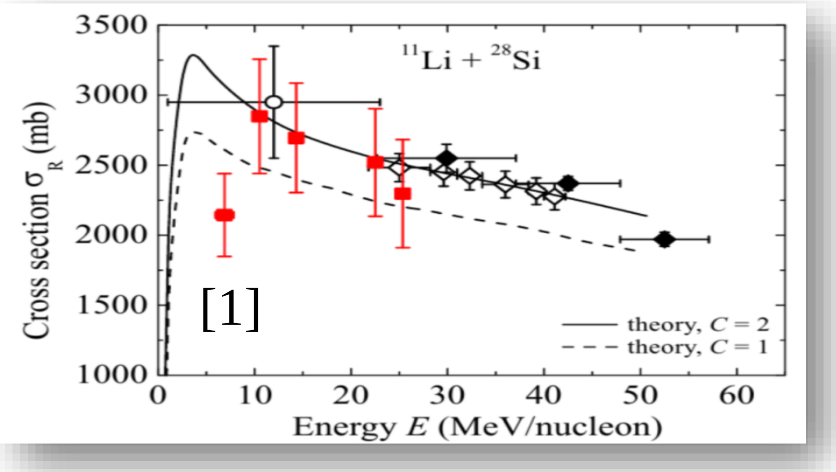


The probabilities of -2*n*

Evaluation of the significance of spin-orbit interaction in description of external neutrons during grazing collisions of light weakly bound ^{11}Li nucleus with ^{28}Si

1. One of the criteria for the applicability and degree of accuracy of theoretical models is the quantitative agreement between the values of the calculated and experimentally measured reaction cross sections. In [1], calculations were performed within the framework of the time-dependent approach taking into account the spin-orbit interaction. This work was perfect for comparing our results of calculating the cross sections and probabilities without taking into account the spin orbital interaction.

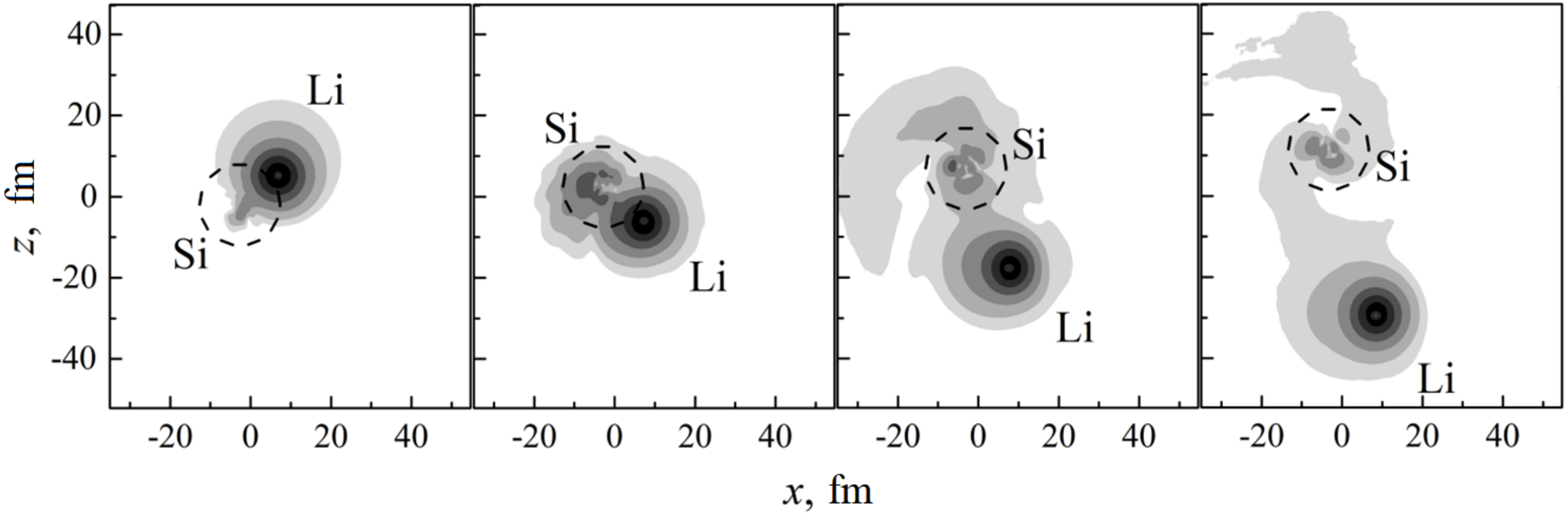




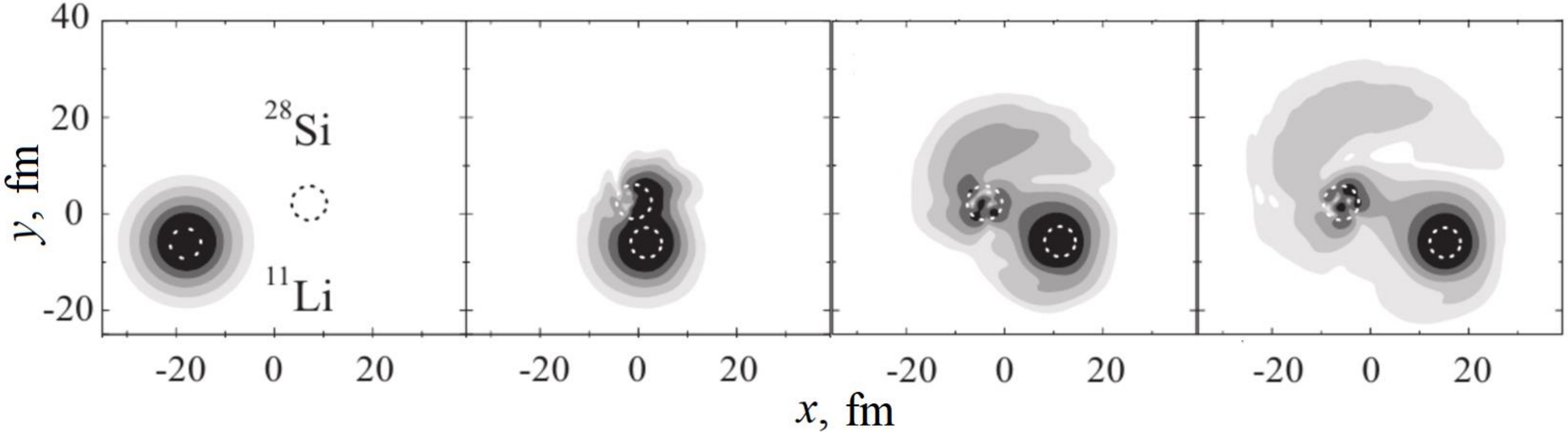
2. Single-particle energy levels of neutrons in the 11Li nucleus and the upper levels in the 28Si nucleus in the shell model without spin-orbit interaction (a, c) and taking into account the spin-orbit interaction (b, d). The following results of our calculations were performed with the parameters of the Woods-Saxon potential from [1]. Solid lines are occupied levels, dashed lines are unoccupied levels.

[1] Penionzhkevich Yu.E. et al., Phys.Rev. C. 99, 014609 (2019).

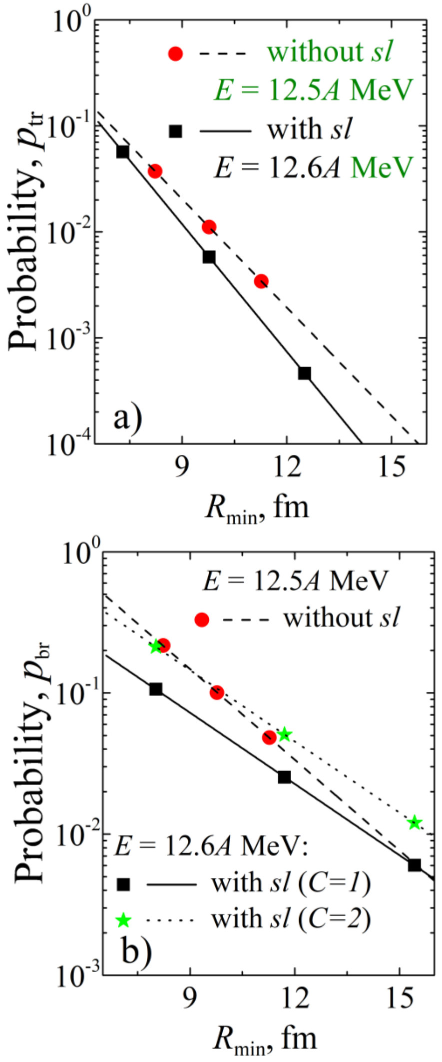
Probabilities neutron transfer (a) and breakup (b)



Without spin-orbit interaction, 12.6 MeV/nucleon



Taking into account the spin-orbit interaction, 12.5 MeV / nucleon [1]



Our results matches the value that was calculated in [1].

[1] Penionzhkevich Yu.E. et al., Phys.Rev. C. 99, 014609 (2019).

Conclusion

- ✓ A good description of the experimental data on two-neutron removal reaction ^{28}Si(^{11}Li, ^{9}Li) has been obtained.
- ✓ In the experimental work [1] on the two-neutron removal reaction, the removal mechanism was not determined. Our calculations show that the main contribution is made by the neutron transfer channel for the two-neutron removal cross section ^{28}Si(^{11}Li, ^{9}Li) in the low-energy region. The nuclear breakup channel gives a large contribution to the two-neutron removal cross section with increasing projectile energy.
- ✓ The use of the form of the interaction potential of the ^{28}Si target with external neutrons of the ^{11}Li projectile corresponding to the charge distribution of ^{28}Si made it possible to obtain fairly agreement between theoretical calculations and experimental data for the reaction of removing two neutrons ^{28}Si(^{11}Li, ^{9}Li).
- ✓ Excluding of the spin-orbit interaction made it possible to reduce the time of the numerical solution of the TDSE by an order of magnitude.
- ✓ There are not significant differences in the results of calculating the probabilities based on the numerical solution of the TDSE for external weakly bound neutrons without and with the spin-orbit interaction.

[1] Warner R.E., Patty R.A., Voyles P.M. et al. Total Reaction and 2n-Removal Cross Sections of 20-60A MeV ^{4,6,8}He, $^{6-9,11}$Li, and 10Be on Si // Phys.Rev.C. – 1996. – Vol. 54. – P. 1700-1709.

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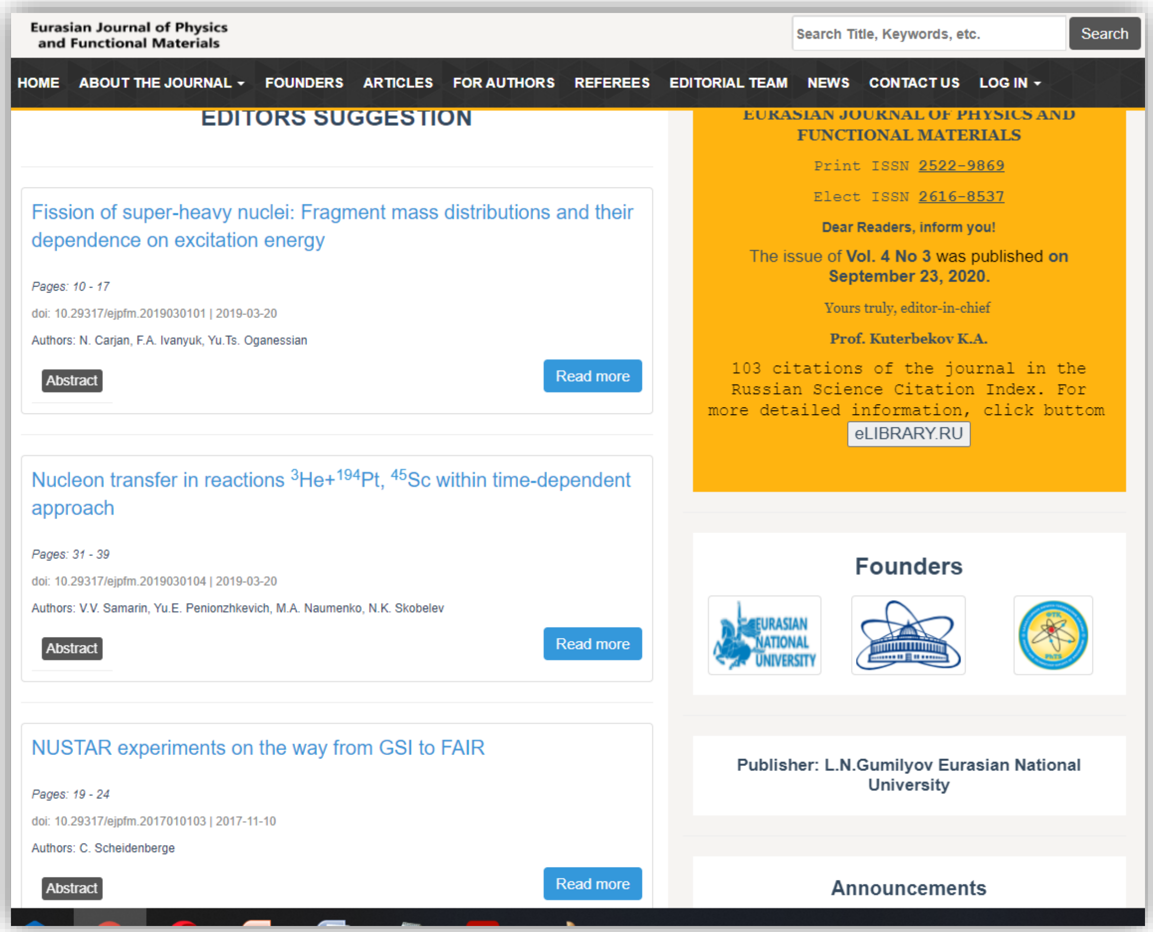

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