THE QUATERNARY FISSION AS A VIRTUAL PROCESS

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The aim is to generalize the approaches of the quantum theory of the ternary fission [*Kadmensky S.G. // Phys. At. Nucl. 2003. V. 66. P. 1739.*], and to find an answer if the description of the spontaneous and low-energy induced quaternary fission is possible on the base the description of two-proton decay [*S.G. Kadmensky, Yu. V. Ivankov, Phys. At. Nucl. 2014. V. 77. P. 1605; P. 1075.*] and ternary fission [*S.G. Kadmensky, L.V. Titova, D.E. Lyubashevsky// Phys. At. Nucl. 2020. V. 83. P. 326.*] as virtual processes using the representation of the statistical decay theory in the chaines of the genetically connected nuclei [*S.G. Kadmensky, O.A. Bulychev / Bull. of RAS: Physics. 2016. V. 80. P. 921.*]

Task

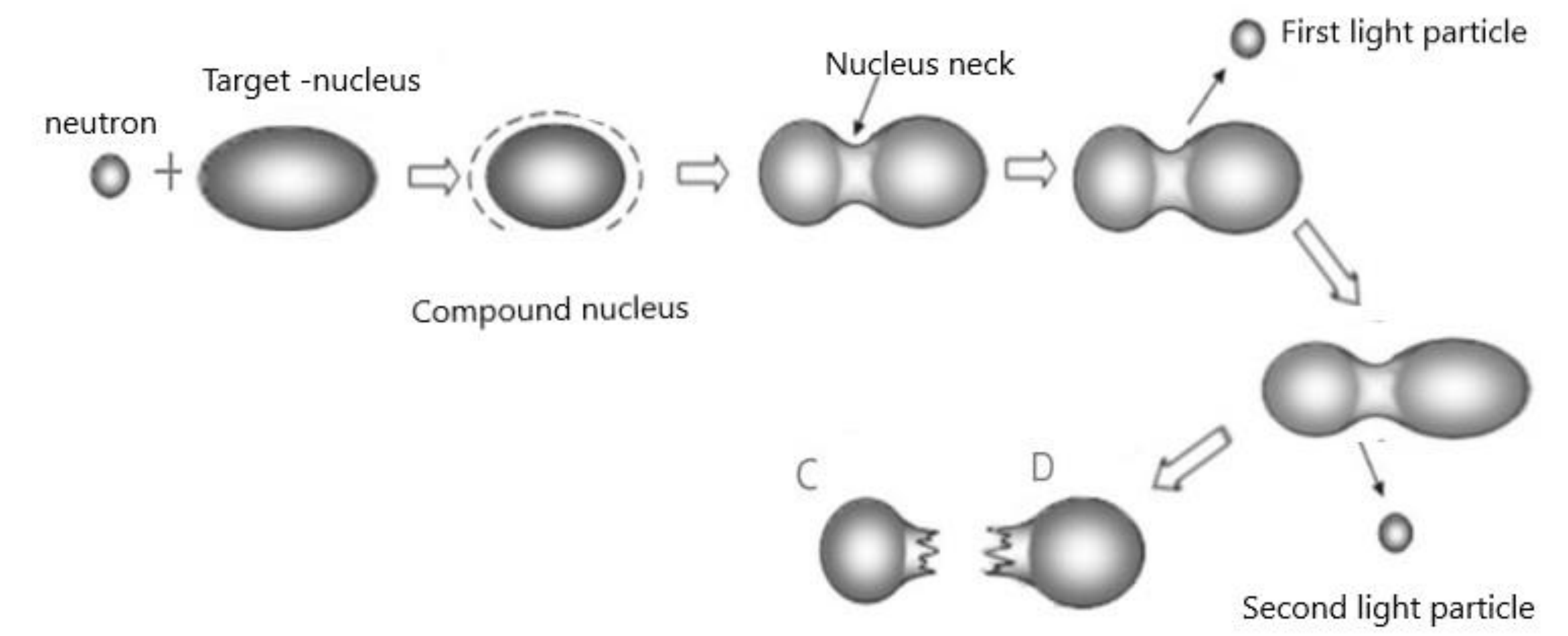
- to construct the quaternary fission widths considering the fission as three-step sequential process;
- to get the ratio of the Coulomb barrier penetrabilities for the first and second light particles of the quaternary fission.

Object : Quaternary spontaneous fission of ^{248}Cm, ^{252}Cf

Quaternary fission of ^{233}U, ^{235}U induced by thermal neutrons

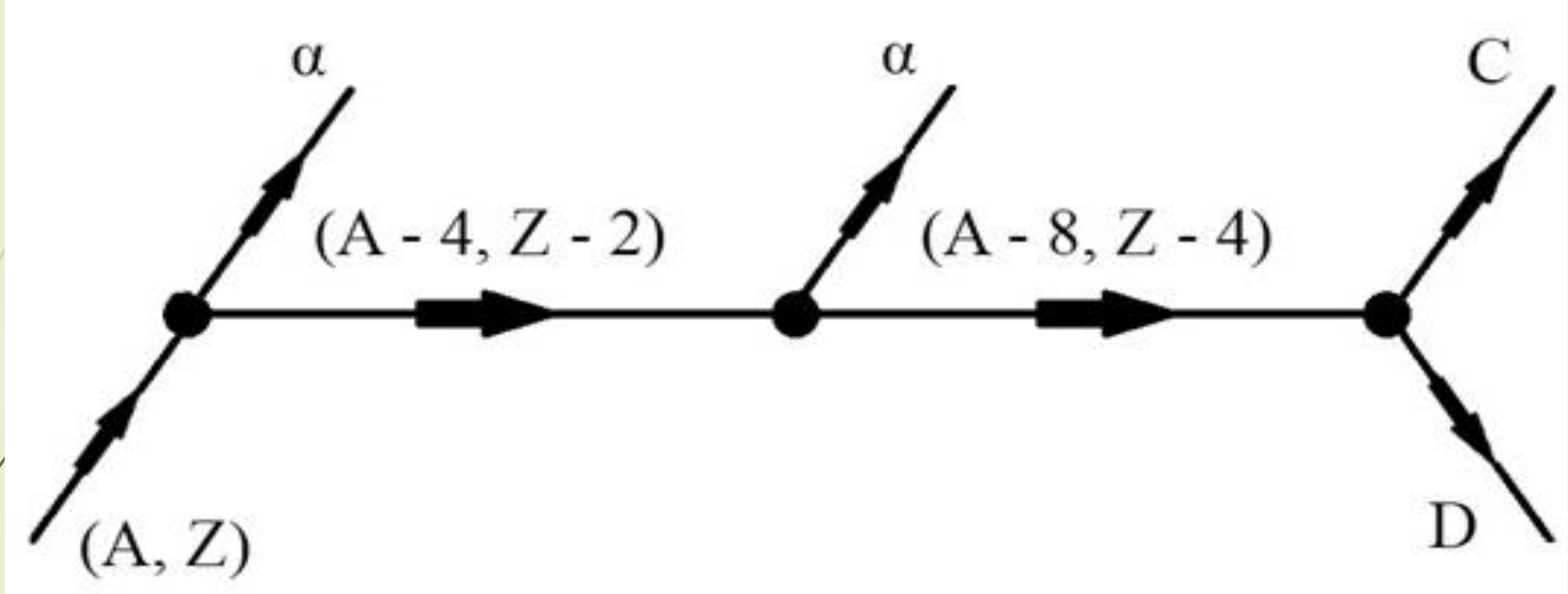
Quaternary fission as a three-step sequential process

S.G. Kadmensky, L.V. Titova, A.O. Bulychev // Phys. At. Nucl. – 2015. – V.78, №7-8. – P.716-724.
S.G. Kadmensky, L.V. Titova // Phys. At. Nucl. . – 2013. – V. 76, № 1. – P. 18-26.



Fomichev A.S. et al. // Nucl. Instr. and Meth. in Phys. Research. A. 1997. V. 384. P. 519.
Jesinger P. et al. //The European Physical Journal A. 2005. P. 379.
Kamanin D.V. et al. // Eurasian Journ. of Phys. and Funct. Mat. 2019. P. 139.

Feinman diagram for quaternary fission as virtual process



Horizontal arrows with indexes indicate the Green function.

Shaded circles are nodes of the Feynman diagram.

The virtual width of the quaternary spontaneous fission

$$\Gamma_{QF} = \frac{1}{(2\pi)^2} \iint \frac{\Gamma_1^{A_1}(T_1)\Gamma_2^{A_2}(T_2)\Gamma_3^{A_{LF}}(Q - T_1 - T_2)}{(Q_1 - T_1)^2(Q_2 - T_2)^2} dT_1 dT_2$$

Q_1 is the decay energy of the nucleus A_1 ;

Q_2 is the decay energy of the intermediate nucleus A_2 , which is formed after the first light particles flight;

Q is the quaternary fission energy;

T_1, T_2 are kinetic energies of the first and second light particles ;

$\Gamma_1^{A_1}, \Gamma_2^{A_2}, \Gamma_3^{A_3}$ are partial decay widths for parent and intermediate nuclei.

Energy distribution for α –particle in spontaneous quaternary fission and decay widths for nuclei A and (A-4) from the configuration with neck

$$W_{\alpha,\alpha} = \frac{1}{(2\pi)^2} \frac{\Gamma_{\alpha 1}^{A}(T_1)\Gamma_{\alpha 2}^{A-4}(T_2)\Gamma_3^{A-8}}{(Q_1 - T_1)^2 (Q_2 - T_2)^2 \Gamma_f^A}$$

Lets consider $\frac{\Gamma_f^{(A-8)}}{\Gamma_f^A} \approx 1$

$$\Gamma_{\alpha_1}^{(A)}(T_{\alpha_1}) = 2\pi N_{\alpha_1} W_{\alpha_1}(T_{\alpha_1})\left(Q_{\alpha_1}^{A} - T_{\alpha_1}\right)^2;$$

$$\Gamma_{\alpha_2}^{(A-4)}(T_{\alpha_2}) = 2\pi N_{\alpha_2} W_{\alpha_2}(T_{\alpha_1})\left(Q_{\alpha_2}^{(A-4)} - T_{\alpha_2}\right)^2$$

$$\Gamma_{\alpha_1}^{(A)}(T_{\alpha_1}) = \omega_{\alpha_1} \frac{\hbar c\sqrt{2T_{\alpha_1}}}{2R_A\sqrt{\mu_{\alpha_1} c^2}} P(T_{\alpha_1}); \quad \Gamma_{\alpha_2}^{(A-4)}(T_{\alpha_2}) = \omega_{\alpha_2} \frac{\hbar c\sqrt{2T_{\alpha_2}}}{2R_{A-4}\sqrt{\mu_{\alpha_2} c^2}} P(T_{\alpha_2})$$

where $P(T_\alpha)$ is Coulomb barrier penetrability

Energy distribution for α – particle in quaternary fission

$$W_\alpha(T_\alpha) = \frac{1}{\sqrt{2\pi}\sigma_{T_\alpha}} exp\left(-\frac{(T_\alpha - \langle T_\alpha \rangle)^2}{2\sigma_{T_\alpha}^2}\right)$$

where $\langle T_\alpha \rangle$ is average kinetic energy,

$\sigma_{T_\alpha} = \frac{FHWM}{2\sqrt{2\ln 2}}$ is standard deviation,

$FWHM$ – the width at half-height of the Gaussian distribution.

Energy distribution of the second α-particle: $W_{\alpha 2} = 2W_{\alpha\alpha} - W_{\alpha 1}$

S. Vermote et al. // Nucl. Phys. A. 2010. V. 837. P. 176.
M. Mutterer and J. P. Theobald, Dinuclear Decay Modes, Bristol: IOP Publ., 1996, Chap. 12.
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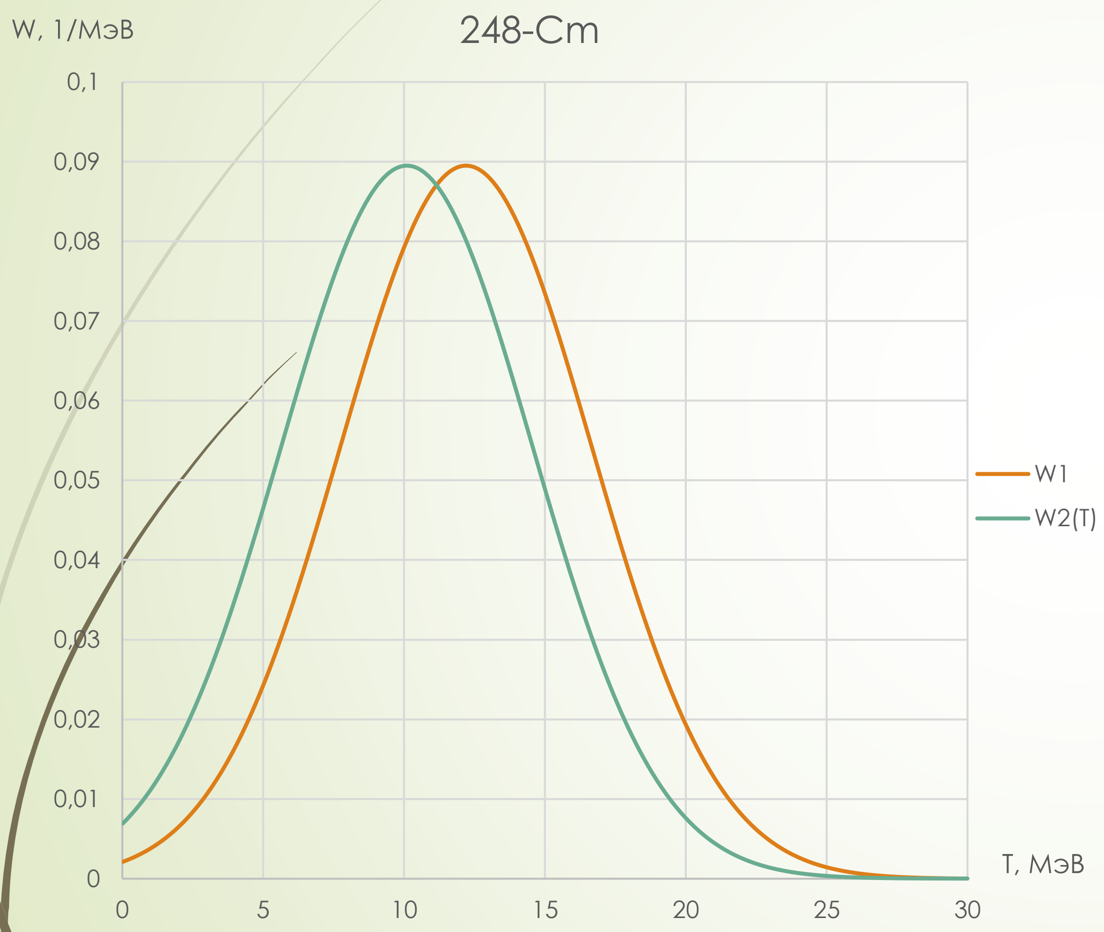
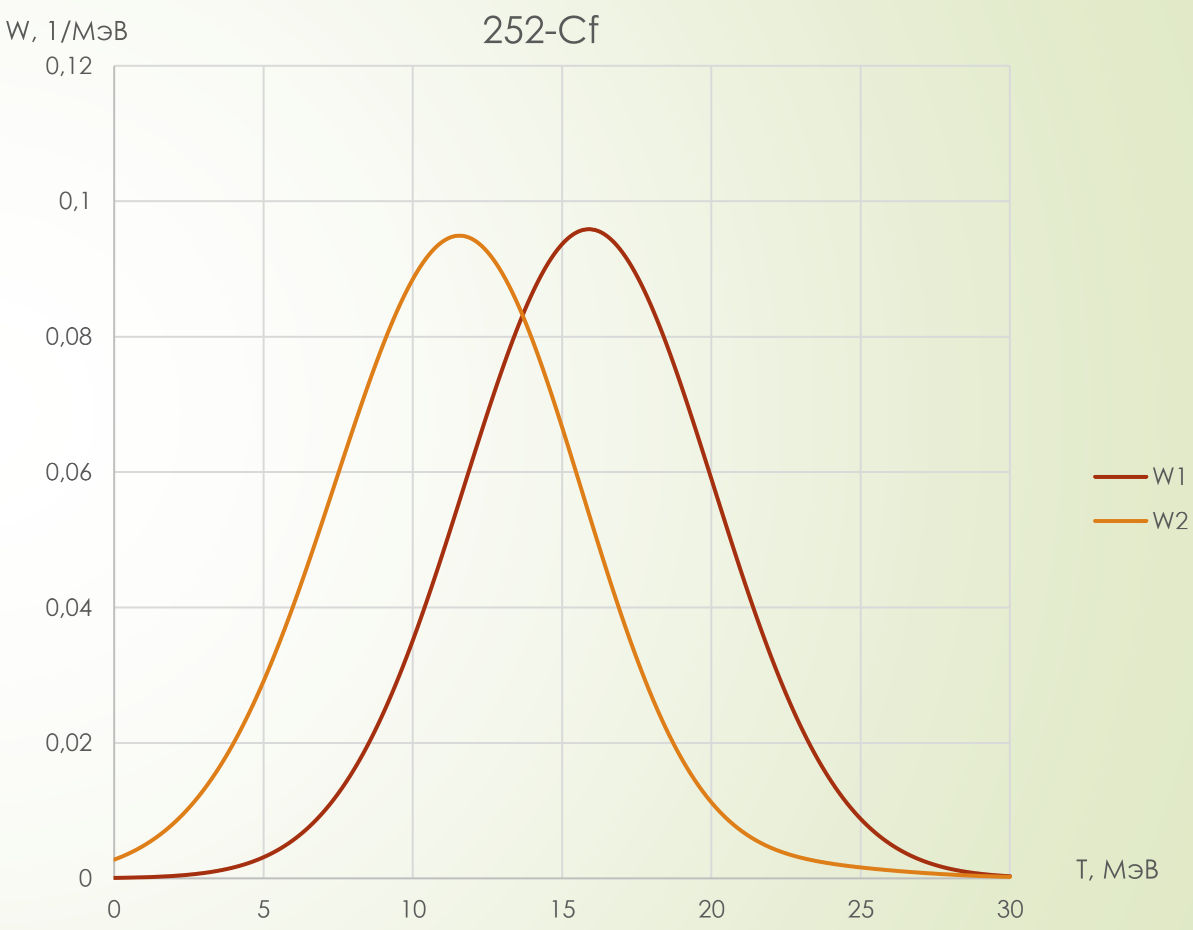
Energy distributions of alpha– particles in quaternary fission of ^{248}Cm, ^{252}Cf





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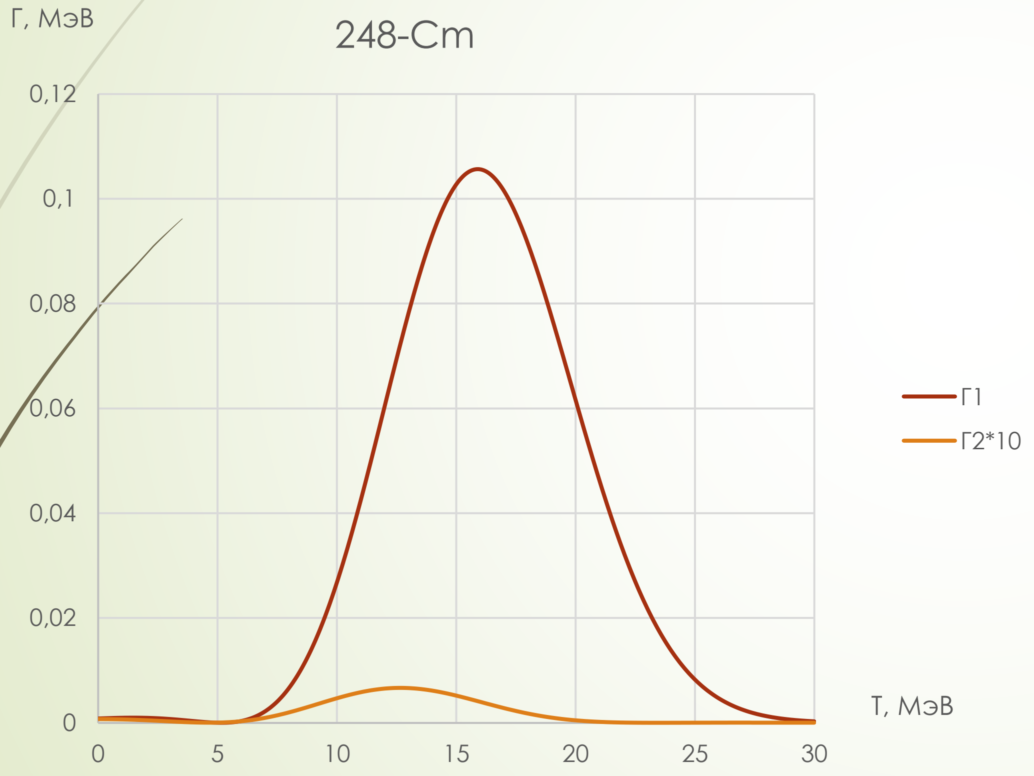
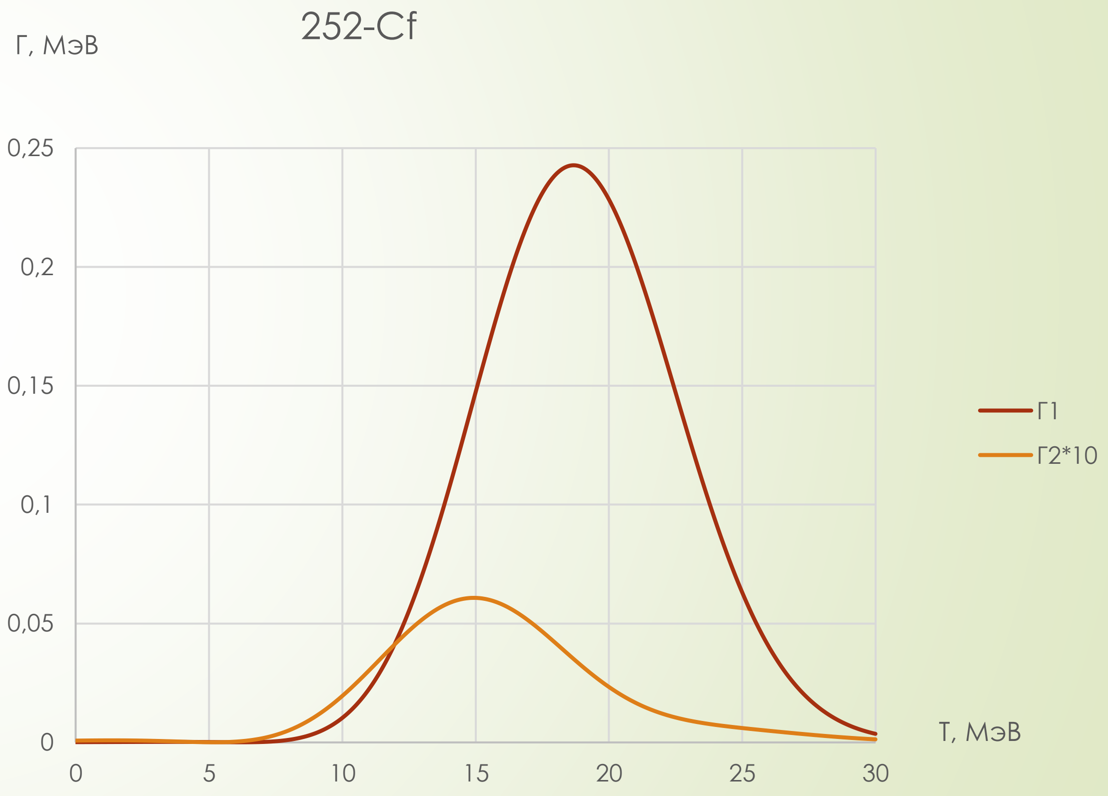
Widths $(\Gamma_\alpha^A(T_\alpha))^0$ and $(\Gamma_\alpha^{A-4}(T_\alpha))^0$ for α −decays, connecting with the first and second α-particle flight from fissile nucleus neck in spontaneos fission ^{248}Cm and ^{252}Cf





The ratio of the Coulomb barrier penetrabilities for the first and the second α –particles in spontaneos fission

$$\frac{P(T_{\alpha_2})}{P(T_{\alpha_1})} = \frac{\sqrt{(T_{\alpha_1})_{max}}}{\sqrt{(T_{\alpha_1})_{max}}} \frac{\left\{N_{\alpha_2} W_{\alpha_2}(T_{\alpha_2}) \left(Q_{\alpha_2}^{(A-4)} - T_{\alpha_2}\right)^2\right\}_{max}}{\left\{N_{\alpha_1} W_{\alpha_1}(T_{\alpha_1}) (Q_{\alpha_1}^{A} - T_{\alpha_1})^2\right\}_{max}}$$

$Q_{\alpha_1}^{(A)}$ is α –decay energy for nucleus A;

$Q_{\alpha_2}^{(A-4)}$ is α –decay energy for nucleus $(A-4)$,

$W_{\alpha_1}, W_{\alpha_2}$ are energy distribution for the first and second α –particles,

$T_{\alpha_1}, T_{\alpha_2}$ are kinetic energies of the first and second α –particles,

$N_{\alpha_1}, N_{\alpha_2}$ are probabilities of the α – particles emission,

$P(T_{\alpha_1}), P(T_{\alpha_2})$ are Coulomb barriers penetrabilities for the first and second α –particles.

Yields of the α – particle in the spontaneous quaternary fission ^{248}Cm, ^{252}Cf

$$N_{\alpha,\alpha}=2N_{\alpha_1}N_{\alpha_2}$$

Ядро	$N_{\alpha,\alpha}$ [1]	N_{α_1} [2]	N_{α_2} [3]
^{248}Cm	$(1,4 \pm 0,3) \times 10^{-7}$	$(2,3 \pm 0,3) \times 10^{-3}$	$(3,04 \pm 0,24) \times 10^{-5}$
^{252}Cf	$(9,72 \pm 3,26) \times 10^{-7}$	$(3,24 \pm 0,12) \times 10^{-3}$	$(15,0 \pm 5,1\ ) \times 10^{-5}$

[1] Fomichev A.S. et al. // Nucl. Instr. and Meth. in Phys. Research. A. 1997. V. 384. P. 519.
[2] S. Vermote et al. // Nucl. Phys. A. 2010. V. 837. P. 176.
[3] S. G. Kadmensky, L. V. Titova, Phys. At. Nucl. 2016. V. 76. P. 18.]

Coulomb barrier heights and penetrabilities ratio for the first and second α –particles in prescission configuration of the fissile nucleus with neck

	^{248}Cm	^{252}Cf
$(T_{\alpha_1})_{max}$, MeV	12,2	15,9
$(T_{\alpha_2})_{max}$, MeV	10,1	12,7
$\frac{P_{\alpha_2}}{P_{\alpha_1}}$	0,006	0,025

The virtual width for induced by thermal neutrons quaternary fission

$$\Gamma_{QF} = \frac{1}{(2\pi)^2} \iint \frac{\Gamma_{\alpha 1}^{A}(T_1)\Gamma_{\alpha 2}^{A-4}(T_2)\Gamma_{f}^{A-8}(Q - T_{\alpha 1} - T_{\alpha 2})}{(Q_{\alpha_1}^{(A)} + B_n - T_{\alpha 1})^2 \left(Q_{\alpha_2}^{(A-4)} - T_{\alpha 2}\right)^2} dT_{\alpha 1} dT_{\alpha 2}$$

$Q_{\alpha_1}^{(A)}$ *is* the decay energy of the nucleus A;

$Q_{\alpha_2}^{(A-4)}$ *is* the decay energy of the intermediate nucleus (*A-4*), which is formed after the first light particles flight;

$\boldsymbol{Q}$ *is the* quaternary fission energy;

$\boldsymbol{T_{\alpha 1}}, \boldsymbol{T_{\alpha 2}}$ are kinetic energies of the first and second light particles ;

$\boldsymbol{\Gamma_{\alpha 1}^{A}}, \boldsymbol{\Gamma_{\alpha 2}^{A-4}}, \boldsymbol{\Gamma_{f}^{A-8}}$ are partial decay widths for parent and intermediate nuclei.

$\boldsymbol{B_n}$ is neutron binding energy;

The ratio of the Coulomb barrier penetrabilities for the first and the second α –particles in induced fission

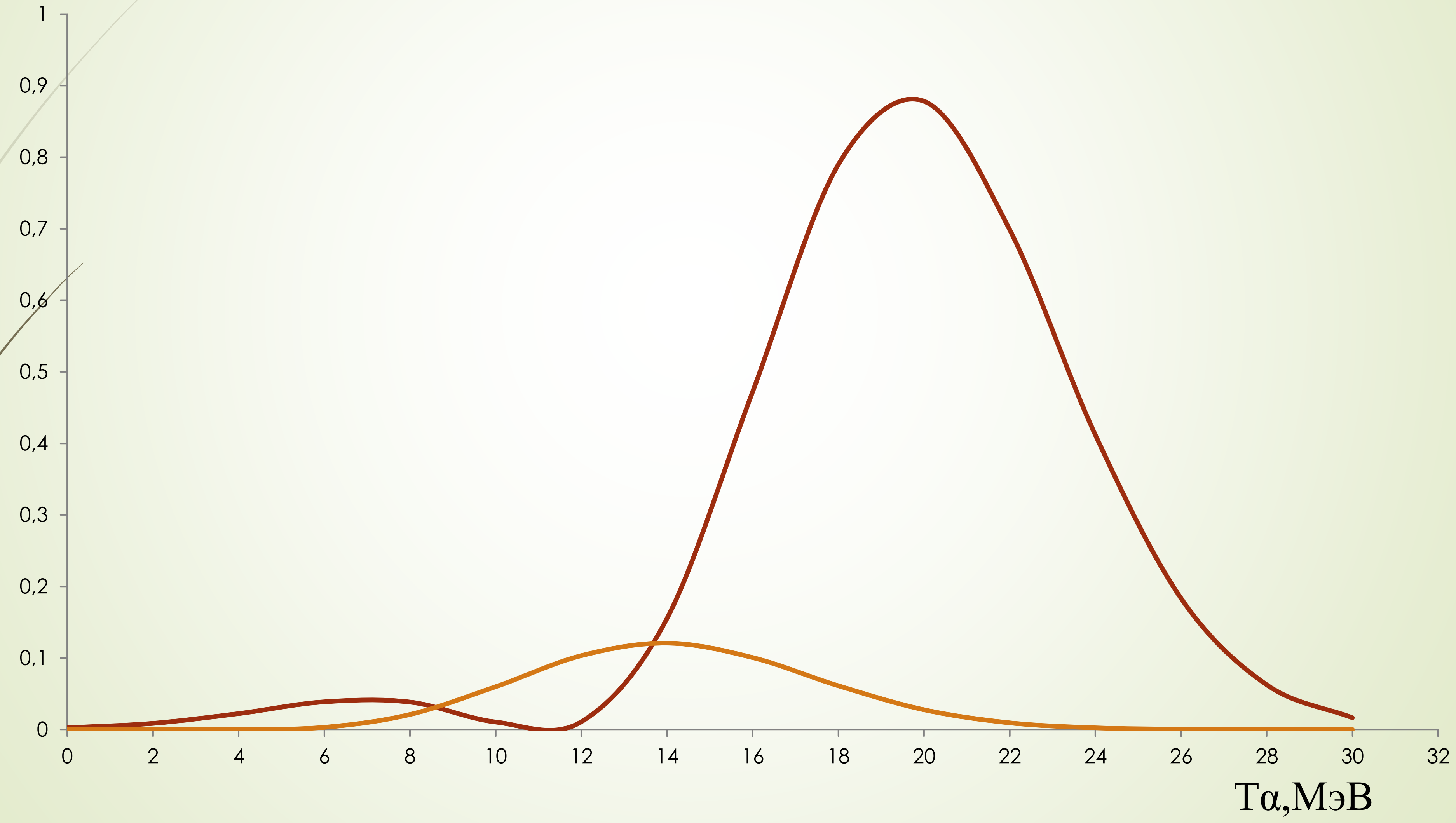
$$\frac{P(T_{\alpha_2})}{P(T_{\alpha_1})} = \frac{\sqrt{(T_{\alpha_1})_{max}}}{\sqrt{(T_{\alpha_1})_{max}}} \frac{\left\{N_{\alpha_2} W_{\alpha_2}(T_{\alpha_2}) \left(Q_{\alpha_2}^{(A-4)} - T_{\alpha_2}\right)^2\right\}_{max}}{\left\{N_{\alpha_1} W_{\alpha_1}(T_{\alpha_1}) \left(Q_{\alpha_1}^{A} + B_n - T_{\alpha_1}\right)^2\right\}_{max}}$$

Yields of the α– particle in the quaternary fission of compound nuclei ^{234}U, ^{236}U

Ядро	$N_{\alpha,\alpha}$	N_{α_1}	N_{α_2}
^{234}U	$(0.89 \pm 0.28) \times 10^{-7}$	$(2.17 \pm 0.07) \times 10^{-3}$	$(2.05 \pm 0.65) \times 10^{-5}$
^{236}U	$(0.54 \pm 0.17) \times 10^{-7}$	$(1.70 \pm 0.03) \times 10^{-3}$	$(1.6 \pm 0.5) \times 10^{-5}$

[1] M. Mutterer and J. P. Theobald, Dinuclear Decay Modes, Bristol: IOP Publ., 1996, Chap. 12.
[2] Jesinger P. et al. //The European Physical Journal A. 2005. P. 379.
Kamanin D.V. et al. // Eurasian Journ. of Phys. and Funct. Mat. 2019. P. 139.
[3] С. Г. Кадменский, Л. В. Титова, ЯФ. 2016. Т. 76. С. 18.]

Widths $(\Gamma_\alpha^A(T_\alpha))^0$ and $(\Gamma_\alpha^{A-4}(T_\alpha))^0$, connected with the first and second α-particles flight from the fissile nucleus neck for the fission of compound nuclei U^{234} и U^{236}



Coulomb barrier heights and penetrabilities ratio for the first and second α –particles in prescission configuration of the fissile nucleus with neck

	^{234}U	^{236}U
$(T_{\alpha_1})_{max}$, MeV	19.9	19.6
$(T_{\alpha_2})_{max}$, MeV	14.0	13.9
$\frac{P_{\alpha_2}}{P_{\alpha_1}}$	0.026	0.025

The decrease in the probability of the second alpha particle flight is related to:

- with a change in the shell structure of the neck of the fissile nucleus after the first particle is emitted from it, which affects the probability of the appearance of the second particle, since after the departure of the first particle, this neck does not have time to go to an equilibrium state and "remembers" the quantum characteristics of the nucleons forming the first particle. The first particle is formed taking into account Cooper pairing mainly from the outer nucleons of the neck of the dividing nucleus, which have minimal binding energy, in contrast to the second particle, which will be formed from more deeply bound neck nucleons and therefore will have a noticeably higher separation energy and a lower probability of departure under the influence of the shake effect.
- By total barrier formed by the Coulomb and nuclear potentials of the interaction of this particle with the remaining fissile nucleus, it has a lower height than the height of the potential barrier that the first particle escapes. This is due not only to the fact that the charges of the remaining fissionable nuclei differ after the flight of the first and second particles, but also to the fact that the flight of the second particle occurs from a more elongated neck of the fissile nucleus, which corresponds to a greater distance between the fission pre-fragments. This factor may lead to a slight increase in the probability of the second particle appearing compared to the first.

S.G. Kadmensky, L.V. Titova, A.O. Bulychev // Phys. At. Nucl. – 2015. – V.78, №7-8. – P.716-724.
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Conclusions:

- The formulae for the virtual widths for the spontaneous fission of ^{248}Cm and ^{252}Cf, so as for induced by the thermal neutrons fission of ^{233}U and ^{235}U were constructed.
- The yields for the second alpha-particle in the quaternary fission of ^{248}Cm and ^{252}Cf are less than the yields of the first alpha-particle, the energy distribution maximum for the second alpha-particle is shifted to the region of the less energies in comparison with the first alpha-particle energy distribution maxima.
- The ratios of the Coulomb barrier penetrabilities for the first and second alpha-particles in quaternary fission ^{248}Cm and ^{252}Cf, are 0.003 and 0.027 accordingly, and in fission of compound nuclei ^{234}U and ^{236}U, are 0.026 and 0.025 accordingly, that indicates a decrease in the probability of the second alpha particle's flight.
- The Coulomb barrier heights for the first and second alpha-particle flight from the configuration of the fissile nucleus neck are 15,9 MэB, 10,1 MэB and 15,9 MэB, 13 MэB for nuclei ^{248}Cm and ^{252}Cf accordingly, and 19.9 MэB, 14.0 MэB and 19,6 MэB, 13,9 MэB, for nuclei ^{234}U и ^{236}U.

Thank you for your attention!