# Relativistic mean-field effective NN forces in dynamical modeling of heavy-ion fusion


**<u>Maria Chushnyakova</u>**

Omsk State Technical University, Russia

**Igor Gontchar, Natalya Khmyrova**

Omsk State Transport University, Russia

**Anna Klimochkina**

Lomonosov Moscow State University, Russia



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# Plan

- **1. Introduction**
- **2. TMSF** (Trajectory model with surface friction)
- **3. RMF NN forces**
  - **3.1 Potential**
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  - **3.3 Resulting fusion CS vs experiment**
  - **3.4 Dissipation strength $K_R$**
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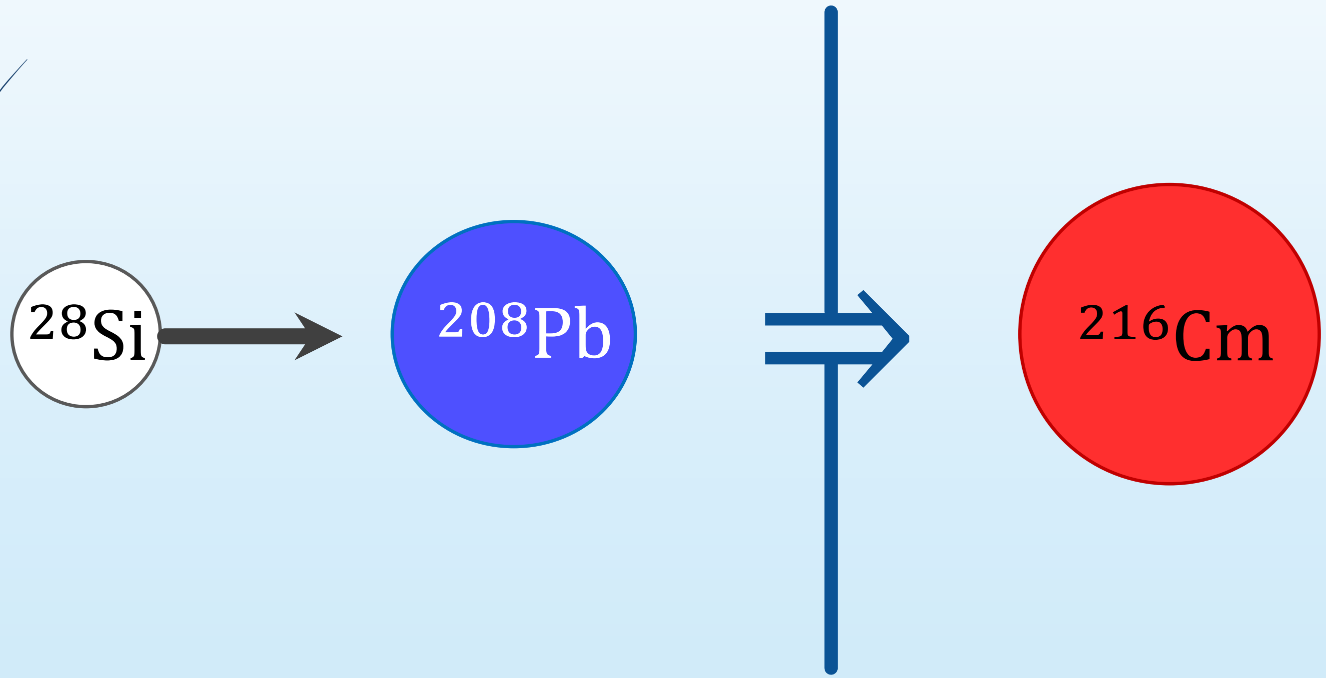
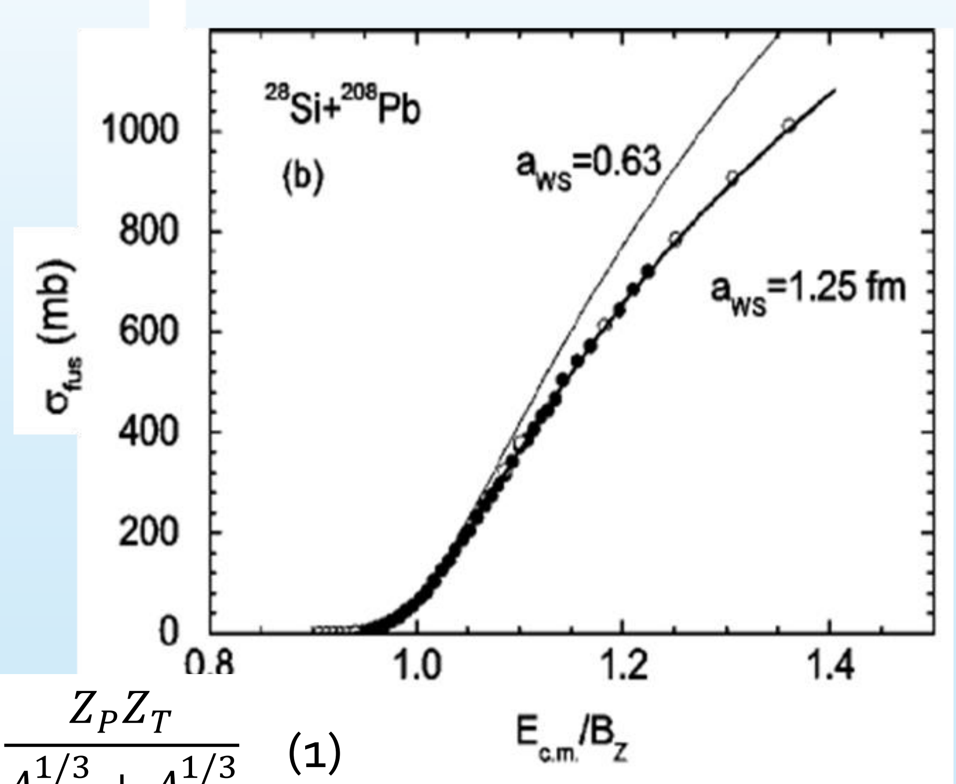
# 1. Introduction



# The problem of the apparently large diffuseness

[1] Newton et al. Phys. Lett. B 586 (2004) 219:
«Systematics of precise nuclear fusion cross sections: the need for a new dynamical treatment of fusion?»

[2] Newton et al. Phys. Rev. C 70 (2004) 024605

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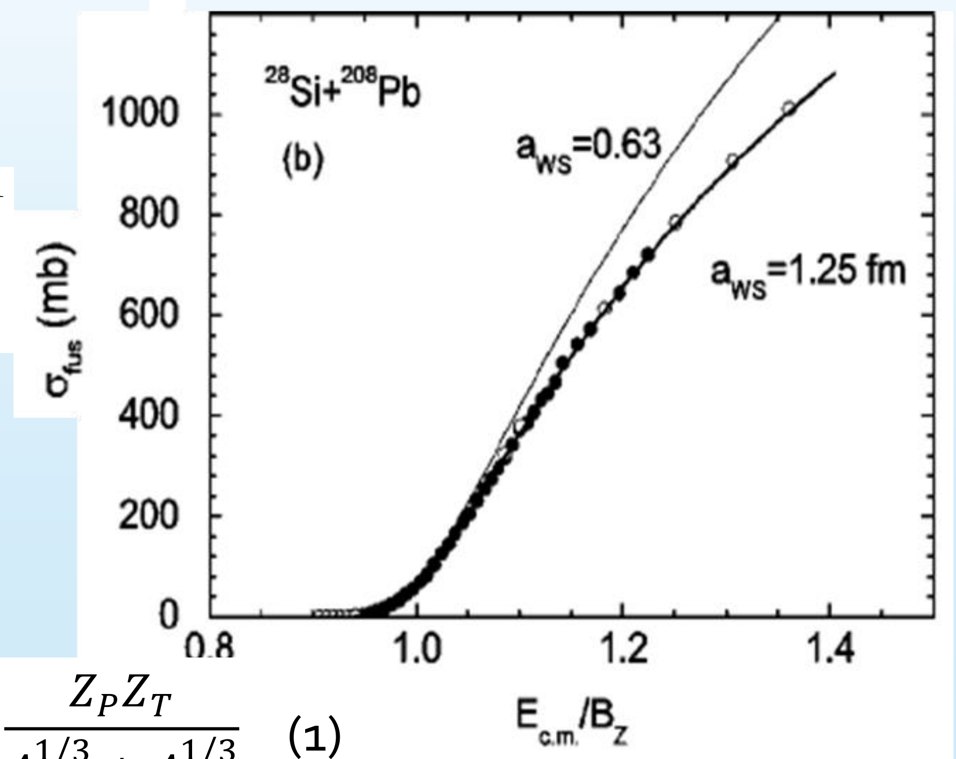
$$U_n(R) = V_{WS}\left\{1 + \exp\left(\frac{R - r_{WS}\left(A_P^{1/3} + A_T^{1/3}\right)}{a_{WS}}\right)\right\}^{-1}$$

Woods-Saxon profile (2)

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$$F_U = -\frac{dU_{tot}}{dq} \qquad (12)$$

$$F_{cen} = \frac{\hbar^2 L^2}{m_q q^3} \qquad (13)$$

$$F_{Dq} = -\frac{p}{m_q} K_R \left[\frac{dU_n}{dq}\right]^2 \qquad (14)$$

Einstein relation:

$$D_q = \theta\eta \quad \Rightarrow \quad D_q = \theta K_R \left[\frac{dU_n}{dq}\right]^2 \qquad (15)$$

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[4] Fröbrich, Phys. Rep. 116 (1984) 337

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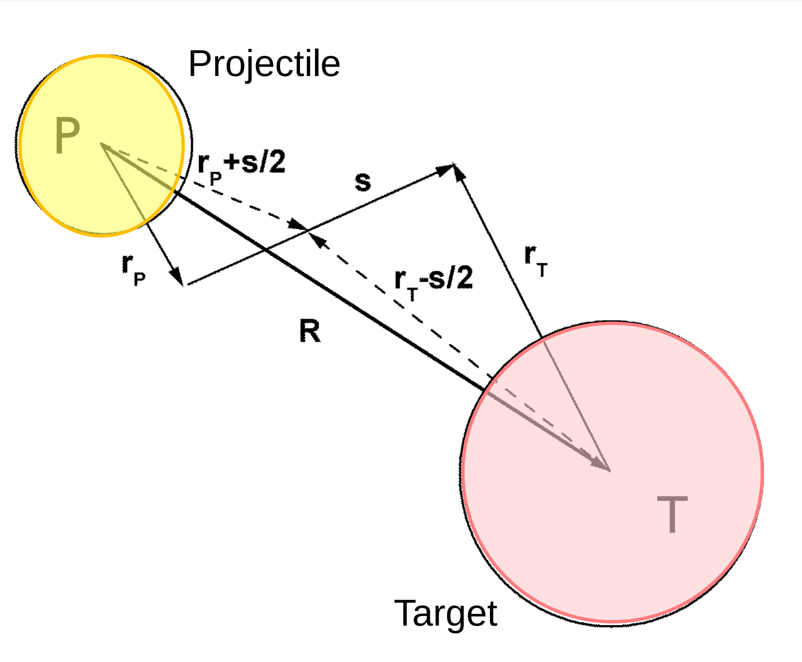
# Dissipation strength

$$F_{Dq} = -\frac{p}{m_q} K_R \left[\frac{dU_n}{dq}\right]^2 \qquad (14)$$

The only varying parameter of the TMSF

# Double-folding model:

$$U_n(R) = \int d\vec{r}_P \int d\vec{r}_T \, \rho_{AP}(\vec{r}_P) \mathrm{v}_{NN}(s) \rho_{AT}(\vec{r}_T) \tag{20}$$




[5] Satchler, Love, Phys. Rep. 55 (1979) 183
[6] Gontchar, MC, Comp. Phys. Comm. 181 (2010) 168–182

# 3. Relativistic mean-field Lagrangian density

$$\mathcal{L} = \bar{\psi}_i \{i\gamma^{\mu}\partial_{\mu} - M\}\psi_i +$$

$$\frac{1}{2}\partial^{\mu}\sigma\,\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} - g_{s}\bar{\psi}_i\psi_i\sigma$$

$$-\frac{1}{4}\Omega^{\mu\vartheta}\Omega_{\mu\vartheta} + \frac{1}{2}m_{\omega}^{2}V^{\mu}V_{\mu} - g_{\omega}\bar{\psi}_i\gamma^{\mu}\psi_iV_{\mu} - \qquad (21)$$

$$\frac{1}{4}\mathbf{B}^{\mu\vartheta}\cdot\mathbf{B}_{\mu\vartheta} - -\frac{1}{2}m_{\rho}^{2}\mathbf{R}^{\mu}\cdot\mathbf{R}_{\mu} - g_{\rho}\bar{\psi}_i\gamma^{\mu}\boldsymbol{\tau}\psi_i\cdot\mathbf{R}_{\mu} -$$

$$-\frac{1}{4}F^{\mu\vartheta}F_{\mu\vartheta} - e\,\bar{\psi}_i\gamma^{\mu}\frac{(1-\tau_{3i})}{2}\psi_iA_{\mu}$$

[7] MC, Bhuyan et al, Nucl. Phys. A 994 (2020) 121657
[8] Lalazissis et al, Phys. Rev. C 55 (1997) 540

# Effective RMF *NN* forces

| | NL1 [10] | NL2 [10] | NL3 [11] | HS [12] |
|---|---|---|---|---|
| $m_\omega c^2$ /MeV | 795.359 | 780.0 | 782.501 | 783 |
| $m_\rho c^2$ /MeV | 763.0 | 763.0 | 763.000 | 770 |
| $m_\sigma c^2$ /MeV | 492.25 | 504.89 | 508.194 | 520 |
| $g_\omega/\sqrt{\hbar c}$ | 13.285 | 11.493 | 12.868 | 13.8 |
| $g_\rho/\sqrt{\hbar c}$ | 4.975 | 5.507 | 4.474 | 8.08 |
| $g_\sigma/\sqrt{\hbar c}$ | 10.138 | 9.111 | 10.271 | 10.47 |
| $g_2$ /fm $/\sqrt{\hbar c}$ | -12.172 | -2.304 | -10.431 | 0 |
| $g_3/\sqrt{\hbar c}$ | -36.265 | 13.783 | -28.885 | 0 |

[10] Hirata et al, Phys. Rev. C 44 (1991) 1467
[11] Lahiri et al, Int. J. Mod. Phys. E 25 (2016) 1650015
[12] Horowitz, Serot, Nucl. Phys. A 368 (1981) 503

# Effective RMF *NN* forces



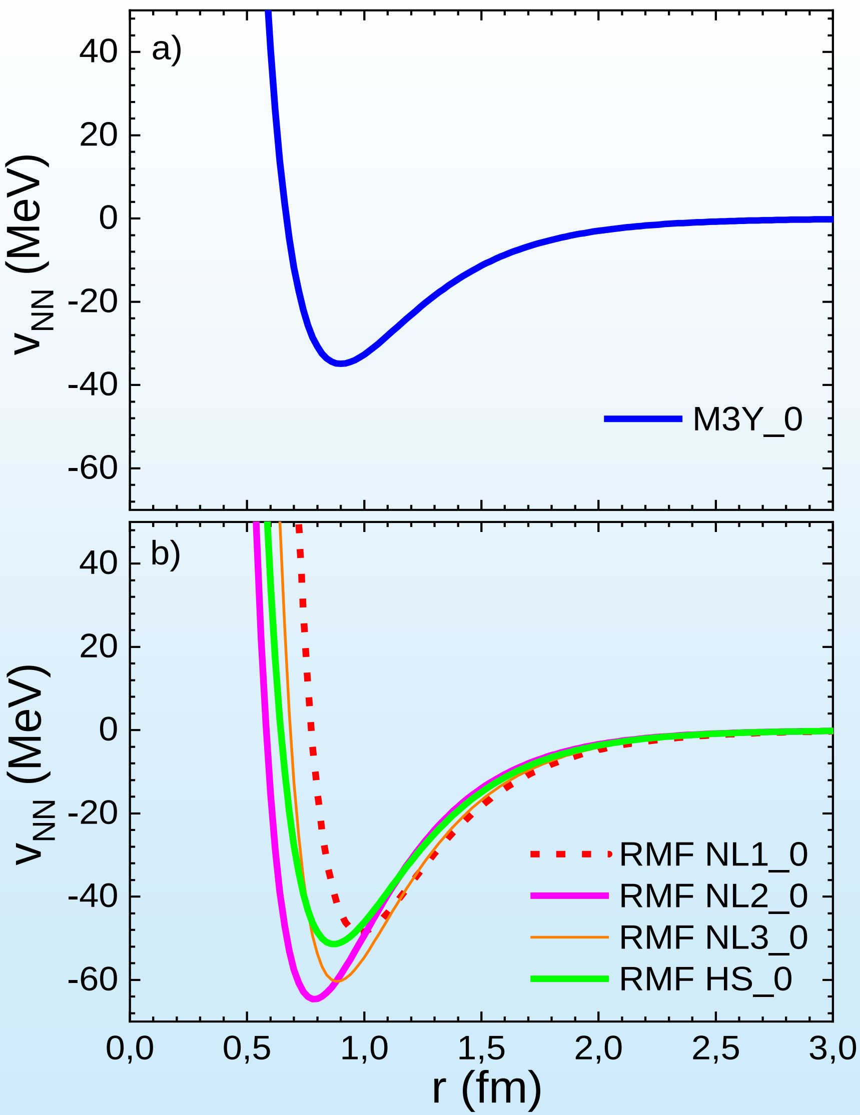
$$
\begin{aligned}
&\mathrm{v}_{NN}(r) = \\
&= \frac{g_\omega^2}{4\pi}\,\frac{\exp(-m_\omega rc/\hbar)}{r} + \frac{g_\rho^2}{4\pi}\,\frac{\exp\left(-m_\rho rc/\hbar\right)}{r} \\
&- \frac{g_\sigma^2}{4\pi}\,\frac{\exp(-m_\sigma rc/\hbar)}{r} + \frac{g_2^2}{4\pi} r \exp(-2m_\sigma rc/\hbar) \\
&+ \frac{g_3^2}{4\pi}\,\frac{\exp(-3m_\sigma rc/\hbar)}{r} - J_{00}\delta(\vec{r})
\end{aligned} \tag{22}
$$

**M3Y:**

$$\mathrm{v}_{NN}(r) = G_1 \frac{\exp(-r/r_1)}{r/r_1} - G_2 \frac{\exp(-r/r_2)}{r/r_2} - J_{00}\delta(\vec{r}) \tag{23}$$

[11] Lahiri et al, Int. J. Mod. Phys. E 25 (2016) 1650015

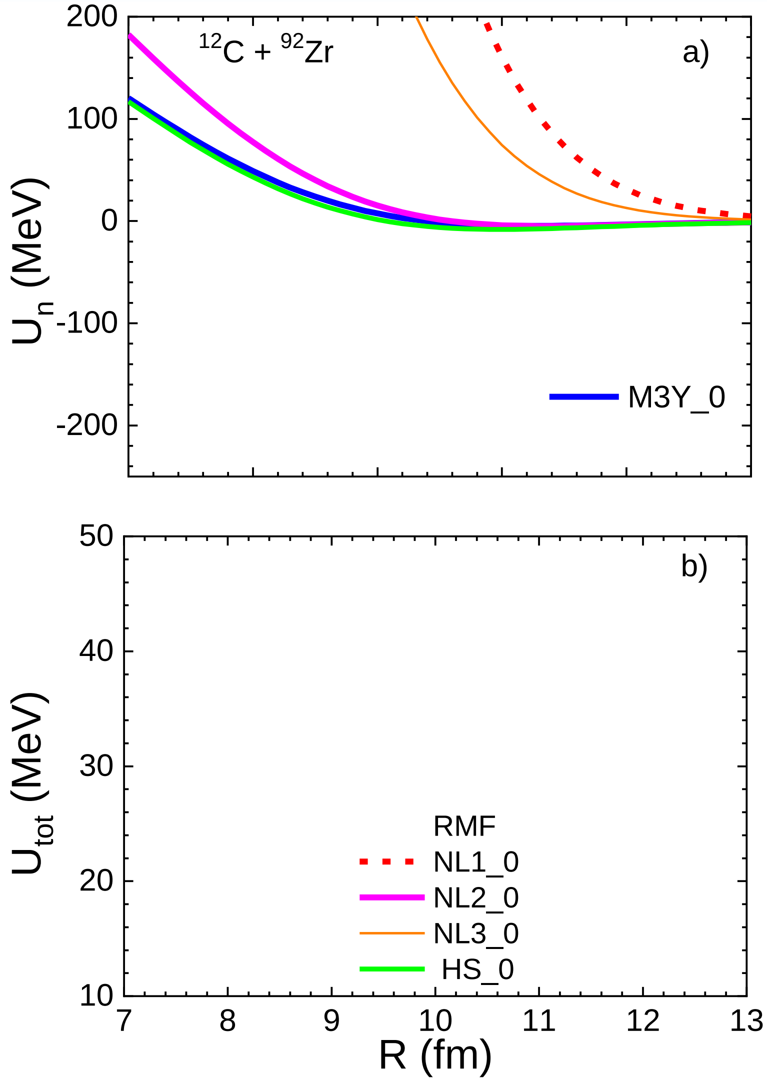
# Effective *NN* forces (direct part)




**M3Y**

**RMF**

# Nucleus-nucleus interaction potentials (including <u>direct</u> part of NN forces)




**Nuclear term**

**Total potential**

(without exchange part in RMF NN forces)

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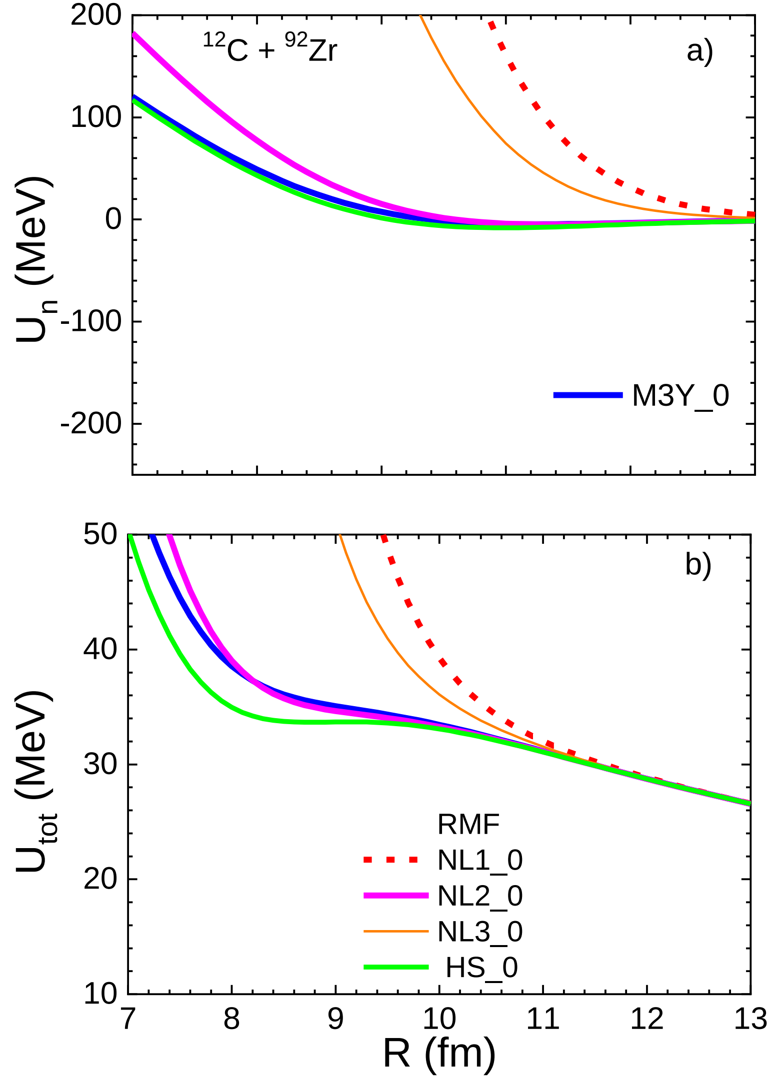


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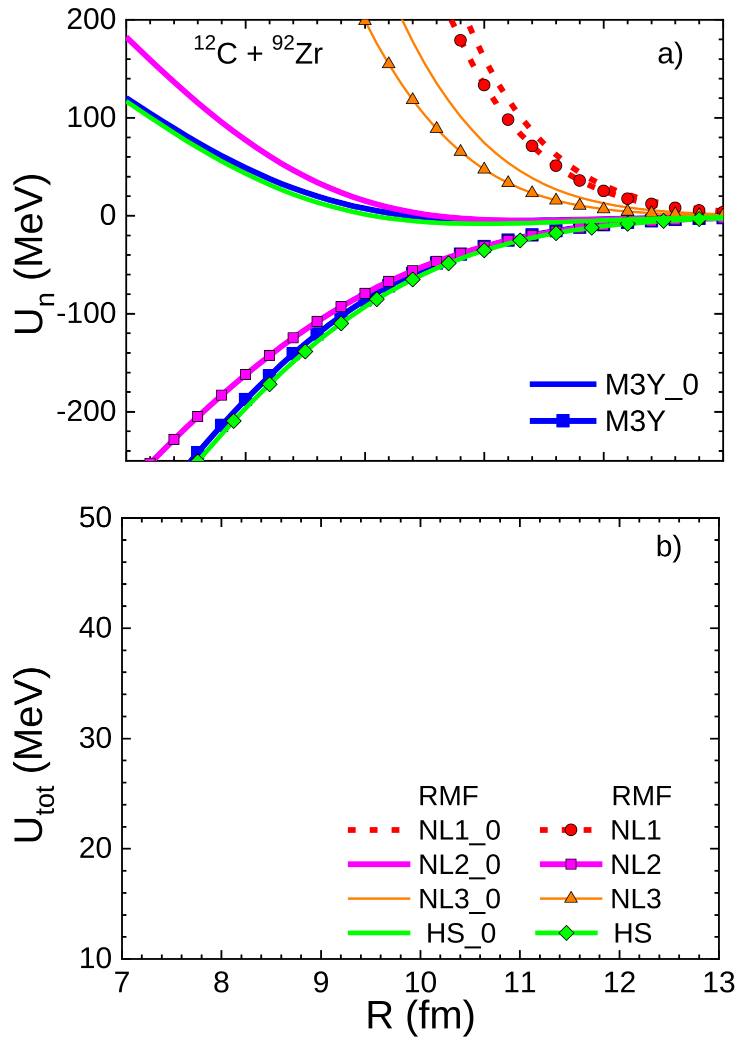
# Nucleus-nucleus interaction potentials



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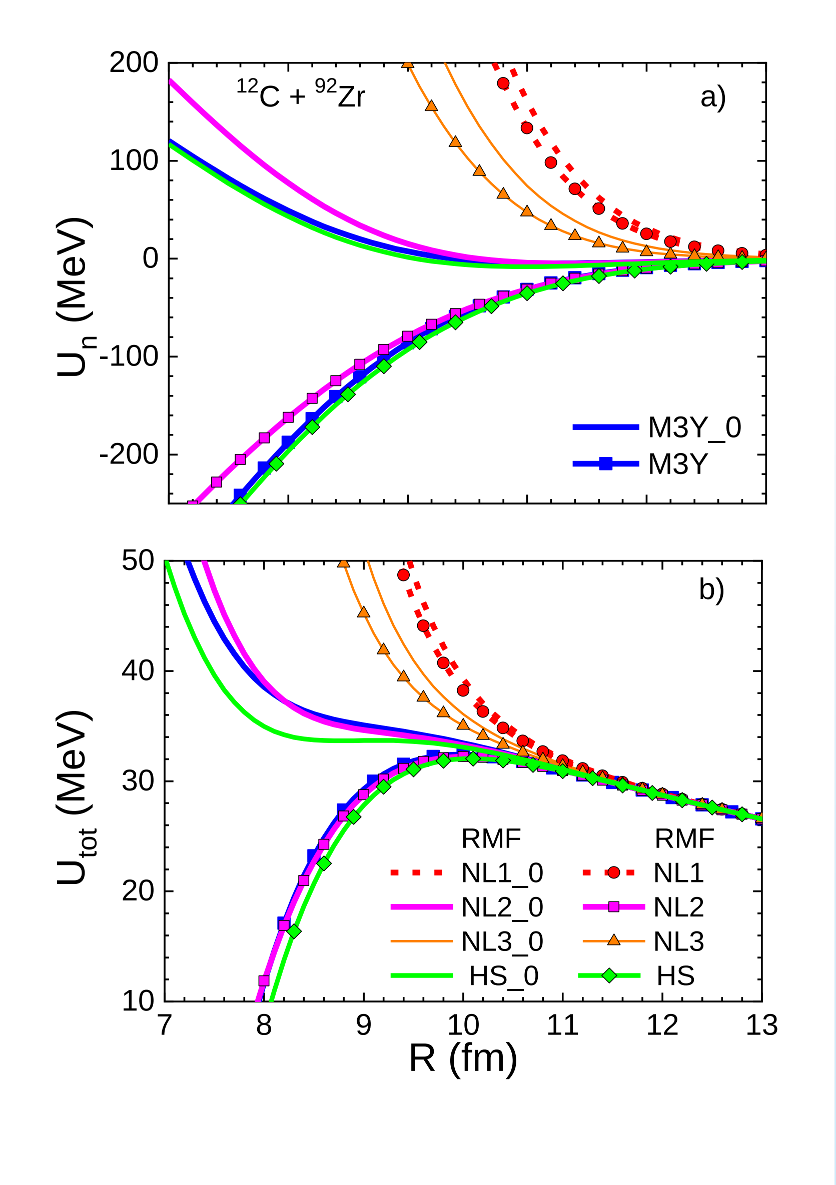
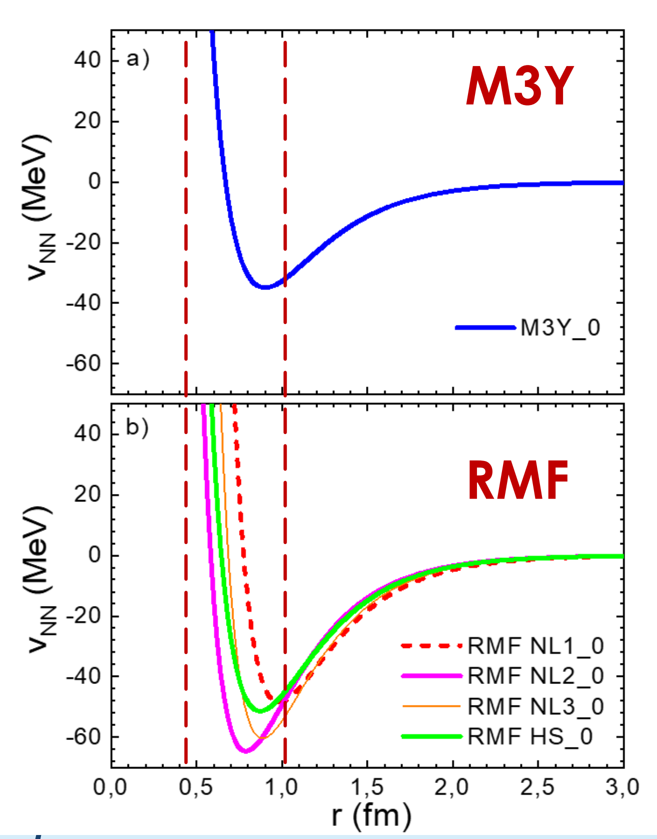
# Nucleus-nucleus interaction potentials

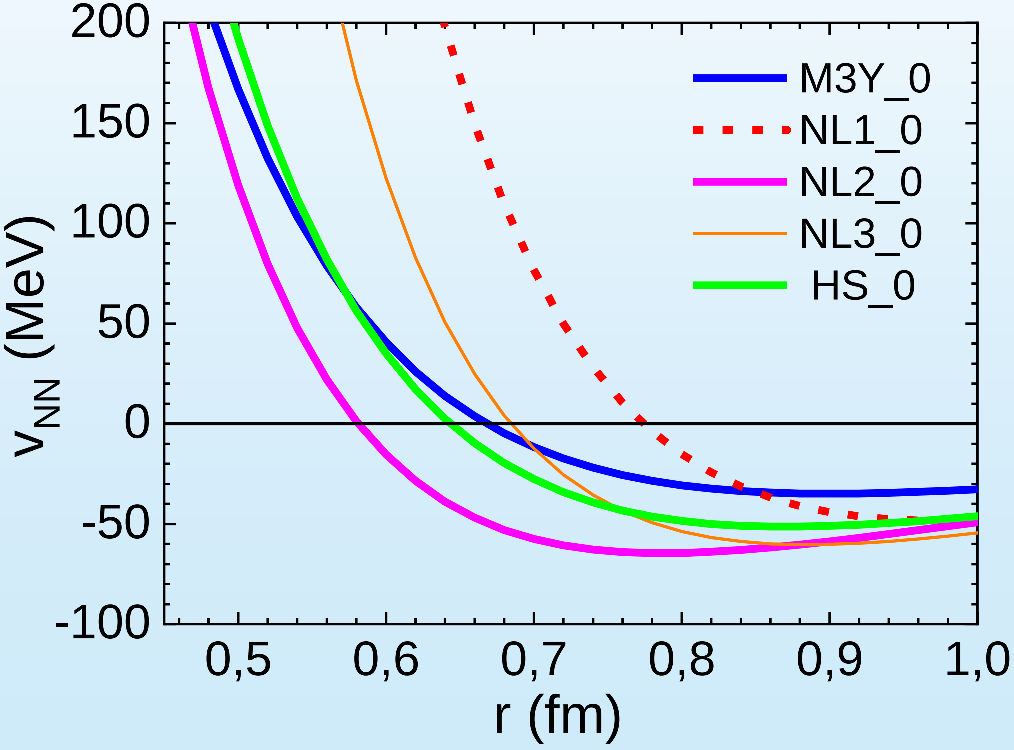



**Nuclear term**

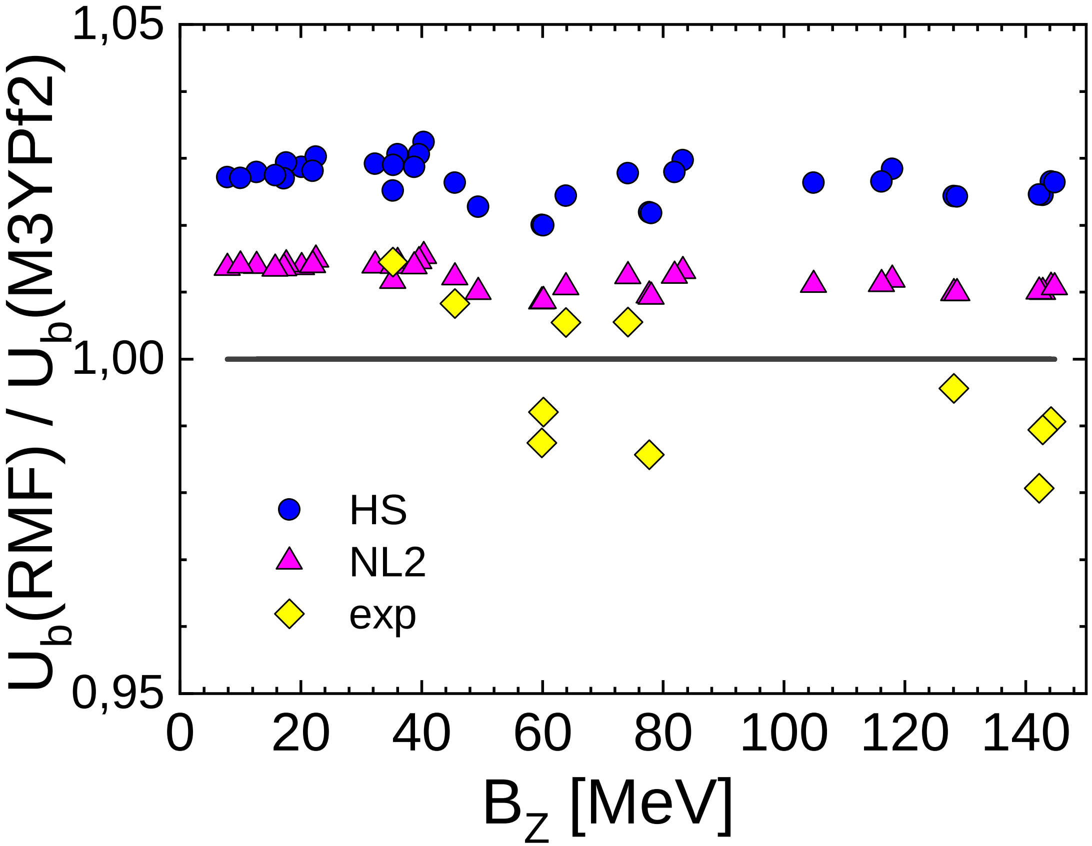
**Total potential**

# Effective *NN* forces: domain of small *r*-values (direct part)



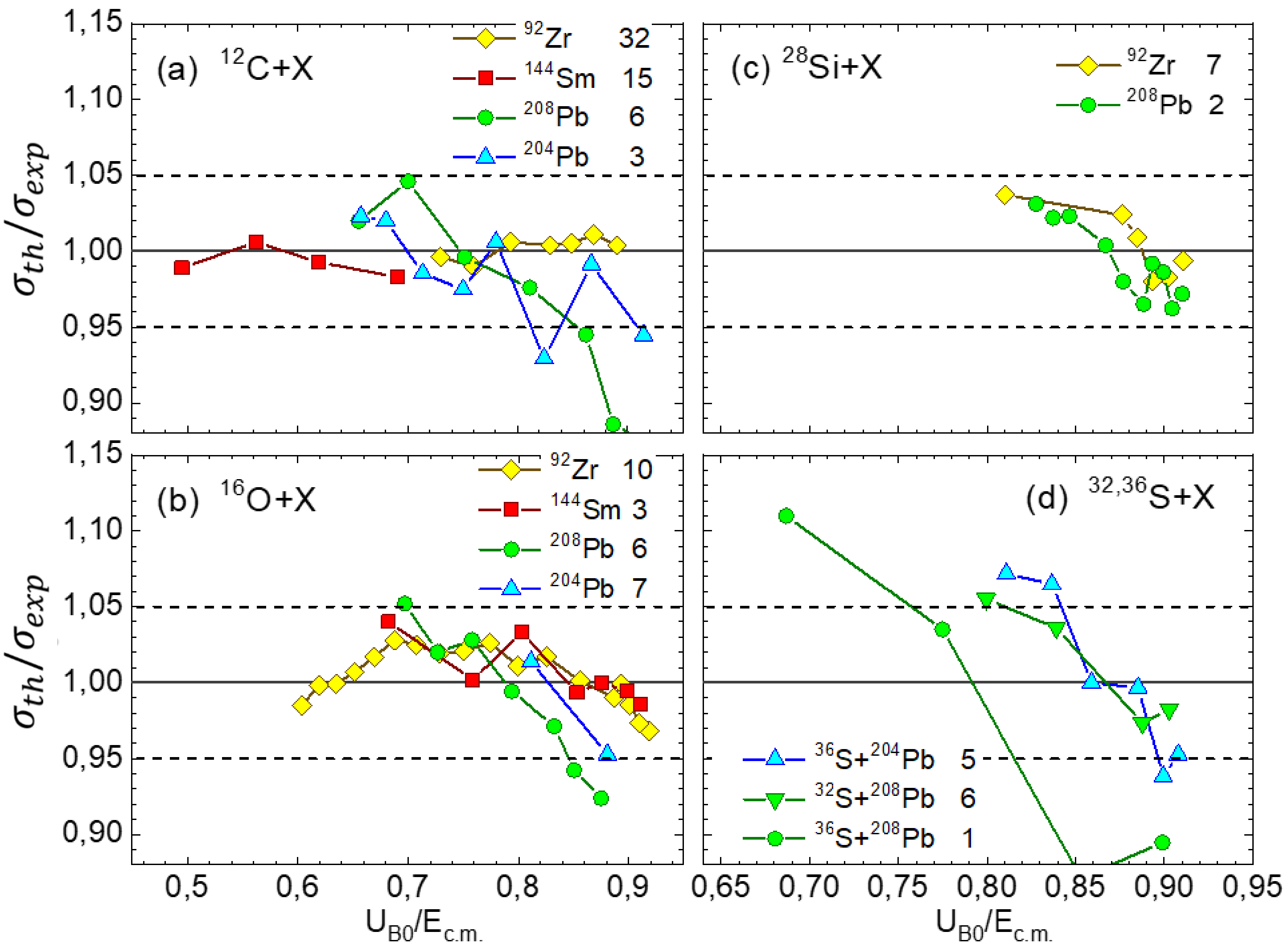
# Barrier heights




$$B_Z = \frac{Z_P Z_T}{A_P^{1/3} + A_T^{1/3}} \quad (1)$$


[13] MC, Gontchar, J. Phys. G (2020) doi: 10.1088/1361-6471/ab907a
[2] Newton et al., Phys. Rev. C 70 (2004) 024605

# Resulting TMSF fusion cross-sections vs experiment with NL2 *NN* forces

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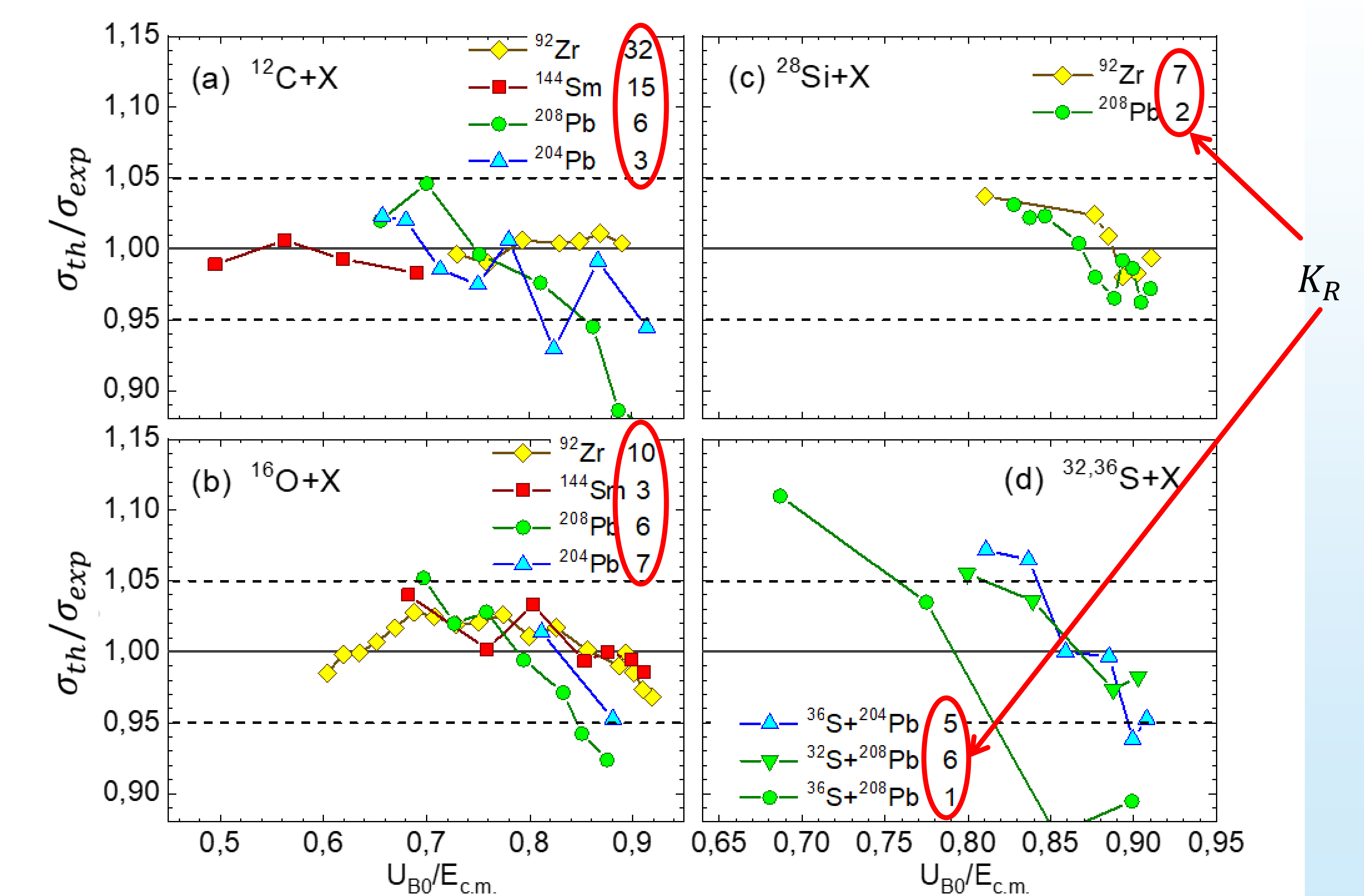


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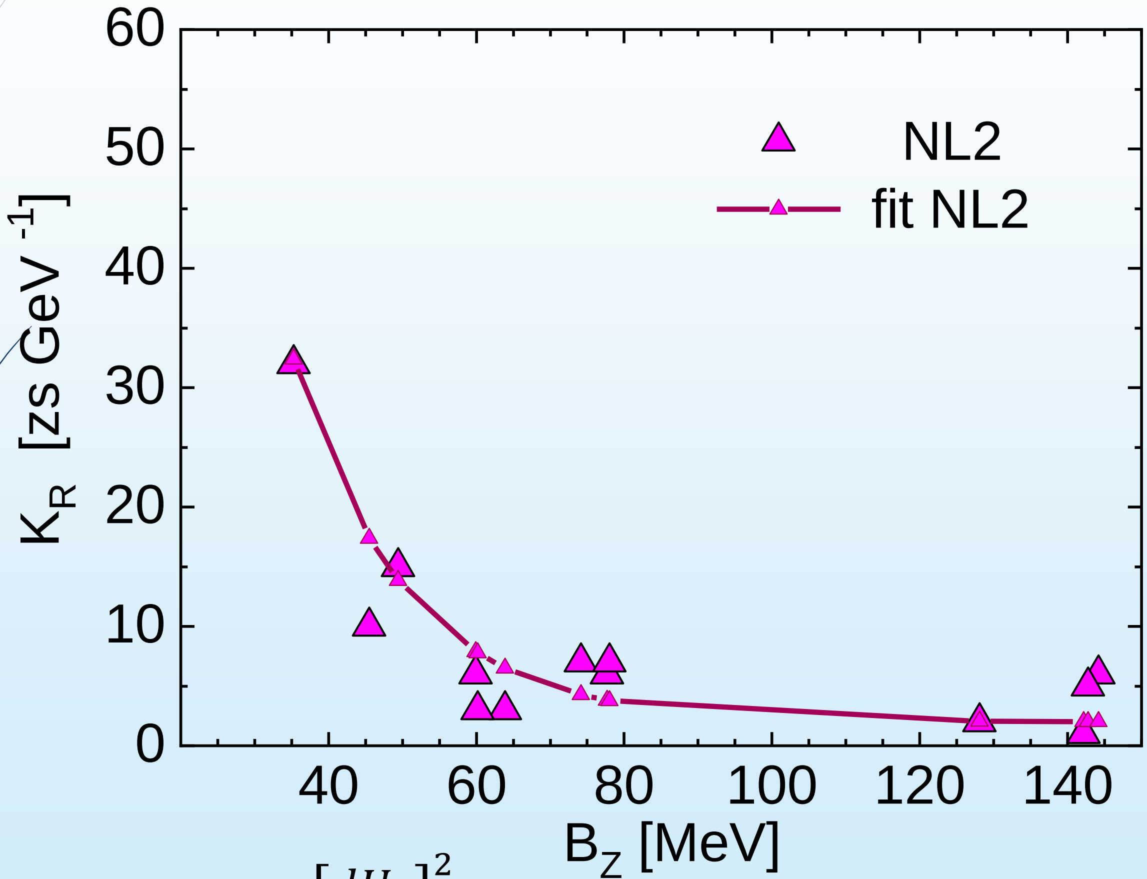
# The dissipation strength extracted from the analysis

## RMF NN forces

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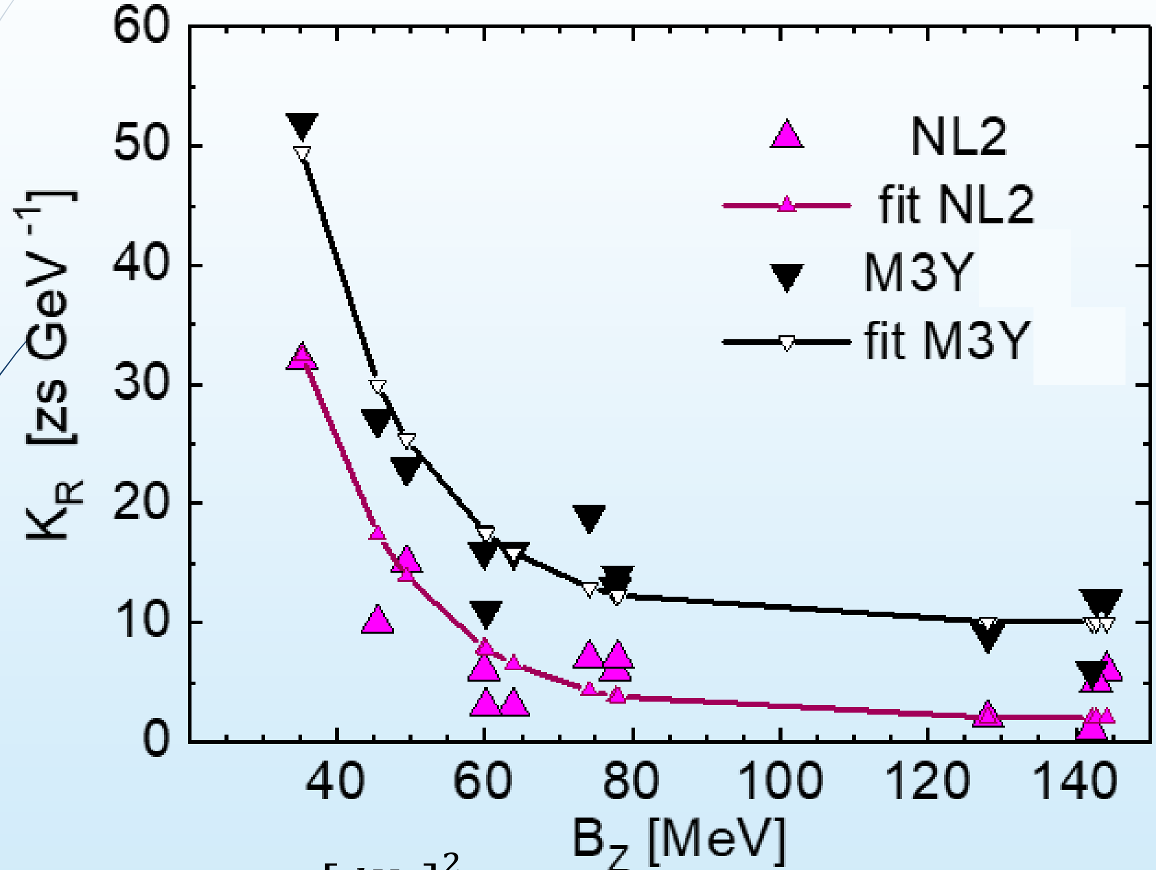
# The dissipation strength extracted from the analysis

**RMF vs M3Y NN forces**

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# Conclusions



- The relativistic mean-field (**RMF**) effective nucleon-nucleon (NN) forces are implemented to the double-folding potential
- The characteristic of the resulting Coulomb barriers are evaluated for NL2 and HS parameter sets - NL1 and NL3 sets do not result in the barrier
- Using **RMF** NN forces, the capture (fusion) cross-sections are calculated within the dynamical trajectory fluctuation-dissipation model (**TMSF**) and compared with the experimental data:
  - Most of the calculated CS values are in 5% agreement with the data
  - The only varying parameter of TMSF – dissipation strength $K_R(B_Z)$ resembles this dependence for the M3Y NN forces but has smaller values

# Thank you for your attention!

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Omsk State Technical University, Russia
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