TAGS SPECTRA ANALYSIS AND BETA DECAY STRENGTH FUNCTION STRUCTURE

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The β–transition probability is proportional to the product of the lepton part described by the Fermi function $f(Q_\beta - E)$ and the nucleon part described by the β-decay strength function $S_\beta(E)$, where E is the excitation energy in daughter nuclei and Q_β is the total energy of β-decay. The strength function $S_\beta(E)$ governs the nuclear energy distribution of elementary charge-exchange excitations and their combinations like proton particle (πp)–neutron hole (νh) coupled into a momentum I^π : $[\pi p \text{ x } \nu h]I^\pi$ and neutron particle (νp)–proton hole (πh) coupled into a momentum I^π : $[\nu p \text{ x } \pi h)]I^\pi$. The strength function of Fermi-type β-transitions takes into account excitations $[\pi p \text{ x } \nu h]0^+$ or $[\nu p \text{ x } \pi h]0^+$. Since isospin is a quite good quantum number, the strength of the Fermi-type transitions is concentrated in the region of the isobar-analogue resonance (IAR). The strength function for β-transitions of the Gamow–Teller (GT) type describes excitations $[\pi p \text{ x } \nu h]1^+$ or $[\nu p \text{ x } \pi h]1^+$. At excitation energies E smaller than Q_β (total β-decay energy), $S_\beta(E)$ determines the characters of the β-decay. For higher excitation energies that cannot be reached with the β-decay, $S_\beta(E)$ determines the charge exchange nuclear reaction cross sections, which depend on the nuclear matrix elements of the β-decay type.

Information on the structure of $S_\beta(E)$ is important for many nuclear physics areas. Reliable experimental data on the structure of $S_\beta(E)$ are necessary for predicting half-lives of nuclei far from the stability line, verifying completeness of decay schemes, calculating energy release from decay of fission products in nuclear reactors, calculating spectra of delayed particles, calculating the delayed fission probability and evaluating fission barriers for nuclei far from the β stability line, calculating production of various elements in astrophysical processes, and developing microscopic models for calculation of $S_\beta(E)$, especially in deformed nuclei.

Until recently, experimental investigations of the $S_\beta(E)$ structure were carried out using total absorption gamma-ray spectrometers (TAGS) and total absorption spectroscopy methods, which had low energy resolution. With TAGS spectroscopy, it became possible to demonstrate experimentally the resonance structure of $S_\beta(E)$ for Gamow–Teller β transitions. However, TAGS methods have some disadvantages arising from low energy resolution of NaI-based spectrometers. Modern experimental instruments allow using nuclear spectroscopy methods with high energy resolution to study the fine structure of $S_\beta(E)$.

High-resolution nuclear spectroscopy methods, like total absorption gamma spectroscopy (TAGS) methods, give conclusive evidence of the resonance structure of $S_\beta(E)$ for GT transitions in both spherical and deformed nuclei. High-resolution nuclear spectroscopy methods made it possible to demonstrate experimentally the resonance nature of $S_\beta(E)$ for FF transitions and reveal splitting of the peak in the strength function for the GT β^+/EC decay of the deformed nucleus into two components. This splitting indicates anisotropy of oscillation of the isovector density component.

For the GT-β transitions, FF-β transitions in the ξ approximation (Coulomb approximation), and unique FF-β transitions the $T_{1/2}$, ft, level populations I(E), $S_\beta(E)$ and reduced probabilities B(GT), $[B(\lambda\pi = 0^-)+B(\lambda\pi=1^-)]$, $[B(\lambda\pi = 2^-)]$ are related as follows :

$$d(I(E))/dE = S_\beta(E)\, T_{1/2}\, f(Q_\beta - E), \quad (1)$$

$$(T_{1/2})^{-1} = \int S_\beta(E) f(Q_\beta - E)\, dE, \quad (2)$$

$$\int_{\Delta E} S_\beta(E)dE = \sum_{\Delta E} 1/(ft). \quad (3)$$

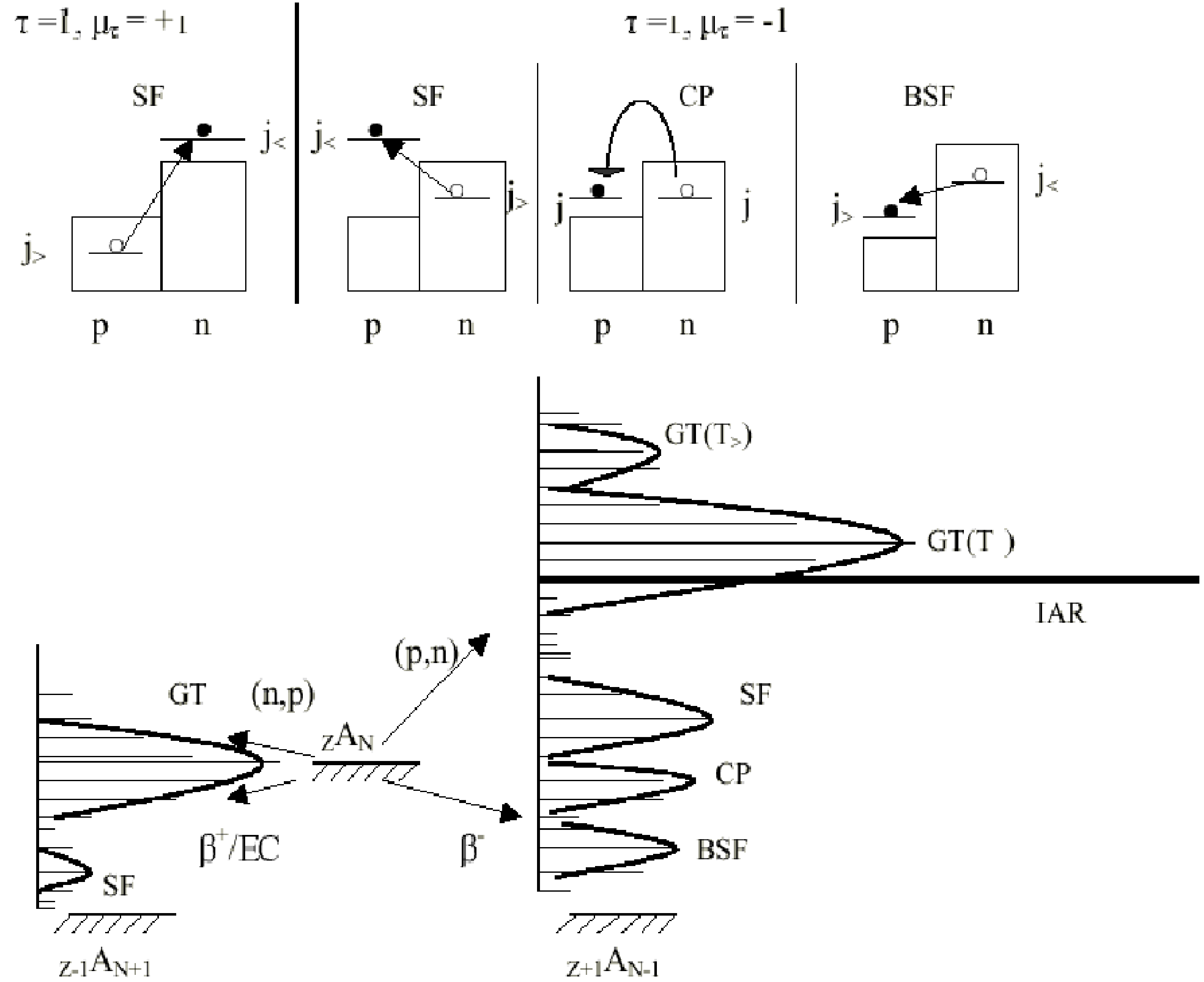
$$B(GT,E)=(g^{eff}{}_A)^2/4\pi \left| \langle I_f \| \sum t_\pm(k)\, \sigma_\mu(k) \| I_i \rangle \right|^2 /(2I_i+1), \quad (4)$$

$$B(GT,E)=[D(g^2{}_V/4\pi)]/ft, \quad (5)$$

$$[B(\lambda\pi =0^-) + B(\lambda\pi =1^-)] = [D\, g^2{}_V/4\pi]/ft, \quad (6)$$

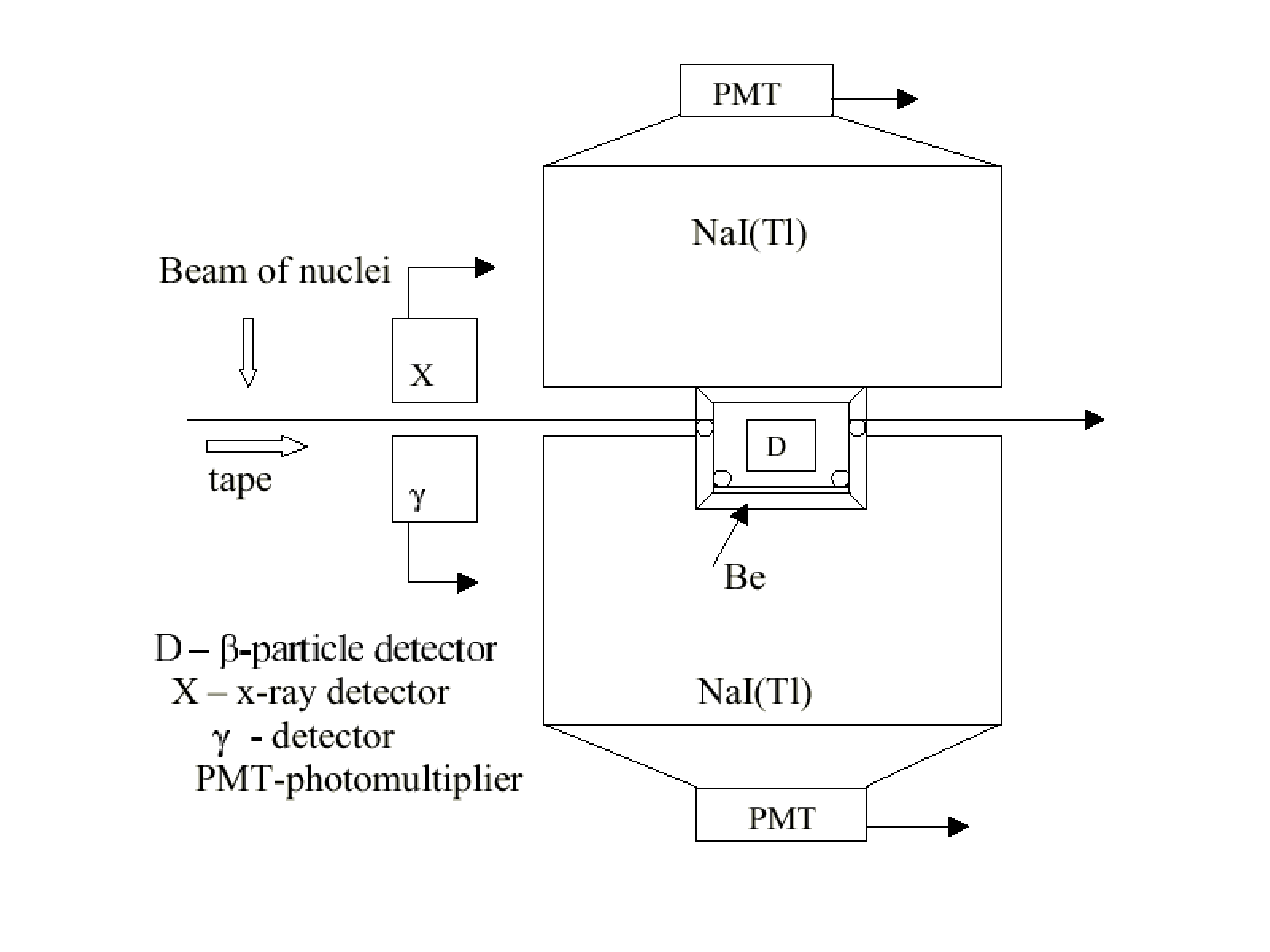
$$[B(\lambda\pi=2^-)]=3/4[Dg^2{}_V/4\pi]/ft, \quad (7)$$

where $S_\beta(E)$ – the beta decay strength function which describe the nuclear part of transition, $f(Q - E)$ – the Fermi function which describe the lepton part of transition and Q – is the total energy of the beta decay.



τ =1 , μτ = +1
τ =1 , μτ = -1
SF
SF
CP
BSF
j<
j>
j
p
n
GT(T>)
GT(T)
IAR
SF
CP
BSF
GT
(n,p)
(p,n)
ZAN
β+/EC
β-
SF
Z-1AN+1
Z+1AN-1

The operating principle of a total-absorption γ-spectrometer (TAGS) is based on summation of the energies of the cascade γ-rays produced after β-decay to excited levels of the daughter nucleus in 4π-geometry. TAGS can't distinguish the GT and FF transitions and don't take into account the conversion electron emission, which give the systematic uncertainties, especially for high *Z*. TAGS does not measure beta strength to the g.s. $S_\beta(0)$.



There are two methods of the TAGS spectra analysis.

In the first one it is necessary to identify the total absorption peaks in TAGS spectra and have 4π-spectrometer with exponential energy dependence of the photoefficiency (i.e., the ratio of the number of pulses in the total absorption peak to the number of γ-ray incident on the detector) for γ-ray registration. Only in this case the efficiency of TAGS peak registration does not depend on the details of decay scheme. This method gives the good results, but can be applied for nuclei with total β-decay energy Q_β less than 5-6 MeV. **Quantitative** characteristics may be obtain as a rule only for **one (β^-- decay) peak** and for **two peaks (β^+/EC-decay)** in $S_\beta(E)$.

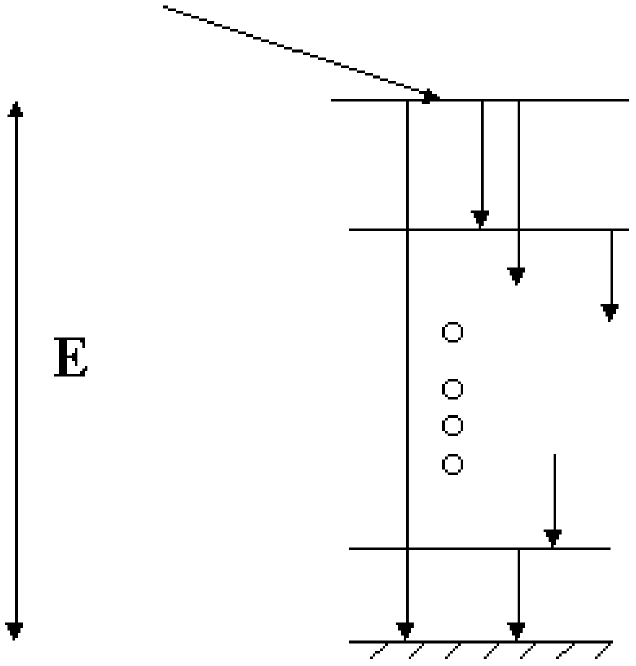
The second method is based on so called **response function application**, but a lot of assumption must be done for extraction the $S_\beta(E)$ shape from the TAGS spectrum shape. Analysis depends on the assumptions about the decay scheme which as a rule is not known. It is very difficult to estimate the associated systematic errors of such analysis and **only qualitative** information about $S_\beta(E)$ may be obtained.

In the first one it is necessary to identify the total absorption peaks in TAGS spectra and have 4π-spectrometer with **exponential** energy dependence of the photoefficiency

$S_\beta(E)$ study by TAS spectroscopy:

1. Direct measurement of the levels populations after β-decay **I(E)** ⟹ **$S_\beta(E)$**
3. 4π geometry
4. Photoefficiency $\varepsilon_{ph} = \exp(-\alpha E)$

⟹ for total-absorption peaks the total absorption efficiency **ε_{ta}**: **$\varepsilon_{ta}(E) = \varepsilon_{ph}(E)$**, and not depend on decay scheme details.



Total absorption spectrometer

ε_{ph} = exp(-αE) for 100 keV < E < 4500 keV (ε_{ph} _ 46% at E_γ = 1MeV)

$\alpha = (9.0 \pm 0.4) \cdot 10^{-4}\, keV^{-1}$ (NaI O 160 x h110 mm + NaI O 210 x h140 mm)

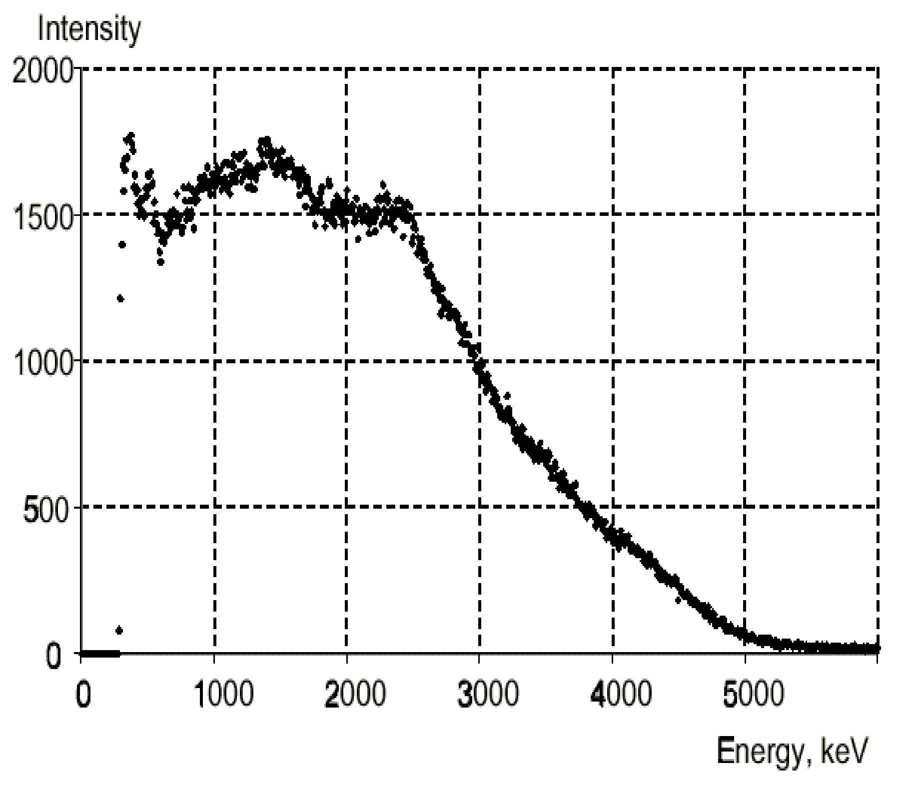
$\alpha = (7.8 \pm 0.3) \cdot 10^{-4}\, keV^{-1}$ (NaI O 200 x h110 mm + NaI O 210 x h140 mm)

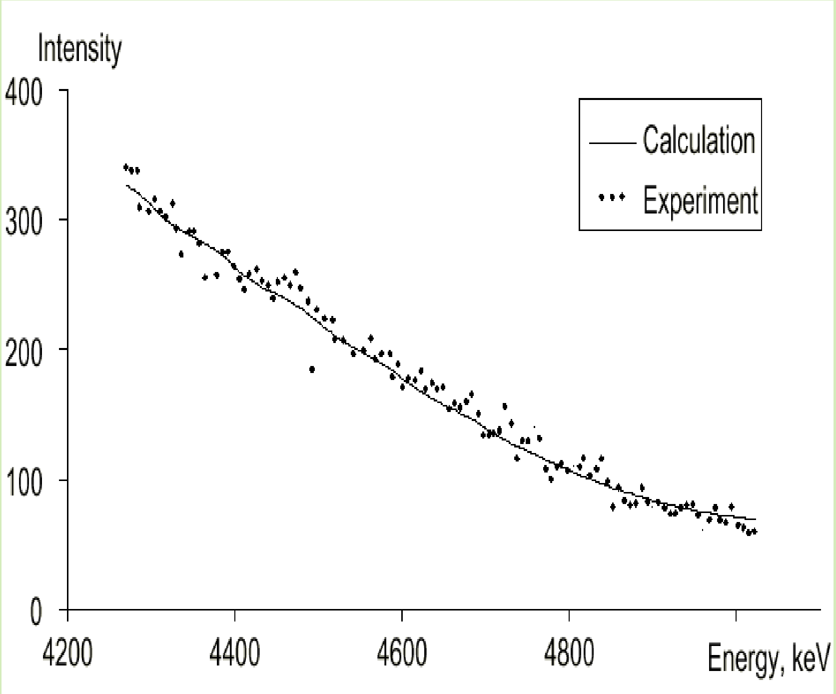
Indeed, if we have a deexcitation scheme for a level with energy E populated by the β decay, then, if relation (**4.**) holds true, the detection efficiency for the total absorption peak for a cascade of n γ rays with the total energy $E = E_{\gamma 1} + \ldots + E_{\gamma n}$ is defined as

$$\varepsilon_{\mathrm{tot}}(E) = \exp(-\alpha E_{\gamma 1}) \times \ldots \times \exp(-\alpha E_{\gamma n})$$
$$= \exp(-\alpha(E_{\gamma 1} + \ldots + E_{\gamma n})) = \exp(-\alpha E), \quad (7)$$

and does not depend on the scheme of γ transitions

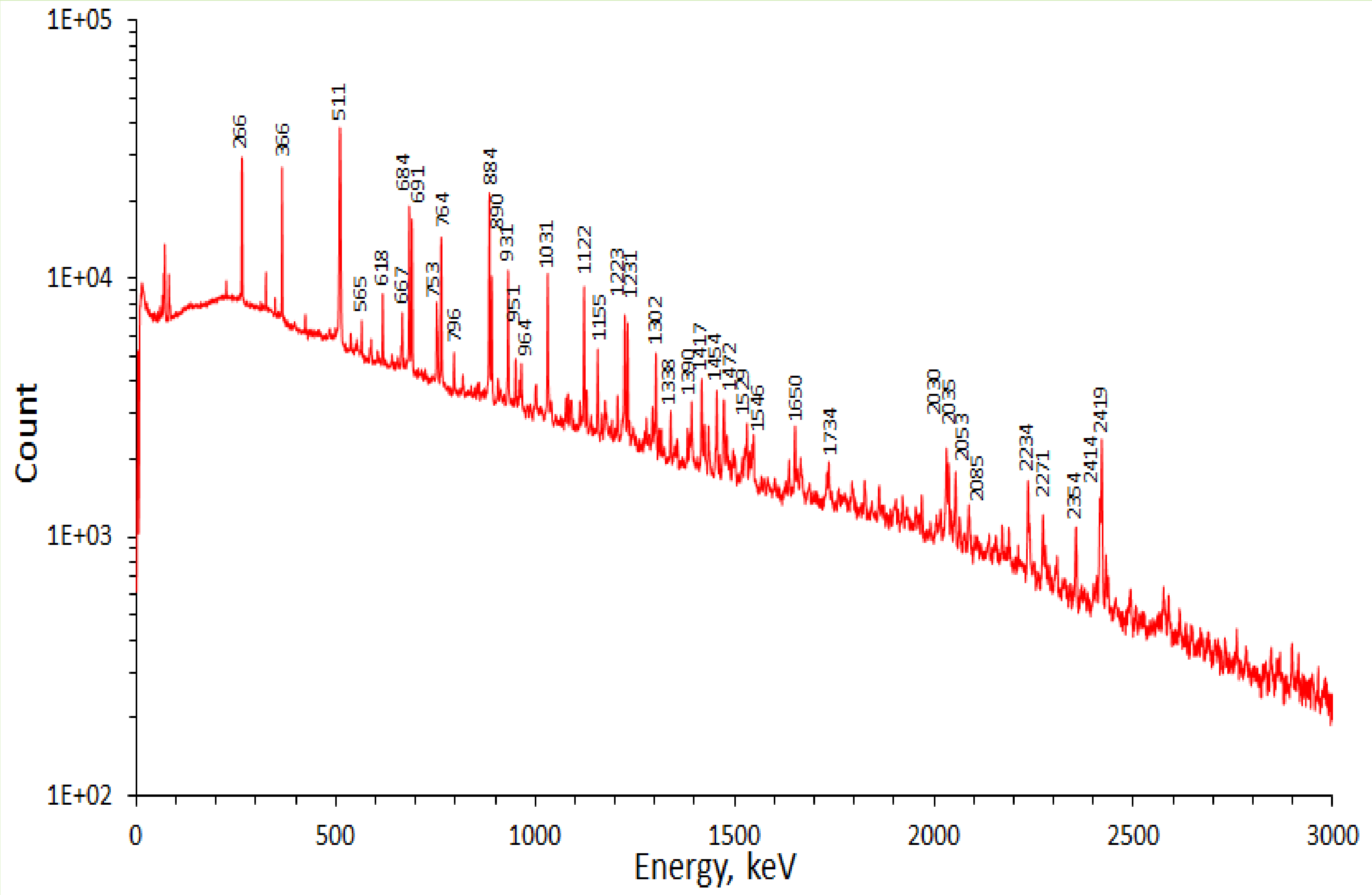
…but, conversion electrons not detected,
at *Eγ* > 5MeV no exponential dependence of efficiency from Eγ,
TAS spectra analysis may give not correct result,
can not separate GT and FF beta transitions, can not measure fine structure,…. *etc*.
…BUT, in combination with high resolution methods it is very useful



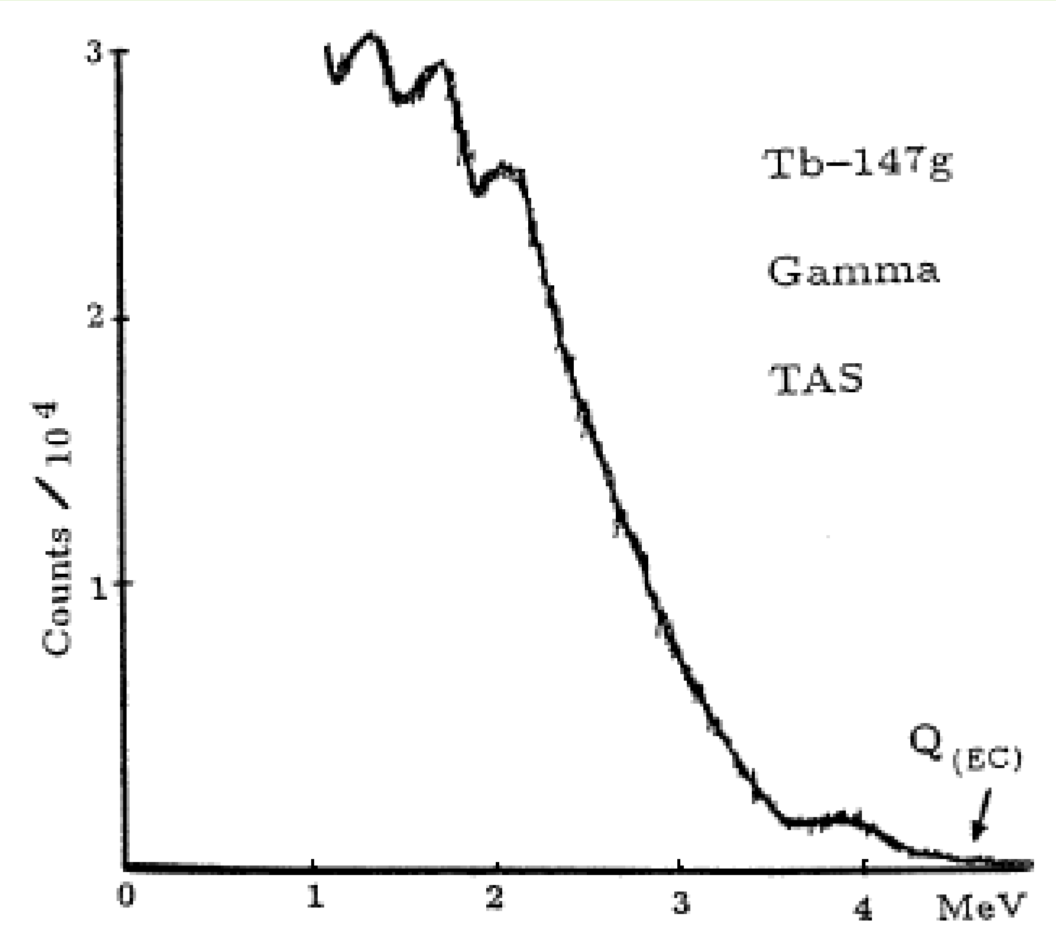


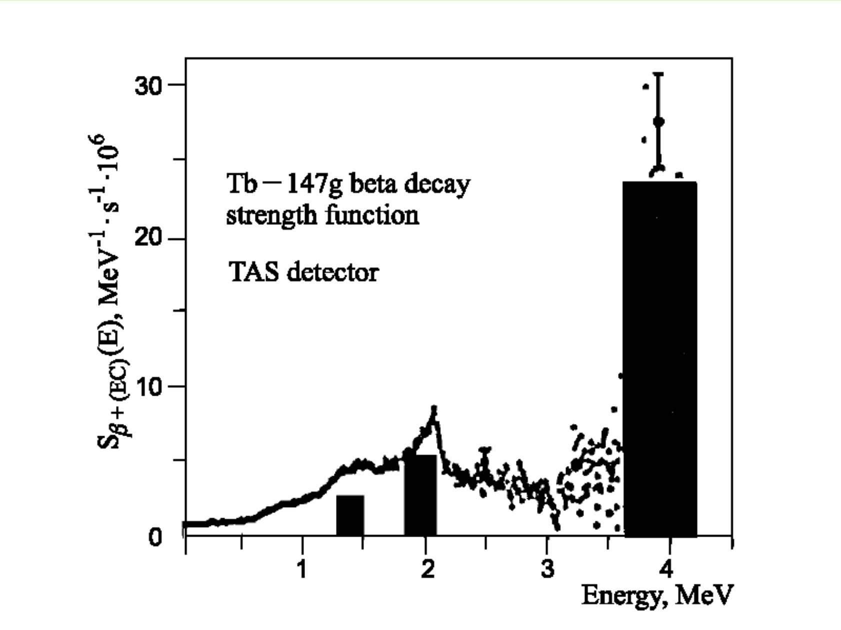
Experimental (*a*) and fitted (*b*) TAS spectra of 156-Ho ($T_{1/2}$ ≈*56min)*

Q_{EC}=(5.05±0.07)MeV

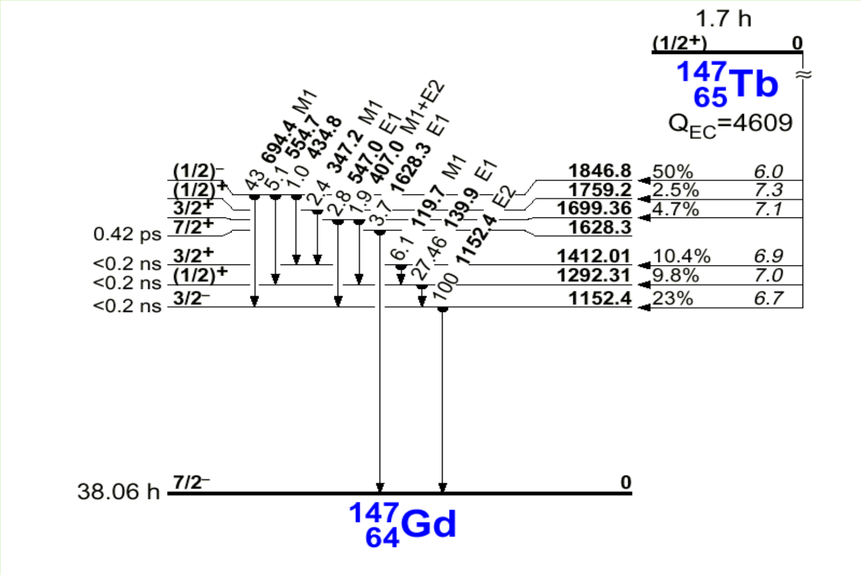


γ-spectrum, ^{156g}Ho(56 min), HpGe (50%)-detector


Tb–147g
Gamma
TAS
Counts / 10⁴
Q (EC)
MeV
0
1
2
3
4
1
2
3



Tb − 147g beta decay
strength function
TAS detector
$S_{\beta+(EC)}(E)$, MeV^{-1}·s^{-1}·10^6
Energy, MeV
0
10
20
30
1
2
3
4



1.7 h
(1/2+) 0
$^{147}_{65}$Tb
Q_{EC}=4609
(1/2)− 1846.8 50% 6.0
(1/2)+ 1759.2 2.5% 7.3
3/2+ 1699.36 4.7% 7.1
0.42 ps 7/2+ 1628.3
<0.2 ns 3/2+ 1412.01 10.4% 6.9
<0.2 ns (1/2)+ 1292.31 9.8% 7.0
<0.2 ns 3/2− 1152.4 23% 6.7
38.06 h 7/2− 0
$^{147}_{64}$Gd
43 694.4 M1
5.1 554.7
1.0 434.8
2.4 347.2 M1
2.8 547.0 E1
1.9 407.0 M1+E2
3.7 1628.3 E1
6.1 119.7 M1
27.46 139.9 E1
100 1152.4 E2

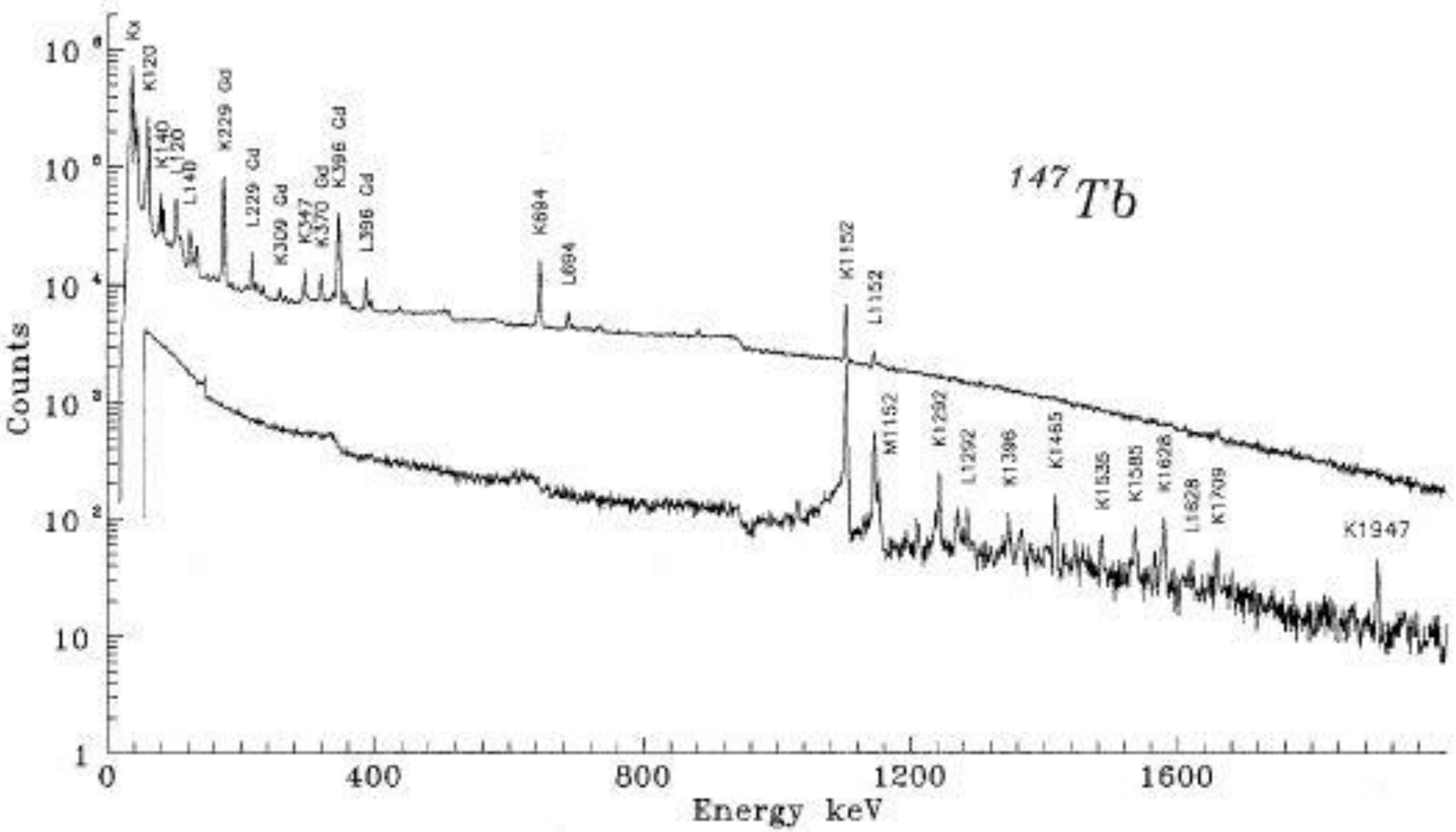
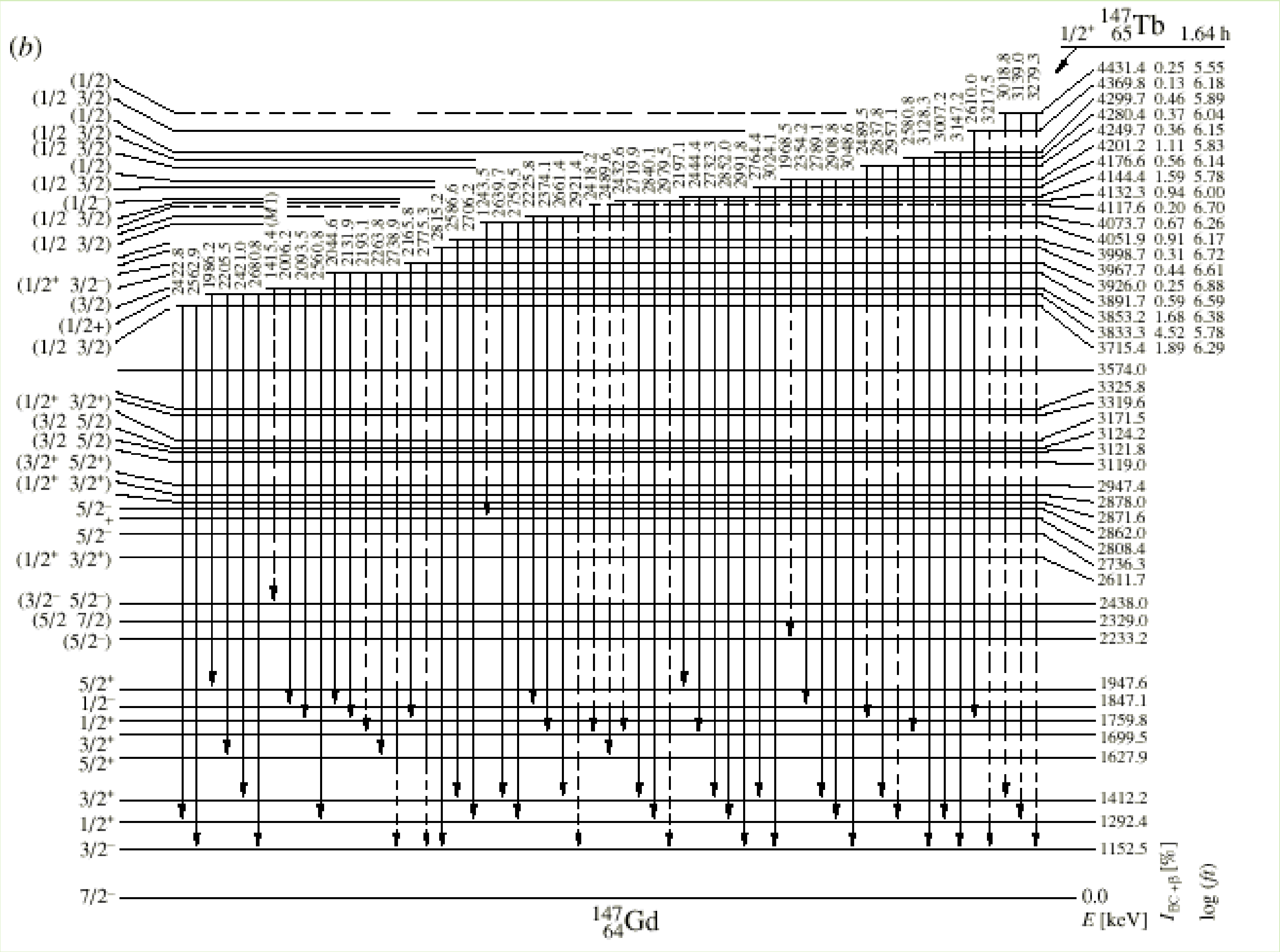
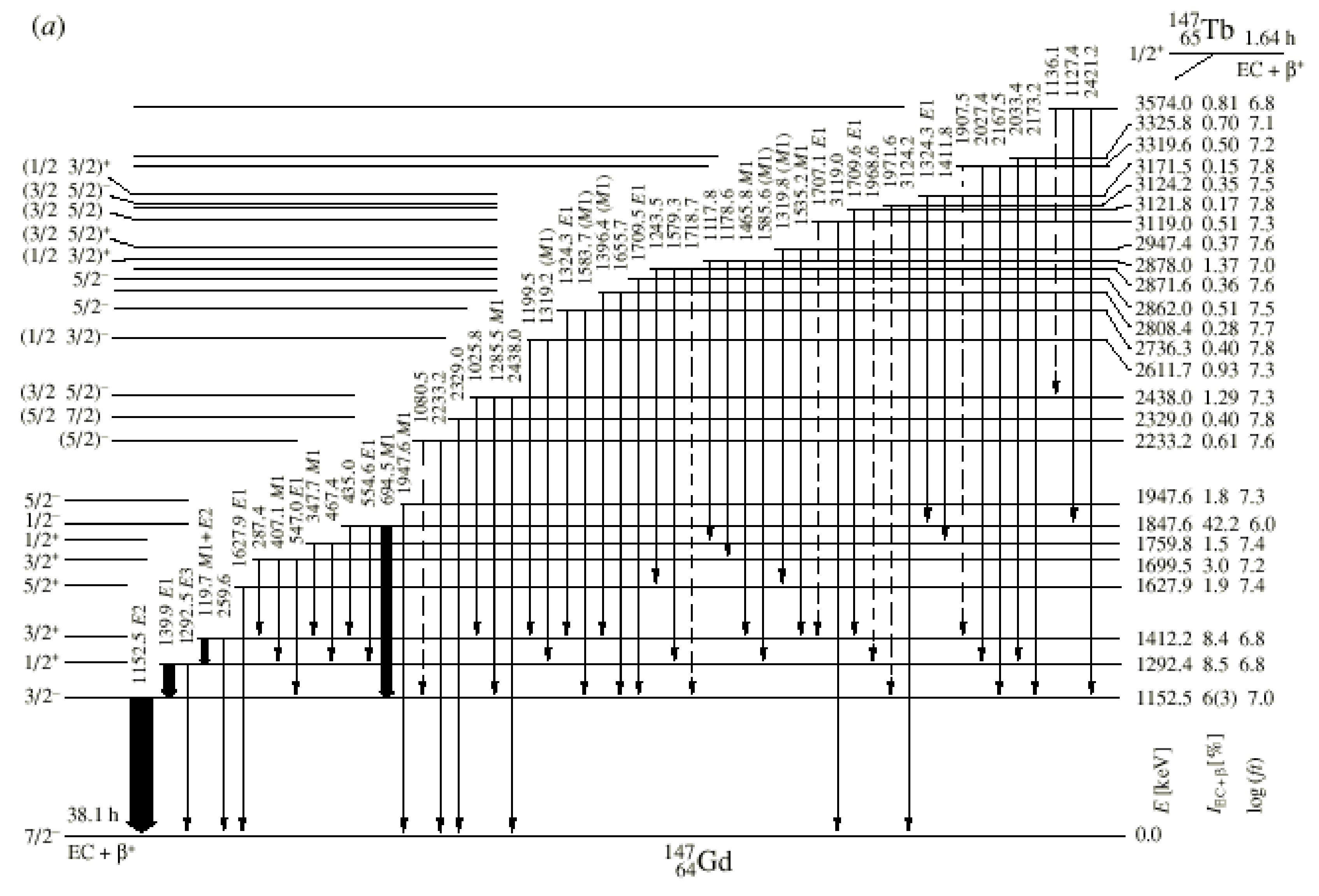
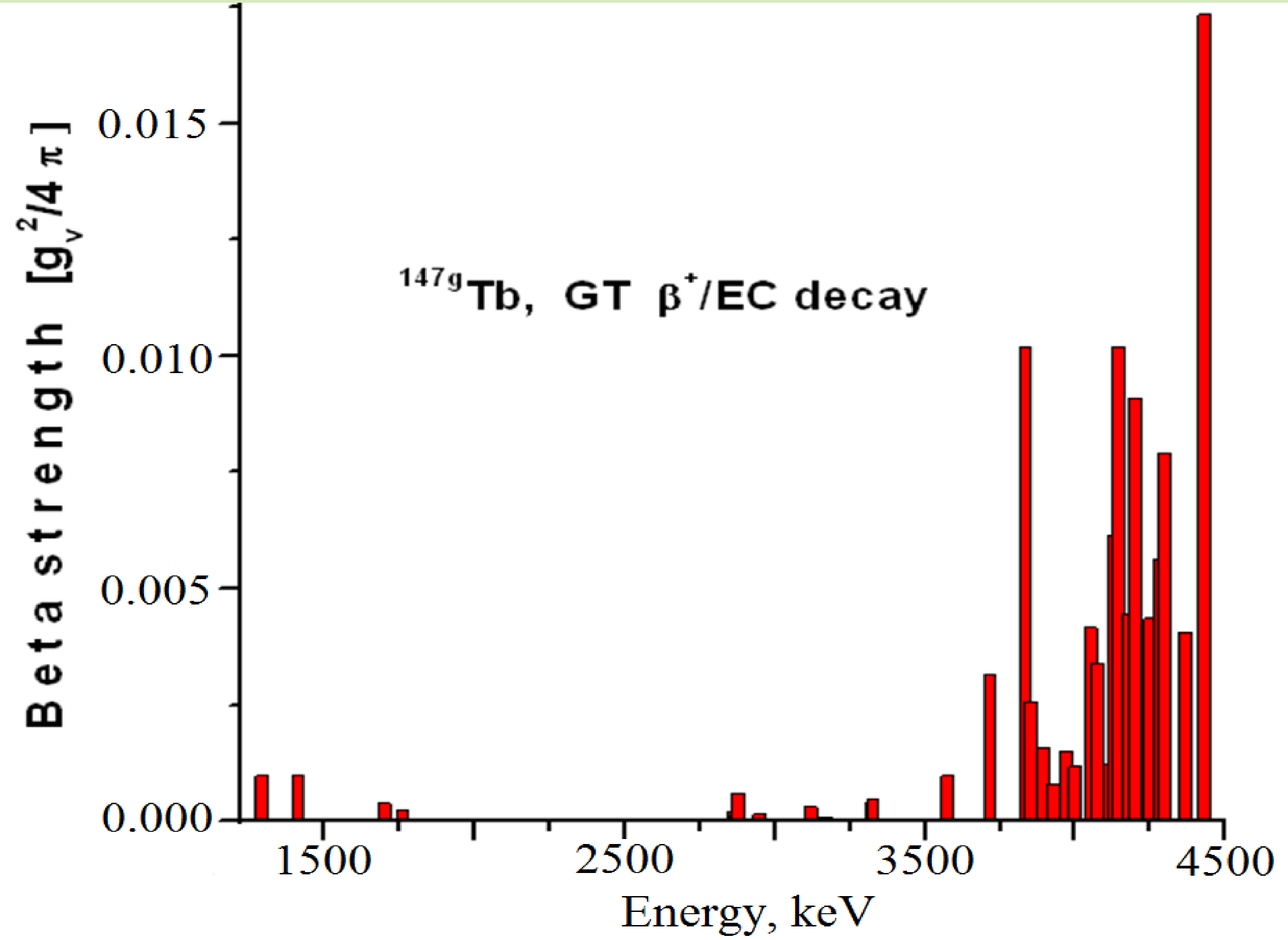


Fig. 2. Spectra of the internal conversion electrons at the decay of ^{147g}Tb, measured with a miniorange magnetic filter (*bottom*) and without it (*top*)

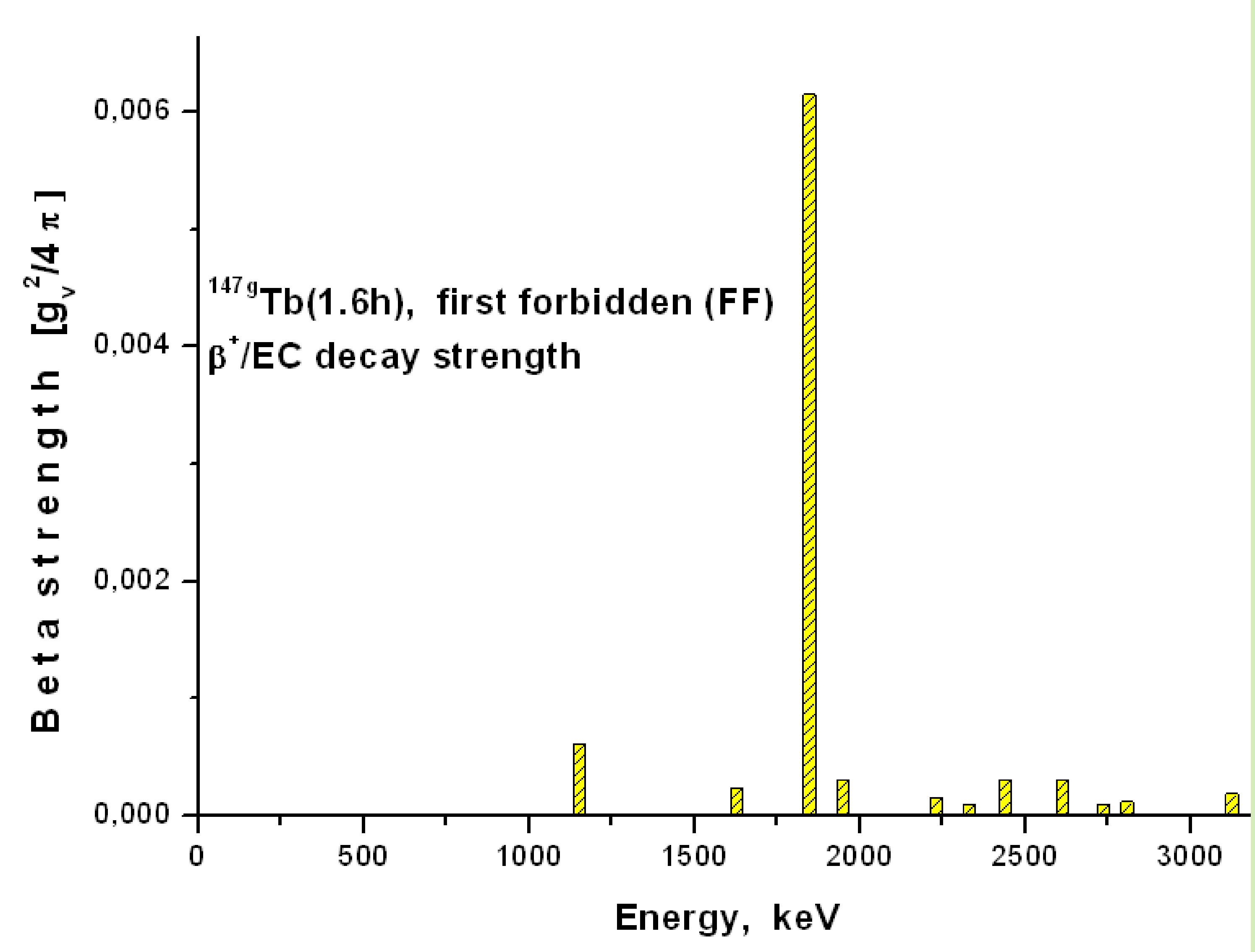


(b)
1/2+ 147 65 Tb 1.64 h
4431.4 0.25 5.55
4369.8 0.13 6.18
4299.7 0.46 5.89
4280.4 0.37 6.04
4249.7 0.36 6.15
4201.2 1.11 5.83
4176.6 0.56 6.14
4144.4 1.59 5.78
4132.3 0.94 6.00
4117.6 0.20 6.70
4073.7 0.67 6.26
4051.9 0.91 6.17
3998.7 0.31 6.72
3967.7 0.44 6.61
3926.0 0.25 6.88
3891.7 0.59 6.59
3853.2 1.68 6.38
3833.3 4.52 5.78
3715.4 1.89 6.29
3574.0
3325.8
3319.6
3171.5
3124.2
3121.8
3119.0
2947.4
2878.0
2871.6
2862.0
2808.4
2736.3
2611.7
2438.0
2329.0
2233.2
1947.6
1847.1
1759.8
1699.5
1627.9
1412.2
1292.4
1152.5
0.0
E [keV]
I_EC+β [%]
log (ft)
147 64 Gd
5/2+
1/2−
1/2+
3/2+
5/2+
3/2+
1/2+
3/2−
7/2−
(3/2− 5/2−)
(5/2 7/2)
(5/2−)
(1/2+ 3/2+)
(3/2 5/2)
(3/2 5/2)
(3/2+ 5/2+)
(1/2+ 3/2+)
5/2−
5/2−
(1/2+ 3/2+)
(1/2+ 3/2−)
(3/2)
(1/2+)
(1/2 3/2)
(1/2)
(1/2 3/2)
(1/2)
(1/2 3/2)
(1/2 3/2)
(1/2)
(1/2 3/2)
(1/2−)
(1/2 3/2)
(1/2 3/2)

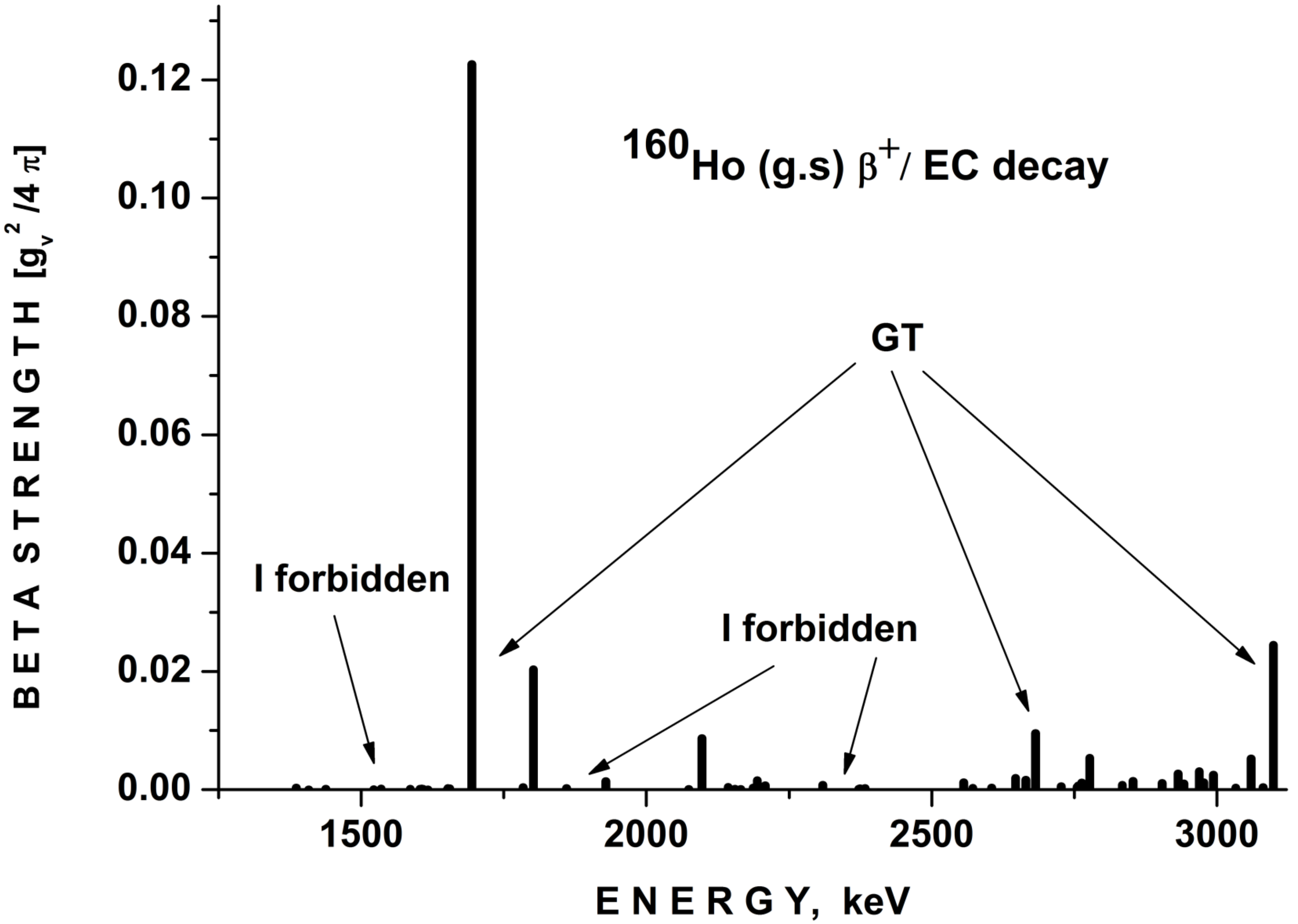

(a)
$^{147}_{65}$Tb 1.64 h
1/2⁺
EC + β⁺
1152.5 E2
139.9 E1
1292.5 E3
119.7 M1+E2
259.6
1627.9 E1
287.4
407.1 M1
547.0 E1
347.7 M1
467.4
435.0
534.6 E1
694.5 M1
1947.6 M1
1080.5
2233.2
2329.0
1025.8
1285.5 M1
2438.0
1199.5
1319.2 (M1)
1324.3 E1
1583.7 (M1)
1396.4 (M1)
1655.7
1709.5 E1
1243.5
1579.3
1718.7
1117.8
1178.6
1465.8 M1
1585.6 (M1)
1319.8 (M1)
1535.2 M1
1707.1 E1
3119.0
1709.6 E1
1968.6
1971.6
3124.2
1324.3 E1
1411.8
1907.5
2027.4
2167.5
2033.4
2173.2
1136.1
1127.4
2421.2
(1/2 3/2)⁺
(3/2 5/2)⁻
(3/2 5/2)
(3/2 5/2)⁺
(1/2 3/2)⁺
5/2⁻
5/2⁻
(1/2 3/2)⁻
(3/2 5/2)⁻
(5/2 7/2)
(5/2)⁻
5/2⁻
1/2⁻
1/2⁺
3/2⁺
5/2⁺
3/2⁺
1/2⁺
3/2⁻
7/2⁻
38.1 h
EC + β⁺
3574.0 0.81 6.8
3325.8 0.70 7.1
3319.6 0.50 7.2
3171.5 0.15 7.8
3124.2 0.35 7.5
3121.8 0.17 7.8
3119.0 0.51 7.3
2947.4 0.37 7.6
2878.0 1.37 7.0
2871.6 0.36 7.6
2862.0 0.51 7.5
2808.4 0.28 7.7
2736.3 0.40 7.8
2611.7 0.93 7.3
2438.0 1.29 7.3
2329.0 0.40 7.8
2233.2 0.61 7.6
1947.6 1.8 7.3
1847.6 42.2 6.0
1759.8 1.5 7.4
1699.5 3.0 7.2
1627.9 1.9 7.4
1412.2 8.4 6.8
1292.4 8.5 6.8
1152.5 6(3) 7.0
0.0
E [keV]
$I_{EC+\beta}$ [%]
log (ft)
$^{147}_{64}$Gd



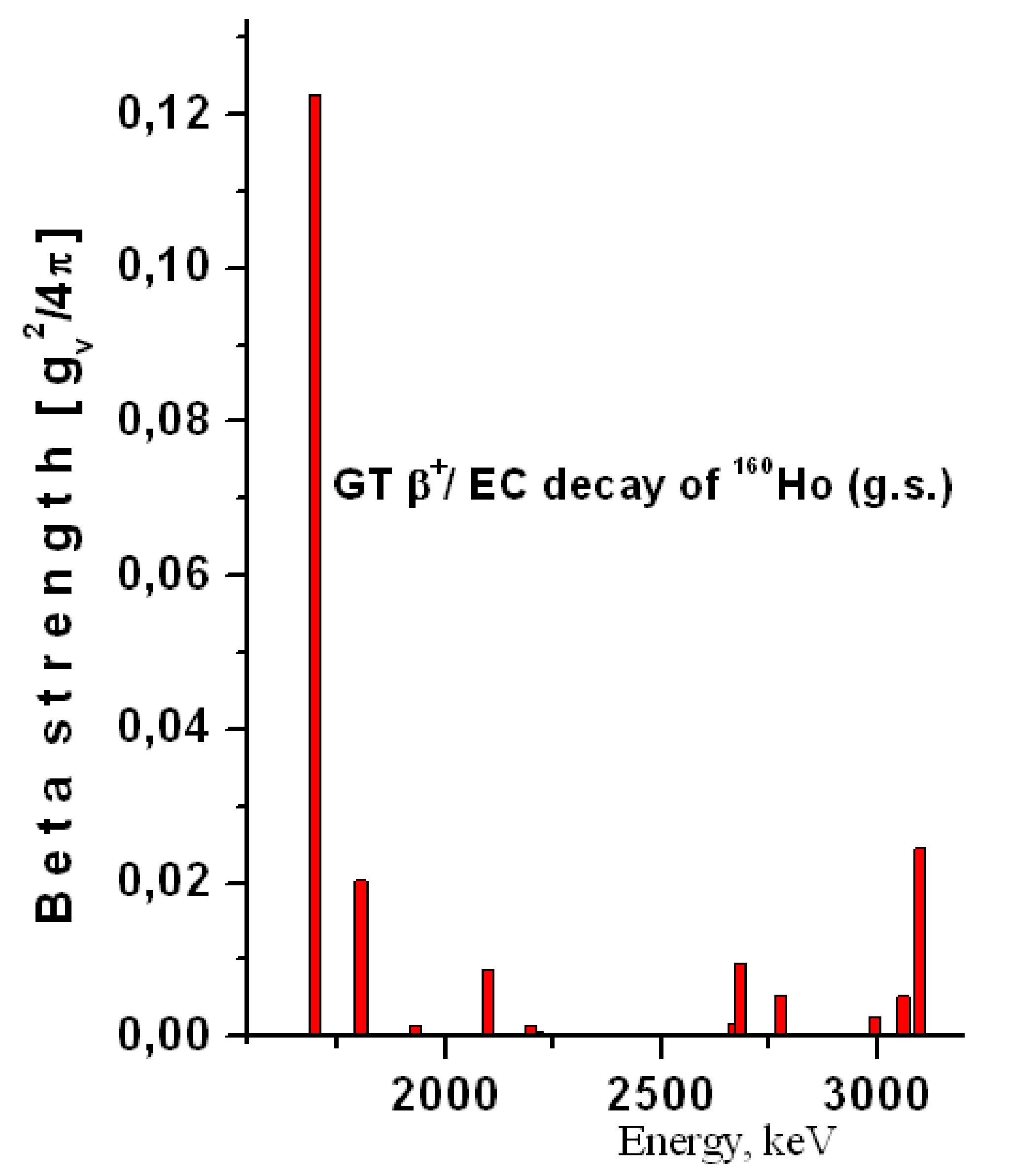
$S_β(E)$ for GT transitions in the β+/EC decay of the spherical nucleus ^{147g}Tb ($1/2^+$; $T_{1/2}$ = 1.6 h, Q_{EC} = 4.6 MeV).



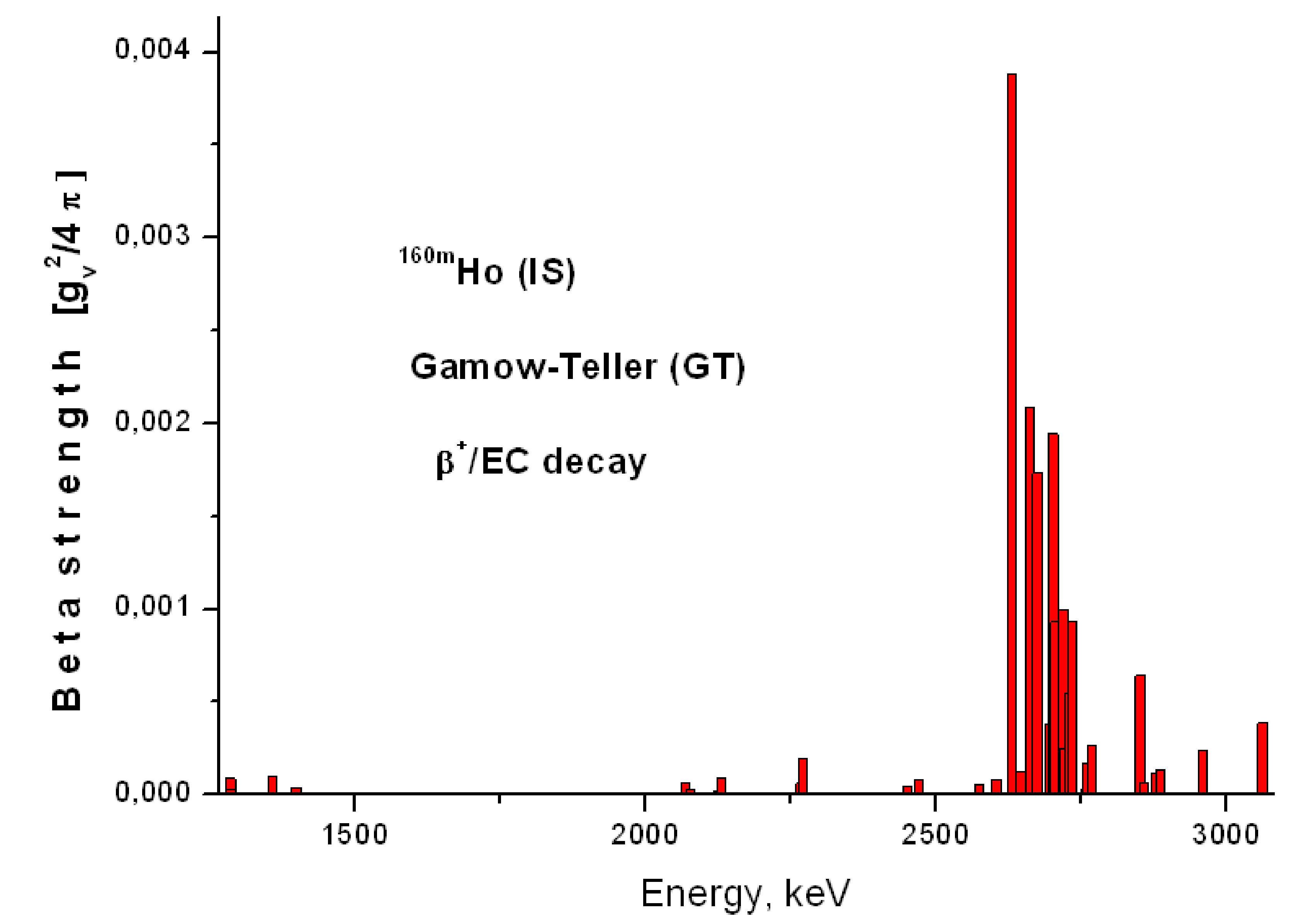
$S_\beta(E)$ for first-forbidden transitions in the β+/EC decay of the spherical nucleus ^{147g}Tb ($T_{1/2}$ = 1.6 h, Q_{EC} = 4.6 MeV).



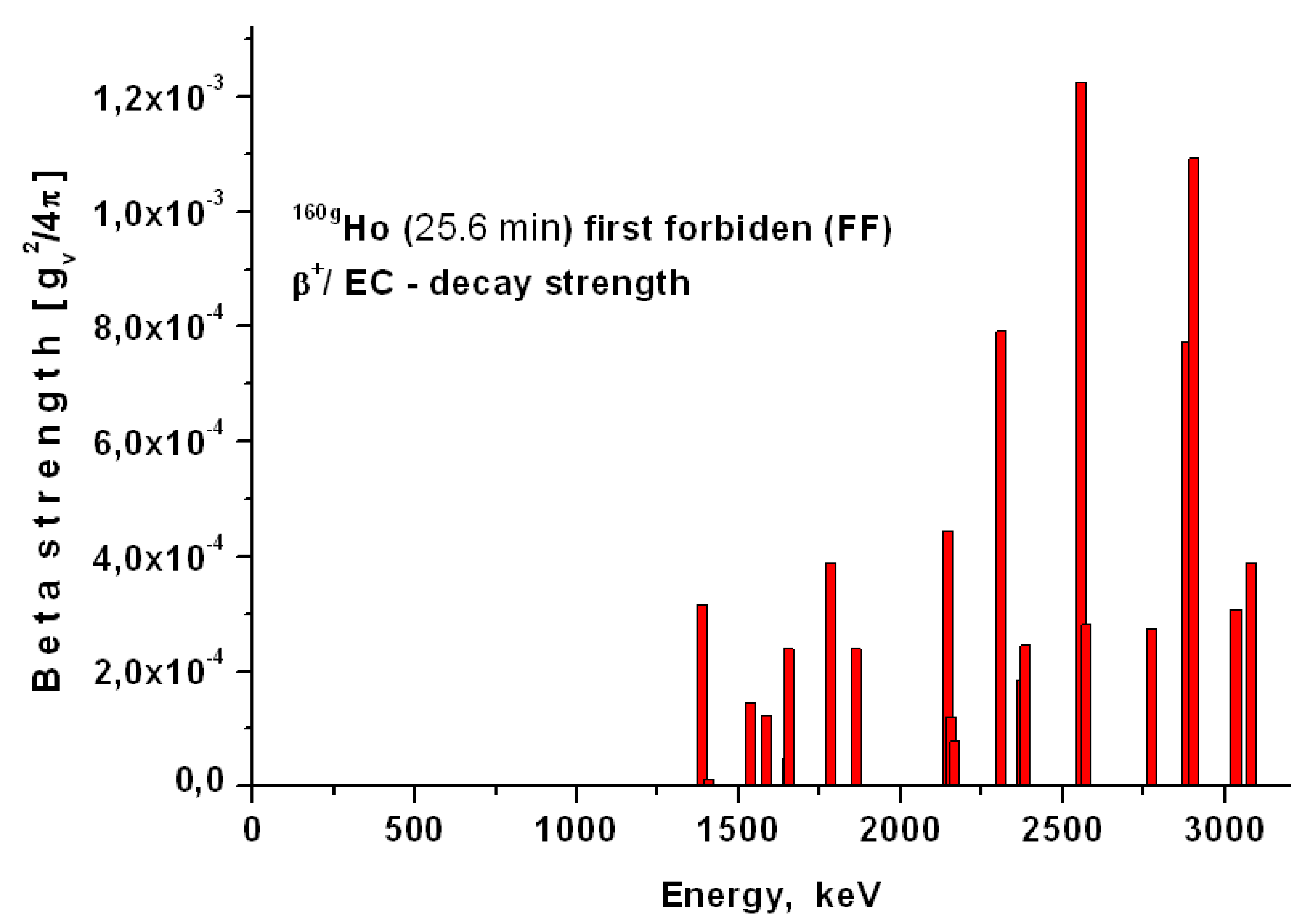
160Ho (g.s) β+/ EC decay
GT
I forbidden
I forbidden
BETA STRENGTH [gv2 /4 π]
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1500
2000
2500
3000
ENERGY, keV



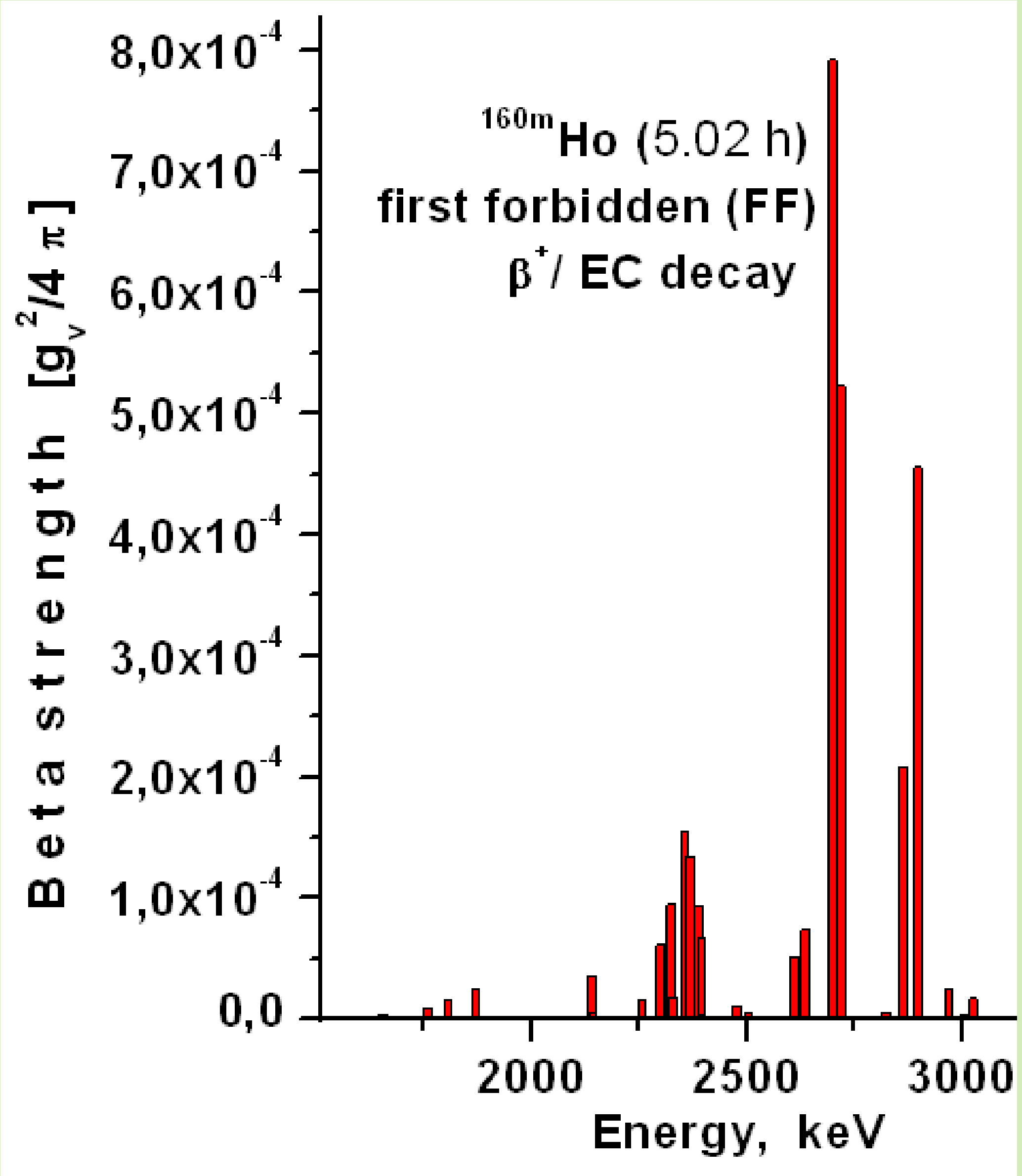
$S_\beta(E)$ for Gamow–Teller transitions in the β+/EC decay of the deformed nucleus ^{160g}Ho (5^+ ;25.6 min), Q_{EC} = 3286(15) keV.



$S_{\beta}(E)$ for Gamow–Teller transitions in the β+/EC decay of the deformed nucleus of the isomer ^{160m}Ho (2^{-}; 5.02 h), Q_{EC} = 3346 keV



$S_\beta(E)$ for Gamow–Teller transitions in the β+/EC decay of the deformed nucleus of the isomer ^{160m}Ho (2^-; 5.02 h), Q_{EC} = 3346 keV



$S_\beta(E)$ for first-forbidden transitions in the β+/EC decay of the deformed nucleus of the isomer ^{160m}Ho (5.02 h)

Bohr A., Mottelson B. // Nuclear Structure V. 1. 1969. Benjamin, New York

$$B^{\pm}(\mathrm{GT},E)=((g^{eff}_{A})^2/4\pi)\,|\langle I_f \| \sum t_{\pm}(k)\sigma(k) \| I_i\rangle|^2/(2I_i+1), \quad (1)$$

$$B^{\pm}(\mathrm{GT},E) = [D(g^2_{V}/4\pi)]/ft,\ D = (6144 \pm 2)\ \mathrm{sec} \quad (2)$$

$$S^{-} - S^{+} = 3(N\text{-}Z),\ \ \text{(Ikeda sum rule)} \quad (3)$$

$$S^{\pm} = \Sigma_f |\langle I_f \| \sum\ t_{\pm}(k)\,\sigma(k) \| I_i\rangle|^2/(2I_i+1), \quad (4)$$

$$\sum_j B^{-}(\mathrm{GT},E_j) - \sum_k B^{+}(\mathrm{GT},E_k) = 3(N\text{-}Z)(g^{eff}_{A})^2/4\pi, \quad (5)$$

$$d(I(E))/dE = S_{\beta}(E)\ T_{1/2}\, f(Q_{\beta} - E), \quad (6)$$

$$(T_{1/2})^{-1} = \int\ S_{\beta}(E) f(Q_{\beta} - E)\, dE, \quad (7)$$

$$\int_{\Delta E} S_{\beta}(E)dE = \sum_{\Delta E} 1/(ft), \quad (8)$$

where $S_{\beta}(E)$ is in units $\mathrm{Mev}^{-1}\ \mathrm{s}^{-1}$, and *ft* is in seconds.

$$\sum_j D/ft_j = 3(N\text{-}Z)\ (g^{eff}_{A}/g_{V})^2 \quad \text{(Ikeda sume rule if ALL GT strength is in } Q_{\beta} \text{ window)} \quad (9)$$

Instead of $B^{\pm}$(GT,E) (usually given in units of $(g^{eff}_A)^2/4\pi$) or in $(g^2_V/4\pi)$, and $(g^2_A/4\pi)$,) the quantities $B(\mathrm{GT},E)=|\langle I_f \| \sum t_{\pm}(k)\sigma(k) \| I_i\rangle|^2/(2I_i+1)$and $B'(\mathrm{GT},E)=4\pi/\, g^2_A\, B(\mathrm{GT},E)$ are often used in the literature.

Some times there are errors in due to not proper using $B^{\pm}$(GT,E), B'(GT,E), and B(GT,E).

The test of error absent is: in ALL cases one must obtain the formula:

$\sum_j D/ft_j = 3(N\text{-}Z)\,(g^{eff}_A/g_V)^2$ (Ikeda sume rule if ALL GT strength is in Q_β window)

The **second** method is based on so called **response function application**, but a lot of assumption must be done for extraction the $S_{\beta}(E)$ shape from the TAGS spectrum shape. Analysis depends on the assumptions about the decay scheme which as a rule is not known. It is very difficult to estimate the associated systematic errors of such analysis and **only qualitative information** about $S_{\beta}(E)$ may be obtained.

TAGS can't distinguish the GT and FF transitions and don't take into account the conversion electron emission, which give the **systematic uncertainties**, especially for high *Z*.

CONCLUSION

Only combination of TAGS with high resolution nuclear spectroscopy methods may give the quantitative information about $S_{\beta}(E)$.