CORRELATIONS BETWEEN PROPERTIES OF NUCLEAR MATTER AND NEUTRON STARS

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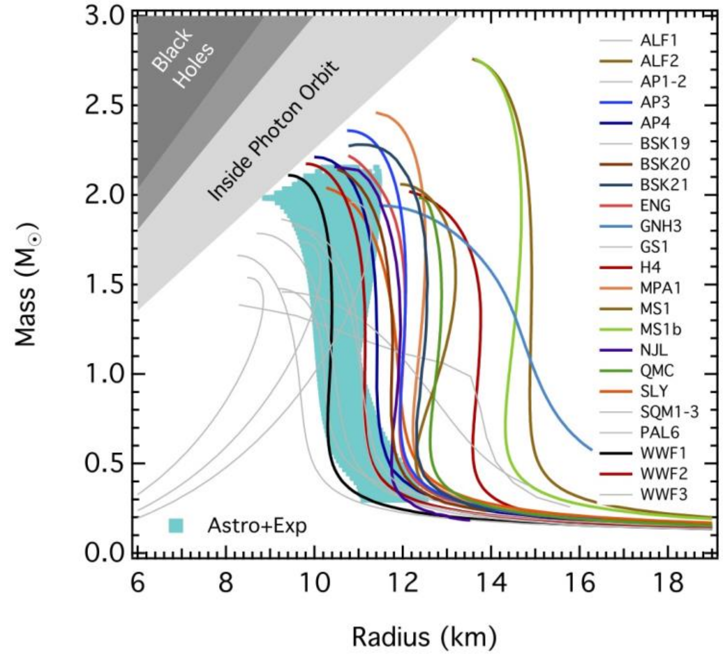
INTRODUCTION

Over the last decades, our knowledge on Neutron Stars (NS) has been greatly advanced: NS with large masses were discovered, radii of a number of NS were measured, and a gravitational signal from the merger of two NS was observed.

Many authors have been suggested the interplay between the properties of effective nucleon interactions used to calculate the equation of state and characteristics of NS. In this work, an attempt is made to put the study of such interplay on a quantitative footing. We analyze a large number of sets of parameters of the Skyrme nucleon-nucleon potential and calculate the coefficients of correlation between the saturation quantities of the nuclear matter and the NS properties.

Mass-radius relation for NS



SKYRME INTERACTION

The Skyrme interaction is used to calculate equation of state of nuclear matter.

It is a nonrelativistic self-consistent mean field model based on effective energy density functionals and the properties of the N-N interaction.

$$
\begin{aligned}
V(\vec{r_1}, \vec{r_2}) = & \, t_0(1 + x_0 P_\sigma)\delta(\vec{r}) \\
& + \frac{1}{2} t_1 (1 + x_1 P_\sigma)[\vec{P}'^2 \delta(\vec{r}) + \delta(\vec{r})\vec{P}^2] \\
& + t_2 \vec{P}' \delta(\vec{r}) \vec{P} \\
& + \frac{1}{6} t_3 (1 + x_3 P_\sigma)[\rho(\vec{R})]^\sigma \delta(\vec{r}) \\
& + i W_0 \vec{\sigma} [\vec{P}' \times \delta(\vec{r}) \vec{P}]
\end{aligned}
$$

$$
\vec{r} = \vec{r}_1 - \vec{r}_2, \; \vec{R} = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2), \; \vec{P} = \tfrac{1}{2i}(\nabla_1 - \nabla_2), \quad \vec{\sigma} = \vec{\sigma}_1 + \vec{\sigma}_2, \; P_\sigma = \tfrac{1}{2}(1 + \vec{\sigma}_1 \vec{\sigma}_2)
$$

NUCLEAR MATTER

Energy and mass density

$$\epsilon(n_e, n_p, n_n, n_\mu) = n_b\varepsilon + n_n m_n c^2 + n_p m_p c^2 + \epsilon_e(n_e) + \epsilon_\mu(n_\mu)$$

$$\rho(n) = \frac{\epsilon(n)}{c^2}$$

Symmetry energy (a_s),
The slope of a_s (L)
The symmetry incompressibility (K_{sym})

$$a_s = \frac{1}{8}\frac{\partial^2\varepsilon}{\partial Y_p^2}_{Y_p=1/2}$$

$$L = 3n_0(\frac{\partial a_s}{\partial n})_{n=n_0}$$

$$K_{sym} = 9n_0^2(\frac{\partial^2 a_s}{2})_{n=n_0}$$

Energy per particle

$$\begin{aligned}\varepsilon(Y_p, n) = \frac{E}{A} = \frac{H}{n} = &\frac{3}{5}\frac{\hbar^2}{2m}(\frac{3\pi^2}{2})^{2/3} n^{2/3} F_{5/3} \\ &+ \frac{1}{8} t_0 n[2(x_0+2) - (2x_0+1)F_2] \\ &+ \frac{1}{48} t_3 n^{\sigma+1}[2(x_3+2) - (2x_3+1)F_2] \\ &+ \frac{3}{40}(\frac{3\pi^2}{2}) n^{5/3}[[t_1(x_1+2) + t_2(x_2+2)]F_{5/3} \\ &+ \frac{1}{2}[t_2(2x_2+1) - t_1(2x_1+1)]F_{8/3}],\end{aligned}$$

$$F_m(Y_p) = 2^{m-1}[Y_p{}^m + (1-Y_p)^m]$$

Incompressibility (K_{inf})

$$K_{inf} = 9n^2\frac{\partial^2\varepsilon}{\partial n^2}$$

Isospin dependence of incompressibility at saturation density

$$K_{\tau,v} = \left(K_{\rm sym} - 6L - \frac{Q_o}{K_o}L\right)$$

NEUTRON STAR MATTER

β-equilibrium matter

$$\begin{cases} \mu_p(Y_p) + \mu_e(Y_e) = \mu_n(Y_p) \\ \mu_\mu(Y_p, Y_e) = \mu_e(Y_e) \end{cases}$$

Chemical potentials

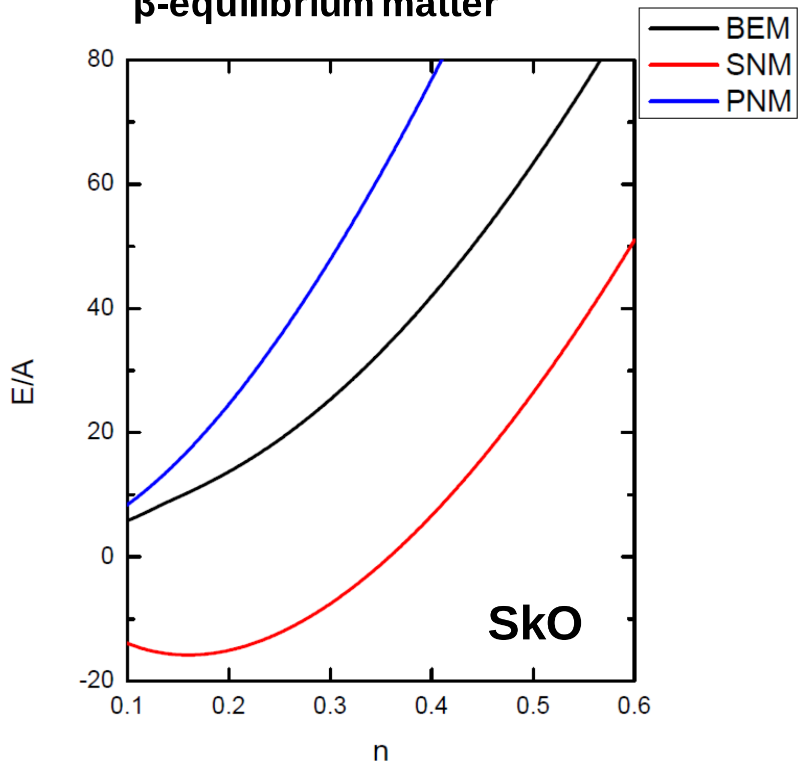
$$\mu_p = \varepsilon(n, Y_p) + n\frac{\partial \varepsilon}{\partial n}(n, Y_p) + (1 - Y_p)\frac{\partial \varepsilon}{\partial Y_p}(n, Y_p)$$

$$\mu_n = \varepsilon(n, Y_p) + n\frac{\partial \varepsilon}{\partial n}(n, Y_p) - Y_p\frac{\partial \varepsilon}{\partial Y_p}(n, Y_p)$$

$$\mu_e = \sqrt{{m_e}^2 + (3\pi^2 Y_e n)^{2/3}}$$

$$\mu_\mu = \sqrt{{m_\mu}^2 + (3\pi^2 Y_\mu n)^{2/3}}$$

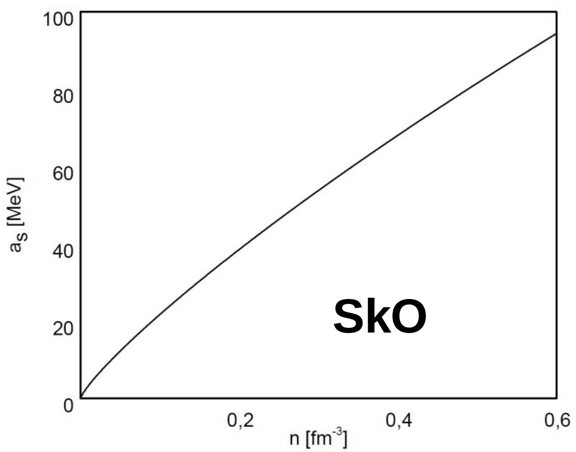
Energy per particle for symmetric nuclear matter, pure nuclear matter and β-equilibrium matter

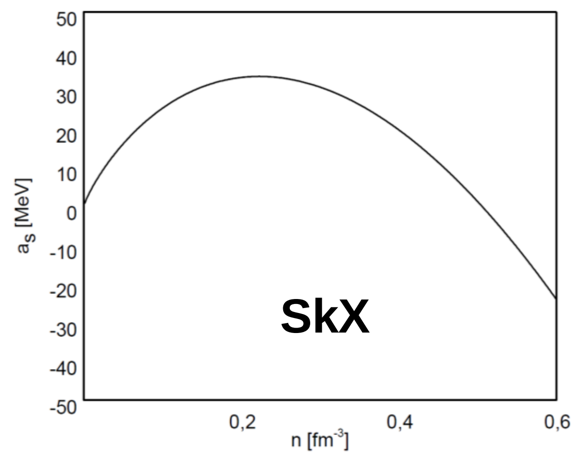


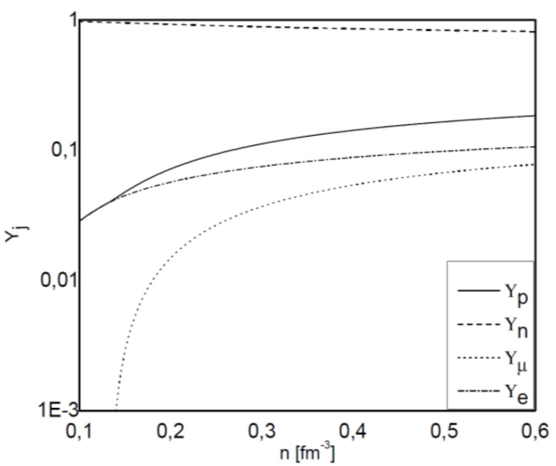
Characteristics for parameters fitting: $\mathbf{E_0}$, $\mathbf{a_{s0}}$

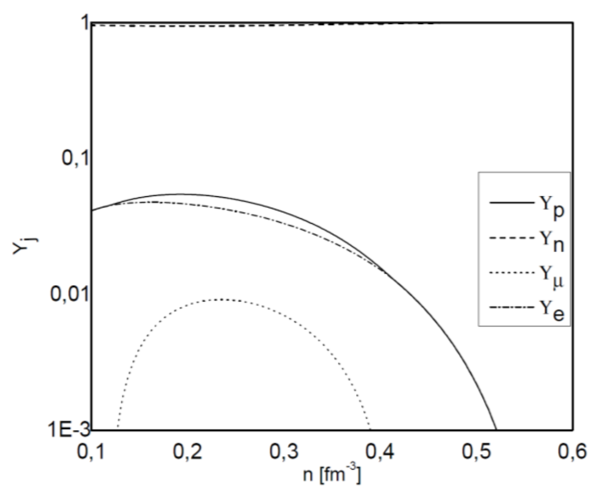
SYMMETRY ENERGY

For different Skyrme parametrizations the dependence of the symmetry energy on the density can be radically different. There are two main options for the behavior of the symmetry energy (**a_s**).









Models using parameterizations of the second group (on the right) predict significantly lower maximum mass of the neuron star and they are considered less suitable for this than parametrizations of the first group. However, some of them can be used to describe neutron stars. We used only those parameterizations of the second group for which the predicted maximum mass exceeds 1.4 solar masses (**1.4 M_s**).

NEUTRON STAR

Tolman-Oppenheimer-Volkov equation

$$\frac{dP}{dr} = \frac{G}{r^2}\frac{[\rho(r) + P(r)/c^2][m(r) + (4\pi r^3 P(r)/c^2)]}{1 - (2Gm(r)/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

Mass of the neutron star

$$M = \int_0^R 4\pi r^2 \rho dr$$

Baryon number

$$A = \int_0^R \frac{4\pi r^2 n dr}{(1 - (2Gm(r)/rc^2)^{1/2})}$$

Calculated properties of neutron stars:

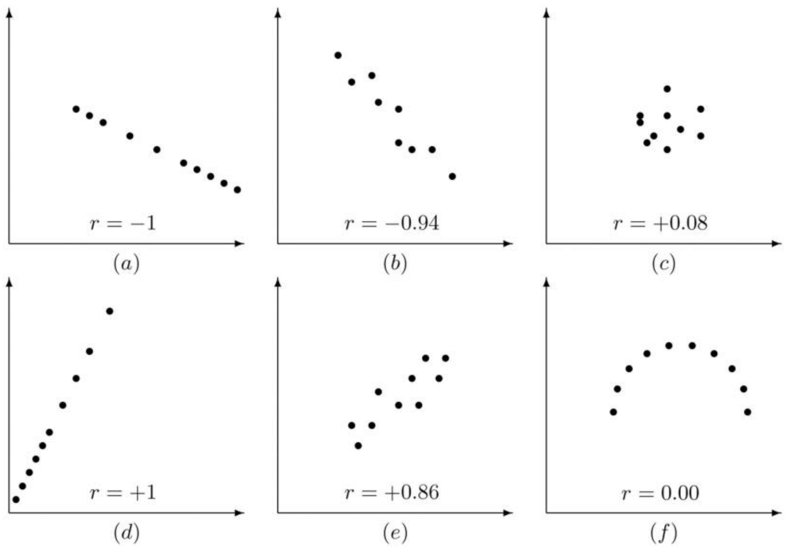
- Mass, radius, baryon number and central density for a star of ***maximum mass*** – **M_{max}, $R(M_{max})$, $A(M_{max})$** and **$n(M_{max})$.**
- Radius, baryon number and central density for a star with "canonical" ***$M=1.4\ M_s$*** – **$R(1.4\ M_s)$, $A(1.4\ M_s)$** and **$n(1.4\ M_s)$.**

CORRELATIONS

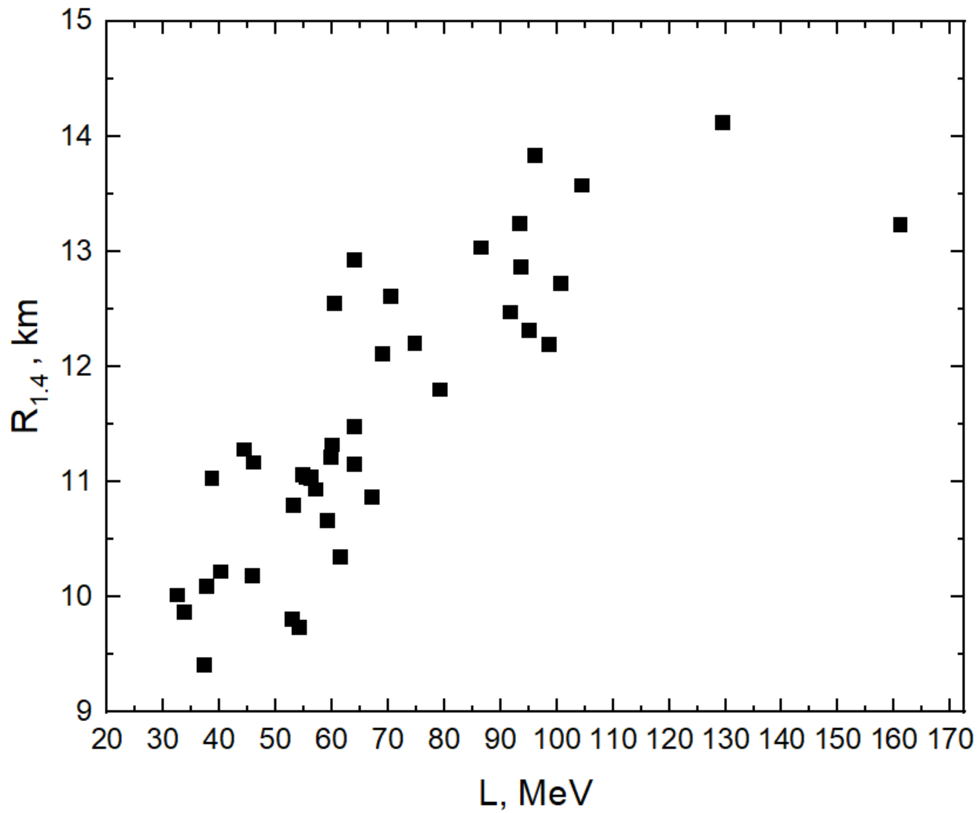
For numerical estimation of correlations the Pearson coefficient was used.

$$r_{XY} = \frac{\sum(X-\overline{X})(Y-\overline{Y})}{\sqrt{\sum(X-\overline{X})^2\sum(Y-\overline{Y})^2}}$$

The correlation coefficient ranges from −1 to 1 and measures **linear** correlation between two variables



Correlations between **R(1.4 M$_s$)** and **L**

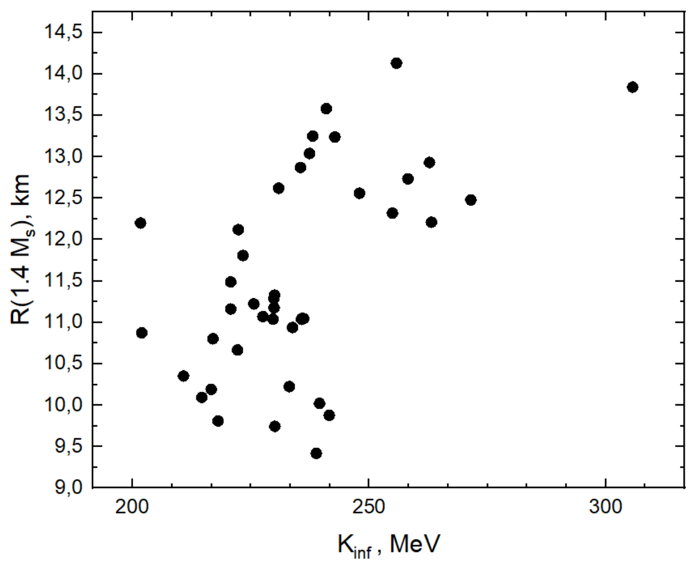


CORRELATIONS BETWEEN PROPERTIES OF NEUTRON STARS MATTER AND PROPERTIES OF NEUTRON STARS

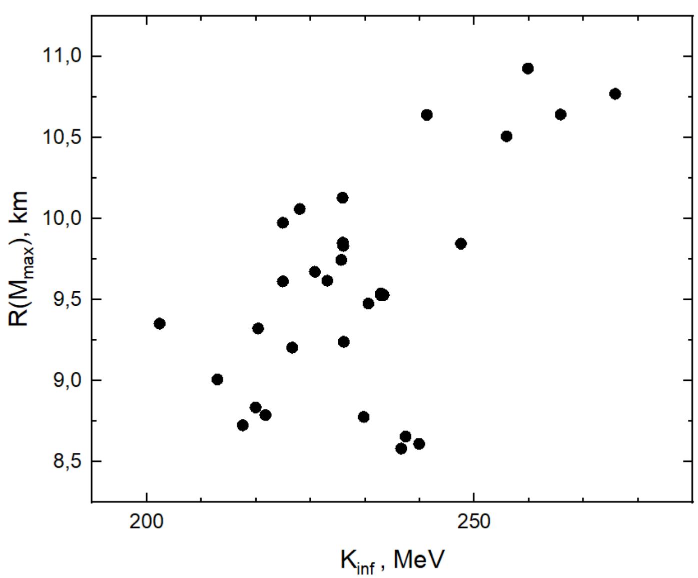
	N (1,4 Ms)	R(1,4 Ms)	A(1,4 Ms)	n (Mmax)	Mmax	R(Mmax)
E_0	0,18	-0,09	0,30	0,24	-0,30	-0,49
K_{inf}	-0,46	0,55	-0,62	-0,63	0,64	0,58
a_S	-0,35	0,39	-0,52	-0,38	0,29	0,65
L	-0,63	0,82	-0,76	-0,68	0,53	0,72
K_{sym}	-0,73	0,88	-0,72	-0,76	0,65	0,75
Q_{sym}	0,45	-0,71	0,66	0,51	-0,33	-0,46
$K_{\tau,v}$	0,34	-0,55	0,71	0,48	-0,33	-0,52
m*	0,28	-0,37	0,31	0,43	-0,48	-0,29

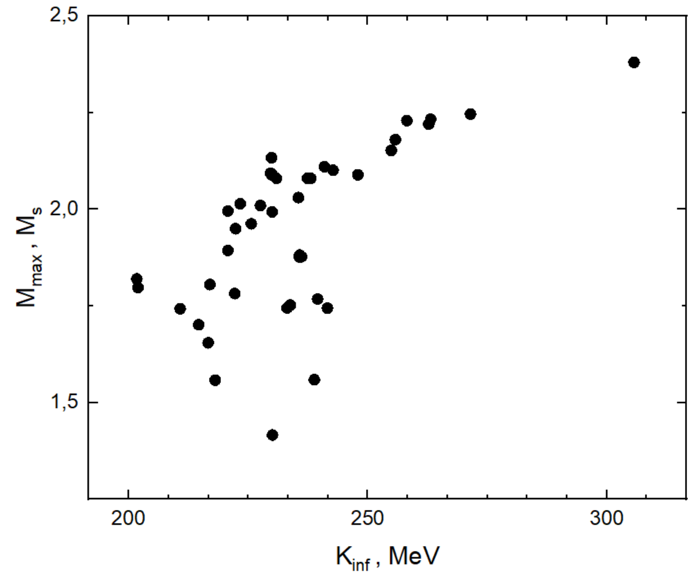
CORRELATIONS BETWEEN INCOMPRESSIBILITY (K_{INF}) AND NEUTRON STAR PROPERTIES

	N (1,4 Ms)	R(1,4 Ms)	A(1,4 Ms)	n (Mmax)	Mmax	R(Mmax)
E_0	0,18	-0,09	0,30	0,24	-0,30	-0,49
K_{inf}	-0,46	0,55	-0,62	-0,63	0,64	0,58
a_S	-0,35	0,39	-0,52	-0,38	0,29	0,65
L	-0,63	0,82	-0,76	-0,68	0,53	0,72
K_{sym}	-0,73	0,88	-0,72	-0,76	0,65	0,75
Q_{sym}	0,45	-0,71	0,66	0,51	-0,33	-0,46
$K_{\tau,v}$	0,34	-0,55	0,71	0,48	-0,33	-0,52
m*	0,28	-0,37	0,31	0,43	-0,48	-0,29

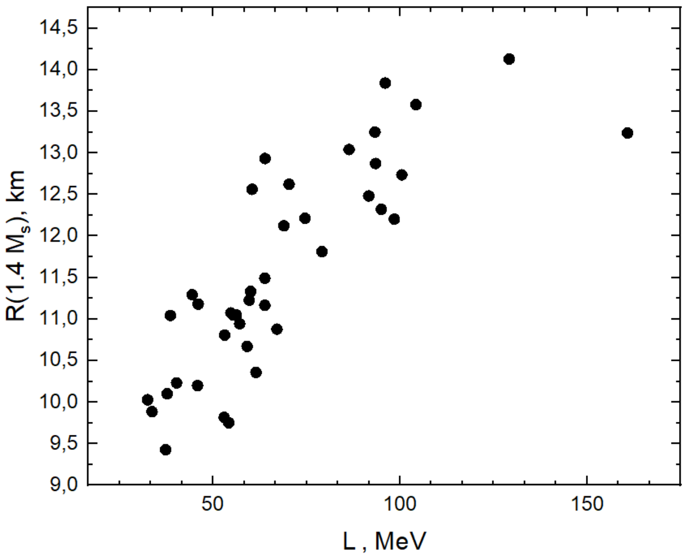


$$K_{inf} = 9n^2 \frac{\partial^2 \varepsilon}{\partial n^2}$$



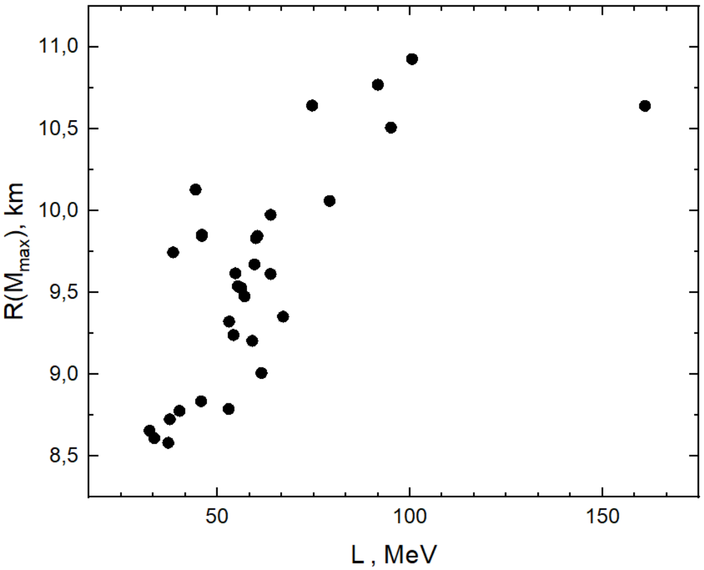


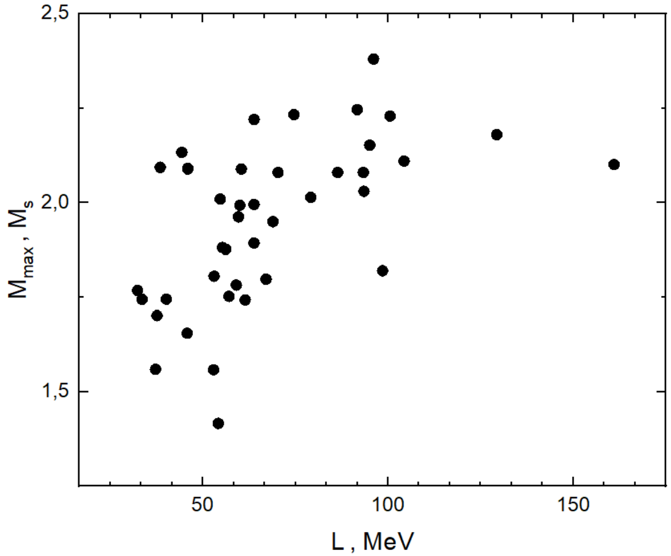
CORRELATIONS BETWEEN THE SLOPE OF SYMMETRY ENERGY (L) AND NEUTRON STAR PROPERTIES



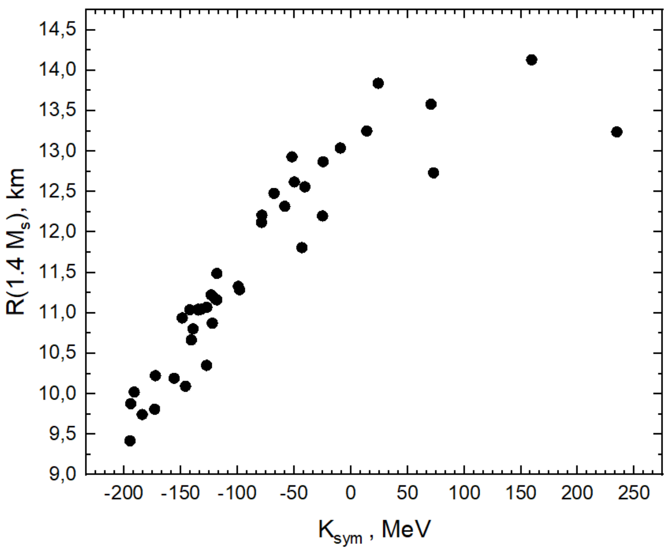
	N (1,4 Ms)	R(1,4 Ms)	A(1,4 Ms)	n (Mmax)	Mmax	R(Mmax)
E_0	0,18	-0,09	0,30	0,24	-0,30	-0,49
K_{inf}	-0,46	0,55	-0,62	-0,63	0,64	0,58
a_s	-0,35	0,39	-0,52	-0,38	0,29	0,65
L	-0,63	0,82	-0,76	-0,68	0,53	0,72
K_{sym}	-0,73	0,88	-0,72	-0,76	0,65	0,75
Q_{sym}	0,45	-0,71	0,66	0,51	-0,33	-0,46
$K_{\tau,v}$	0,34	-0,55	0,71	0,48	-0,33	-0,52
m*	0,28	-0,37	0,31	0,43	-0,48	-0,29

$$L = 3n_0\left(\frac{\partial a_s}{\partial n}\right)_{n=n_0}$$





CORRELATIONS BETWEEN SYMMETRY INCOMPRESSIBILITY (K_{SYM}) AND NEUTRON STAR PROPERTIES



	N (1,4 Ms)	R(1,4 Ms)	A(1,4 Ms)	n (Mmax)	Mmax	R(Mmax)
E_0	0,18	-0,09	0,30	0,24	-0,30	-0,49
K_{inf}	-0,46	0,55	-0,62	-0,63	0,64	0,58
a_s	-0,35	0,39	-0,52	-0,38	0,29	0,65
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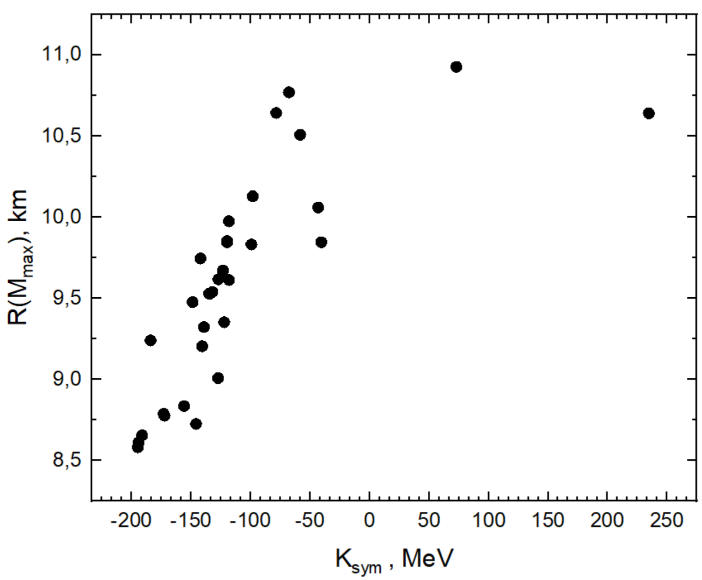
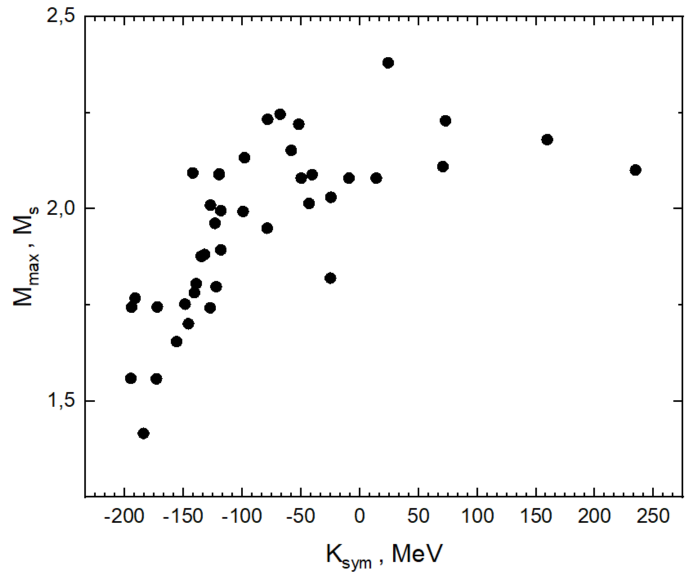
K_{sym}	-0,73	0,88	-0,72	-0,76	0,65	0,75
Q_{sym}	0,45	-0,71	0,66	0,51	-0,33	-0,46
$K_{\tau,v}$	0,34	-0,55	0,71	0,48	-0,33	-0,52
m*	0,28	-0,37	0,31	0,43	-0,48	-0,29

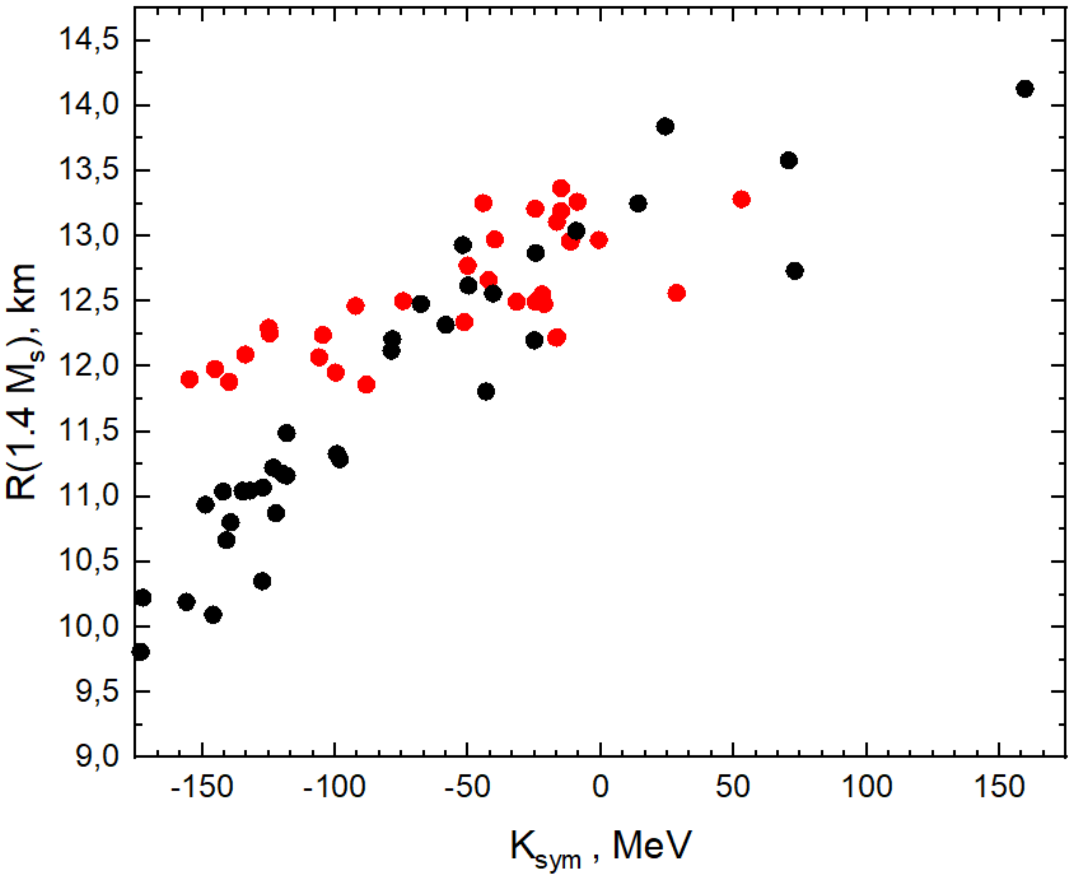
$$K_{sym} = 9n_0^2\left(\frac{\partial^2 a_s}{2}\right)_{n=n_0}$$





RELATIVISTIC MEAN FIELD MODEL

We have also considered some relativistic mean field models and got correlations for similar properties. (red - RMF, black - Skyrme)



CONCLUSION

- Using Skyrme parametrization for NN-interaction nuclear matter characteristics and mass-radius dependence for neutron stars are calculated.

- We use 42 different Skyrme parametrizations and calculate Pearson coefficients for correlations between nuclear matter characteristics and properties of neutron stars (M,R for a star of maximum mass and R (M=1.4 Ms)).

- The strongest effect on neutron star properties is provided by the slope of symmetry energy (L) and symmetry incompressibility (K_{sym}). Also weaker correlations are observed for incompressibility (K_{inf}) and effective mass (m*).

- As for compressive properties it turns out, that incompressibility dependence from isospin asymmetry is more important for neutron stars than the value of incompressibility of symmetric nuclear matter at saturation.

- Similar correlations are observed in some data from relativistic mean field model calculations, which indicates that deduced features are model independent.