

PROPERTIES OF COLLECTIVE STATES OF ISOTOPES ^{156}Gd

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Introduction

The gadolinium isotope with a mass A=156 is one of the most studied nuclei. The main reason is the large cross section (n, γ) reaction in ^{156}Gd , which provides big opportunities for studying the emission spectra in this reaction. Full results on this nucleus are given in references [1,2]. Several other nuclear processes supplement information on the levels and rotation bands in ^{156}Gd . In the reaction of $(\alpha, 2n)$ data were obtained on the states of rotational bands with $K^\pi = 0_1^+$ to $I = 26^+$, $K^\pi = 0_2^+$ to $I = 14^+$, $K^\pi = 0_3^+$ to $I = 10^+$, $K^\pi = 0_4^+$ to $I = 6^+$, $K^\pi = 0_5^+$ to $I = 4^+$, $K^\pi = 2_1^+$ to $I = 15^+$ and $K^\pi = 2_2^+$ to $I = 4^+$.

In the present work, the having the experimental information of the rotational bands of ^{156}Gd is discussed. And also influence of mixture of $K^\pi=0^+$, 2^+ , and 1^+ bands to the energy levels and reduced probabilities of $E2-$ transitions and their ratio to nuclides of $^{156,158,160}\text{Gd}$ is discussed are analysed.

1 Model

To analyze the properties of low-lying positive parity states in Gd isotopes, the phenomenological model of [3] is used. This model takes into account the mixing of states of the $gr-$, $\beta-$, $\gamma-$ and $K^\pi = 1^+$ bands. The Hamiltonian model is

$$H = H_{rot} + H_{KK'} \quad (1)$$

$$H_{KK'} = \omega_K \delta_{K,K'} - \omega_{rot}(I) \cdot (j_x)_{KK'} \chi(I, K) \delta_{K,K'+1} \quad (2)$$

where $(j_x)_{K,K'} = \langle K | j_1 | K' \rangle$ – is the matrix element of the Coriolis coupling of the rotational band members, $\omega_{rot}(I)$ – is the rotational frequency of the core ($\omega_{rot}(I) = dE_{cor}(I)/dI$), ω_K – is the energies of the band heads, and

$$\chi(I,0) = 1, \quad \chi(I,1) = \left[1 - \frac{2}{I(I+1)} \right]^{\frac{1}{2}}.$$

The eigenfunction of Hamiltonian model (1) reads

The Eigen wave function of Hamiltonian is

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \left\{ \sqrt{2} \Psi_{gr,K}^I D_{M,0}^I(\theta) + \sum_{K'} \frac{\Psi_{K',K}^I}{\sqrt{1 + \delta_{K',0}^{(3)}}} \times \right. \\ \left. \times \left[D_{M,0}^I(\theta) b_{K'}^+ + (-1)^{I+K'} D_{M,-K'}^I(\theta) b_{-K'}^+ \right] \right\} |0\rangle$$

where $\Psi_{K',K}^I$ – is the amplitudes of basis states mixture. The states consist of $(4+\nu)$ bands, where ν – is the number of included 1^+ – states. It includes ground $|0\rangle$ state bands and the single – phonon $b_{\lambda=2,K}^+ |0\rangle = b_K^+ |0\rangle$ with $K^\pi = 0_m^+, 2_\ell^+$ and $K^\pi = 1_\nu^+$ – rotational bands.

By solving the Schrödinger equation

$$H_{K,n}^\sigma \Psi_{K,n}^I = \varepsilon_n^\sigma \Psi_{K,n}^I \quad (4)$$

we obtained wave function and energy of states with positive parity.

The energy of states is defined by equation

$$E_n^\sigma(I) = E_{rot}(I) + \varepsilon_n^\sigma(I). \quad (5)$$

Where energy of rotational core $E_{rot}(I)$ can be determine by different methods, for example, by Harris parameterization [4] of the angular moment and energy.

$$\sqrt{I(I+1)} = \mathfrak{J}_0 \omega_{rot}(I) + \mathfrak{J}_1 \omega_{rot}^3(I) \quad (6)$$

$$E_{rot}(I) = \frac{1}{2} \mathfrak{J}_0 \omega_{rot}^2 + \frac{3}{4} \mathfrak{J}_1 \omega_{rot}^4(I) \quad (7)$$

Where \mathfrak{J}_0 and \mathfrak{J}_1 – parameters of inertia of rotational core. The rotational frequency of the core $\omega_{rot}(I)$ is found by solving cubic equation (7). This equation has two imaginary roots and one real root. The real root is

$$\omega_{rot}(I) = \left\{ \frac{\tilde{I}}{2J_1} + \left[\left(\frac{J_0}{3J_1} \right)^3 + \left(\frac{\tilde{I}}{2J_1} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} + \left\{ \frac{\tilde{I}}{2J_1} - \left[\left(\frac{J_0}{3J_1} \right)^3 + \left(\frac{\tilde{I}}{2J_1} \right)^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \quad (8)$$

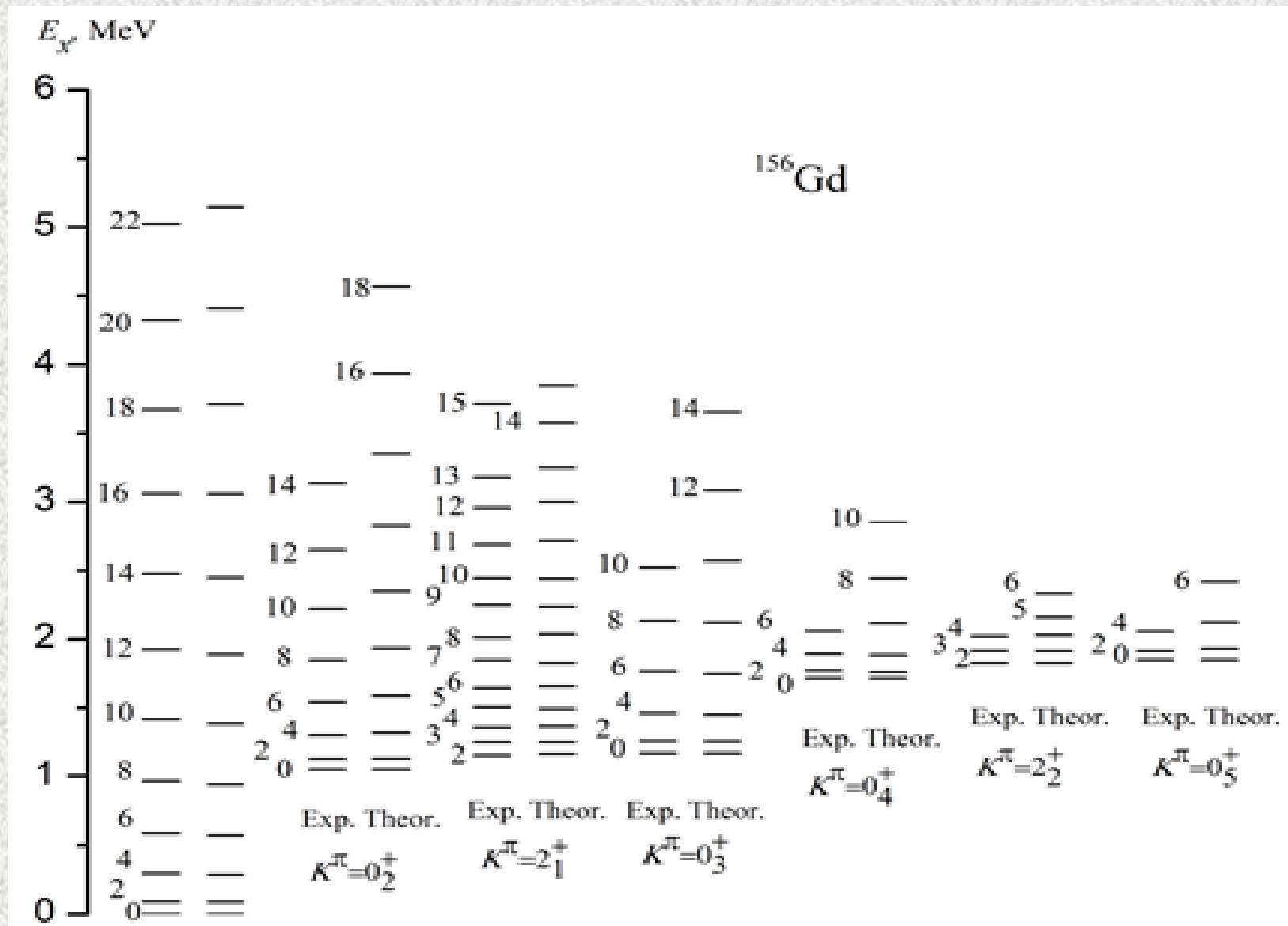
where $\tilde{I} = \sqrt{I(I+1)}$. Equation (8) gives $\omega_{rot}(I)$ at the given spin I of the core

The matrix elements of Coriolis mixture of rotational band states $(j_x)_{K,K'}$ and bandhead energy of gamma band (ω_γ) , where determined by fitting the calculated energy spectra with positive parity states from experimental data. In this case $(j_x)_{K,1_1} = (j_x)_{K,1_2}$ are equal each other. The value of parameters of calculation is given in Table 1.

| \mathfrak{I}_0 | \mathfrak{I}_1 | ω_{2_1} | ω_{2_2} | $\langle 0_1 \hat{j}_x 1_v^+ \rangle$ | $\langle 0_2 \hat{j}_x 1_v^+ \rangle$ | $\langle 0_3 \hat{j}_x 1_v^+ \rangle$ | $\langle 0_4 \hat{j}_x 1_v^+ \rangle$ | $\langle 0_5 \hat{j}_x 1_v^+ \rangle$ | $\langle 2_1^+ \hat{j}_x 1_v^+ \rangle$ | $\langle 2_2^+ \hat{j}_x 1_v^+ \rangle$ |
|------------------|------------------|----------------|----------------|---|---|---|---|---|---|---|
| 33.3 | 169.47 | 1.076 | 1.739 | 0.416 | 0.45 | 0.25 | 0.91 | 0.42 | 0.40 | 0.1 |

NOTE; $\mathfrak{I}_0(\hbar / MeV)$, $\mathfrak{I}_1(\hbar^4 / MeV^3)$ are the inertial parameters of rotating core (Harris parameters), $\omega_{2_{1,2}}$ (MeV) are energies of the $K^\pi = 2_{1,2}^+$ band heads, and $\langle K | \hat{j}_x | K' \rangle$ (MeV) are the matrix elements of the Coriolis interaction.

The comparison of theoretical and experimental data of energy spectra for the ^{156}Gd isotopes are given in *Figures 1* respectively.



Electrical Quadrupole γ - transitions

Electromagnetic transitions are important to understand the nature and analysis the various modes of nuclear excitations.

Using the wave functions which is calculated by solving the Schrodinger Eq. (4) , we can calculate E2- transitions from the $|I_i K_i\rangle$ state to the level of ground band states $|I_f 0_1\rangle$.

The formula of reduced probabilities of E2- transitions is follows as

$$B(E2; I_i K_i \rightarrow I_f 0_1) = \frac{1}{2I_i + 1} \left| \langle I_f 0_1 | \hat{m}(E2) | I_i K_i \rangle \right|^2 \quad (9)$$

where $\hat{m}(E2)$ is matrix element the operator of electric quadrupole transitions and it is obtained as

$$\begin{aligned}
\langle I_f 0_1 | \hat{m}(E2) | I_i K_i \rangle = & \sqrt{2I_i + 1} \left\{ \sqrt{\frac{5}{16\pi}} Q_0 \left[\Psi_{0_1, 0_1}^{I_f} \Psi_{0_1, K_i}^{I_i} C_{I_i 0; 20}^{I_f 0} \right. \right. \\
& + \sum_n \Psi_{K_n, 0_1}^{I_f} \Psi_{K_n, K_i}^{I_i} C_{I_i K_n; 20}^{I_f K_n} \Big] \\
& + \sqrt{2} \left[\Psi_{0_1, 0_1}^{I_f} \sum_n \frac{(-1)^{K_n} m_{K_n}}{\sqrt{1 + \delta_{K_n, 0}}} \Psi_{K_n, K_i}^{I_i} C_{I_i K_n; 2-K_n}^{I_f 0} \right. \\
& \left. \left. + \Psi_{0_1, K_i}^{I_i} \sum_n \frac{m_{K_n}}{\sqrt{1 + \delta_{K_n, 0}}} \Psi_{K_n, 0_1}^{I_f} C_{I_i 0; 2K_n}^{I_f K_n} \right] \right\} \quad (10)
\end{aligned}$$

where m_{K_n} in Eq. (10) is matrix elements between intrinsic wave function of ground bands $K_v^\pi = 0_1^+$ and other state bands, which is including to the basis state of Hamiltonian (1), Q_0 – intrinsic quadrupole moment of nuclei and $C_{I_i K_i; 2K_i + K_f}^{I_f K_f}$ – is a Clebsch – Gordon coefficients.

In the adiabatic approximation, the following equations are valid for $B(E2)$ factors from the states of the $\beta-$ and $\gamma-$ bands:

$$B^{rot}(E2; I_i \beta \rightarrow I_f gr) = \left| m_0 C_{I_i 0; 20}^{I_f 0} \right|^2 \quad (11)$$

$$B^{rot}(E2; I_i \gamma \rightarrow I_f gr) = 2 \left| m_2 C_{I_i 2; 2-2}^{I_f 0} \right|^2 \quad (12)$$

The Coriolis mixture do not played important rule in low spin $I=2$ and the values of parameters m_{β_1} , m_{β_2} and m_{γ} can be obtained from the adiabatic formula (11) and (12).

The comparison of theoretical calculation and experimental data of reduced probabilities of $B(E2)-$ transitions between the different state bands to the ground bands is given in Table 3.

Table 2. The values of the parameters m_K and the internal quadrupole moment Q_0 used in the calculations for the isotopes ^{156}Gd (in units of $e\text{Fm}^2$).

| Q_0 [6] | m_{0_2} | m_{0_3} | m_{0_4} | m_{0_5} | m_{1_ν} | m_{2_1} | m_{2_2} |
|-----------|-----------|-----------|-----------|-----------|-------------|-----------|-----------|
| 687 | -14.0 | 14.4 | 10.0 | -2.0 | -13.0 | 25.0 | 8.0 |

Table 3. The reduced probabilities of $E2 -$ transitions from states $K^\pi = 2_1^+ - 0_2^+ -$ and $0_3^+ -$ bands.

| $I_i K_i$ | $I_f K_f$ | $B(E2; I_i K_i \rightarrow I_f K_f), e^2 \text{ Fm}^4$ | | | |
|-----------|-----------|--|----------------|----------|-----------|
| | | exp. | exp. [1] | exp. [8] | This work |
| $2^+ 2_1$ | $0^+ 0_1$ | 222(11) [5,6] | 233(8) | 175(35) | 237 |
| $2^+ 2_1$ | $2^+ 0_1$ | 355(19) [5,6] | 361(13) | 255(55) | 354 |
| $2^+ 2_1$ | $4^+ 0_1$ | 32(3) [5,6] | 38(2) | 40(9) | 43 |
| $3^+ 2_1$ | $2^+ 0_1$ | 364(17) [6,7] | 364(70) | 385(75) | 379 |
| $3^+ 2_1$ | $4^+ 0_1$ | 280(60) [6,7] | 255(50) | 255(45) | 264 |
| $4^+ 2_1$ | $2^+ 0_1$ | 78(9) [6,7] | 90(+20–25) | 95(25) | 151 |
| $4^+ 2_1$ | $4^+ 0_1$ | 460(50) [6,7] | 509(+115–145) | 565(155) | 346 |
| $5^+ 2_1$ | $4^+ 0_1$ | 295 [6,7] | 399(+1000–250) | 500(380) | 283 |
| $5^+ 2_1$ | $6^+ 0_1$ | 410(40) [6,7] | 549(+1700–400) | 745(565) | 369 |
| $2^+ 0_2$ | $0^+ 0_1$ | 31,6(18) [5,6] | 31,4(30) | 43(15) | 51 |
| $2^+ 0_2$ | $2^+ 0_1$ | 164(16) [6,7] | 165(15) | 235(75) | 106 |
| $2^+ 0_2$ | $4^+ 0_1$ | 181(17) [6,7] | 205(20) | 275(95) | 33 |
| $4^+ 0_2$ | $2^+ 0_1$ | 61(7) [6,7] | 65(+25–35) | 79(32) | 81 |
| $4^+ 0_2$ | $4^+ 0_1$ | 140(13) [6,7] | – | 159(61) | 162 |
| $4^+ 0_2$ | $6^+ 0_1$ | 91(14) [6,7] | 105(+35–55) | 130(50) | 5 |
| $2^+ 0_3$ | $0^+ 0_1$ | 15,4(14) [5,6] | 15,4(2) | 11(4) | 25 |
| $2^+ 0_3$ | $2^+ 0_1$ | 8,4(+24–19) [6,7] | 21(+3–2) | 4,3(17) | 57 |
| $2^+ 0_3$ | $4^+ 0_1$ | 210(25) [6,7] | 215(25) | 153(58) | 153 |
| $4^+ 0_3$ | $2^+ 0_1$ | 15(4) [6,7] | – | – | 26 |
| $4^+ 0_3$ | $4^+ 0_1$ | 230(20) [6,7] | – | – | 75 |
| $4^+ 0_3$ | $6^+ 0_1$ | 370(30) [6,7] | – | – | 167 |

Table 4. Quadrupole electrical transitions $B(E2; I_i 0_1 \rightarrow I_f 0_1)$ in $0_1 -$ band ($e^2 b^2$).

| I_i | I_f | Exp. | This work |
|-------|-------|-----------------|-----------|
| 2 | 0 | 0,92(3) [9] | 0,94 |
| 4 | 2 | 1,29(2) [9] | 1,32 |
| 6 | 4 | 1,47(4) [9] | 1,46 |
| 8 | 6 | 1,57(15) [9,10] | 1,53 |
| 10 | 8 | 1,59(9) [9,10] | 1,57 |

Table 5. Theoretical and experimental values of the ratios $R_{IK} = B(E2; IK \rightarrow I_1 0_1) / B(E2; IK \rightarrow I_2 0_1)$ for transitions from the levels of $K = 2_1^+, 2_2^+, 0_2^+$ – and 0_3^+ – bands to the levels of the ground band.

| IK | I ₁ 0 ₁ | I ₂ 0 ₁ | Exp. | This work | Alaga rule [12] |
|-------------------------------|-------------------------------|-------------------------------|--|-----------|-----------------|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 2 ⁺ 2 ₁ | 2 ⁺ 0 ₁ | 0 ⁺ 0 ₁ | 1,55(1) [1] 1,54(5) [6] 1,56(17) [6] 1,75(55) [11] | 1,50 | 1,43 |
| 2 ⁺ 2 ₁ | 4 ⁺ 0 ₁ | 2 ⁺ 0 ₁ | 0,106(3) [1] 0,101(6) [6] 0,105(3) [6] | 0,122 | 0,05 |
| 3 ⁺ 2 ₁ | 4 ⁺ 0 ₁ | 2 ⁺ 0 ₁ | 0,70(3) [1] 0,67(18) [6] 0,77(15) [7] 0,56(21) [11] | 0,70 | 0,40 |
| 4 ⁺ 2 ₁ | 4 ⁺ 0 ₁ | 2 ⁺ 0 ₁ | 6,03(12) [1] 5,38(29) [6] 5,81(24) [6] 5,9(6) [7] | 2,29 | 2,95 |
| 4 ⁺ 2 ₁ | 6 ⁺ 0 ₁ | 4 ⁺ 0 ₁ | 0,046(8)[1] 0,030(6) [6] | 0,33 | 0,09 |
| 5 ⁺ 2 ₁ | 6 ⁺ 0 ₁ | 4 ⁺ 0 ₁ | 1,44(15) [1] 1,41(16) [6] 1,45(19) [6] 1,40(16) [7] | 1,30 | 0,57 |

| | | | | | |
|-----------------------------------|------------------------------------|-----------------------------------|---|-------------|-------------|
| 6⁺2₁ | 6⁺0₁ | 4⁺0₁ | 3,7 (3) [1] 5,9(14) [11] | 2,05 | 3,71 |
| 7⁺2₁ | 8⁺0₁ | 6⁺0₁ | 2,0(12) [11] | 1,92 | 0,67 |
| 9⁺2₁ | 10⁺0₁ | 8⁺0₁ | 2,5(12) [11] | 2,57 | 0,73 |
| 2⁺0₂ | 2⁺0₁ | 0⁺0₁ | 5,50(38) [1] 5,26(25) [6] 5,06(51) [7] | 2,10 | 1,43 |
| 2⁺0₂ | 4⁺0₁ | 2⁺0₁ | 1,18(8) [1] 1,17(5) [6] 1,10(11) [7] | 0,31 | 1,8 |
| 4⁺0₂ | 4⁺0₁ | 2⁺0₁ | 2,20(17) [1] 2,94(35) [6] 2,30(22) [7] | 2,01 | 0,91 |
| 4⁺0₂ | 6⁺0₁ | 4⁺0₁ | 0,71(33) [1] 0,7(3) [6] 0,65(9) [7] | 0,2 | 1,75 |
| 6⁺0₂ | 6⁺0₁ | 4⁺0₁ | 1,59(50) [1] 1,2(8) [11] | 1,76 | 0,81 |

| 1 | 2 | 3 | 4 | 5 | 6 |
|-------------------------|-------------------------|------------------------|---|-------------|-------------|
| 8+0₂ | 8+0₁ | 6+0₁ | 1,98(22) [1] | 1,32 | 0,59 |
| 10+0₂ | 10+0₁ | 8+0₁ | 18,5 (13) [1] >1,7 [11] | 0,95 | 0,74 |
| 2+0₃ | 2+0₁ | 0+0₁ | 3,94(18) [1] 0,50(15) [6] 0,55(17) [7] | 2,30 | 1,43 |
| 2+0₃ | 4+0₁ | 2+0₁ | 3,58(12) [1] 28(8) [6] 25(8) [7] | 2,69 | 1,8 |
| 4+0₃ | 4+0₁ | 2+0₁ | 16(5) [6] 15(4) [7] | 2,84 | 0,91 |
| 4+0₃ | 6+0₁ | 4+0₁ | 3,0(3) [6] 1,6(2) [7] | 2,23 | 1,75 |
| 3+2₂ | 4+0₁ | 2+0₁ | 1,41(12) [1] | 0.95 | 0.40 |
| 4+2₂ | 4+0₁ | 2+0₁ | 7,19(102) [1] | 4.53 | 2.95 |

Magnetic Dipole Transitions

The multipole mixing coefficients $\delta(E2/M1)$ are calculated for the transitions $I_i K_i \rightarrow I_f gr$ from the β_v- , $\gamma-$ and 1^+ bands for the ^{156}Gd .

$$\begin{aligned}\delta(I_i K_i \rightarrow I_f K_f) &= 0.834 \cdot E_\gamma(MeV) \\ &\times \frac{\langle I_f K_f | \hat{m}(E2) | I_i K_i \rangle}{\langle I_f K_f | \hat{m}(M1) | I_i K_i \rangle} \left(\frac{e \cdot b}{\mu_0} \right)\end{aligned}\quad (13)$$

The reduced matrix element of $M1-$ transition in Eq. (13) is determined as follows:

$$\begin{aligned} \langle I'0_1 | \hat{m}(M1) | IK \rangle &= \sqrt{\frac{3(2I+1)}{4\pi}} \\ &\times \left(\sum_{K_1=1}^2 (g_{K_1} - g_R) K_1 \Psi_{K_1, K}^I \Psi_{K_1, 0_1}^{I'} C_{IK_1; 10}^{IK_1} \right. \\ &\quad \left. + \frac{\sqrt{6}}{10} \sum_{\nu} m_{1_{\nu}}^+ (\Psi_{0_1, 0_1}^I \Psi_{1_{\nu}, K}^{I'} - \Psi_{1_{\nu}}^I \Psi_{0_1, K}^I) C_{I1; 1-1}^{I0} \right) \end{aligned} \quad (14)$$

where $m_{1_{\nu}}^+ = \langle 0_1 | \hat{m}(M1) | 1_{\nu}^+ \rangle$ is matrices elements between the wave functions of ground 0_1 and 1_{ν}^+ - bands, g_K – intrinsic g - factor of band with $K \neq 0$. From the systematic of g_R – factors of deformed nuclei rear – earth and transuranium region is follows:

$$g_R \approx 0.4 \pm 0.1 \quad (g_R = Z/A).$$

The values of parameter $m_{1_\nu}^{'}$ is given in Table 5, which is determined using the experimental data [3] for the probability of $M1-$ transitions from the state $I^\pi = 1_\nu^+$ to the level of ground bands. The adiabatical formula for the $M1-$ transitions has form:

$$B(M1; I1_\nu \rightarrow I'0_1) = \frac{3}{4\pi} \cdot 0,06 \cdot \left(m_{1_\nu}^{'} \cdot C_{I1;1-1}^{I'0} \right)^2 \quad (15)$$

Table 6. Characteristics of the 1^+_v state in ^{156}Gd . E_{1_v} denotes the energies of the 1^+_v levels.

| 1^+_v | $E_{1_v^+}$ (keV) | E_γ (keV) | I_γ | $m'_{1_v} (\mu_N)$ | Exp. | | $B(\text{M1}) (\mu_N)$ |
|------------|-------------------|------------------|------------------------|--------------------|--|--|--|
| | | | | | [1] | [2] | |
| 1^+_1 | 1965 | 1877 1965 | 15.39(12) 39.90(20) | 11.25 | - - | - - | 0.303 0.604 |
| 1^+_2 | 2027 | 1938 2027 | 59.4(4) 100.0(5) | -7.82 | 0.014^{+3}_{-5} $0.037(11)$ | 0.025^{+6}_{-9} $0.066(20)$ | 0.145 0.292 |
| 1^+_3 | 2187 | 2098 2187 | 100.0(5) 91.5(5) | -3.88 | - - | - - | 0.036 0.072 |
| 1^+_4 | 2270 | 2187 2270 | 100.0(6) 48.1(4) | -7.34 | - - | - - | 0.128 0.257 |
| 1^+_5 | 2301 | 2211 2301 | 100.0(24) 10.6(9) | -0.25 | - - | - - | $1 \cdot 10^{-4}$ $3 \cdot 10^{-4}$ |
| 1^+_6 | 2361 | 2292 2361 | -100(5) | -0.25 | - - | - - | $1 \cdot 10^{-4}$ $3 \cdot 10^{-4}$ |
| 1^+_7 | 2402 | 2314 2403 | 53 100 | -4.8 | 0.036^{+11}_{-14} 0.061^{+12}_{-19} | 0.064^{+20}_{-25} 0.109^{+22}_{-34} | 0.055 0.110 |
| 1^+_8 | 2785 | 2696 2785 | 55 100 | -4.09 | 0.027^{+8}_{-9} 0.044^{+10}_{-12} | 0.048^{+14}_{-16} 0.079^{+18}_{-22} | 0.040 0.080 |
| 1^+_9 | 2974 | 2885 2974 | 52(10) 100 | -4.87 | $0.036(6)$ $0.063(22)$ | $0.064(11)$ $0.113(16)$ | 0.056 0.113 |
| 1^+_{10} | 3010 | 2921 3010 | 55(20) 100 | -0.79 | 0.010^{+5}_{-6} $0.017(4)$ | 0.018^{+9}_{-11} $0.030(7)$ | 0.0015 0.030 |
| 1^+_{11} | 3050 | 2961 3050 | 26(16) 100 | -2.77 | 0.008^{+4}_{-5} $0.020(6)$ | 0.014^{+7}_{-9} $0.036(11)$ | 0.018 0.037 |
| 1^+_{12} | 3070 | 2981 3070 | 57(5) 100 | -9.23 | 0.142^{+17}_{-19} $0.227(17)$ | 0.254^{+30}_{-34} $0.406(30)$ | 0.203 0.407 |
| 1^+_{13} | 3122 | 3033 3122 | 50(20) 100 | -2.64 | $0.010(1)$ $0.018(4)$ | $0.018(2)$ $0.032(7)$ | 0.021 0.029 |
| 1^+_{14} | 3158 | 3069 3158 | - | -4.87 | -0.063(9) | -0.113(39) | 0.056 0.113 |
| 1^+_{15} | 3218 | 3121 3218 | 44(10) 100 | -4.80 | 0.029^{+8}_{-9} $0.061(7)$ | 0.052^{+14}_{-16} $0.109(13)$ | 0.055 0.110 |

Table 7. Reduced probabilities of M1 transitions from the states of the $K^\pi = 2_1^+$ and 0_3^+ bands

| IK_i | E_I | E_γ | Exp. | | Theor. |
|-------------------|---------|------------|------------------------|------------------------------------|----------------------|
| | | | $B(M1)$ W.u | $B(M1)(\mu_N)$ | $B(M1)(\mu_N)$ |
| $2^+2_{\gamma_1}$ | 1154 | 1065.2 | $6 \cdot 10^{-5}(4)$ | $1.07 \cdot 10^{-4}(72)$ | $1.7 \cdot 10^{-3}$ |
| $3^+2_{\gamma_1}$ | 1248 | 959.8 | $6 \cdot 10^{-5}(4)$ | $1.07 \cdot 10^{-4}(72)$ | $6.76 \cdot 10^{-4}$ |
| | | 1159 | $1.4 \cdot 10^{-4}(3)$ | $2.51 \cdot 10^{-4}(54)$ | $9.80 \cdot 10^{-4}$ |
| $4^+2_{\gamma_1}$ | 1355 | 1067.2 | $0.014(+7, -8)$ | $2.51 \cdot 10^{-3}(+1.25, -1.43)$ | $4.10 \cdot 10^{-3}$ |
| 2^+0_3 | 1258.01 | 1169.1 | $0.0078(+9, -7)$ | $1.40 \cdot 10^{-2}(+16, -13)$ | $2.9 \cdot 10^{-3}$ |

Table 8. Multipole mixture coefficients $\delta(E2/M1)$ for ^{156}Gd . $\langle E2 \rangle_{if}$ and $\langle M1 \rangle_{if}$ are the reduced matrix elements of the and M1 transitions, respectively, and E_γ is the energy of transition

| $I_i K_i$ | $I_f K_f$ | E_γ, MeV | $\langle E2 \rangle_{if} e\text{Fm}^2$ | $\langle M1 \rangle_{if}, \mu_N$ | $\delta_{\text{exp.}} [1]$ | $\delta_{\text{theor.}}$ | $\delta_{\text{adiab.}} [12]$ |
|-----------------|-----------------|------------------------|--|----------------------------------|--|--------------------------|-------------------------------|
| 22 ₁ | 20 ₁ | 1.0652 | -18.81 | 0.0412 | -16(5) | -4.1 | - |
| 32 ₁ | 20 ₁ | 1.159 | 19.46 | -0.0313 | -11.8(+6,-7) | -6.0 | - |
| 32 ₁ | 40 ₁ | 0.9598 | -16.23 | 0.0260 | -12(+13,-5) | -5.0 | - |
| 42 ₁ | 40 ₁ | 1.0672 | -18.60 | 0.0639 | +4.0(+9,-16) | -2.6 | - |
| 52 ₁ | 40 ₁ | 1.2187 | -16.83 | 0.0488 | $\delta > 7$ | -3.5 | - |
| 52 ₁ | 60 ₁ | 0.922 | 19.21 | -0.0417 | - | -3.5 | - |
| 62 ₁ | 60 ₁ | 1.060 | 17.00 | -0.063 | $\delta < -0.8 \text{ or } > 2.5$ | -2.4 | - |
| 72 ₁ | 60 ₁ | 1.2648 | 15.06 | -0.0634 | - | -2.5 | - |
| 82 ₁ | 80 ₁ | 1.0457 | 15.84 | -0.0584 | $\delta < -0.6 \text{ or } \delta > 1.6$ | -2.4 | - |
| 92 ₁ | 80 ₁ | 1.2843 | -13.73 | 0.0758 | $\delta < -0.8$ 0.39(6) | -1.9 | - |
| 20 ₂ | 20 ₁ | 1.0405 | 10.31 | -0.1011 | +5.9(+14,-28) | -0.9 | - |
| 40 ₂ | 40 ₁ | 1.0106 | -12.75 | 0.2176 | - | 0.49 | - |
| 11 ₁ | 20 ₁ | 1.876 | 14.66 | 0.5503 | +0.41(+25,-14) +0.35(4) | 0.41 | 0.37 |
| 11 ₂ | 20 ₁ | 1.938 | 14.61 | -0.3812 | -0.55(3) | -0.63 | -0.5 |
| 11 ₃ | 20 ₁ | 2.0977 | 14.49 | -0.1888 | -1.2(2) or -1.08(+0.03,-0.22) | -1.34 | -1.20 |
| 11 ₄ | 20 ₁ | 2.1807 | 14.44 | -0.3579 | -0.66(+0.06,-0.08) | -0.73 | -0.66 |
| 20 ₃ | 20 ₁ | 1.1691 | -7.53 | -0.0539 | 0.38(6) | 1.4 | - |
| 40 ₃ | 40 ₁ | 1.1741 | -8.65 | -0.0934 | - | 0.91 | - |

Table 9. Magnetic moments $\mu(I^+0_1)$ of the states of the main rotational bands of ^{156}Gd nuclei

| I | Exp. [1] | Theor. | Adiab. |
|--------|-----------------|-----------------|-----------------|
| | $\mu(I)(\mu_N)$ | $\mu(I)(\mu_N)$ | $\mu(I)(\mu_N)$ |
| 2^+ | +0.774(8) | 0.768 | 0.82 |
| 4^+ | +1.24(8) | 1.54 | 1.64 |
| 6^+ | +1.5(13) | 2.31 | 2.46 |
| 8^+ | - | 3.09 | 3.28 |
| 10^+ | +3.4(5) | 3.87 | 4.10 |
| 12^+ | - | 4.65 | 4.92 |

CONCLUSIONS

- Theoretical calculations were carried out for ^{156}Gd nucleus in the framework of the phenomenological model taking into account Coriolis mixing of low-lying rotation bands with positive parity.
- The nonadiabaticities observed in the energies, electromagnetic characteristics of the excited states are explained by the Coriolis mixing of low excited rotational states. To describe the all adiabatic rotational bands has been used the same moments of inertia. The reduced probabilities of $E2-$ and $M1-$ transitions are calculated. The calculated theoretical values of the probabilities of electromagnetic transitions from the states of $K^\pi = 0_2^+$, $K^\pi = 0_3^+$, $K^\pi = 2_1^+$, $K^\pi = 2_2^+$ bands and the ratio of transitions from them are compared with existing experimental results and their compatibility is shown.
- The theoretical values of the multipole mixture coefficients $\delta(E2/M1)$ for transitions from the levels of $K^\pi = 0_{2,3}^+, 2_1^+, 1_\nu^+$ bands to the states of the ground band, and the values $B(M1)$ and magnetic moments of the states of the main bands $\mu(I^+ 0_1)$, were in satisfactory agreement with the available experimental data.

• REFERENCES

- [1] Reich C W 2012 Nuclear Data Sheets, 113 2537
- [2] Pitz H H, Berg U E P, Heil R D, Knaissl U and Stock R 1989 Nucl. Phys. A, 492 411
- [3] Usmanov P N, Mikhailov I N 1997 Phys. Part. Nucl. Lett, 28 348; Usmanov P N, Mikhailov I N 1997 Fiz. Elem. Chastits At. Yadra, 28 887
- [4] Reich C. W. // Nucl. Data Sheets 2005. V. 105. P. 557-774.
- [5] McGowan F K, Milner W T 1981 Phys. Rev. C, 23 1926
- [6] Begzhanov R B, Belenkiy V M, Zalyubovskiy I I, Kuznichenko A B 1989 Handbook on Nuclear Physics Tashkent FAN 1 2
- [7] Backlin A, Hedin G, Fogelberg B et al. 1982 Nucl. Phys. A, 380 189
- [8] Aprahamian A, de Haan R C, Lesher S R et al. 2018 Phys. Rev. C, 98 034303
- [9] Sie S H, Ward D, Geiger J S et al. 1977 Nucl. Phys. A, 291 443
- [10] Kearns F, Varley G, Dracoulis G D et al. 1977 Nucl. Phys. A, 278 109
- [11] Konijn J, Be Boer F W N, Van Poelgeest A et al. 1981 Nucl. Phys. A, 352 191
- [12] Alaga G 1957 Nucl. Phys. 4 625

THANKS FOR YOUR ATTENTION!