Observation of sterile antineutrino oscillation in Neutrino-4 experiment at SM-3 reactor

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Neutrino-4 collaboration

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Reactor antineutrino anomaly

- Observed/predicted averaged event ratio: $R = 0.927 \pm 0.023$ (3.0 $\sigma$)

\[
P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{14} \sin^2(1.27 \frac{\Delta m_{14}^2 [\text{eV}^2]}{E_{\bar{\nu}} [\text{MeV}]} L [\text{m}])
\]

The first observation of effect of oscillation in Neutrino-4 experiment on search for sterile neutrino

The period of oscillation for neutrino energy 4 MeV is 1.4 m

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Reactor antineutrino anomaly with oscillation curve obtained in experiment Neutrino-4.
Due to some peculiar characteristics of its construction, reactor SM-3 provides the most favorable conditions to search for neutrino oscillations at short distances. However, SM-3 reactor, as well as other research reactors, is located on the Earth’s surface, hence, cosmic background is the major difficulty in considered experiment.
Movable and spectrum sensitive antineutrino detector at SM-3 reactor

1. detector (5x10 cells)
2. internal active shielding
3. external active shielding
4. steel and lead
5. borated polyethylene
6. moveable platform
7. feed screw
8. step motor
9. shielding

Passive shielding - 60 tons
Range of measurements is 6 - 12 meters

Liquid scintillator detector
50 sections 0.235x0.235x0.85 m³
Gamma background in passive shielding does not depend neither on the power of the reactor nor on distance from the reactor.
The background of fast neutrons in passive shielding does not depend neither on the power of the reactor nor on distance from the reactor.

The background of fast neutrons in passive shielding is 10 times less than outside. The background of fast neutrons outside of passive shielding is defined by cosmic rays and practically does not depend on reactor power.
Absence of noticeable dependence of the background on both distance and reactor power was observed. As a result, we consider that difference in reactor ON/OFF signals appears mostly (>95%) due to antineutrino flux from operating reactor.
Measurements with the detector have started in June 2016. Measurements with the reactor ON were carried out for 720 days, and with the reactor OFF for 417 days. In total, the reactor was switched on and off 87 times.

\[
\frac{\text{(ON – OFF)}}{\text{OFF}} = 50\%
\]
That distribution has the form of normal distribution, but its width exceeds unit by 7%.

Additional dispersion of measurement result which appears due to fluctuations of cosmic background

\[ \sigma = 1.070 \pm 0.045 \]

That distribution has the form of normal distribution, but its width exceeds unit by 7.0 ± 4.5 %.

Broadening in the statistical distribution of the neutrino signal due to fluctuations in the cosmic background will be 5%.
Energy calibration of the full-scale detector

Pu-Be neutron source

22 Na- gamma source

Energy resolution ±250 keV.
There is problems with energy spectrum therefore we proposed the spectrum independent method of the experimental data analysis.

Spectrum of prompt signals in the detector for a total cycle of measurements summed over all distances (average distance — 8.6 meters). The red line shows Monte-Carlo simulation with neutrino spectrum of $^{235}\text{U}$, as the SM-3 reactor works on highly enriched uranium.
SPECTRAL INDEPENDENT METHOD OF DATA TREATMENT AND ANALYSIS OF THE RESULT
Probability of antineutrino disappearance

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{14} \sin^2(1.27 \frac{\Delta m_{14}^2 [eV^2]}{E_{\bar{\nu}} [MeV]}) \] (1)

The spectrum independent method of experimental data analysis

\[ R_{i,k}^{\exp} = \frac{N(E_i, L_k) L_k^2}{K^{-1} \sum_k N(E_i, L_k) L_k^2} = \frac{[1 - \sin^2 2\theta_{14} \sin^2(1.27 \Delta m_{14}^2 L_k / E_i)]}{K^{-1} \sum_k [1 - \sin^2 2\theta_{14} \sin^2(1.27 \Delta m_{14}^2 L_k / E_i)]} = R_{i,k}^{th} \] (2)

The denominator is significantly simplified with a range of measurement distances significantly greater than the characteristic oscillation period:

\[ R_{i,k}^{th} \approx \frac{1 - \sin^2 2\theta_{14} \sin^2(1.27 \Delta m_{14}^2 L_k / E_i)}{1 - 1/2 \sin^2 2\theta_{14}} \]

The method of the analysis of experimental data should not rely on precise knowledge of spectrum. One can carry out model independent analysis using equation (2), where numerator is the rate of antineutrino events with correction to geometric factor $1/L^2$ and denominator is its value averaged over all distances.
MONTE CARLO SIMULATION OF EXPECTED RESULTS WITH EMPLOYING OF SPECTRAL INDEPENDENT METHOD OF DATA ANALYSIS

The source of antineutrino with geometrical dimensions of the reactor core 42x42x35cm$^3$ was simulated, as well as a detector of antineutrino taking into account its geometrical dimensions (50 sections of 22.5x22.5x75cm$^3$).
The expected effect for the different energy resolution from MC calculation

Energy resolution 0.1 MeV

Energy resolution 0.25 MeV
The expected effect for the different energy resolution from MC calculation

Energy resolution 0.5 MeV (our case)

Energy resolution 0.75 MeV
Probability of antineutrino disappearance

\[ P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{14} \sin^2 \left(1.27 \frac{\Delta m_{14}^2 [eV^2] L [m]}{E_{\bar{\nu}} [MeV]} \right) \]  

The spectrum independent method of experimental data analysis

\[ R^\text{exp}_{i,k} = \frac{N(E_i, L_k) L_k^2}{K^{-1} \sum_k N(E_i, L_k) L_k^2} = \frac{[1 - \sin^2 2\theta_{14} \sin^2 \left(1.27 \Delta m_{14}^2 L_k / E_i \right) \right]}{K^{-1} \sum_k [1 - \sin^2 2\theta_{14} \sin^2 \left(1.27 \Delta m_{14}^2 L_k / E_i \right)]} = R^\text{th}_{i,k} \]  

The results of the analysis of optimal parameters \( \Delta m_{14}^2 \) and \( \sin^2 2\theta_{14} \)

using \( \chi^2 \) method

\[ \sum_{i,k} \left[ \left( R^\text{exp}_{i,k} - R^\text{th}_{i,k} \right)^2 / (\Delta R^\text{exp}_{i,k})^2 \right] = \chi^2 \left( \sin^2 2\theta_{14}, \Delta m_{14}^2 \right) \]
The results of the analysis of optimal parameters $\Delta m_{14}^2$ and $\sin^2 2\theta_{14}$ using $\chi^2$ method

$$\sum_{i,k} [(R_{i,k}^{\text{exp}} - R_{i,k}^{\text{th}})^2 / (\Delta R_{i,k}^{\text{exp}})^2] = \chi^2 (\sin^2 2\theta_{14}, \Delta m_{14}^2)$$

We observed the oscillation effect at C.L. 3.2 $\sigma$
in vicinity of:

$\Delta m_{14}^2 \approx 7.25\text{eV}^2$

$\sin^2 2\theta_{14} \approx 0.26$
Results of data analysis with average by energy intervals
125keV, 250keV и 500keV

\[ \Delta m_{14}^2 \approx 7.25 \text{eV}^2 \pm 0.13 \]

\[ \sin^2 2\theta_{14} \approx 0.26 \pm 0.08 \]

C.L. 3.2 σ
METHOD OF COHERENT DATA SUMMATION TO OBTAIN DEPENDENCE FROM RATIO L/E
METHOD OF COHERENT DATA SUMMATION TO OBTAIN DEPENDENCE FROM RATIO L/E
The period of oscillation for neutrino energy 4 MeV is 1.4 m

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arxiv:2003.03199
arxiv:2005.05301
Analysis of the confidence level of the result

We observed the oscillation effect at C.L. (3.2 $\sigma$) in vicinity of:

$$\Delta m^2_{14} \approx 7.25 \text{eV}^2 \pm 0.13 \quad \sin^2 2\theta_{14} \approx 0.26 \pm 0.08$$

It is often discussed that stricter limitations on the confidence level of the result can be obtained using the Feldman-Cousins method. In compliance Wilks theorem $\Delta\chi^2$ method is possible to apply successfully if effect is observed at the level of reliability $3\sigma$ more. The result of processing without taking into account systematic errors with an energy interval of 500 keV is $\sin^2 2\theta_{14} = 0.38 \pm 0.11(3.5\sigma)$, and when averaging data over 125keV, 250keV and 500keV is $\sin^2 2\theta_{14} \approx 0.26 \pm 0.08(3.2\sigma)$. Since the reliability of the effect we observe exceeds $3\sigma$, we do not consider it mandatory to use the Feldman-Cousins method and propose to do another additional analysis of our data.
Dependence of antineutrino flux on the distance to the reactor core

normalized experimental dependence

![Normalized experimental dependence graph](image1)

oscillation curve with the experimental results in range 6-12 m

![Oscillation curve and experimental results graph](image2)

FIG. 58. Representation of experimental results in form of dependence of antineutrino flux on the distance to the reactor core normalized with the law $A/L^2$.

FIG. 59. Oscillation curve and experimental results in range 6-12 m.
Analysis of possible systematic effects
To carry out analysis of possible systematic effects one should turn off antineutrino flux (reactor) and perform the same analysis of background data.

The spectrum for neutrino signal and background signal are similar therefore test for systematic effect have to be adequate.

**The problem of fast neutrons**

**False event**

**Neutrino event**

**Fast neutron**

Neutron scattering imitate neutrino reaction
Test of systematic effects

To carry out analysis of possible systematic effects one should turn off antineutrino flux (reactor) and perform the same analysis of obtained data.

Data analysis using coherent summation method

Analysis of the results on oscillation parameters plane

Thus no instrumental systematic errors were observed.
Stability of the oscillation effect

FIG. 54. Stability of the oscillation effect. Black figures are experimental points, red circles expected dependence.

Stability of the correlated background

FIG. 55. Stability of the correlated background (blue dots). Red line is linear approximation.
SYSTEMATIC ERRORS OF THE EXPERIMENT

\[ \Delta m_{14}^2 = 7.25 \pm 0.13_{stat} \pm 1.08_{syst} = 7.25 \pm 1.09. \]

\[ \sin^2 2\theta = 0.26 \pm 0.08_{stat} \pm 0.05_{syst} = 0.26 \pm 0.09(2.9\sigma) \]

Fig. 5. The significantly magnified central area if prompt spectrum has 500 keV bin width.

FIG. 5.3. Confidence levels of the area around oscillation parameters obtained as the best fit in case of averaging over three data sets.
SYSTEMATIC ERRORS OF THE EXPERIMENT

\[ \Delta m^2_{14} = 7.25 \pm 0.13_{\text{stat}} \pm 1.08_{\text{syst}} = 7.25 \pm 1.09\text{eV}^2 \]

\[ \sin^2 2\theta = 0.26 \pm 0.08_{\text{stat}} \pm 0.05_{\text{syst}} = 0.26 \pm 0.09(2.9\sigma) \]
COMPARISON OF THE RESULT OF EXPERIMENT NEUTRINO-4 WITH REACTOR AND GALLIUM ANOMALIES
Reactor antineutrino anomaly with oscillation curve obtained in experiment Neutrino-4

Fig. 13. Reactor antineutrino anomaly [28] with oscillation curve obtained in experiment Neutrino-4.
COMPARISON OF THE RESULT OF EXPERIMENT NEUTRINO-4 WITH REACTOR AND GALLIUM ANOMALIES

\[ \sin^2 2\theta_{14} \approx 0.26 \pm 0.09 \ (2.9\sigma) \]

Neutrino-4 experiment

\[ \sin^2 2\theta_{14} \approx 0.32 \pm 0.10 \ (3.2\sigma) \]

gallium anomaly

\[ \sin^2 2\theta_{14} \approx 0.13 \pm 0.05 \ (2.6\sigma) \]

reactor antineutrino anomaly

Combination of these results gives an estimation for mixing angle

\[ \sin^2 2\theta_{14} \approx 0.19 \pm 0.04 \ (4.6\sigma) \]
COMPARISON WITH OTHER RESULTS OF EXPERIMENTS AT RESEARCH REACTORS AND NUCLEAR POWER PLANTS
The period of oscillation for neutrino energy 4 MeV is 1.4 m.

Comparison of results of the Neutrino-4 experiment with results of other experiments – sensitivities of the experiments.

In experiments on nuclear power plants sensitivity to identification of effect of oscillations with large is considerably suppressed because of the big sizes of an active zone (3.7 m). Experiment Neutrino-4 has some advantages in sensitivity to large values of $\Delta m_{14}^2$ owing to a compact reactor core, close minimal detector distance from the reactor and wide range of detector movements.
## COMPARISON WITH OTHER RESULTS OF EXPERIMENTS AT RESEARCH REACTORS

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Days with reactor ON</th>
<th>Days with reactor OFF</th>
<th>S/B ratio</th>
<th>Number of events, (d^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutrino-4</td>
<td>720 (90 MW)</td>
<td>417</td>
<td>0.5</td>
<td>223 (6-9 m)</td>
</tr>
<tr>
<td>PROSPECT</td>
<td>33 (85 MW)</td>
<td>28</td>
<td>1.3</td>
<td>771 (7-9 m)</td>
</tr>
<tr>
<td>STEREO</td>
<td>179 (58 MW)</td>
<td>235</td>
<td>1.1</td>
<td>366 (9 – 11m)</td>
</tr>
</tbody>
</table>
NEUTRINO MODEL 3+1
THE STRUCTURE OF 3+1 NEUTRINO MODEL AND REPRESENTATION OF PROBABILITIES OF VARIOUS OSCILLATIONS

\[
\begin{pmatrix}
    \nu_e \\
    \nu_\mu \\
    \nu_\tau \\
    \nu_s
\end{pmatrix}
= \begin{pmatrix}
    U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
    U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
    U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
    U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix}
\begin{pmatrix}
    \nu_1 \\
    \nu_2 \\
    \nu_3 \\
    \nu_4
\end{pmatrix}
\]

\[|U_{e4}|^2 = \sin^2(\theta_{14})\]
\[|U_{\mu4}|^2 = \sin^2(\theta_{24}) \cdot \cos^2(\theta_{14})\]
\[|U_{\tau4}|^2 = \sin^2(\theta_{34}) \cdot \cos^2(\theta_{24}) \cdot \cos^2(\theta_{14})\]

\[P_{\nu_e\nu_e} = 1 - 4|U_{e4}|^2(1 - |U_{e4}|^2) \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_e}}\right) = 1 - \sin^2 2\theta_{ee} \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_e}}\right)\]

\[P_{\nu_\mu\nu_\mu} = 1 - 4|U_{\mu4}|^2(1 - |U_{\mu4}|^2) \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_\mu}}\right) = 1 - \sin^2 2\theta_{\mu\mu} \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_\mu}}\right)\]

\[P_{\nu_\mu\nu_e} = 4|U_{e4}|^2|U_{\mu4}|^2 \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_e}}\right) = \sin^2 2\theta_{\mu e} \sin^2\left(\frac{\Delta m^2_{14} L}{4E_{\nu_e}}\right)\]
The relations of oscillations parameters required for comparative analysis of experimental results are:

\[
\sin^2 2\theta_{ee} \equiv \sin^2 2\theta_{14} \\
\sin^2 2\theta_{\mu\mu} = 4 \sin^2 \theta_{24} \cos^2 \theta_{14} (1 - \sin^2 \theta_{24} \cos^2 \theta_{14}) \approx \sin^2 2\theta_{24} \\
\sin^2 2\theta_{\mu e} = 4 \sin^2 \theta_{14} \sin^2 \theta_{24} \cos^2 \theta_{14} \approx \frac{1}{4} \sin^2 2\theta_{14} \sin^2 2\theta_{24}
\]

It is an important relation which can be used for experimental verification of 3+1 neutrino model.

It is important that amplitudes of electron and muon oscillations with disappearance determines the amplitude \(\sin^2 2\theta_{\mu e}\) in process with appearance of electron neutrinos in muon neutrino beam.
• Spencer Axani, arXiv:2003.02796

$\Delta m_{14}^2 = 4.47^{+3.53}_{-2.08}\text{eV}^2$

$\sin^2(2\theta_{24}) = 0.10^{+0.10}_{-0.07}$
COMPARISON OF EXPERIMENT NEUTRINO-4 RESULTS WITH RESULTS OF ACCELERATOR EXPERIMENTS MINIBOONE AND LSND


The experiments MiniBooNE and LSND are aimed to search for a second order process of sterile neutrino – the appearance of electron neutrino in the muon neutrino flux \( (\nu_\mu \rightarrow \nu_e) \) through an intermediate sterile neutrino.

A comparison of \( \sin^2 2\theta_{14} \) obtained in MiniBooNE and LSND and \( \sin^2 2\theta_{24} \) obtained in Neutrino-4 can be performed using results of the IceCube experiment:

\[
\sin^2 2\theta_{24} \approx 0.03 \div 0.2
\]

Values of \( \sin^2 2\theta_{\mu e} \) and \( \sin^2 2\theta_{24}, \sin^2 2\theta_{14} \) are related by the expression:

\[
\sin^2 2\theta_{\mu e} \approx \frac{1}{4} \sin^2 2\theta_{14} \sin^2 2\theta_{24}
\]
Pontecorvo–Maki–Nakagawa–Sakata matrix

PMNS matrix for 3 + 1 model

\[
\begin{bmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s
\end{bmatrix} =
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4
\end{bmatrix}
\]

\[|U_{e4}|^2 = \sin^2(\theta_{14})\]
\[|U_{\mu4}|^2 = \sin^2(\theta_{24}) \cdot \cos^2(\theta_{14})\]
\[|U_{\tau4}|^2 = \sin^2(\theta_{34}) \cdot \cos^2(\theta_{24}) \cdot \cos^2(\theta_{14})\]
# Neutrino mass

## 3 complementary methods to measure:

<table>
<thead>
<tr>
<th>Method</th>
<th>Observable</th>
<th>curr. [eV]</th>
<th>near/far [eV]</th>
<th>pro</th>
<th>con</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurie</td>
<td>$\sqrt{\sum</td>
<td>U_{ei}</td>
<td>^2 m_i^2}$</td>
<td>2.3</td>
<td>0.2/0.1</td>
</tr>
<tr>
<td>Cosmo.</td>
<td>$\sum m_i$</td>
<td>0.7</td>
<td>0.3/0.05</td>
<td>best; NH/IH</td>
<td>systemat.; model-dep.</td>
</tr>
<tr>
<td>$0\nu\beta\beta$</td>
<td>$</td>
<td>\sum U_{ei}^2 m_i</td>
<td>$</td>
<td>0.3</td>
<td>0.1/0.05</td>
</tr>
</tbody>
</table>

\[ m_1^2, m_2^2, m_3^2 \ll m_4^2 \]
COMPARISON WITH EXPERIMENT KATRIN ON MEASUREMENT OF NEUTRINO MASS

Limitations on the sum of mass of active neutrinos $\Sigma m = m_1 + m_2 + m_3$ from cosmology are in the range $0.54 \div 0.11$ eV.

$$m_{\nu e}^{\text{eff}} = \sqrt{\sum m_i^2 |U_{ei}|^2}$$

$$\sin^2 2\theta_{14} = 4|U_{14}|^2 (1 - |U_{14}|^2)$$

$$|U_{14}| \approx \frac{1}{4} \sin^2 2\theta_{14}$$

$$m_{14}^2 \approx m_4^2$$

$$m_4 = (2.68 \pm 0.13) \text{ eV}$$

$$\sin^2 2\theta_{14} \approx 0.19 \pm 0.04 (4.6\sigma)$$

$$m_{\nu e}^{\text{eff}} = (0.58 \pm 0.09) \text{ eV}$$

$m_{12}^2, m_{23}^2, m_{34}^2 \ll m_4^2$

$m_{\nu e}^{\text{eff}} \leq 1.1 \text{ eV (CL 90\%)}$
In the same way we can use data about obtained in the IceCube experiment to estimate muon neutrino mass:

\[ m_{\nu_\mu}^{\text{eff}} = (0.42 \pm 0.24) \text{eV} \]

Finally, considering upper limit of \( \sin^2 2\theta_{34} \leq 0.21 \) we can calculate upper limit of tau neutrino mass

\[ m_{\nu_\tau}^{\text{eff}} \leq 0.65 \text{eV} \]
Comparison with neutrino mass constraints from experiments for neutrino less double beta-decay search

This expression for the model 3+1 and with $m_1,m_2,m_3 \ll m_4$ assumption can be simplified:

$$m(0\nu\beta\beta) = \sum_{i=1}^{4} |U_{ei}|^2 m_i$$

The numerical for this with Neutrino-4 and other experiments average result is shown below.

- $m(0\nu\beta\beta) = (0.13 \pm 0.03)\text{eV}$
- $m_{\beta\beta} \leq [0.80-0.182]\text{eV}$

The best restrictions on the Majorana mass were obtained in the GERDA experiment. In these experiments, the half-life of the isotope is measured, which depends on the Majorana mass as follows:

$$1/T_{1/2}^{0\nu} = g_A^4 G_{0\nu}^2 |M_{0\nu}|^2 \frac{(m_{\beta\beta})^2}{m_e^2}$$

The upper limit for the lower limit - the upper limit for the Majorana mass:

- Lower limit for $T_{1/2}/0\nu > 1.8 \times 10^{26}$ years (90% CL)
- Upper limit for $m_{\beta\beta} \leq [80-182]\text{meV}$
\[ \Delta m_{14}^2 = 7.25 \pm 0.13_{stat} \pm 1.08_{syst} = 7.25 \pm 1.09 \]

\[ \sin^2 2\theta = 0.26 \pm 0.08_{stat} \pm 0.05_{syst} = 0.26 \pm 0.09(2.8\sigma) \]

\[ \sin^2 2\theta_{14} \approx 0.19 \pm 0.04(4.6\sigma) \]

\[ m_4 = (2.68 \pm 0.13)\text{eV} \]

\[ m_{\nu e}^{\text{eff}} = (0.58 \pm 0.09)\text{eV} \]

\[ m_{\nu \mu}^{\text{eff}} = (0.42 \pm 0.24)\text{eV} \]

\[ m_{\nu \tau}^{\text{eff}} \leq 0.65\text{eV} \]
Pontecorvo–Maki–Nakagawa–Sakata matrix

PMNS matrix for 3 + 1 model

\[
\begin{bmatrix}
\nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s
\end{bmatrix} =
\begin{bmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{bmatrix}
\begin{bmatrix}
\nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4
\end{bmatrix}
\]

\[
|U_{e4}|^2 = \sin^2(\theta_{14})
\]

\[
|U_{\mu4}|^2 = \sin^2(\theta_{24}) \cdot \cos^2(\theta_{14})
\]

\[
|U_{\tau4}|^2 = \sin^2(\theta_{34}) \cdot \cos^2(\theta_{24}) \cdot \cos^2(\theta_{14})
\]

\[
U_{\text{PMNS}}^{(3+1)} = 
\begin{pmatrix}
0.824_{-0.008}^{+0.007} & 0.547_{-0.011}^{+0.011} & 0.147_{-0.003}^{+0.003} & 0.224_{-0.025}^{+0.025} \\
0.409_{-0.060}^{+0.036} & 0.634_{-0.065}^{+0.022} & 0.657_{-0.014}^{+0.044} & 0.160_{-0.05}^{+0.08} \\
0.392_{-0.048}^{+0.025} & 0.547_{-0.028}^{+0.056} & 0.740_{-0.048}^{+0.012} & < 0.229 \\
< 0.24 & < 0.30 & < 0.26 & > 0.93
\end{pmatrix}
\]
Neutrino flavors mixing scheme including sterile neutrino for normal and inverted mass hierarchy.
\[ \Delta m^2_{14} = 7.25\text{eV}^2, \sin^2\theta_{14} = 0.19, \]
\[ \Delta m^2_{13} = 2.4 \times 10^{-3}\text{eV}^2, \Delta m^2_{12} = 7.4 \times 10^{-5}\text{eV}^2, \]
\[ |U_{e3}|^2 = 0.021, |U_{e2}|^2 = 0.299, |U_{e1}|^2 = 0.679 \]
Thank you for attention

Best regards from Gatchina

Best regards from Dimitrovgrad