

# Measurement of the top-quark mass using t-channel single top events

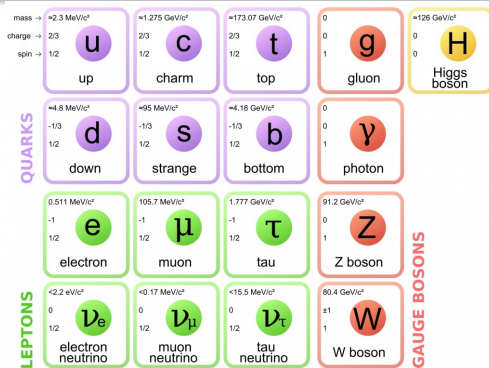
Mintu Kumar, Soureek Mitra, Prof. Shashi Dugad Prof.  
Gagan Mohanty



August 14, 2019

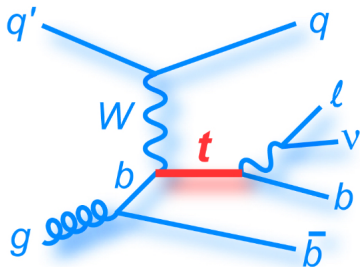
# The Top quark

- Top quark is a most massive elementary particle in the standard model (SM)
- It has the largest coupling with the Higgs boson
- Top and antitop quark pair ( $t\bar{t}$ ) production is the largest contributor to top quark events as well as to top mass measurement
  - **Systematic** uncertainty dominates over the **statistical** one  
 $(m_t = 172.5 \pm 0.20 \pm 0.98 \text{ GeV})$



- t-channel process is the largest contributor to single top quark production
  - This process is cleaner compared to  $t\bar{t}$  due to absence of color reconnection effects

# t-channel process



t-channel process

$$\sigma = 216 \text{ pb @ } 13 \text{ TeV}$$

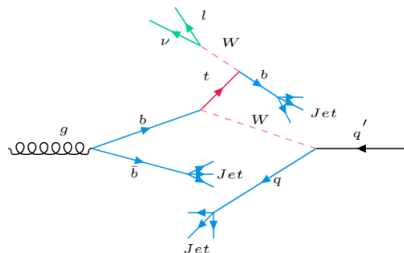
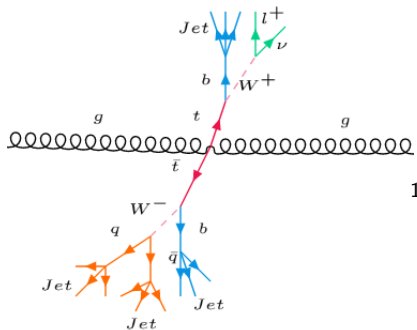
- t-channel is an electroweak process and is considered as signal in our analysis
- This process has two or more jets of which one is **b-tagged**, coming from the top decay
- There is a **light flavor jet** recoiling against the top quark
- We also have an **isolated electron** and **large missing transverse energy** due to escaping neutrino

Parton

# Advantage of t-channel single process

$t\bar{t}$  process

t-channel process



1

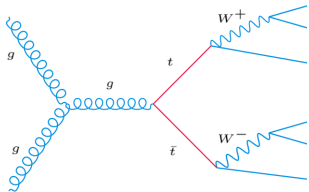
To the total systematic uncertainty (0.98 GeV) in the  $t\bar{t}$  sample, color reconnection contributes at the 20-30% level

provides a statistically independent sample, where color reconnection is less of worry

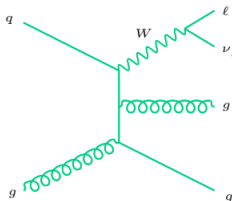
<sup>1</sup>arXiv:1812.10534

# Background processes

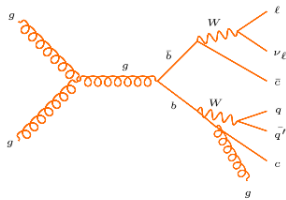
- The most significant background process is  $t\bar{t}$  where one  $t$  undergoes the leptonic decay
- $W$ +jets &  $Z$ +jets are the next dominant contributor followed by QCD multijet process
- Other two single top processes are also considered as background



$t\bar{t}$  process  
 $\sigma = 831$  pb



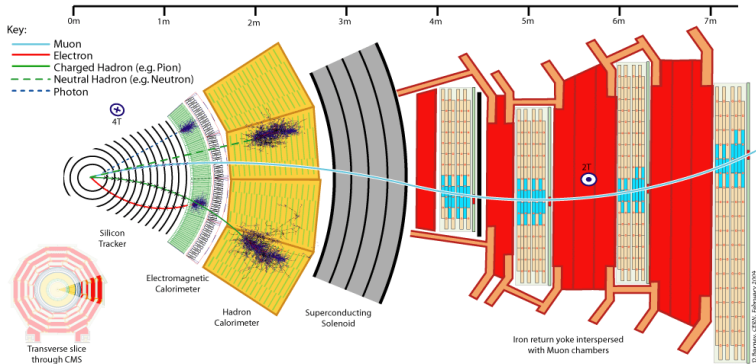
$W$ +Jets process  
 $\sigma = 63$  nb



QCD multijet  
 $\sigma = 302$  nb

cross section

# CMS Detector



- Silicon tracker has two parts: inner pixel and outer strip detector
- Electromagnetic calorimeter (ECAL) is made up of lead tungstate crystals
- Hadron calorimeter is comprised brass and plastic scintillators
- Outside the solenoid, there are gas ionizing detectors embedded in the steel flux return yoke to identify muons
- In our analysis, information from each sub-detector is used

cross view

6/46

- Data sample collected by the CMS detector in proton-proton collisions at center-of-mass energy 13 TeV during 2016 is used; the corresponding luminosity is  $35.9 \text{ fb}^{-1}$
- Signal process t-channel is generated using POWHEG generator
- Among background processes,  $tW$  channel and  $t\bar{t}$  process are also generated by POWHEG
- All other backgrounds are generated with aMC@NLO
- QCD is estimated by a data driven method
- PYTHIA8 is used to model the showering
- All generated events are passed through full simulation of the CMS detector using GEANT4

- **Trigger:**

$\mu$ : HLT\_Iso(Tk)Mu24

e: HLT\_Ele32\_eta2p1\_WPTight\_Gsf

- **Exactly 1 lepton:**

$\mu$ :  $p_T > 26$  GeV,  $|\eta| < 2.4$ , tight ID,  $I_{\text{rel}} < 0.06$  within  $\Delta R = 0.4$

e:  $p_T > 35$  GeV,  $|\eta| < 2.1$ , passes cut-based tight ID, EB-EE transition gap excluded,

IP cuts: for  $|\eta_{\text{SC}}| \leq 1.479$ ,  $|d_{xy}| < 0.05$  cm and  $|d_z| < 0.10$  cm, for  $|\eta_{\text{SC}}| > 1.479$ ,  $|d_{xy}| < 0.10$  cm and  $|d_z| < 0.20$  cm

- **Veto events having another lepton:**

$p_T > 10$  (15) GeV,  $|\eta| < 2.4$  (2.5), loose (veto) ID,  $I_{\text{rel}} < 0.2$  for  $\mu(e)$

- **2 -3 AK4PFJets with:**

$p_T > 40$  GeV,  $|\eta| < 4.7$ , loose ID,  $\Delta R_{(\text{lep}, \text{jets})} > 0.4$

Summer16\_25ns\_v1 corrections applied

- **1- 2 b-tag with**

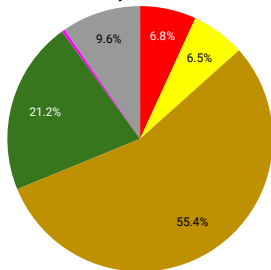
$|\eta| < 2.4$  passing CMVA v2 tight working point

- **$m_t^W > 50$  GeV** Tagging cut flow anti kT Rel.iso trigg



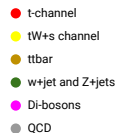
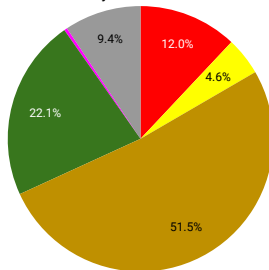
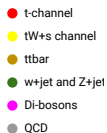
# Event selection

Final Yield Electron +jets



**e + jets**

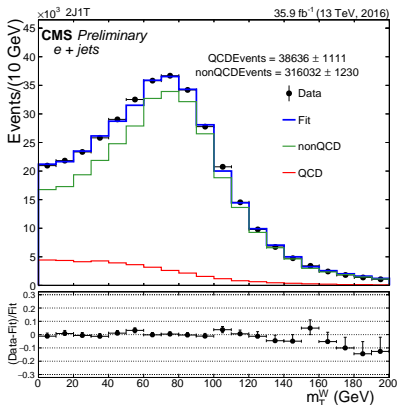
Final Yield Muon +jets



**$\mu$  + jets**

# QCD background estimation

- QCD has high cross section but has a very low selection efficiency
- Require high stat.  $\Rightarrow$  time consuming  $\Rightarrow$  data-driven estimate
  - Go to the sideband (anti-isolation)  $\rightarrow$  Subtract nonQCD from this SB  $\Rightarrow$  data-driven QCD template

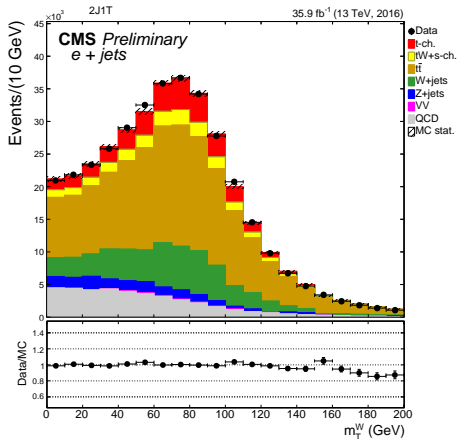


Data driven QCD fit

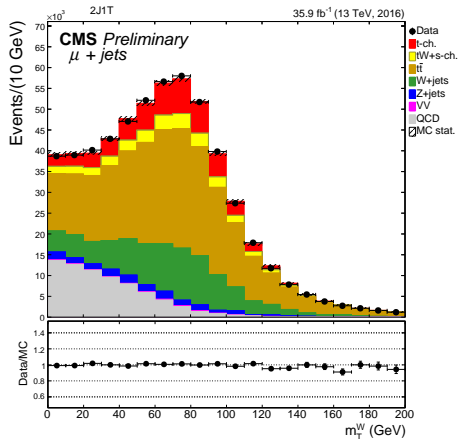
- Maximum likelihood fit to data in the signal region.
- $F(m_t^W) =$   
 $N_{\text{QCD}} \times P(m_t^W) +$   
 $N_{\text{nonQCD}} \times Q(m_t^W)$
- $P(\cancel{p}_T)$  &  $Q(\cancel{p}_T)$  are QCD and nonQCD templates
- $N_{\text{QCD}}$  and  $N_{\text{nonQCD}}$  are the corresponding yields

Met

# Data-MC comparison after the fit

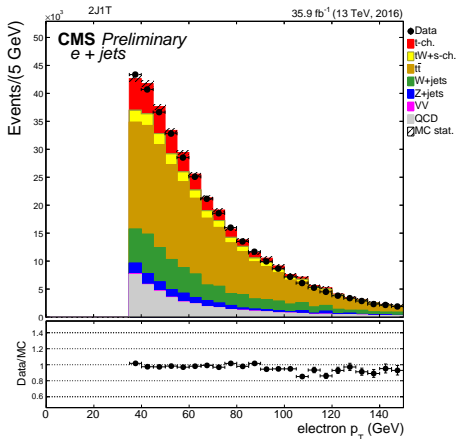


$m_t^W$  distribution of e + jets  
final state

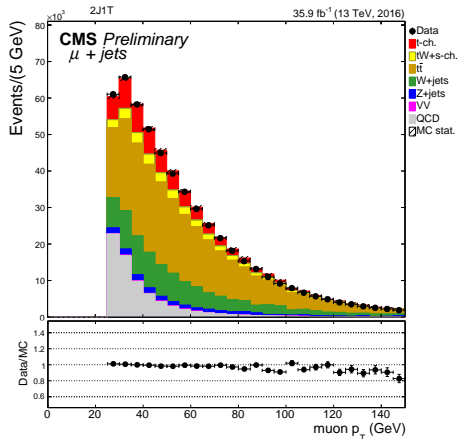


$m_t^W$  distribution of  $\mu$  + jets  
final state

# Data-MC comparison after the fit (contd.)



**Electron  $p_T$  distribution  
of  $e + jets$  final state**



**Muon  $p_T$  distribution of  
 $\mu + jets$  final state** [More](#) [3J1T](#)

# Fox-Wolfram moments in 2J1T region

- Describe correlation between jets based on event topology
- Calculated from all the selected jets in each event
- When there are two jets in the event, we get basic

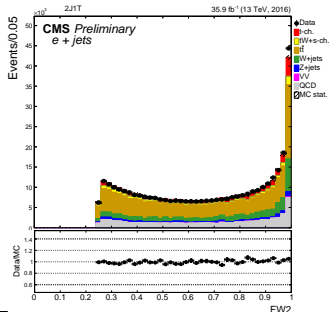
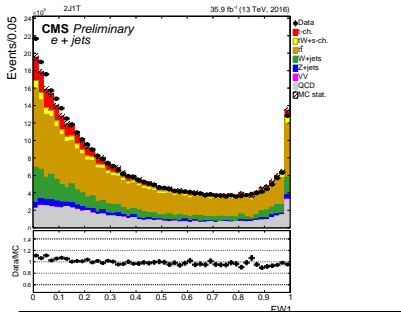
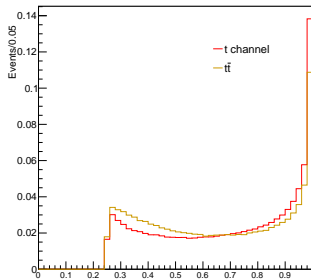
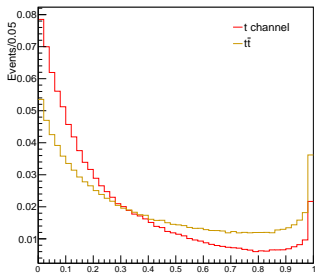
$$H_I = \frac{1 + r^2 + 2rP_I \cos \Omega_{12}}{(1 + r)^2} \quad (1)$$

Here  $r$  is the ratio of  $p_T$  of the jets and  $\Omega_{12}$  is angle between the jets

- First and second Fox-Wolfram moment for 2J1T region are:

$$H_1 = \frac{1 + r^2 + 2r \cos \Omega_{12}}{(1 + r)^2}$$
$$H_2 = \frac{1 + r^2 + 2r(\cos^2 \Omega_{12} - 1)}{(1 + r)^2} \quad (2)$$

# Fox-Wolfram moment distributions

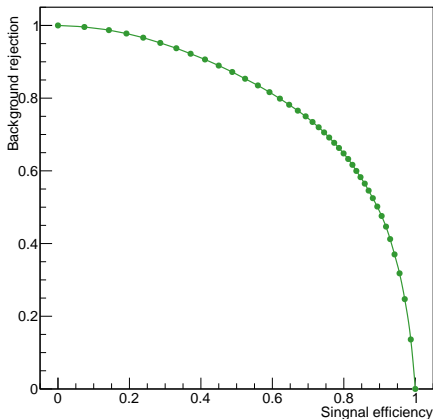


2

# Performance of Fox-Wolfram moments in BDT

Rank	Variable	Separation
1	$\Delta R$ b/w b tag jet & untagged jet	$2.097 \times 10^{-1}$
2	$ \eta $ of untagged jet	$2.047 \times 10^{-1}$
3	Sum of mass of both jet	$1.676 \times 10^{-1}$
4	$\cos \theta^*$ , $\theta$ is angle b/w elec & untagged jet	$7.089 \times 10^{-2}$
5	$m_t^W$ transverse mass of the W	$4.792 \times 10^{-2}$
6	First Fox-Wolfram moment	$2.985 \times 10^{-2}$
7	$ \Delta R $ b/w electron and b tag jet	$1.267 \times 10^{-2}$
8	Sum of $P_T$ 's of both jet	$1.013 \times 10^{-2}$

First Fox-Wolfram moment has best sensitivity among the three and is used in the BDT



## ROC curve

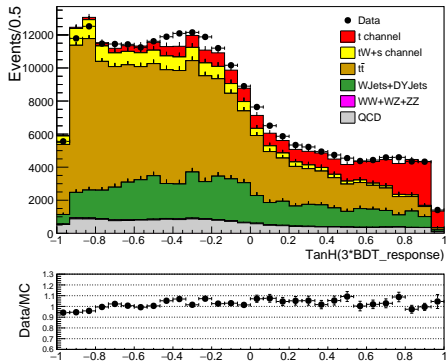
basic

cor

# BDT Response

CMS Preliminary

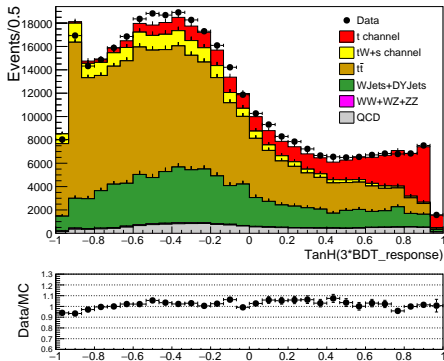
35.9 fb<sup>-1</sup> (13 TeV)



$e + \text{jets}$  final state

CMS Preliminary

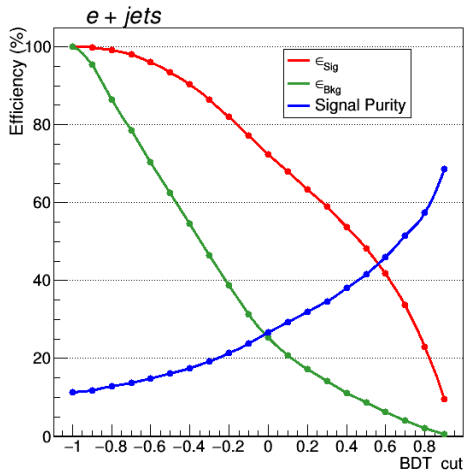
35.9 fb<sup>-1</sup> (13 TeV)



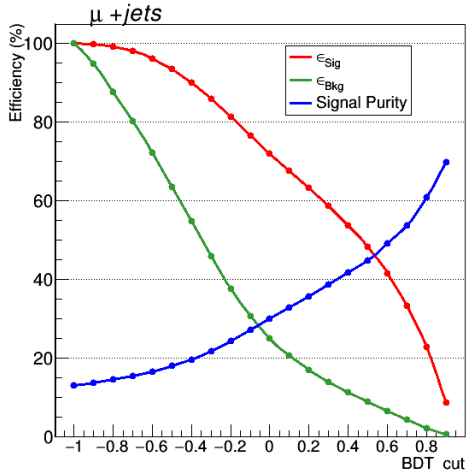
$\mu + \text{jets}$  final state



# Purity, signal efficiency vs. multivariate discriminator



***e + jets* final state**



***$\mu$  + jets* final state**

# Top quark mass reconstruction

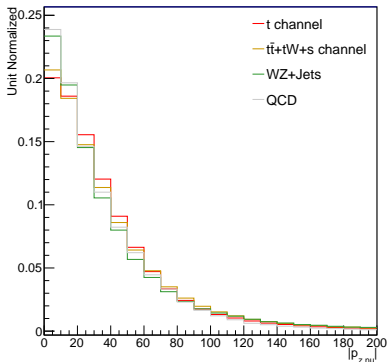
- Top quark mass is reconstructed using the available kinematic information of the event
- The longitudinal momentum of the neutrino is determined from the  $m_W = 80.4$  GeV constraint
- Using energy-momentum conservation we obtain a quadratic equation in  $p_{z\nu}$  with following solutions

$$p_{z,\nu} = \left( \frac{\lambda^2 p_{z,\ell}}{p_{T,\ell}^2} \right) \pm \frac{\sqrt{(\lambda^2 p_{z,\ell}^2 - p_{T,\ell}^2)(\cancel{E}_T E_\ell^2 - \lambda^2)}}{p_{T,\ell}^2}$$

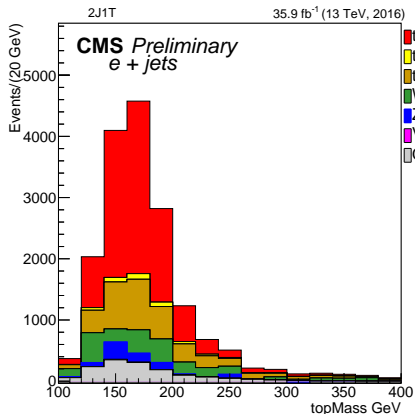
$$\text{Discriminant} \rightarrow (\lambda^2 p_{z,\ell}^2 - p_{T,\ell}^2)(\cancel{E}_T E_\ell^2 - \lambda^2)$$

$$\text{Here } \lambda = \frac{(m_W)^2}{2} + \vec{\cancel{E}}_T \cdot \vec{p}_{T,\ell}$$

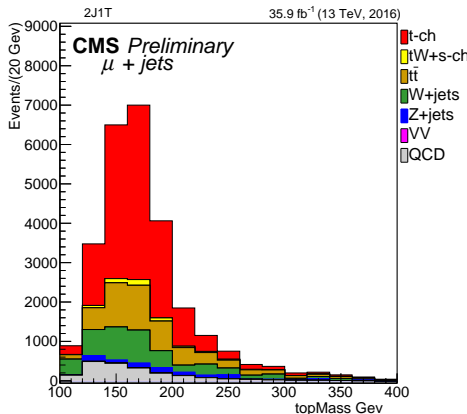
- Discriminant is -ve
  - Put  $m_T^W = m_W$  and make discriminate zero
- Discriminant is +ve
  - choose the solution which have minimum  $|p_{z\nu}|$



# top quark mass after the BDT cut



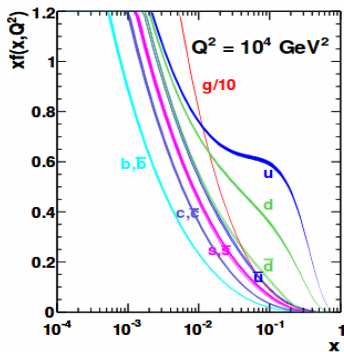
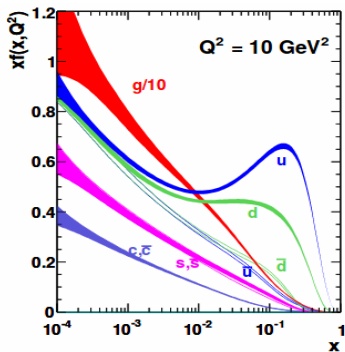
top quark mass distribution  
of  $e + jets$  final state after  
BDT cut



top quark mass distribution  
of  $\mu + jets$  final state after  
BDT cut

Thank you

# Parton Distribution



main

# Jet reconstruction algorithm

- Iterative process to cluster particles based on “closeness”
- For each particle  $i$ , compute

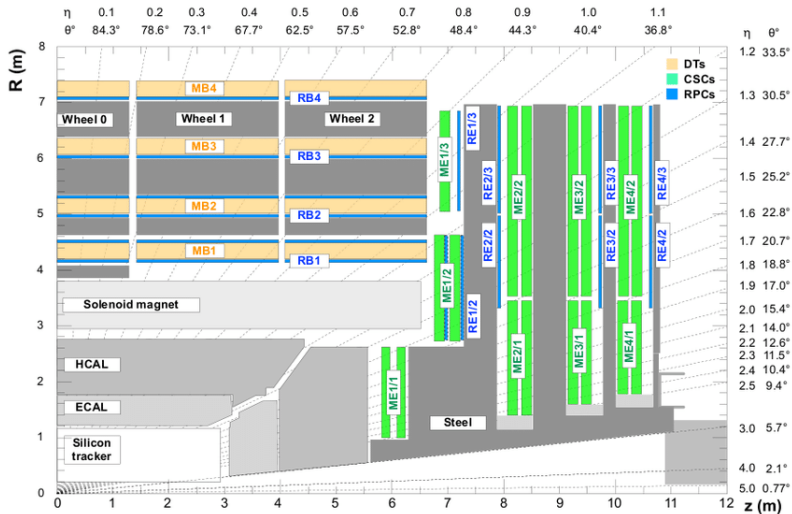
$$d_i = p_{T,i}^{2p}$$

- For each pair of particles  $i$  and  $j$ , compute

$$d_{i,j} = \min(p_{T,i}^{2p}, p_{T,j}^{2p}) \frac{R_{i,j}^2}{R^2} \quad R_{i,j}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- Find minimum of all  $d_i, d_{i,j}$ 
  - If  $d_i$  is minimum,  $i$  is a jet and removed from list of particles
  - If  $d_{i,j}$  is minimum, combine particles  $i$  and  $j$
- Repeat process until all particles are part of a jet with  $R_{i,j} < R$
- Parameter  $p$  governs the relative power of energy vs geometrical scales to distinguish the algorithms:  $1=k_T, -1=Anti-k_T$  main

# Coverage of CMS detector



Source :ResearchGate uploaded by Tamer Elkafrawy

# Data and Simulated samples

Process	$\sigma(\cdot\text{BR})[\text{pb}]$	Generators	Nevents
t-channel top	136.02 (NLO)	powheg-pythia8	67 m
t-channel antitop	80.95 (NLO)	powheg-pythia8	38 m
tw-channel top	35.6 (NLO)	powheg-pythia8	0.9 m
tw-channel antitop	35.6 (NLO)	powheg-pythia8	0.9 m
S channel top+antitop	10.32 (NLO)	aMC@NLO-pythia8	0.6 m
$t\bar{t}$	831.76 (NNLO)	aMC@NLO-pythia8	77 m
$W(\rightarrow l\nu)+0\text{jets}$	49670 (NNLO)	aMC@NLO-pythia8	39 m
$W(\rightarrow l\nu)+1\text{jets}$	8264 (NNLO)	aMC@NLO-pythia8	19 m
$W(\rightarrow l\nu)+2\text{jets}$	2628 (NNLO)	aMC@NLO-pythia8	15 m
$Z/\gamma(\rightarrow ll)+\text{jets}$	5765.4 (NNLO)	aMC@NLO-pythia8	19 m
$WW(\rightarrow l\nu)+\text{jets}$	45.85 (NLO)	aMC@NLO-pythia8	3 m
$WW(\rightarrow 2l2\nu)+\text{jets}$	12.178 (NLO)	powheg-pythia8	1 m
$WZ(\rightarrow l\nu)+\text{jets}$	10.71 (NLO)	aMC@NLO-pythia8	14 m
$WZ(\rightarrow 2l2\nu)+\text{jets}$	5.595 (NLO)	aMC@NLO-pythia8	15 m
$ZZ(\rightarrow l\nu)+\text{jets}$	3.22 (NLO)	aMC@NLO-pythia8	9 m
QCD	302672.16 (LO)	pythia8	22 m

List of MC samples Scaled to  $\mathcal{L}_{\text{int}} = 35.9 \text{ fb}^{-1}$

Run Period	Run Range	Dataset Name	Integrated Luminosity ( $\text{fb}^{-1}$ )
Run B	273158 - 275376	/SingleElectron/Run2016B-03Feb2017.ver2-v2/MINIAOD	5.7
Run C	275657 - 276283	/SingleElectron/Run2016C-03Feb2017-v1/MINIAOD	2.6
Run D	276315 - 276811	/SingleElectron/Run2016D-03Feb2017-v1/MINIAOD	4.2
Run E	276831 - 277420	/SingleElectron/Run2016E-03Feb2017-v1/MINIAOD	4.0
Run F	277981 - 278808	/SingleElectron/Run2016F-03Feb2017-v1/MINIAOD	3.1
Run G	278820 - 280385	/SingleElectron/Run2016G-03Feb2017-v1/MINIAOD	7.6
Run H	281613 - 284044	/SingleElectron/Run2016H-03Feb2017.ver2-v1/MINIAOD	8.4
		/SingleElectron/Run2016H-03Feb2017.ver3-v1/MINIAOD	0.2
Total			35.9

List of data samples

main



24/46



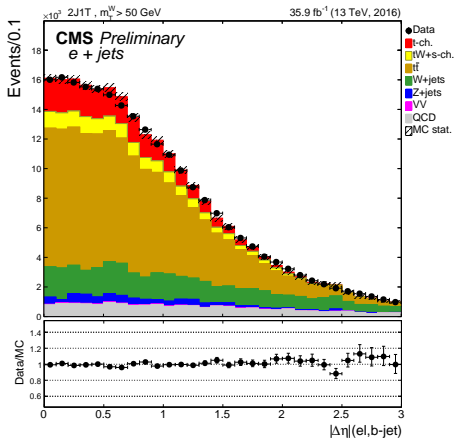
Table: Cut flow for events with the muon final state.

Cut	t-ch.	tW + s-ch.	t $\bar{t}$	W+jets	Z+jets	di-boson	QCD	Total MC	Data	Data/MC(%)
No cut	7779880	2672579	29766400	2213536000	206744000	2781417	1.08533e+10	15538630276	788340964	5.07
Trigger	522486	386189.3	4026120	310895500	44618700	576922.4	1.56252e+08	828871917.7	471040410	56.83
1 tight isolated $\mu$	386696	284249.3	2836410	222101300	21587000	414429.4	2.32383e+07	493565384.7	259002854	52.48
Loose $\mu$ veto	384687	272944.3	2712020	221766800	12231200	387302.6	2.23960e+07	482532953.9	246130600	51.01
e veto	383160	246864.9	244290	221603800	12129600	357592.4	2.23670e+07	481745307.3	245550723	50.97
2 jets	144898.2	89136.9	665306	6059963	544834	76369.4	1.31616e+06	15087927.5	8152312	54.03
1 b-tag	61109.8	32067.6	265547	100684.3	17725.1	2053.3	138056	715988.19	552720	77.20
$m_t^W > 50$ GeV	42180.6	16206.2	181050	68497.3	9156.2	1327.2	32844.4	318495.37	326800	93.04

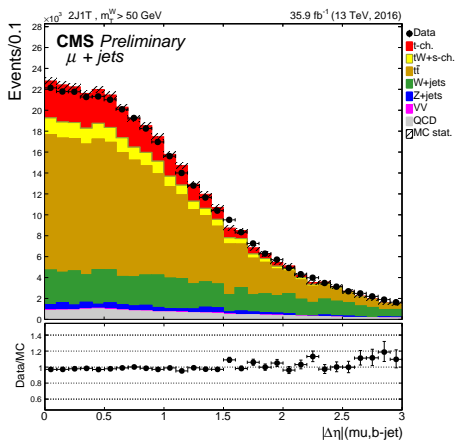
Table: Cut flow for events with the electron final state.

Cut	t-ch.	tW + s-ch.	t $\bar{t}$	W+jets	Z+jets	di-boson	QCD	Total MC	Data	Data/MC(%)
No cut	7717690	2673866	30895200	4.43e+09	2.08e+08	2783180	6.41e+11	6.46e+11	843398435	0.13
Trigger	309594	262230.3	2660710	2.91e+108	2.72e+07	353090.5	9.71e+07	4.19e+08	292170629	69.69
1 Tight isolated e	229830.3	201756.71	2019920	1.77e+08	1.42e+07	251417.7	1.21e+07	2.05e+08	117287424	56.94
Loose e veto	229067.5	192672.2	1920620	1.76e+08	5752430	228387.7	1.21e+07	1.97e+08	108316594	54.91
$\mu$ veto	227904.9	171837.61	1711180	1.76e+07	5709520	208425.8	1.21e+07	1.96e+08	107856987	54.83
2 Jets	86035.3	61974.68	456275	7036850	529504	48973.8	1356662.3	9576275	5918361	61.80
1 b-tag	36664.9	21965.62	182395	116990.6	16016.1	1331.1	28739.4	404102.7	359251	88.90
$m_t^W > 50$ GeV	15852.7	15108.48	128345	41333	7712.2	864	22334.5	271400	238295	87.80

# Data-MC comparison plots 2J1T (contd.)

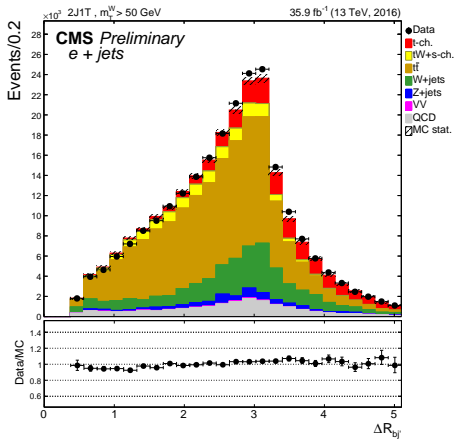


$|\Delta\eta|$  between the  
electron and b tagged jet  
in e+jets final state

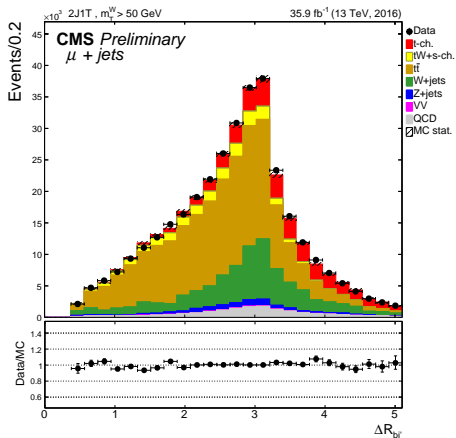


$|\Delta\eta|$  between the muon  
and b tagged jet in  $\mu$ +jets  
final state

# Data-MC comparison plots 2J1T (contd.)



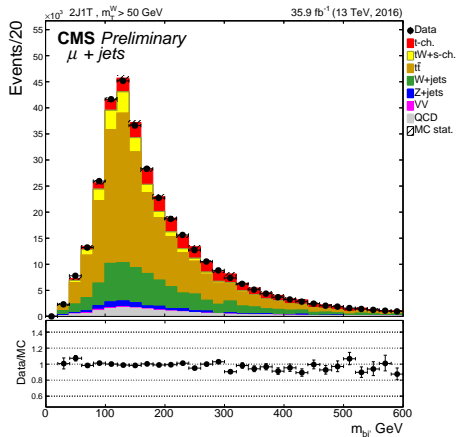
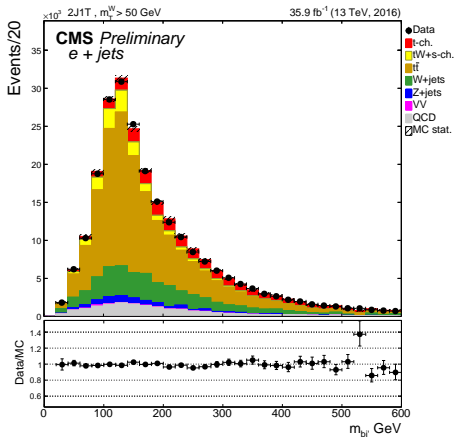
$|\Delta R|$  between the  
electron and b tagged jet in  
 $e+jets$  final state



$|\Delta R|$  between the muon  
and b tagged jet in  $\mu+jets$   
final state

main

# Data-MC comparison plots 2J1T (contd.)

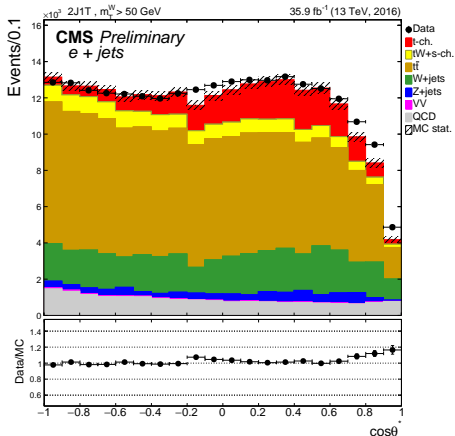


Invariant mass sum of the  
untagged and b tagged jet in  
 $e+jets$  final state

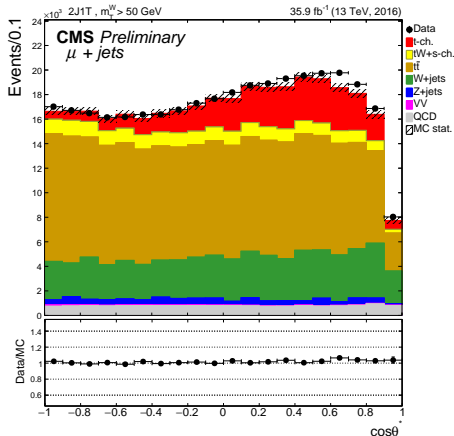
main

Invariant mass sum of the  
untagged and b tagged jet in  
 $\mu+jets$  final state

# Data-MC comparison plots 2J1T (contd.)

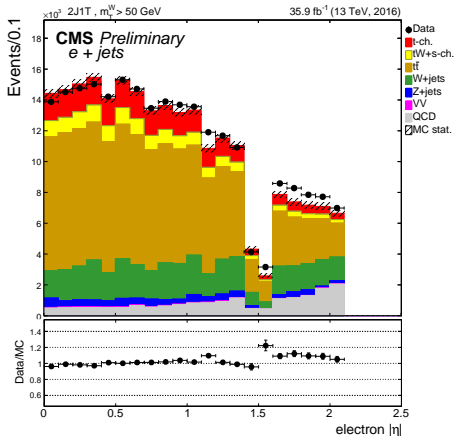


$\theta^*$  is the angle b/w the electron and untagged jet in the top quark frame main

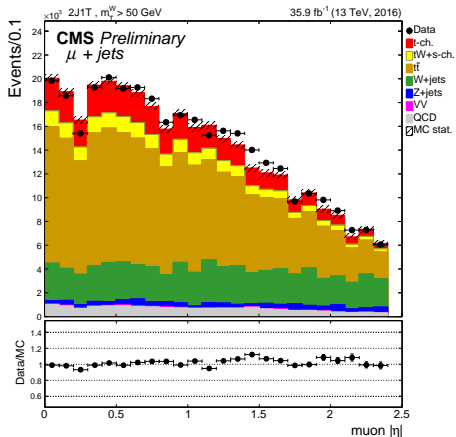


$\theta^*$  is the angle b/w the muon and untagged jet in the top quark frame

# Data-MC comparison plots 2J1T (contd.)

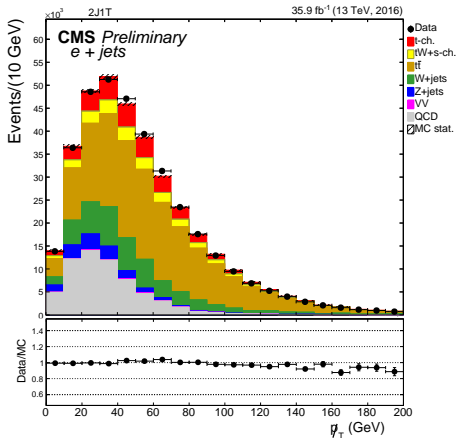


$|\eta|$  of the electron main

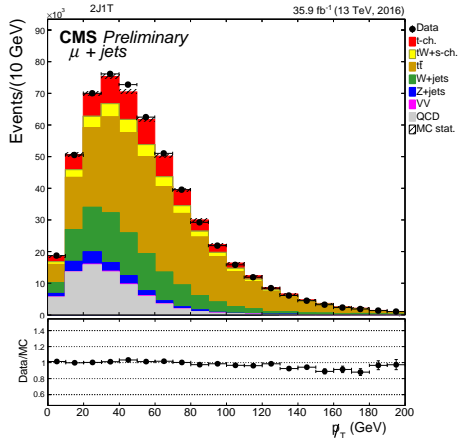


$|\eta|$  of the muon

# Data-MC comparison plots 2J1T (contd.)

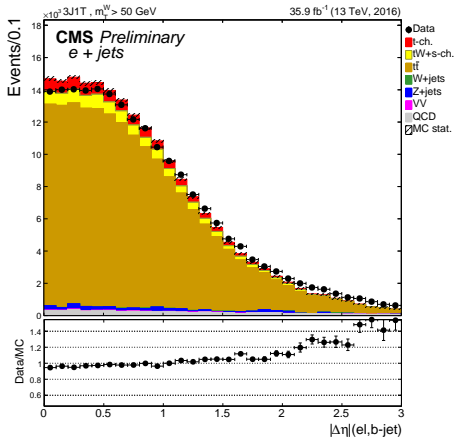


$m_t^W$  transverse mass of W  
boson in e+jets final stat main

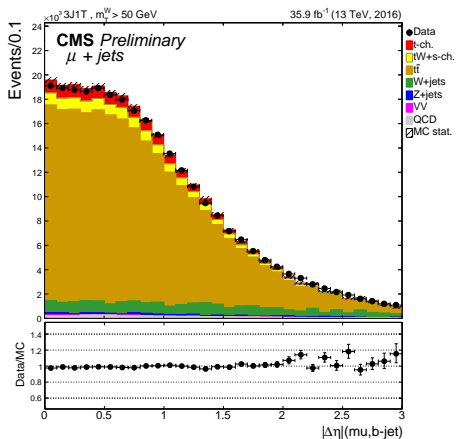


$m_t^W$  transverse mass of W  
boson in  $\mu$ +jets final stat

# Data-MC comparison plots 3J1T



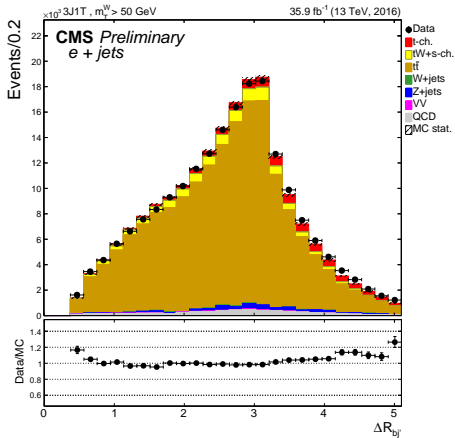
$|\Delta\eta|$  between the  
electron and b tagged jet  
in e+jets final state



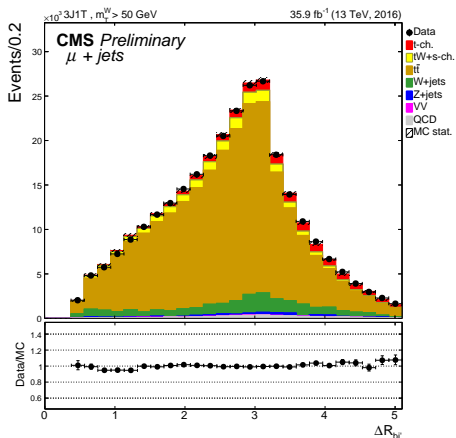
$|\Delta\eta|$  between the muon  
and b tagged jet in  $\mu$ +jets  
final state main



# Data-MC comparison plots 3J1T (contd.)



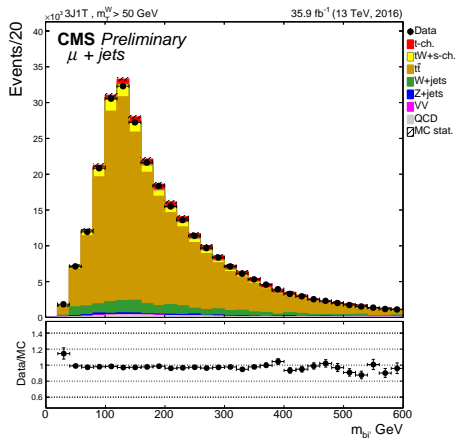
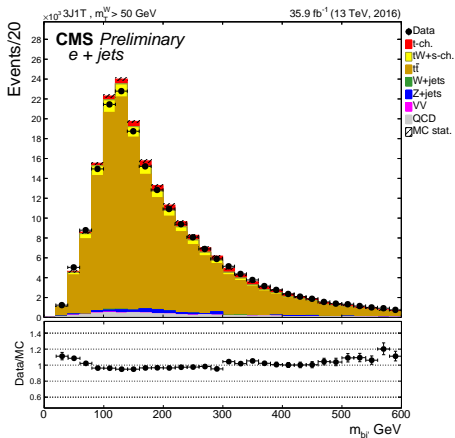
$|\Delta R|$  between the  
electron and b tagged jet in  
*e+jets* final state



$|\Delta R|$  between the muon  
and b tagged jet in *mu+jets*  
final state

main

# Data-MC comparison plots 3J1T (contd.)



Invariant mass sum of the  
untagged and b tagged jet in  
*e + jets* final stat

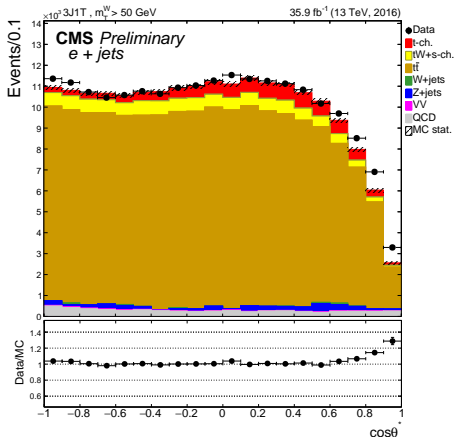
main

Invariant mass sum of the  
untagged and b tagged jet in  
 *$\mu$  + jets* final stat

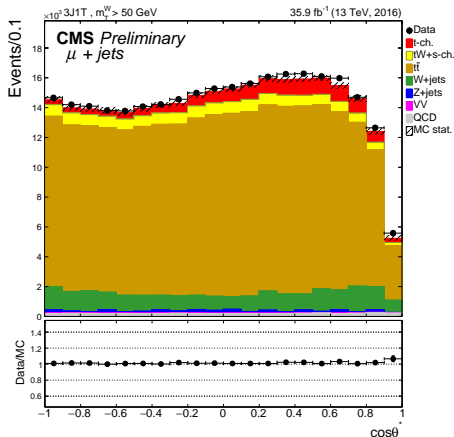
Navigation icons: back, forward, search, refresh, etc.

34/46

# Data-MC comparison plots 3J1T (contd.)

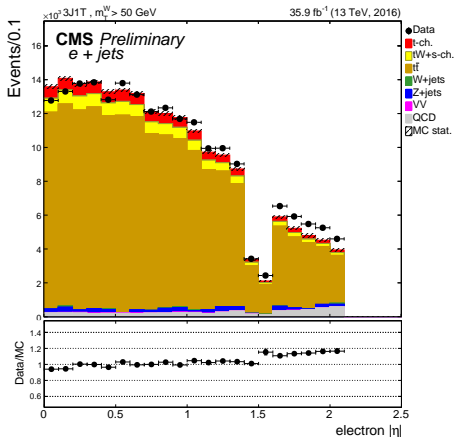


$\theta^*$  is the angle b/w the electron and untagged jet in the top quark frame main

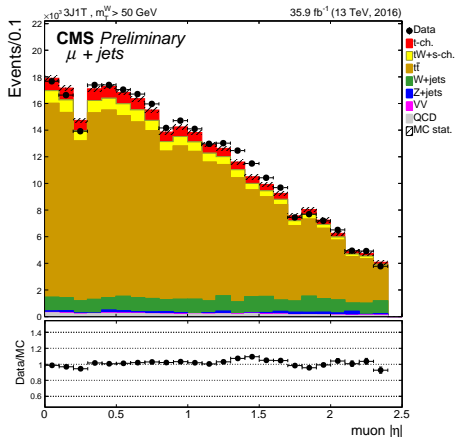


$\theta^*$  is the angle b/w the muon and untagged jet in the top quark frame

# Data-MC comparison plots 3J1T (contd.)



$|\eta|$  of the electron main



$|\eta|$  of the muon

# Fox-Wolfram moments

- Describe correlation between jets using event geometry
- Calculated from all the selected jets in each event
- Moments are defined as the superposition of spherical harmonics  $Y_l^m \cos(\theta)$  :

$$H_l = \frac{4\pi}{2l+1} \sum_{i=-l}^l \left| \sum_{i=1}^N Y_l^m(\Omega) \frac{|p_i|}{\sqrt{s}} \right|^2 \quad (3)$$

Here index  $i$  sums over all the jets and  $\Omega_i$  is the angular distance from the assumed reference axis

- Using an additional theorem for spherical harmonics

$$\frac{4\pi}{2l+1} \sum_{i=-l}^l Y_l^m(\Omega) Y_l^{m*}(\Omega) = P_l(\cos \Omega_{ij}) \quad (4)$$

We can rewrite main

$$H_l = \sum_{i,j=-1}^N \frac{|\vec{p}_i| |\vec{p}_j|}{s} P_l(\cos \Omega_{ij}) \quad (5)$$

- life time of the b quark is large
- Combined Secondary Vertex version 2 (CSVv2):
  - based on secondary vertex and track-based lifetime information
  - combining the variables such as vertex mass, number of tracks at vertex and inside a jet, 2D flight distance significance of the b-flavoured meson, ratio of energy carried by the tracks at the secondary vertex vs all tracks inside the jet etc
- combined MultiVariate Algorithm version 2 (cMVAv2):
  - Algorithm uses a Boosted Decision Tree (BDT)
  - taking as input the different algorithm outputs of CSVv2, a variant of CSVv2 using alternate vertex reconstruction, Jet Probability (JP), Jet B Probability (JBP)
  - Algorithm performs better compared to CSVv2
  - A tight threshold on the b-tagging discriminator value corresponds to mis-tag rate of 0.1% main

$$I_{rel} = \frac{I^{\text{charged hadron}} + \max\left((I^\gamma + I^{\text{neutral hadron}} - I^{\text{pileup}}), 0\right)}{p_{T,\mu}} \quad (6)$$

where  $I$  is the energy deposit by the corresponding particle. [main](#)

# Boosted decision trees

- An advanced multivariate technique to separate signal from backgrounds.
- From input variables find one for which a single cut gives the best improvement in signal purity:

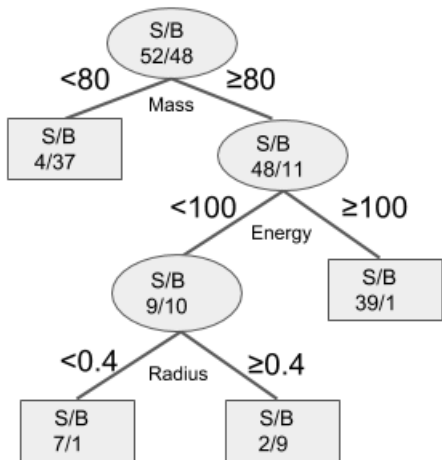
$$P = \frac{S}{S + B}$$

- The set of cuts defines the decision classifier

$$t(x) = \sum_{k=1}^K \alpha_k f_k(x)$$

- separation  $\langle s^2 \rangle$  of a variable  $x$  is given as an integral

$$\langle s^2 \rangle = \frac{1}{2} \int \frac{(\hat{x}_S(x) - \hat{x}_B(x))^2}{(\hat{x}_S(x) + \hat{x}_B(x))} dx$$





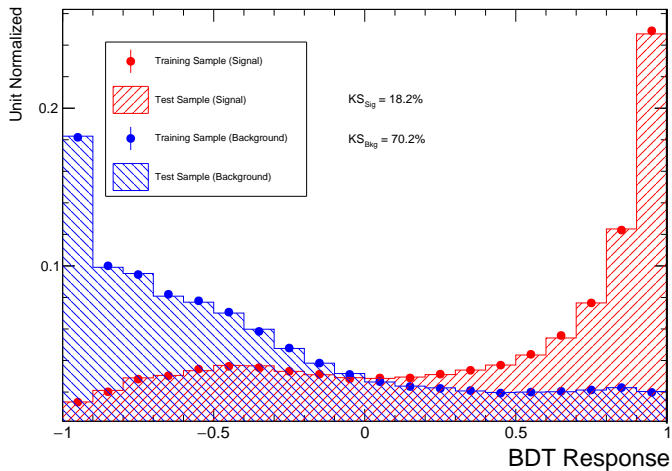
- First initialize the training sample  $T_1$  using the original  
 $x_1, x_2, x_3 \dots x_N$  Variables  
 $y_1, y_2, y_3 \dots y_N$  event data Vector  
 $w_1^1, w_2^1, w_3^1 \dots w_N^1$  event weight

$$\sum_{i=1}^n w_i = 1 \quad (8)$$

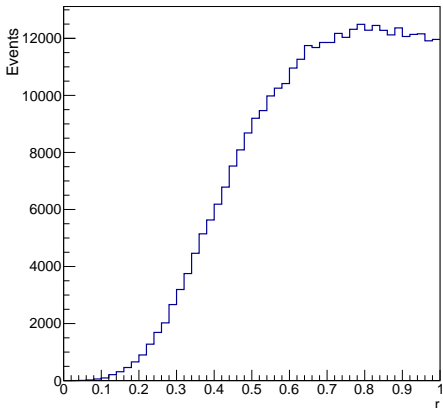
- Then train the classifier  $f_1(x)$  with a method that uses the event weights
- We will define an iterative procedure that gives a series of classifiers  $f_1(x), f_2(x), \dots$
- The classifier at each iterative step is found from an updated training sample, in which the weight of event  $i$  is modified from step  $k$  to step  $k + 1$  according to

$$w = \frac{1 - f_{err}}{f_{err}} \quad \text{where} \quad f_{err} = \frac{\# \text{misclassified event}}{\# \text{all event}} \quad (9)$$

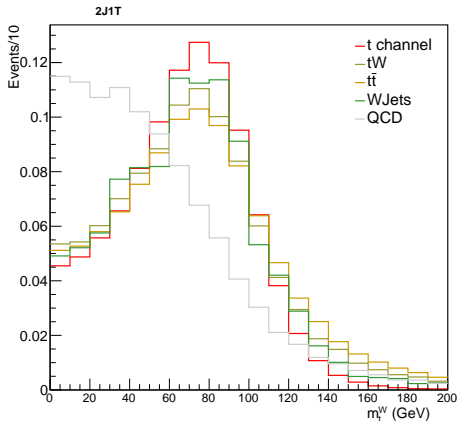
# Over training test



# Data-MC comparison plots (contd.)

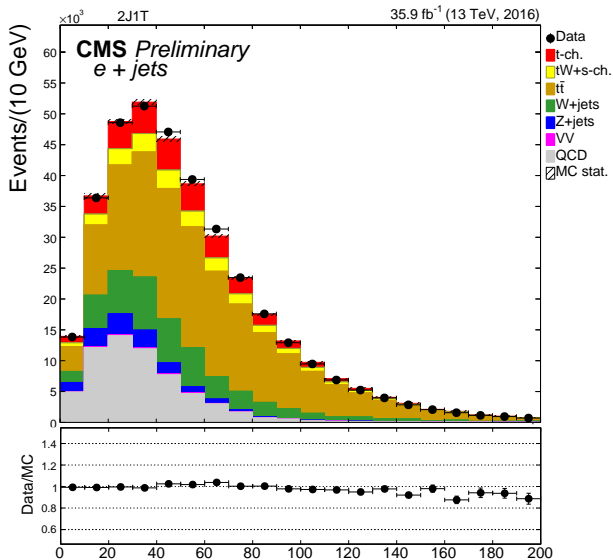


$p_t$  ratio of jets main



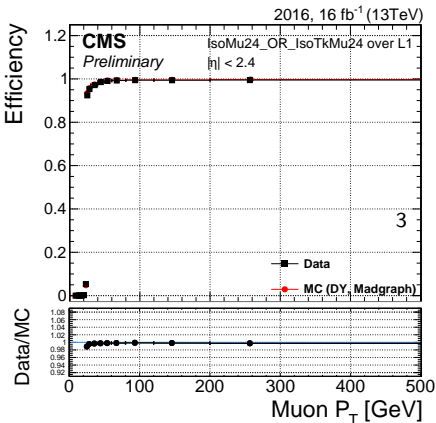
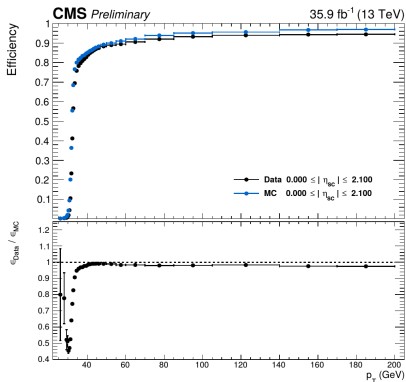
$m_t^W$  transverse mass of W boson

# Met distribution



main

# Trigger efficiency



HLT\_Ele32\_eta2p1\_WPTight

IsoTkMu24\_OR\_IsoTkMu24

<sup>3</sup>Electron trigger performance in CMS with the full 2017 data sample.



45/46

- Electron tight ID
  - $H/E < 0.0414$  (barrel), 0.0641 (endcap),
  - $\sigma_{\eta\eta} < 0.00998$  (barrel), 0.0292 (endcap),
  - $|\Delta\eta\eta| < 0.00308$  (barrel), 0.00605 (endcap),
  - $|\Delta\phi\eta| < 0.0816$ (barrel), 0.0394 (endcap),
  - $|\frac{1}{ESC} - \frac{1}{p}| < 0.0129$ (barrel), 0.0129 (endcap),
  - $I_{rel,corrPF} < 0.0588$  (barrel), 0.0571(endcap).
  - expected missing inner hits  $\leq 1$  (barrel), 1 (endcap)
  - pass conversion veto = yes (barrel), yes (endcap)
- Jet loose ID
  - Neutral Hadron Fraction  $< 0.99$
  - Neutral EM Fraction  $< 0.99$
  - Number of Constituents  $> 1$
  - Charged EM Fraction  $< 0.99$