Event-by-event fluctuation analyses in ALICE and long-term perspectives

- ✓ Why fluctuations?
- ✓ Fluctuation analyses in ALICE
  - ❖ Conserved charge fluctuations
  - ❖ Experimental Challenges
- ✓ Future plans

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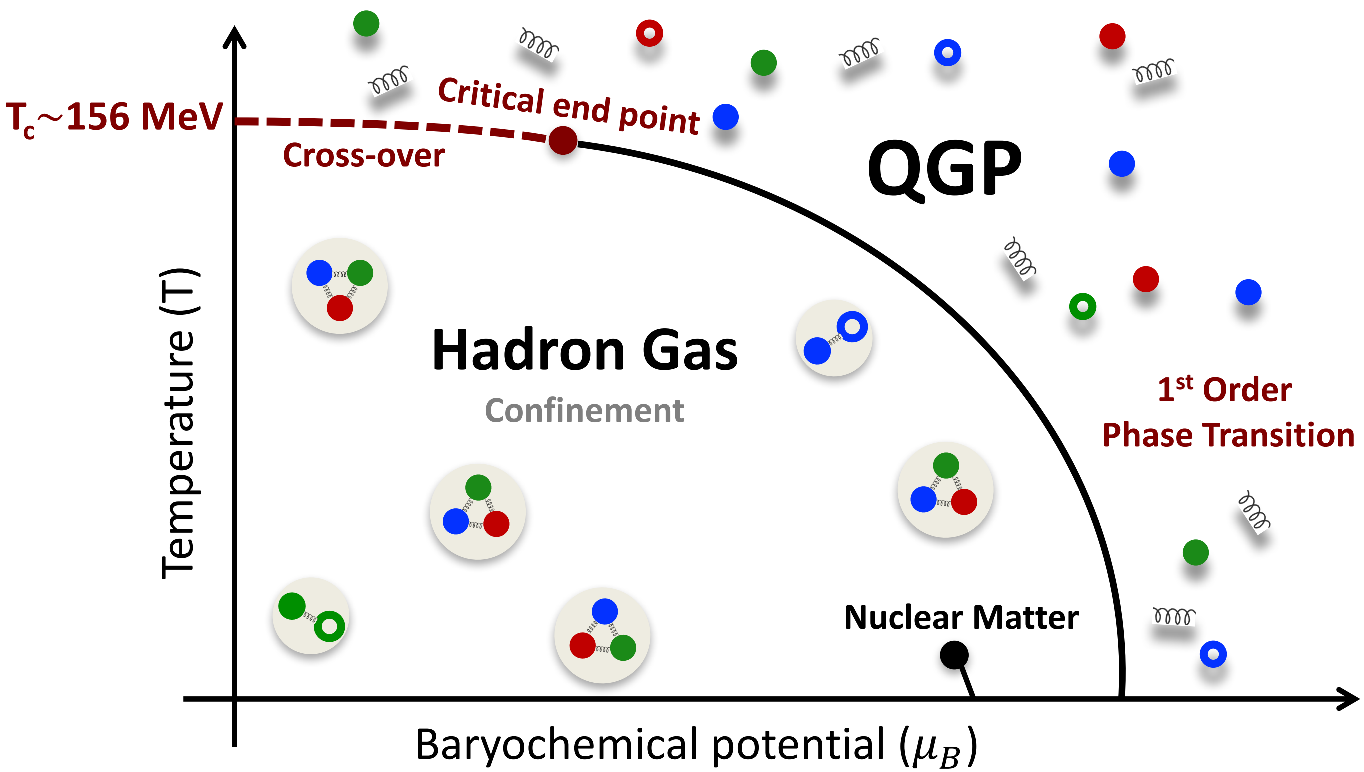
on behalf of the ALICE Collaboration

The 36th Winter Workshop on Nuclear Dynamics (WWND), Puerto Vallarta, Mexico
2 March 2020

QCD phase diagram



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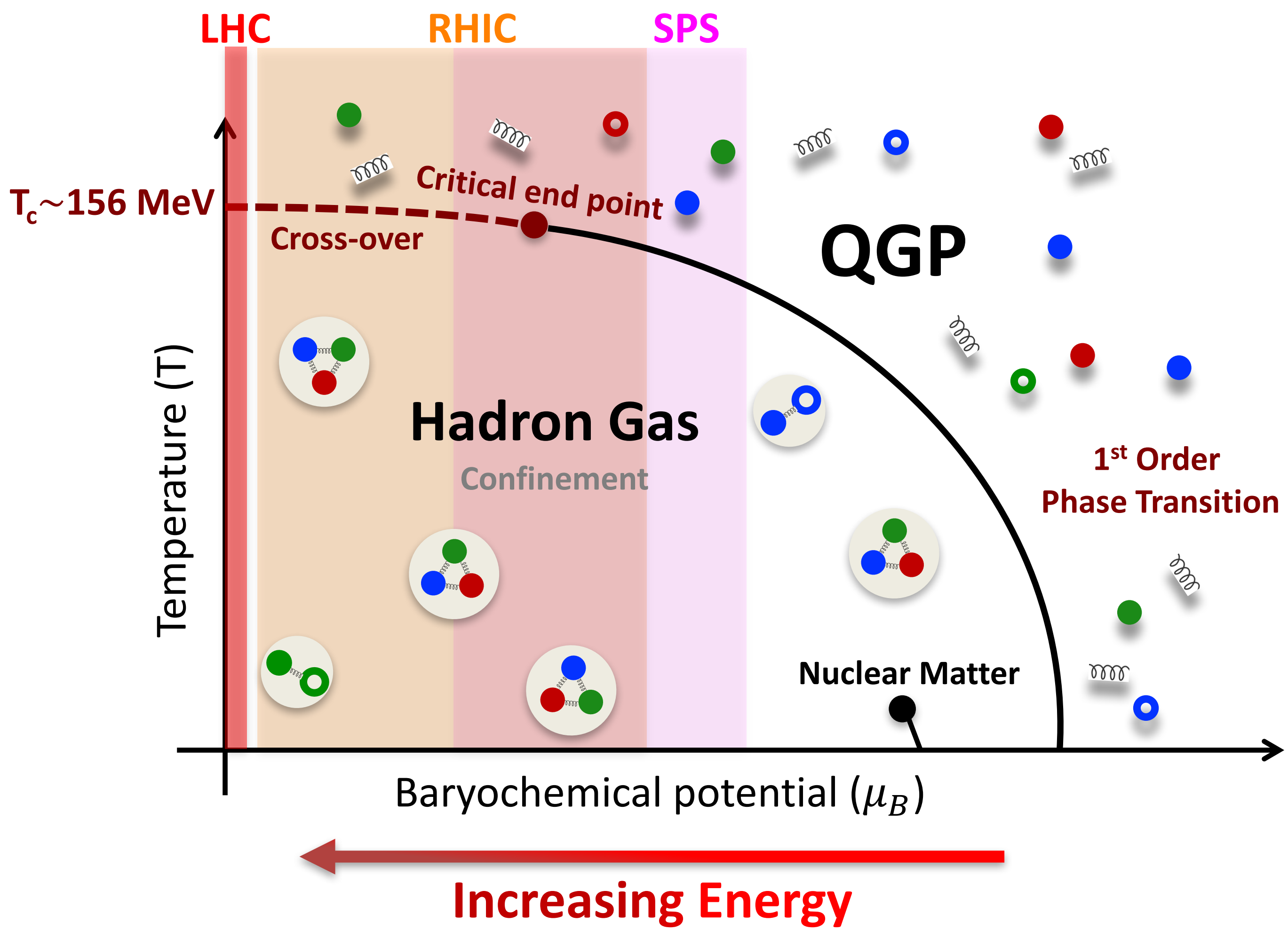
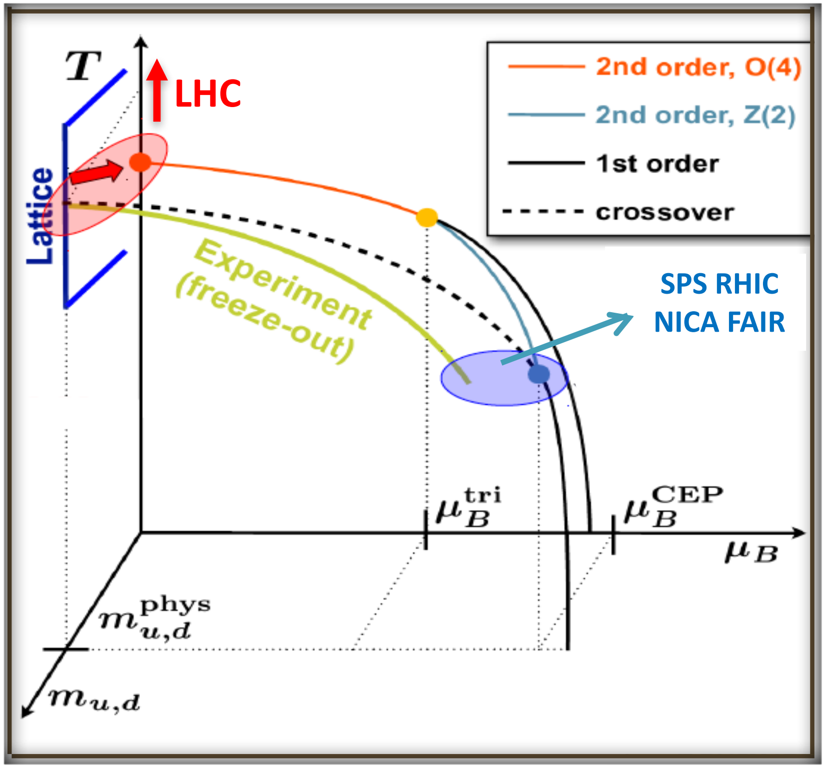


Closer look at QCD Phase diagram: **Nature of chiral phase transition**



F. Karsch, Schleching 2016

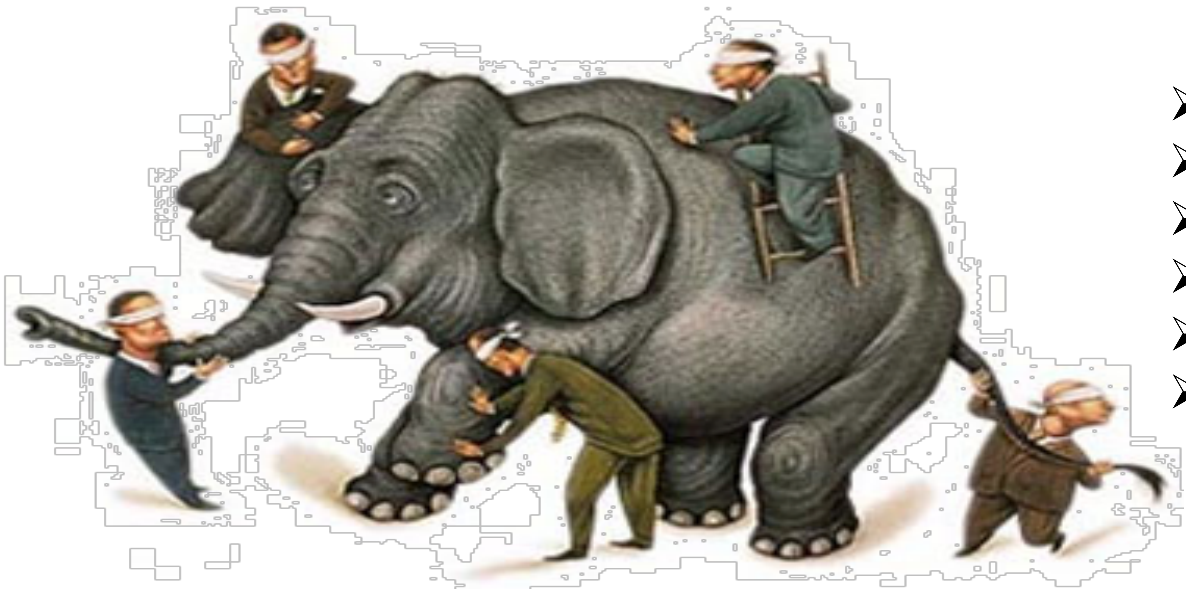
small u, d quark masses

↔

vicinity to 2[nd] order O(4) criticality

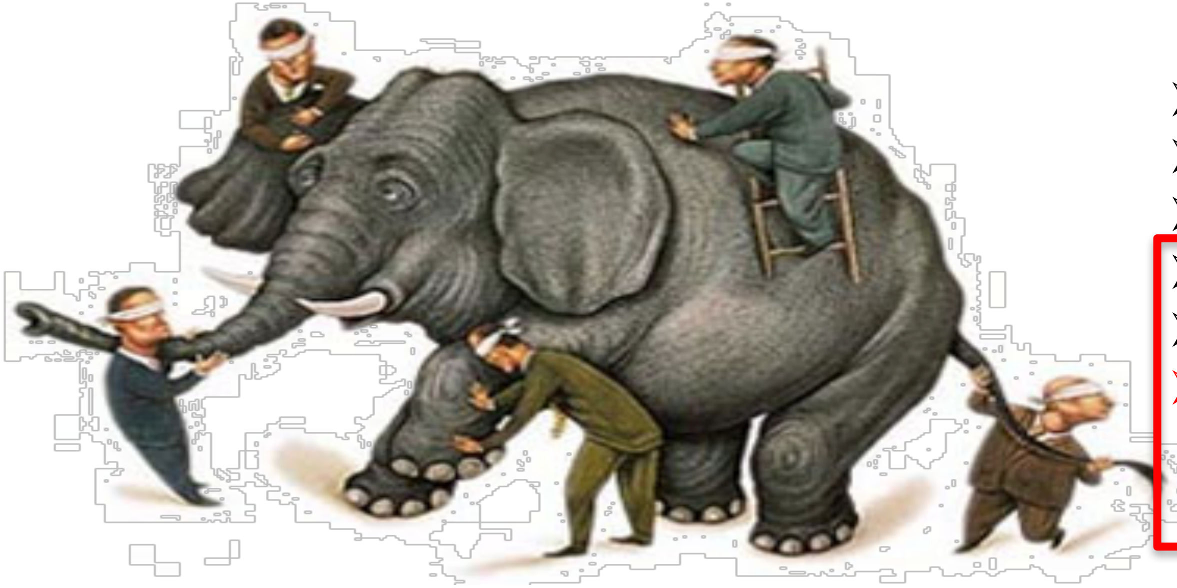
pseudocritical features possible

Fluctuation analyses in ALICE



- Mean-p_T fluctuations
- Intermittency
- Balance Functions
- Strongly intensive Quantities
- Relative particle yield fluctuations
- Conserved-charge fluctuations
  - Net-Lambda
  - Net-baryon

Fluctuation analyses in ALICE



- Mean-p_T fluctuations
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- Balance Functions
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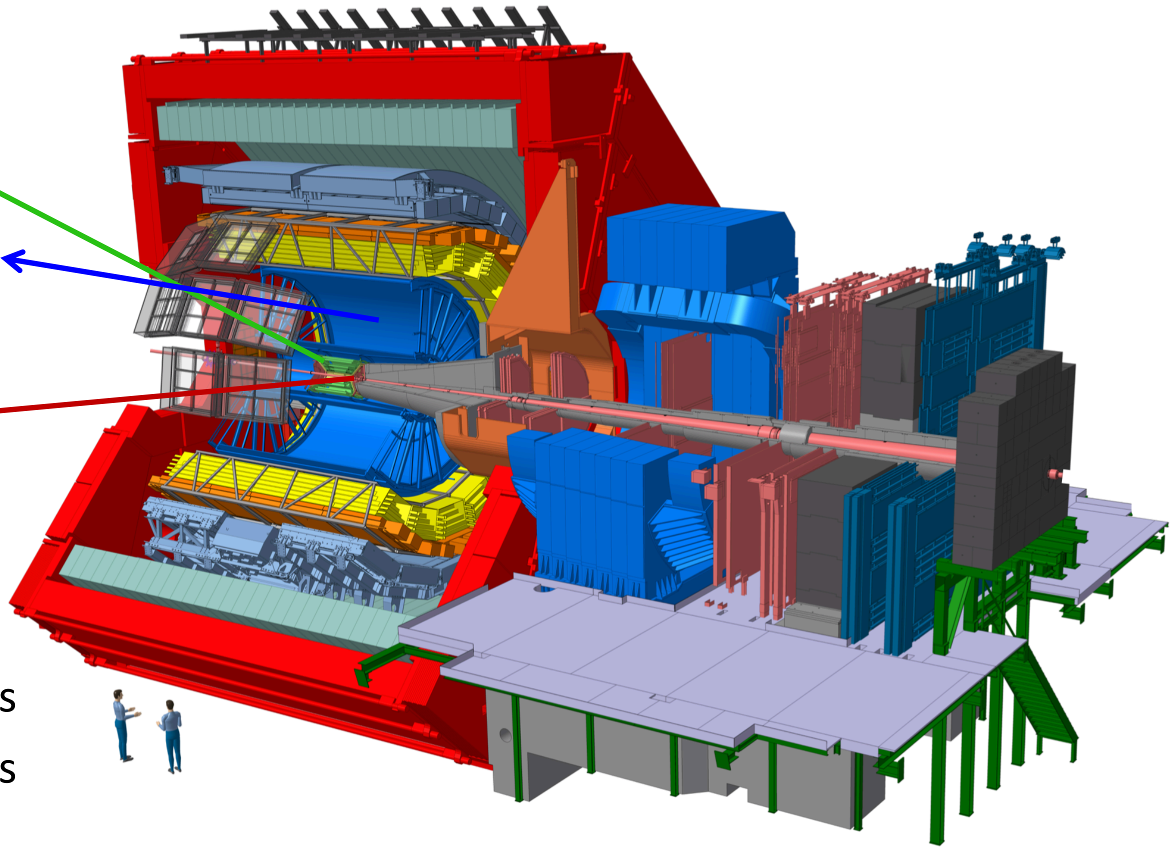
- Relative particle yield fluctuations
- **Conserved-charge fluctuations**
  - **Net-Lambda**
  - **Net-baryon**

Keywords: Correlations, Criticality, Link to Lattice QCD

A Large Ion Collider Experiment

Main detectors used:

- **I**nner **T**racking **S**ystem **(ITS)**
  → Tracking and vertexing
- **T**ime **P**rojection **C**hamber **(TPC)**
  → Tracking and Particle Identification (PID)
- **V**ertex **0** **(V0)**
  → Centrality determination



Data Set:

- $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV, ~78 M events
- $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV, ~12 M events

Correlations: Relative particle yield fluctuations

N_A, N_B: multiplicities for particle types A and B

$$\nu_{dyn} = \frac{\langle N_A(N_A-1)\rangle}{\langle N_A\rangle^2} + \frac{\langle N_B(N_B-1)\rangle}{\langle N_B\rangle^2} - 2\frac{\langle N_A N_B\rangle}{\langle N_A\rangle\langle N_B\rangle}$$

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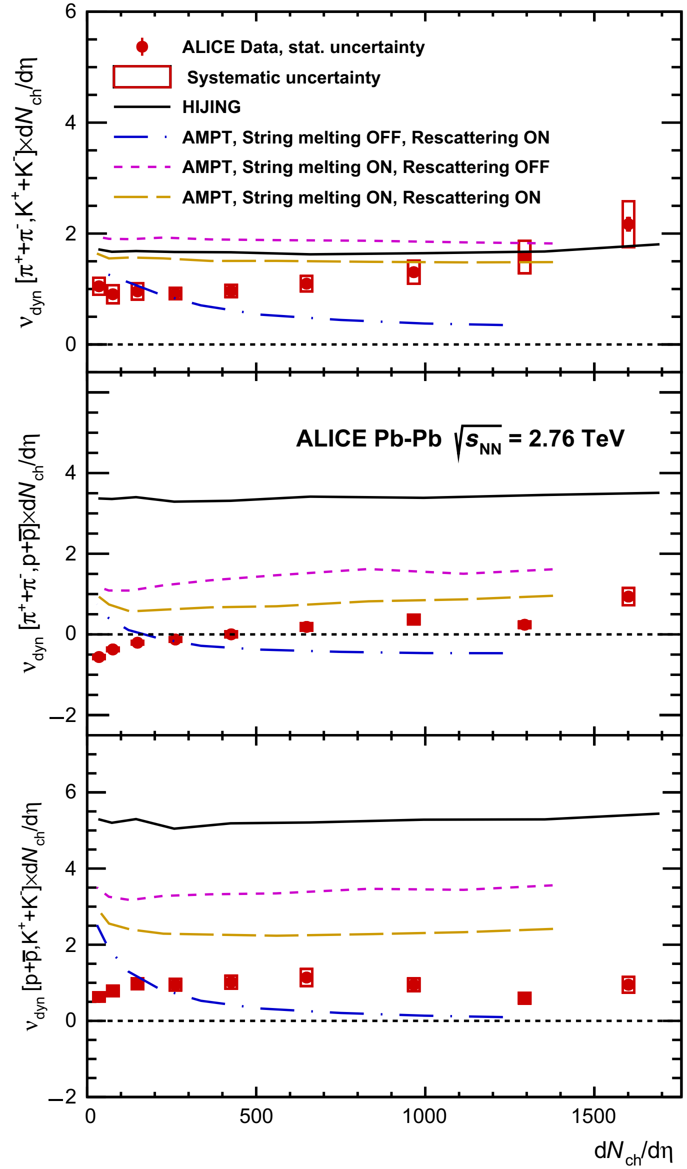
ALICE Collaboration, Eur. Phys. J. C79 (2019) 236



$\nu_{dyn}[\pi,K]$

$\nu_{dyn}[\pi,p]$

$\nu_{dyn}[p,K]$

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$\nu_{dyn}[\pi,p]$: **Increasing correlation** with decreasing centrality

$\nu_{dyn}[\pi,K]$: Increasing anti-correlation between **π and K** or increasing dynamical fluctuations with increasing centrality

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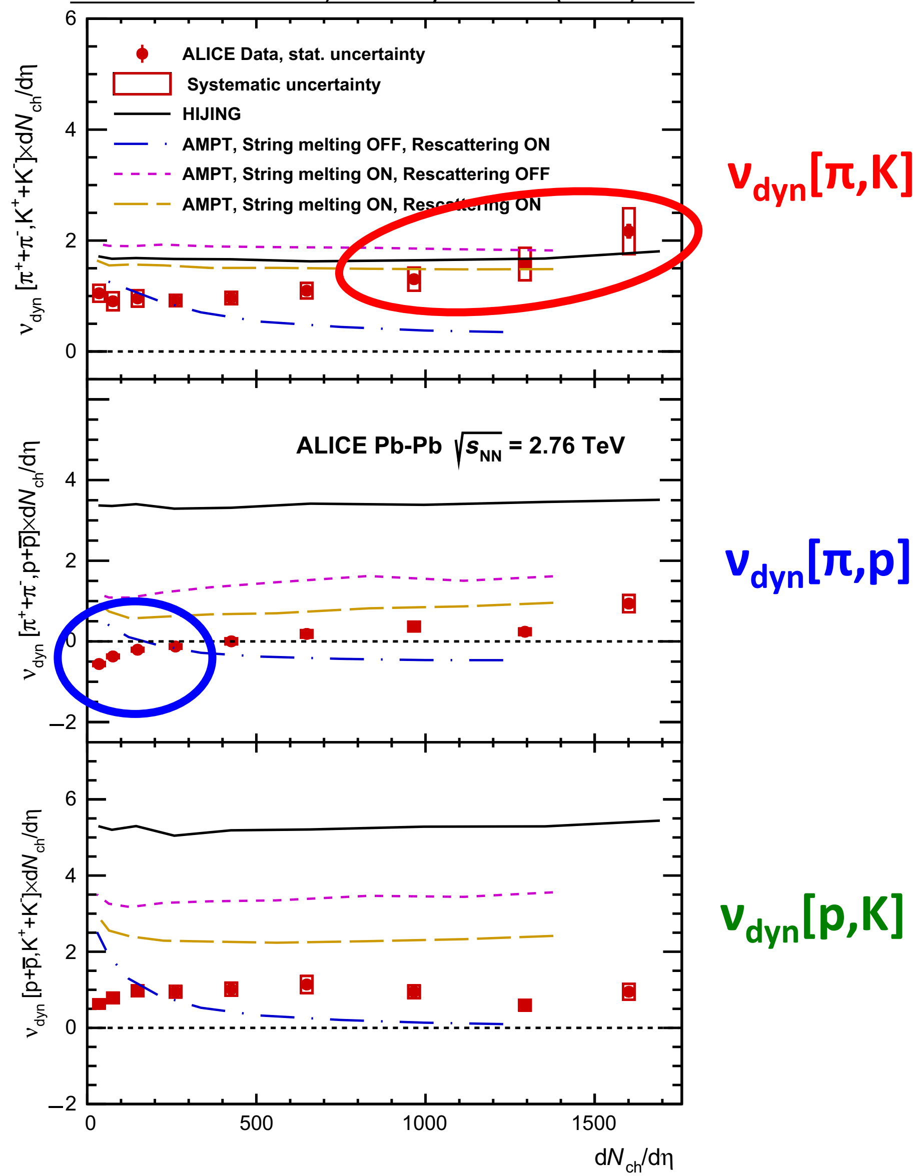

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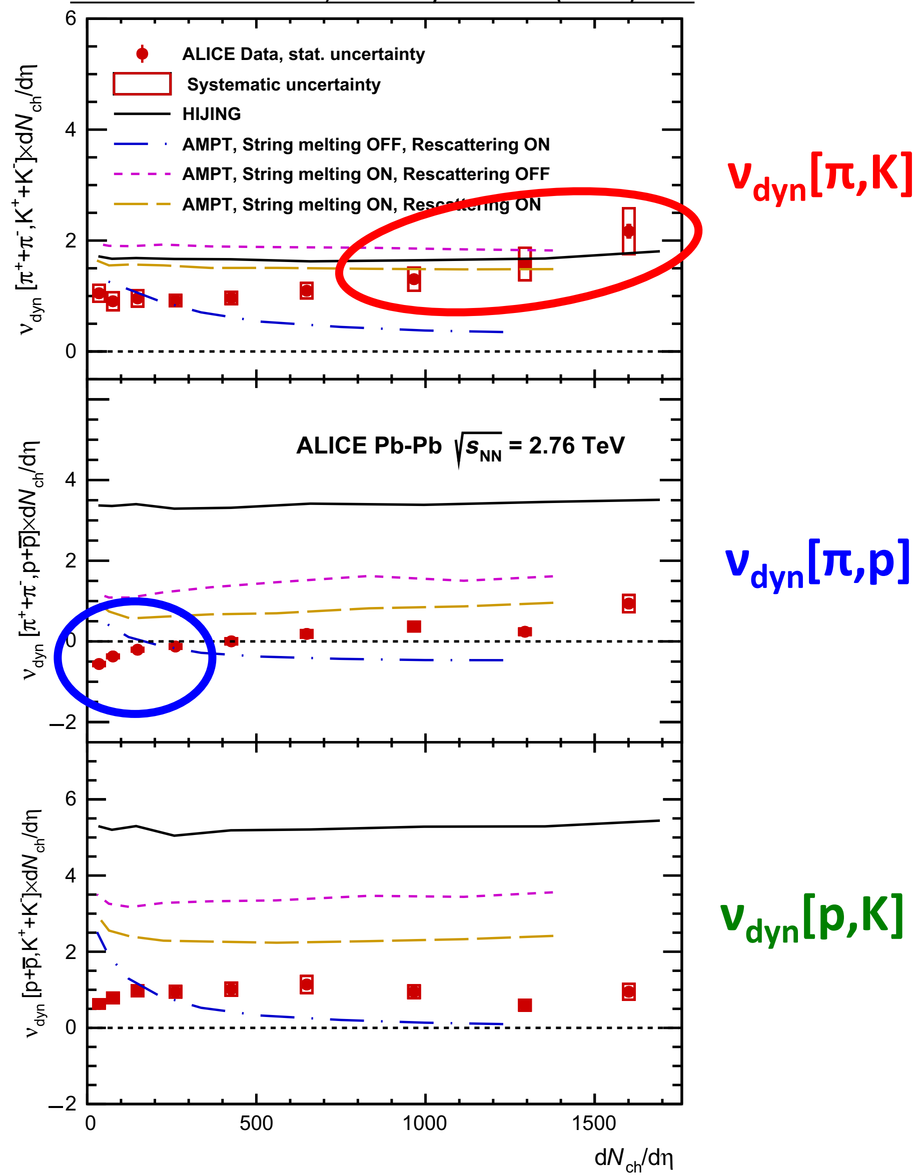
ν_{dyn}[π,p]: Increasing correlation with decreasing centrality

ν_{dyn}[π,K]: Increasing anti-correlation between **π and K** or increasing dynamical fluctuations with increasing centrality

- **HIJING:** no centrality dependence
- **Hadronic rescattering** increases correlations while **String melting** reduces

ALICE Collaboration, Eur. Phys. J. C79 (2019) 236

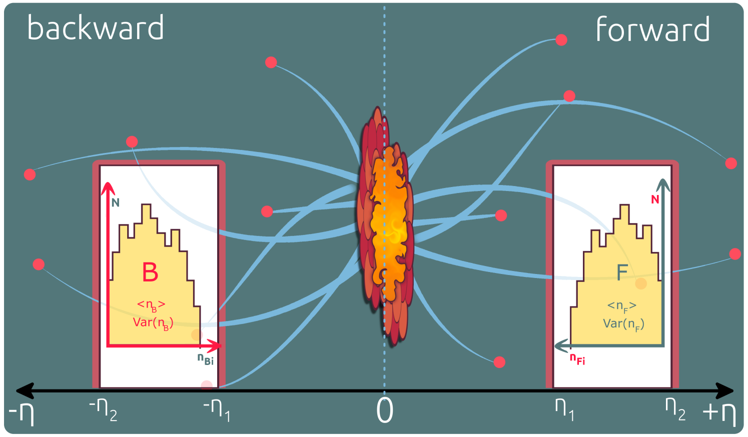


Correlations: Forward and backward correlations

Strongly intensive quantity

$$\Sigma[a,b] - 1 = \frac{\nu_{\mathrm{dyn}}[a,b]}{\dfrac{1}{\langle N_a \rangle} + \dfrac{1}{\langle N_b \rangle}}$$

M. I. Gorenstein and M. Gazdzicki, Phys. Rev. C 84 (2011)



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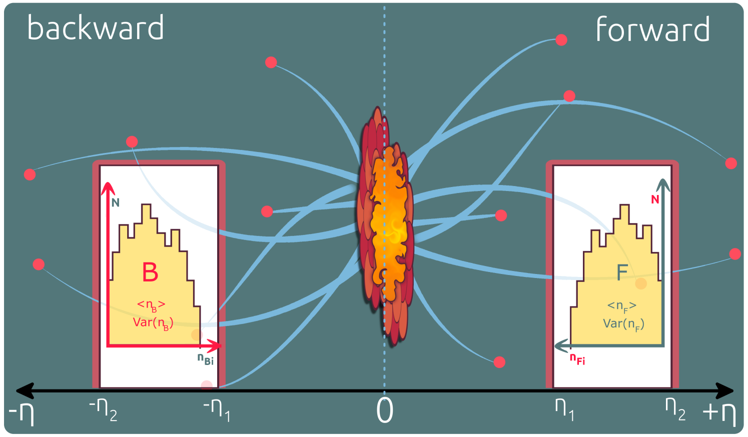

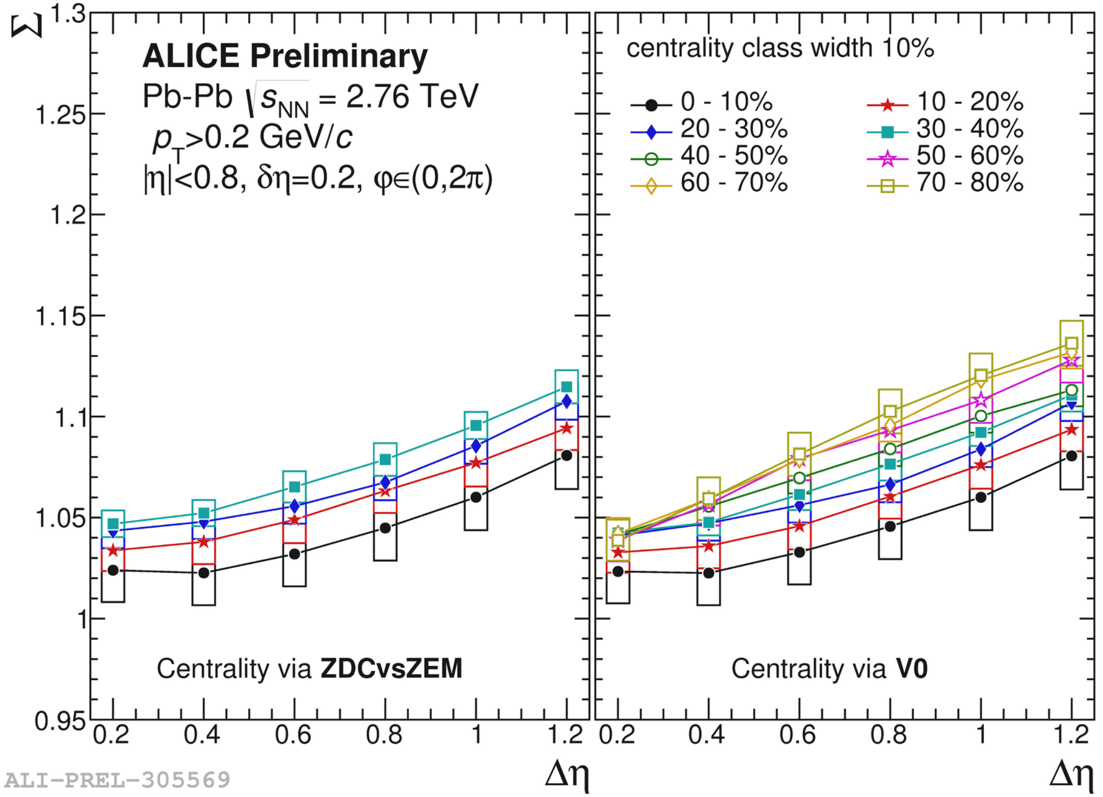

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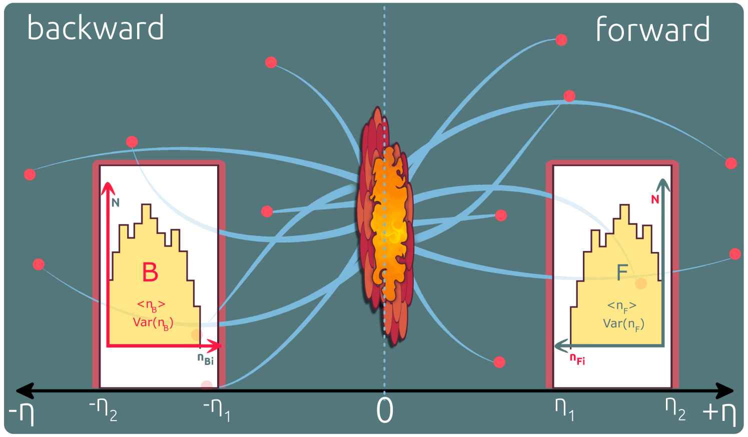
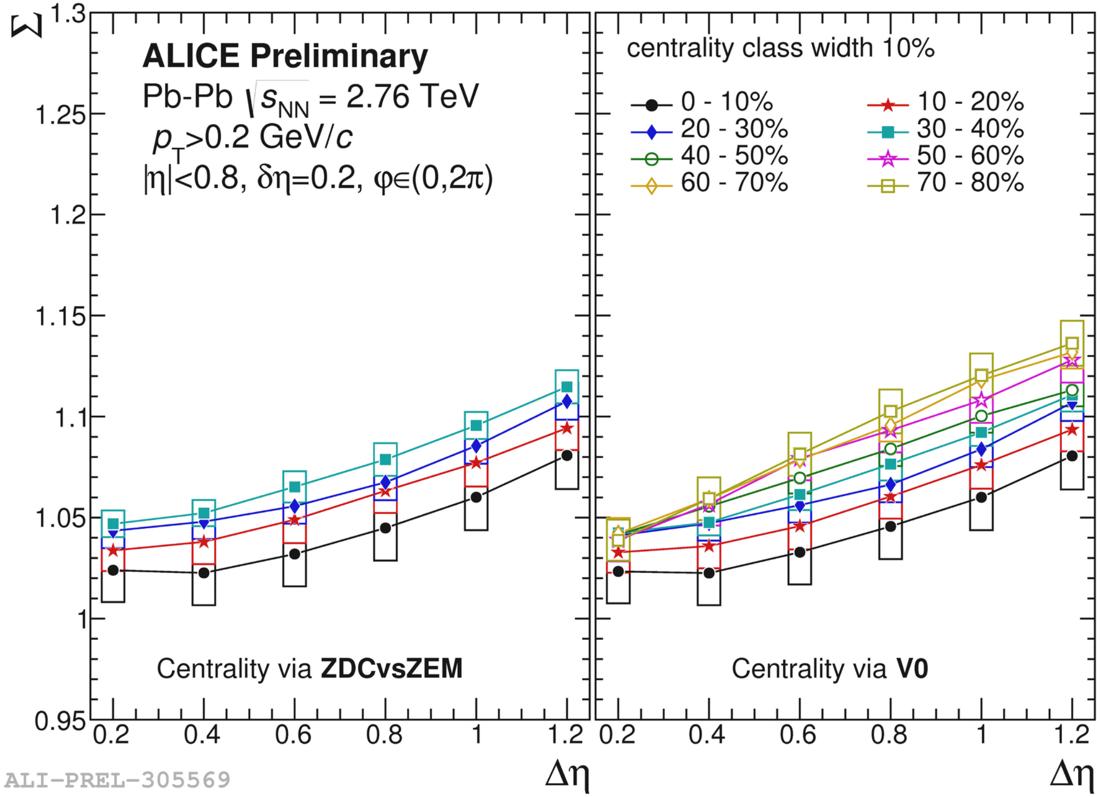




- ❑ **Increasing short range correlations towards central events**
- ❑ **A more differential study including particle identification is needed**

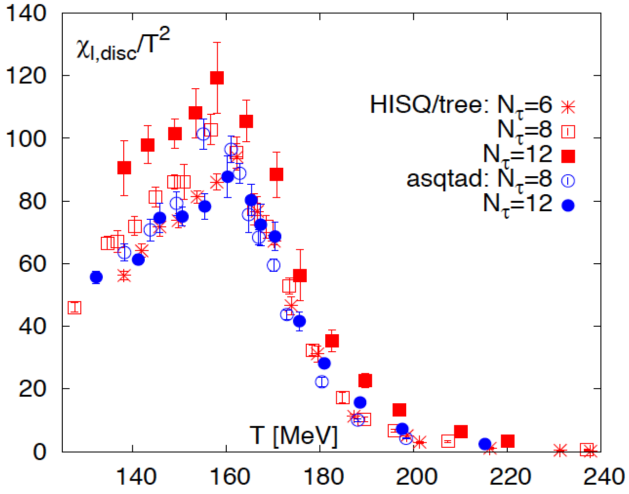
Link experiment to LQCD

⇓

Conserved-charge fluctuations

Criticality & Link to Lattice QCD

HotQCD Collaboration
Phys.Rev. D85 (2012) 054503, Phys.Lett. B795 (2019) 15



$$T_{pc} = 156.5 \pm 1.5 MeV$$

$$\langle\bar{\psi}\psi\rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle_l^{n_f=2}$$

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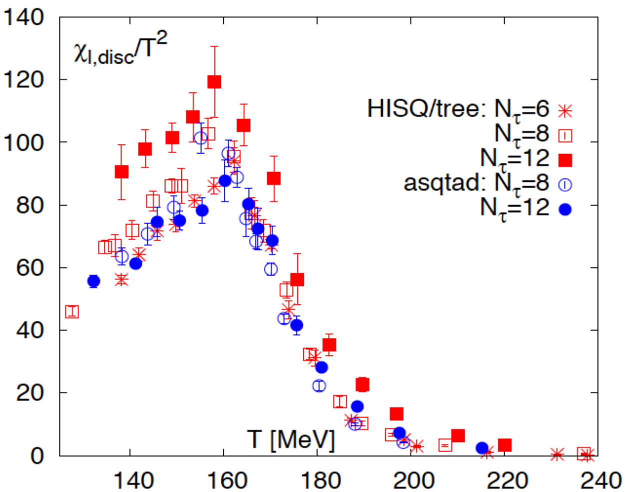

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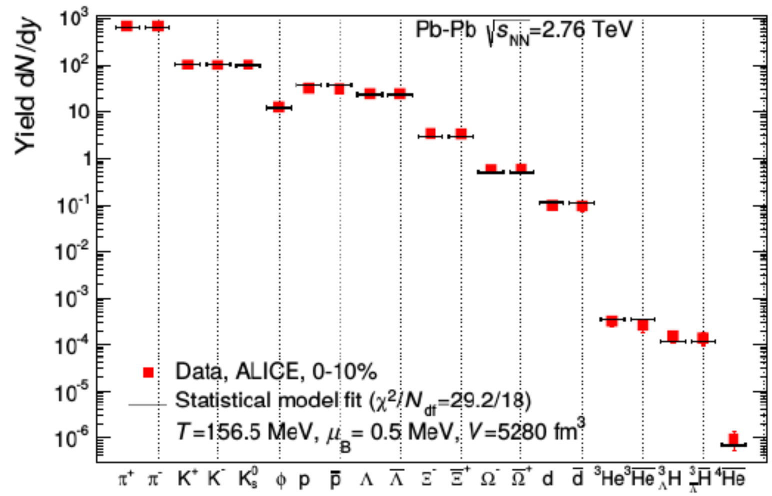

$$T_{fo}^{ALICE} = 156.5 \pm 3 MeV$$

$$\langle N_i \rangle = V\frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^{n} \frac{\left(\langle N_k^{\exp}\rangle - \langle N_k^{HRG}\rangle\right)^2}{\sigma_k^2}$$

Criticality & Link to Lattice QCD

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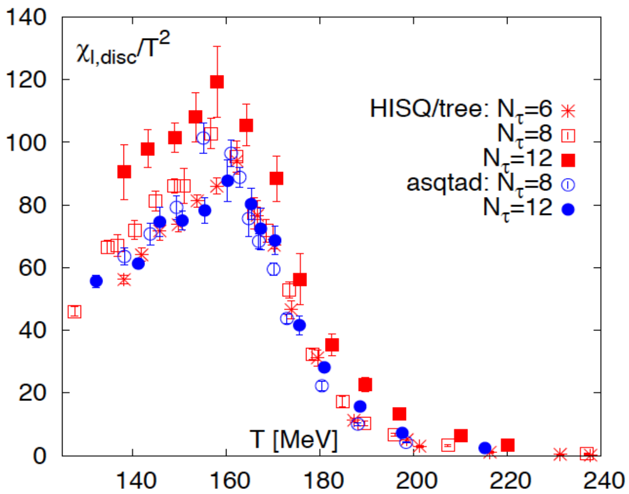

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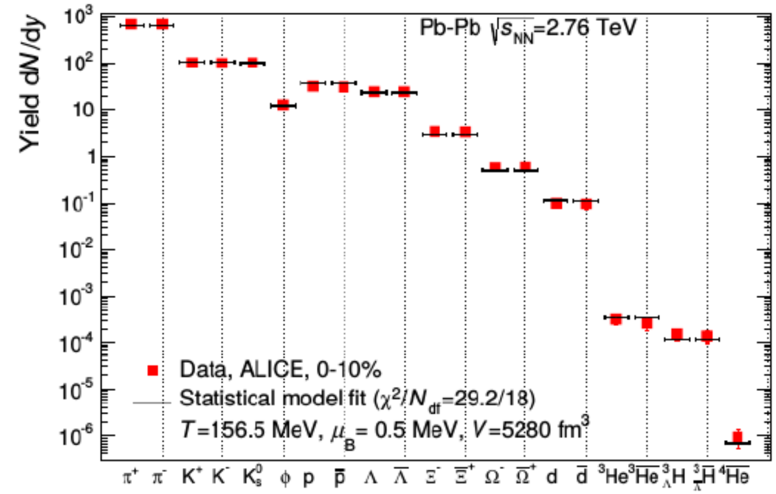

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Chemical freeze-out near T_{pc} → motivation to look for higher order moments

Criticality & Link to Lattice QCD

LQCD

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z\left(V, T, \mu_{B,Q,S}\right) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial\left(\mu_N/T\right)^n}$$

Susceptibilities

Criticality & Link to Lattice QCD

LQCD | **Experiment**

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Susceptibilities

$$\Rightarrow \hat{\chi}_2^B = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

Cumulants

$$\Rightarrow \frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

Higher orders

P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

Criticality & Link to Lattice QCD

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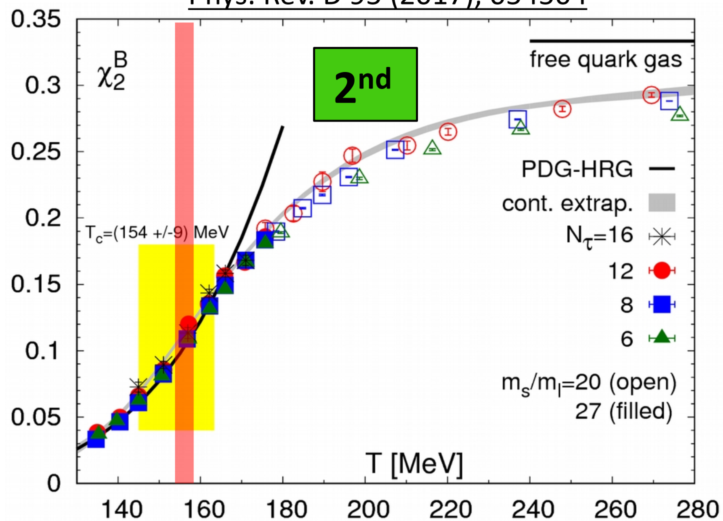
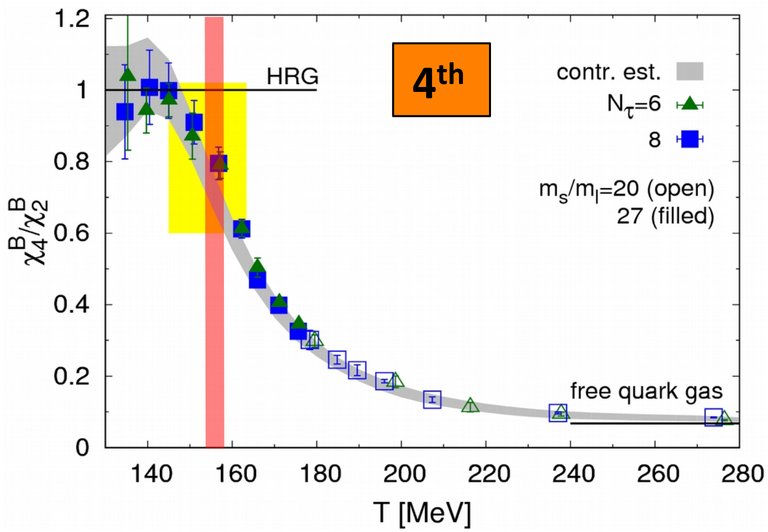


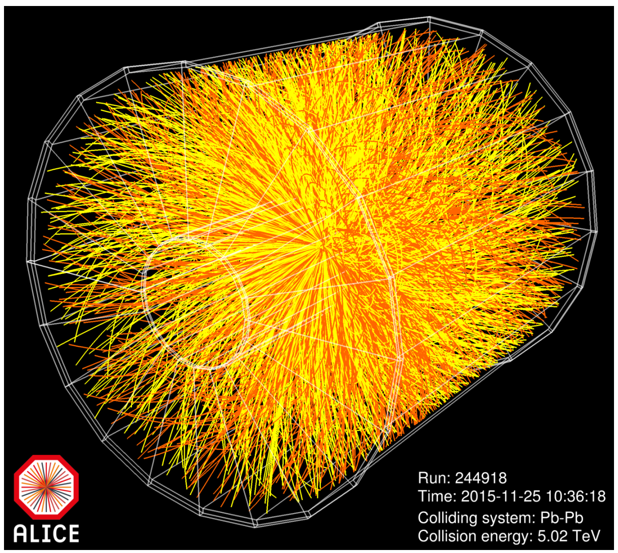


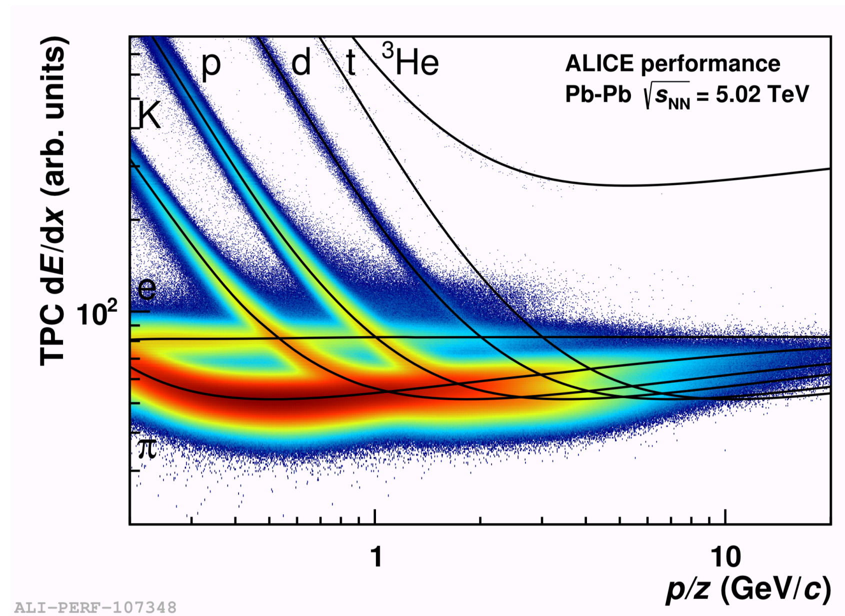
At 4th order LQCD shows a **deviation** **(~25% from unity)** from Hadron Resonance Gas (HRG)

Experimental Challenges

Particle Identification?

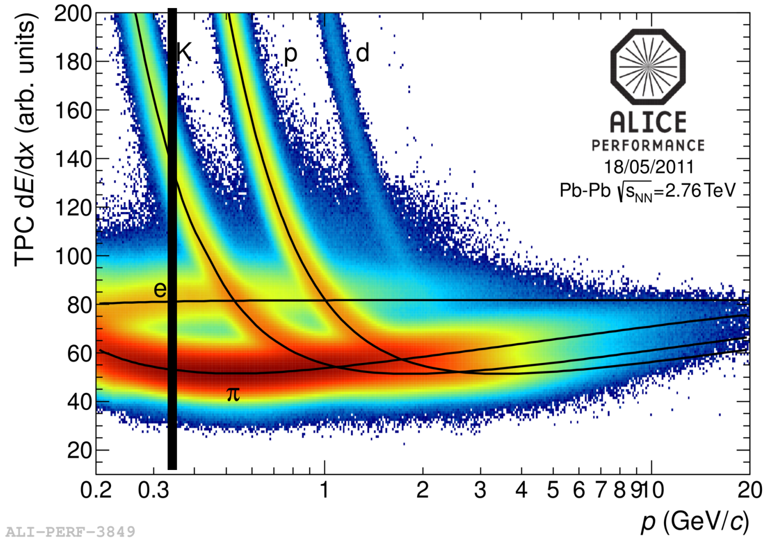
via specific energy loss as function of momentum in the TPC

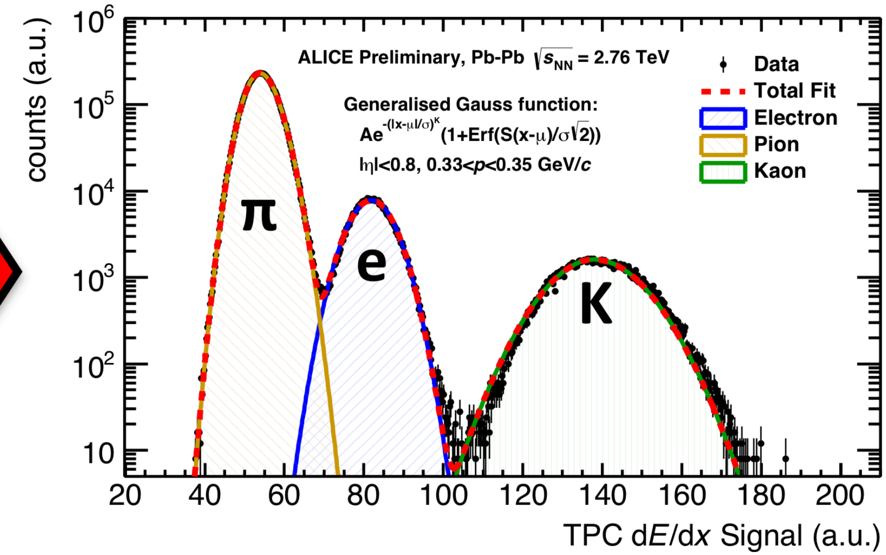




Cut based vs Identity method

Cut-based approach: count tracks of a given particle type





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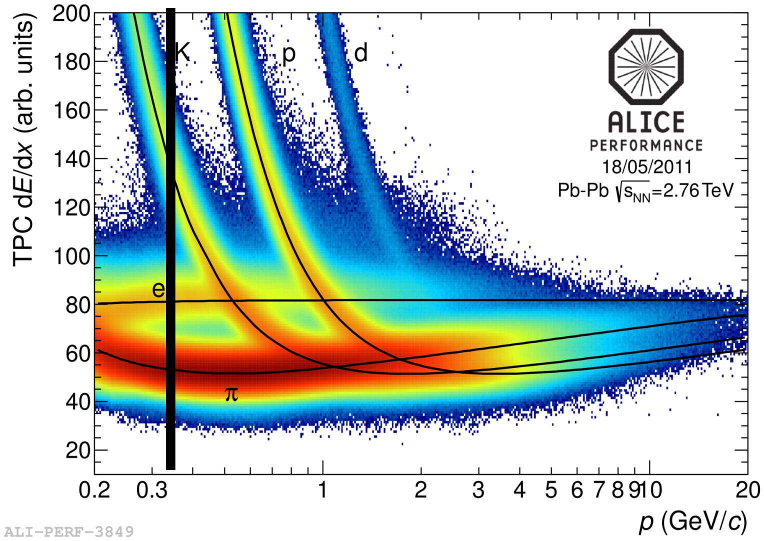

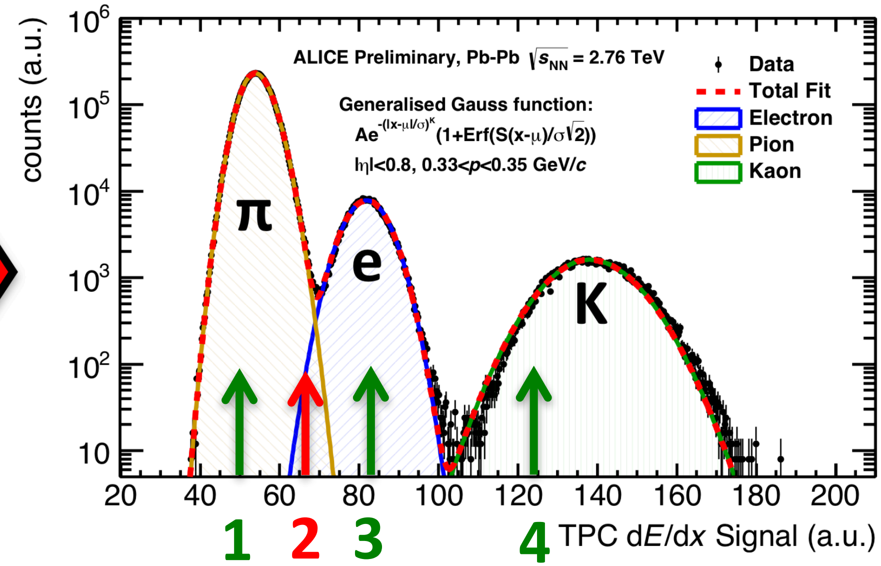

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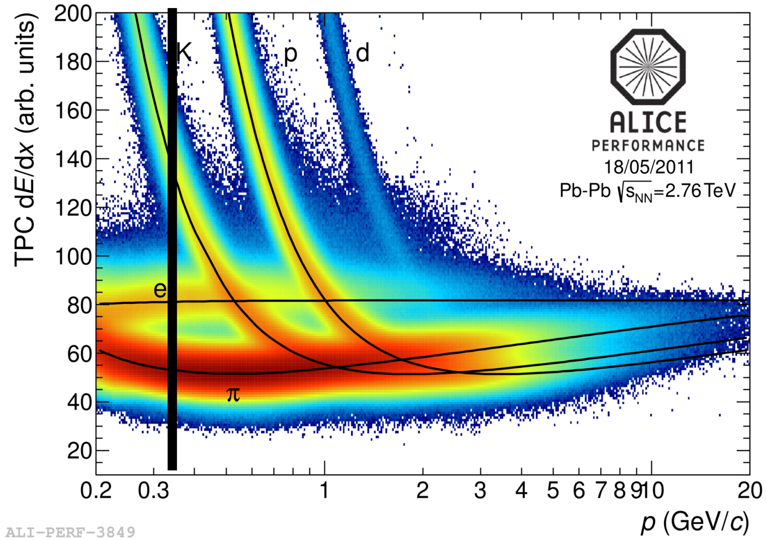

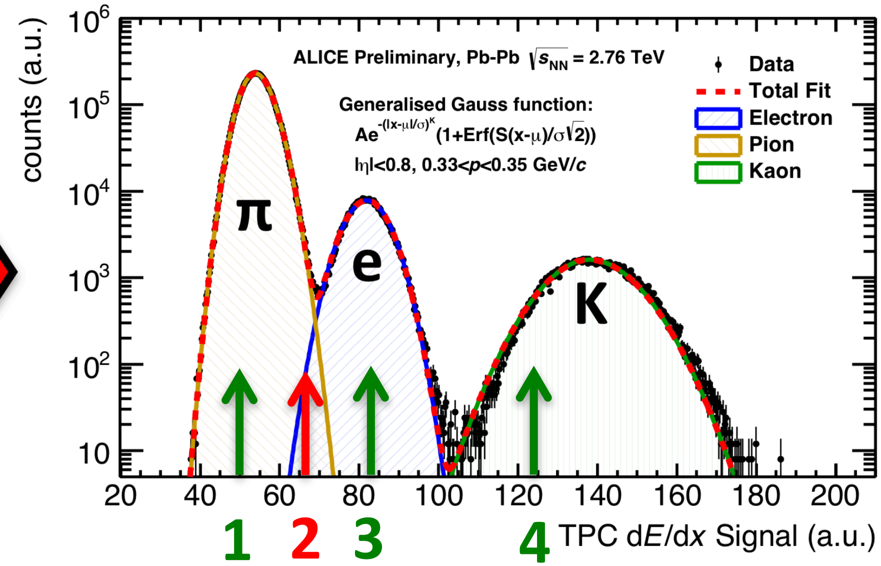

$\omega_\pi^{(1)} = 1, \quad \omega_\pi^{(2)} \cong 0.6, \quad \omega_\pi^{(3)} = 0, \quad \omega_\pi^{(4)} = 0 \quad \Rightarrow \quad W_\pi = 1.6 \neq N_\pi$

$\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$

A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012), PRC 84, 024902 (2011)
M. Arslandok, A. Rustamov, NIM A, 946, (2019), 162622

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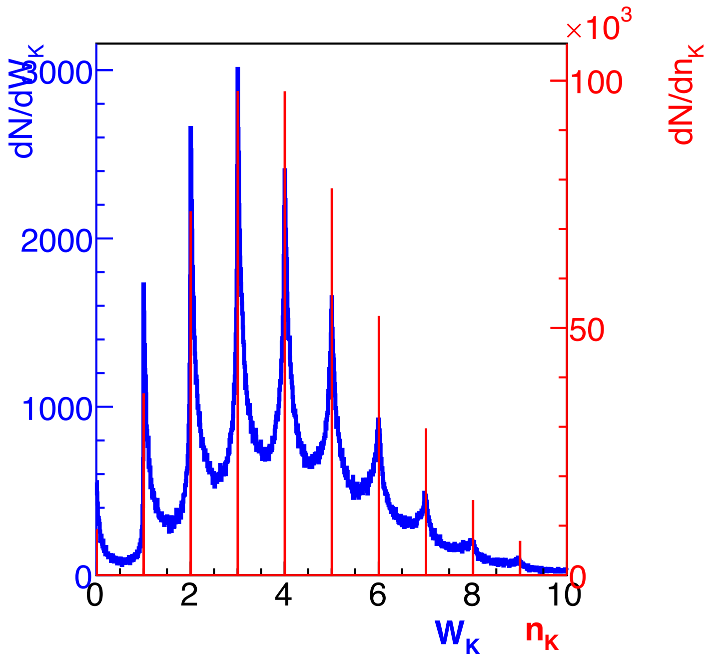

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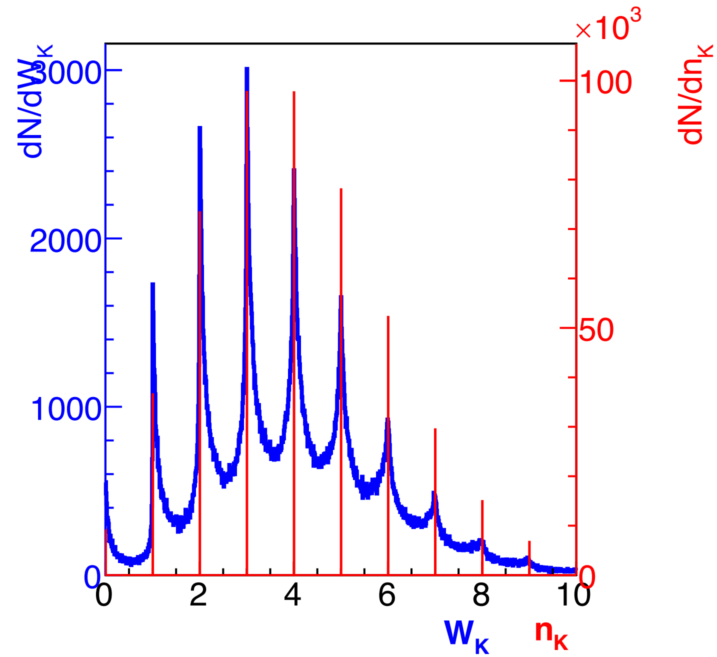
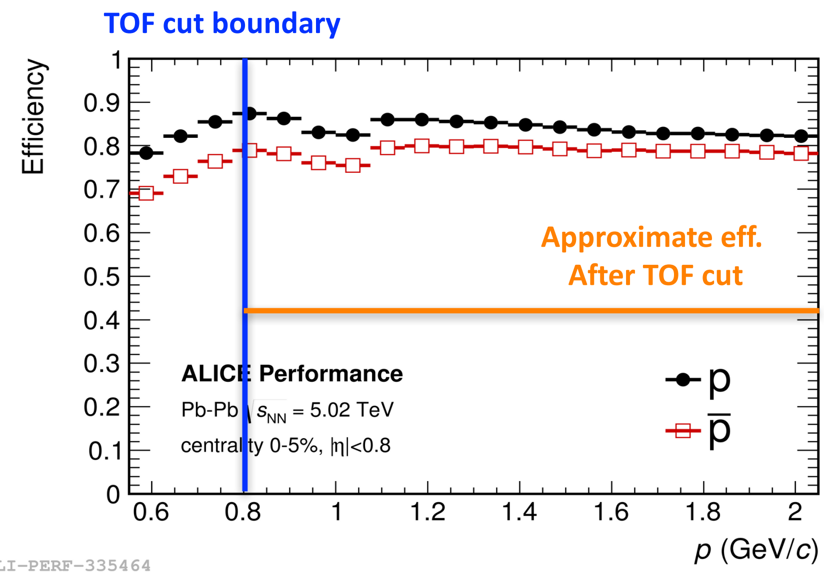


$$\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$$

- **Cut based approach**
  - Use additional detector information or reject a given phase space bin
  - Challenge: efficiency correction and contamination
- **Identity Method**
  - Gives folded multiplicity distribution
  - Easier to correct inefficiencies
  - Ideal approach for low momentum (p<2 GeV/c)



Recent results on conserved-charge fluctuations

Baseline: Skellam distribution

$$X = N_B - N_{\overline{B}}$$

- **rth central moment:**

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

- **First four cumulants**

$$\kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2,$$
$$\kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2$$

- **Uncorrelated Poisson limit:**

$$\langle N_B N_{\overline{B}} \rangle = \langle N_B \rangle \langle N_{\overline{B}} \rangle$$

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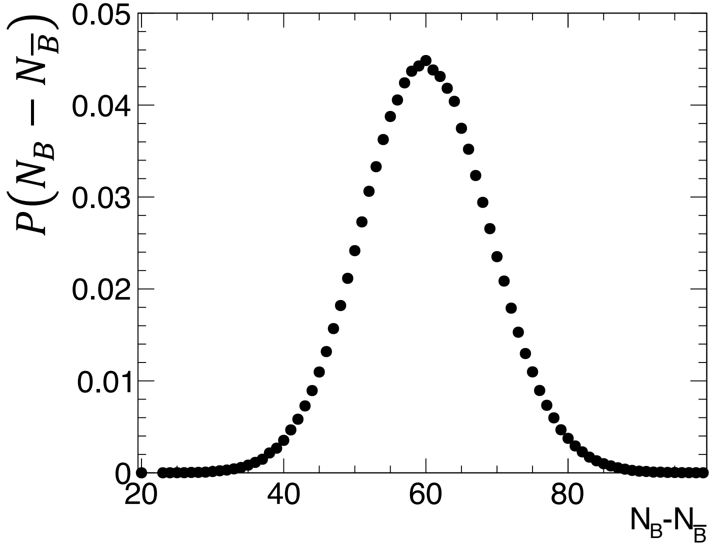


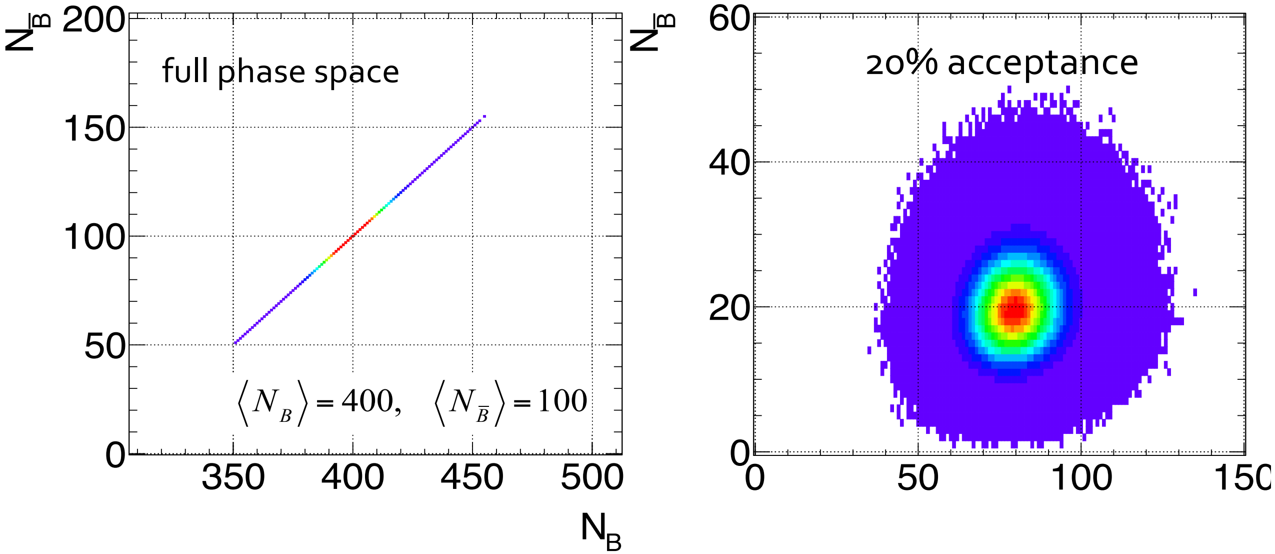
Difference between two independent Poissonian distributions

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\bar{B}} \rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2n}} = \frac{\langle N_B \rangle - \langle N_{\bar{B}} \rangle}{\langle N_B \rangle + \langle N_{\bar{B}} \rangle}$$

Importance of acceptance

- Fluctuations of net-baryons appear only inside **finite acceptance**



P. Braun-Munzinger, A. Rustamov, J. Stachel, QM18, NPA 982 (2019) 307-310

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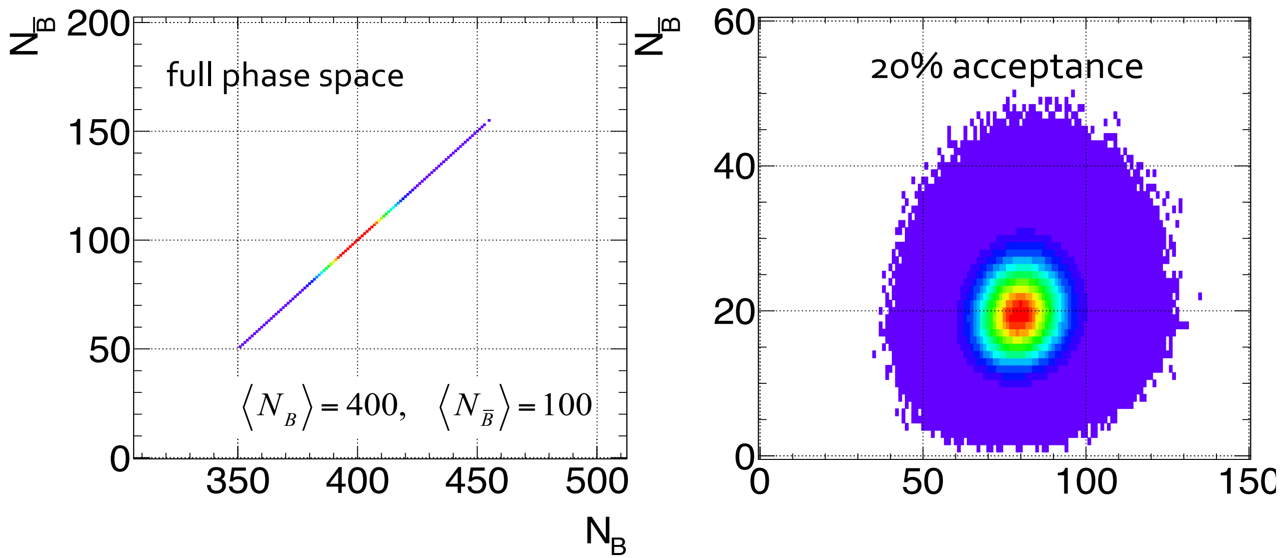

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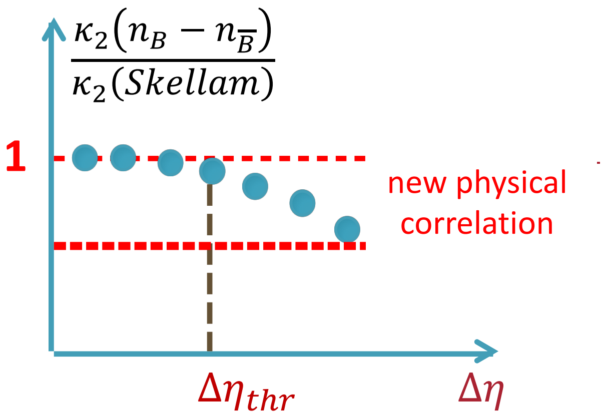
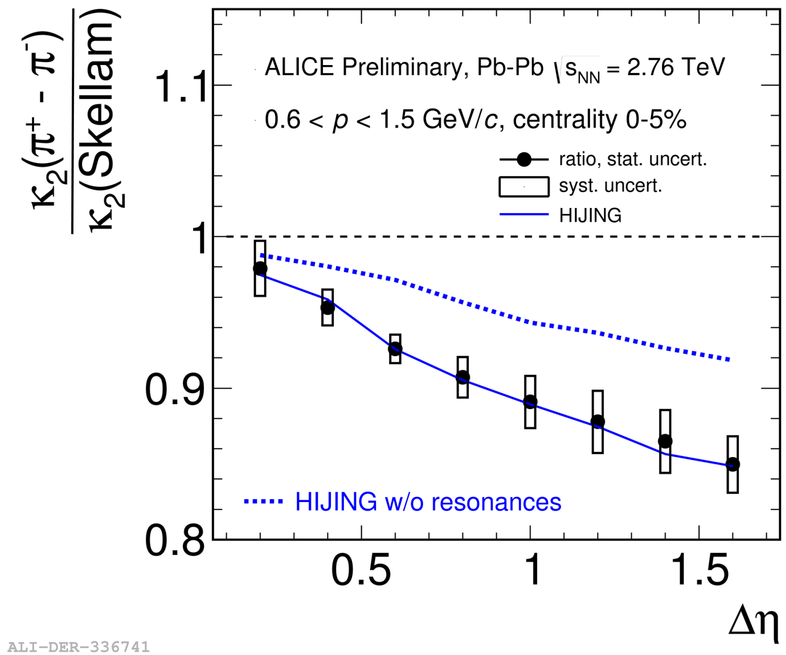


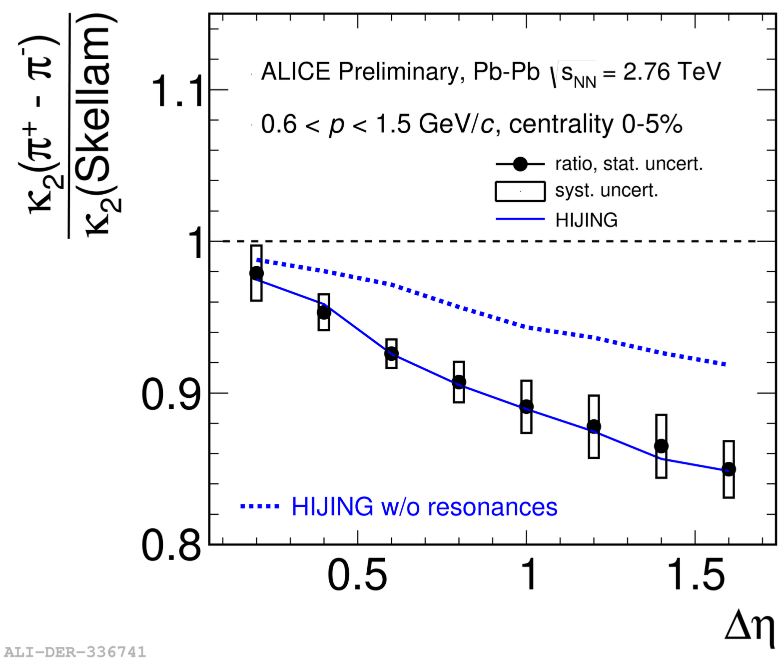
- **In the limit of very small acceptance**
  → only Poissonian fluctuations

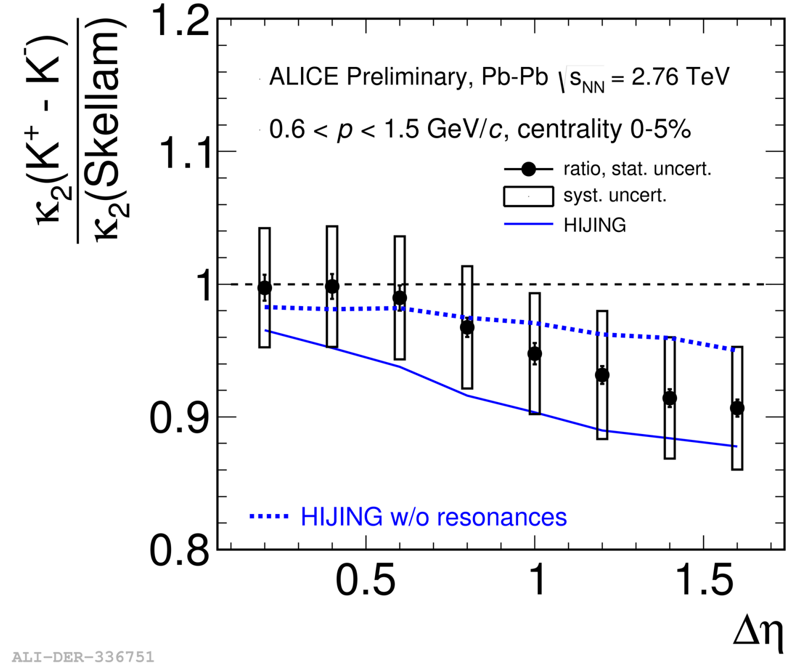
Effect of resonances



- **Net-electric-charge:** → Strongly dominated by **resonance contributions**

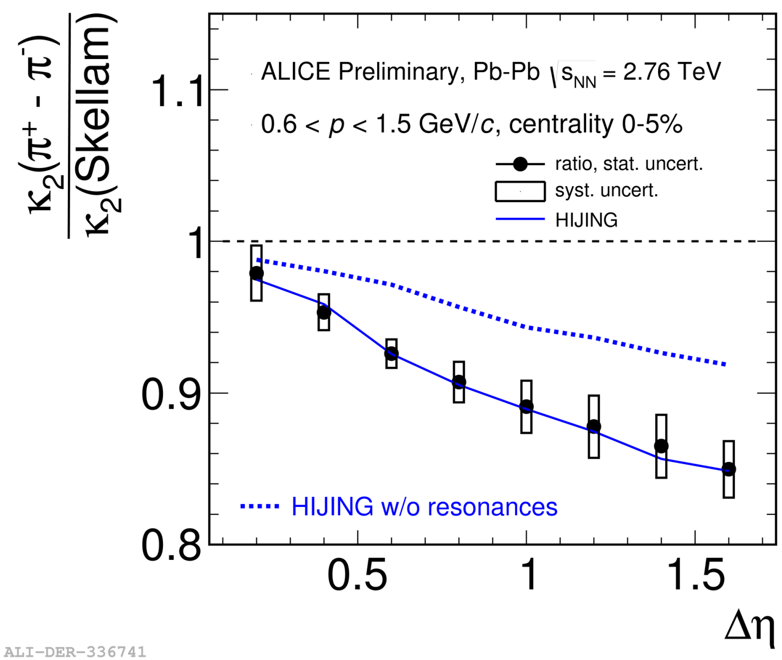
Effect of resonances

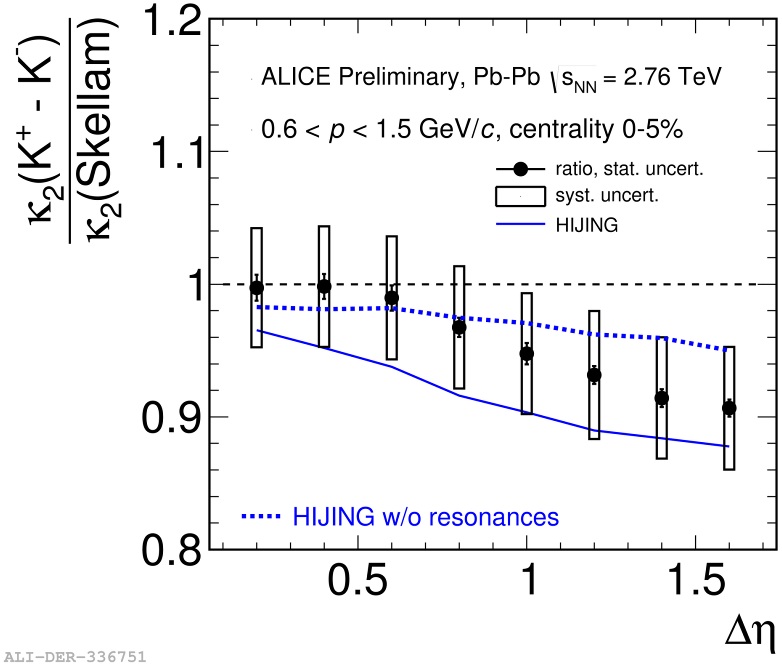




- **Net-electric-charge:** → Strongly dominated by **resonance contributions**
- **Net-strangeness:** → Kaons are dominated by **ϕ-decay**

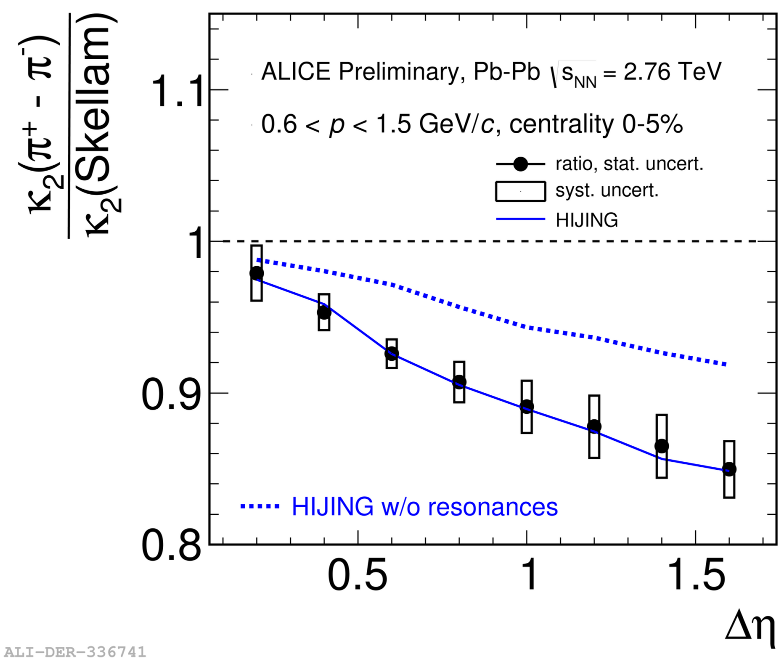
Effect of resonances

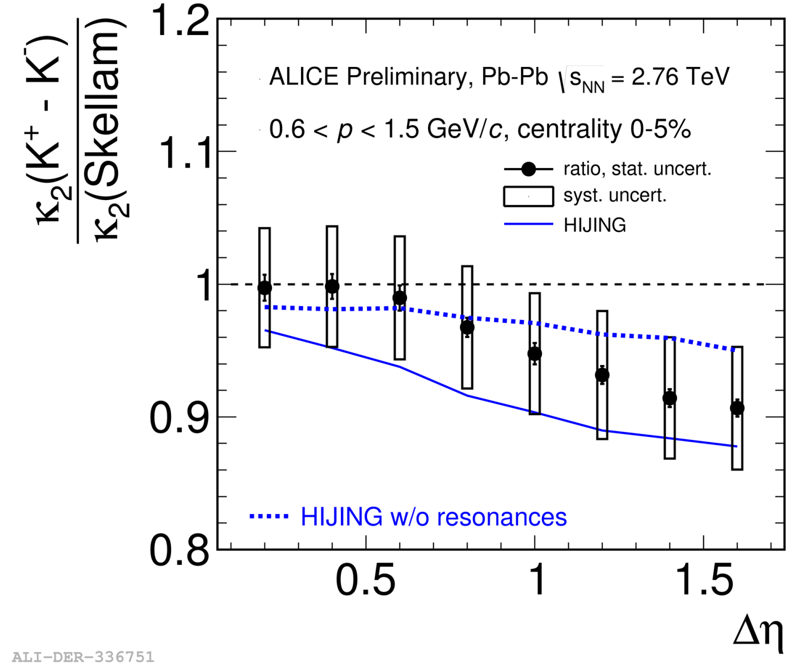




- **Net-electric-charge:** → Strongly dominated by **resonance contributions**
- **Net-strangeness:** → Kaons are dominated by **ϕ-decay**
- **Net-baryon:**
  - → Due to **isospin randomization,** at $\sqrt{s_{NN}}$ > 10 GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))
  - → No resonance feeding $\mathrm{p} + \bar{\mathrm{p}}$
  - → **Best candidate for measuring charge susceptibilities**

Effect of resonances





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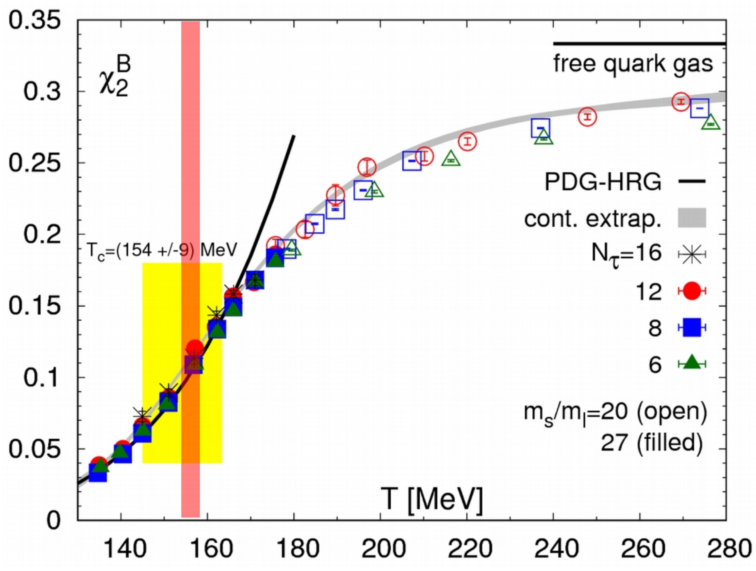
  - → No resonance feeding $\mathrm{p} + \bar{\mathrm{p}}$
  - → **Best candidate for measuring charge susceptibilities**
    - ❖ **Net-Λ:** provides additional information on net-baryon and net-strangeness fluctuations

1st and 2nd order cumulants

LQCD expectations:

- ✓ 1st moments → $T_{pc} = T_{freeze\text{-}out} = \sim 156$ MeV
- ✓ 2nd moments → No deviation from HRG at T_{pc}

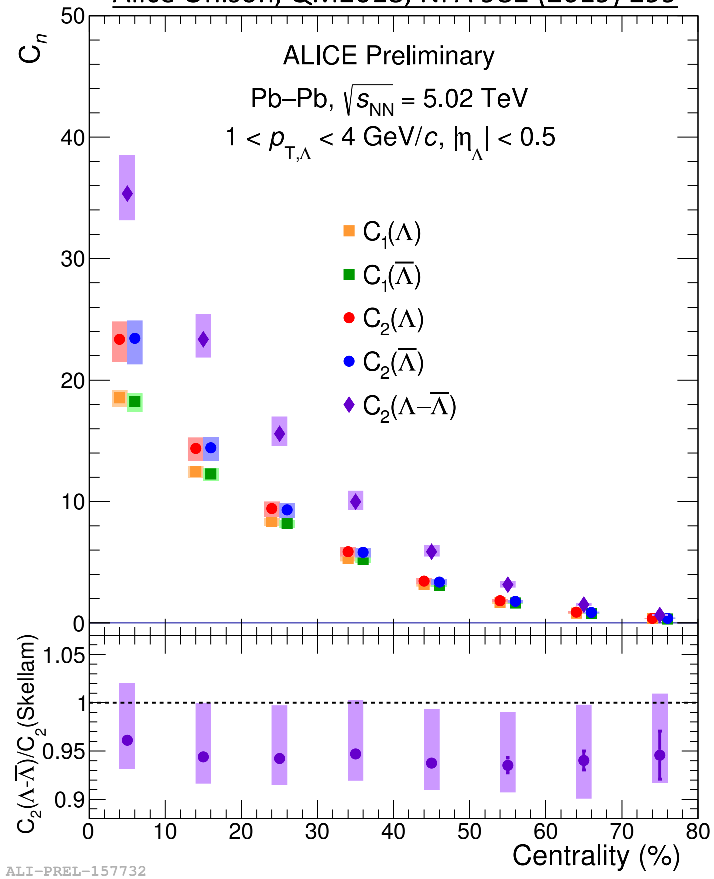


2^{nd} order cumulants: Net-Λ, net-p

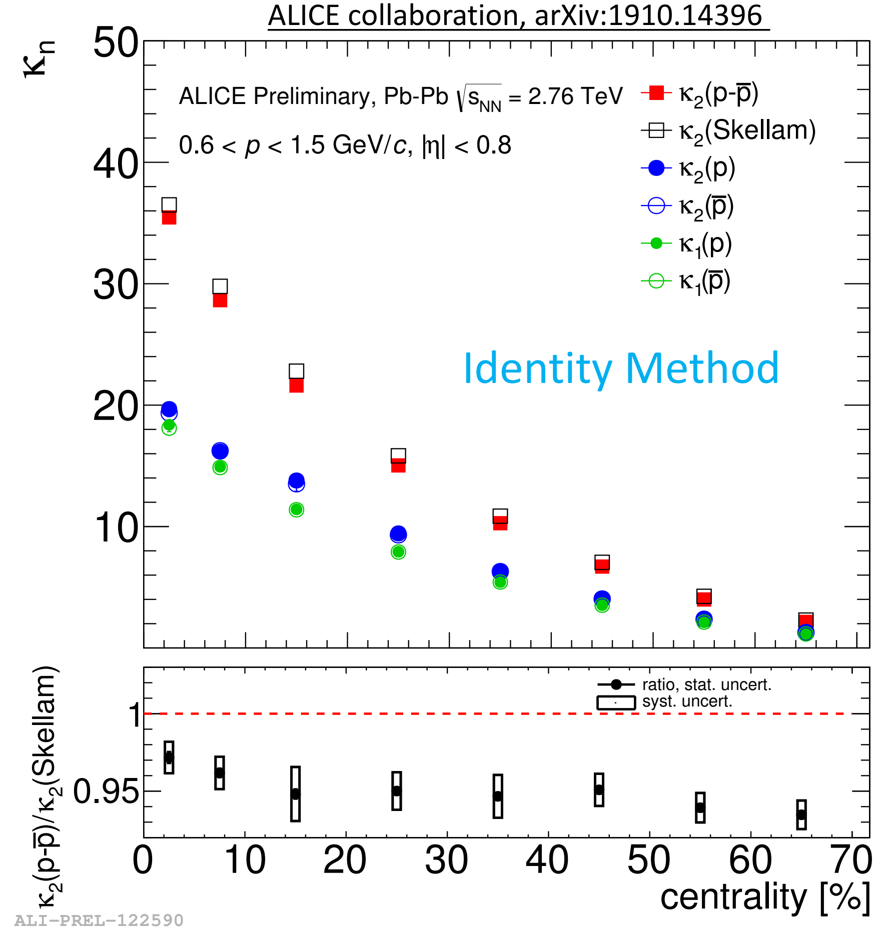
$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

$$\kappa_2(p - \bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - 2(\langle p\bar{p}\rangle - \langle p\rangle\langle\bar{p}\rangle)$$

Alice Ohlson, QM2018, NPA 982 (2019) 299



ALICE collaboration, arXiv:1910.14396

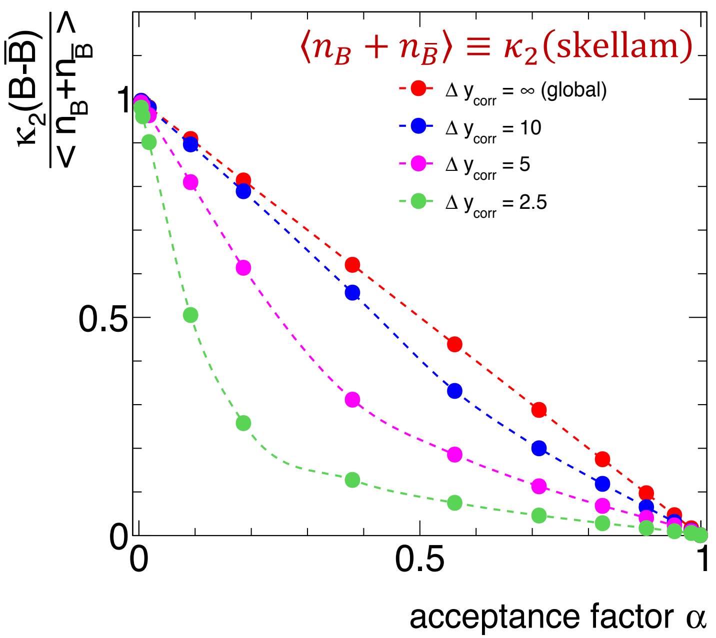


What is the source of deviation/correlation?

Baryon number conservation

- **Baryon number conservation** imposes subtle correlations

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032



$$\alpha = \frac{\langle N_B^{acc} \rangle}{\langle N_B^{4\pi} \rangle} \qquad |y_{\bar{B}} - y_B| < \frac{\Delta y_{corr}}{2}$$

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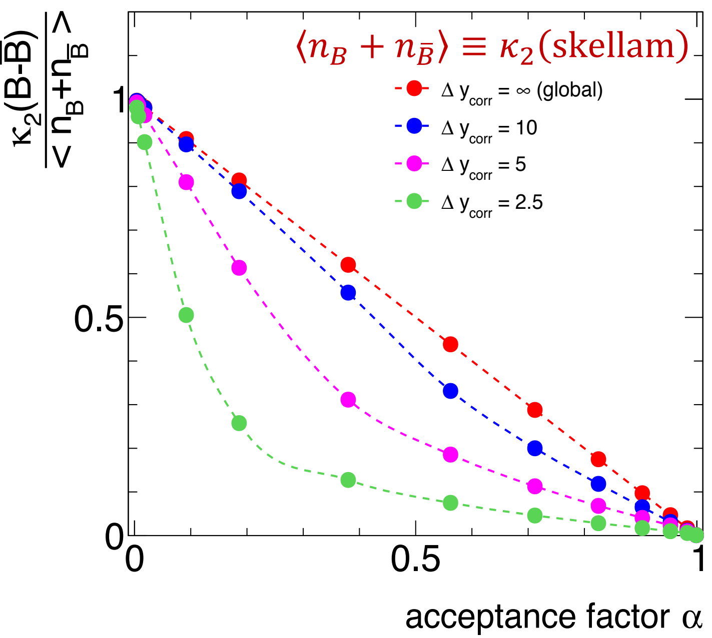

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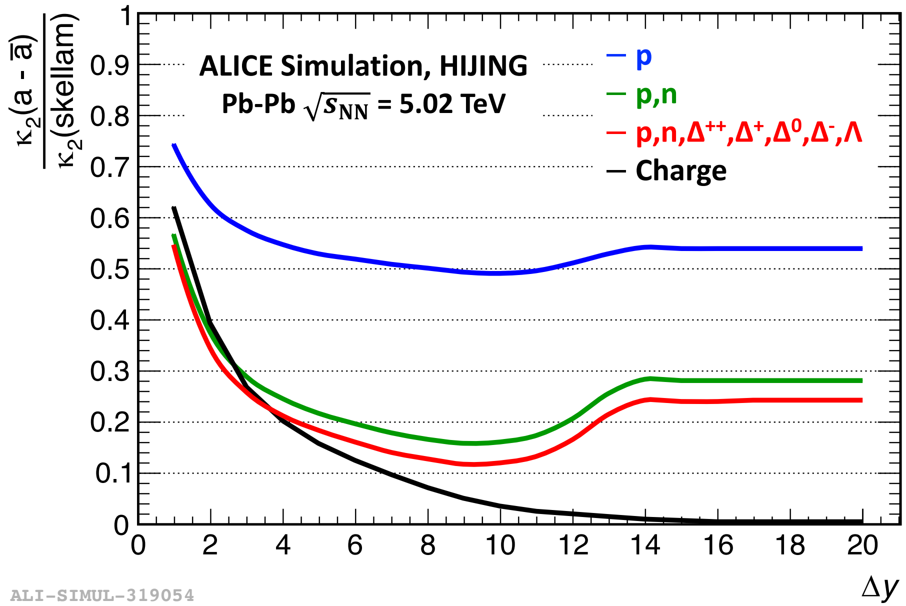
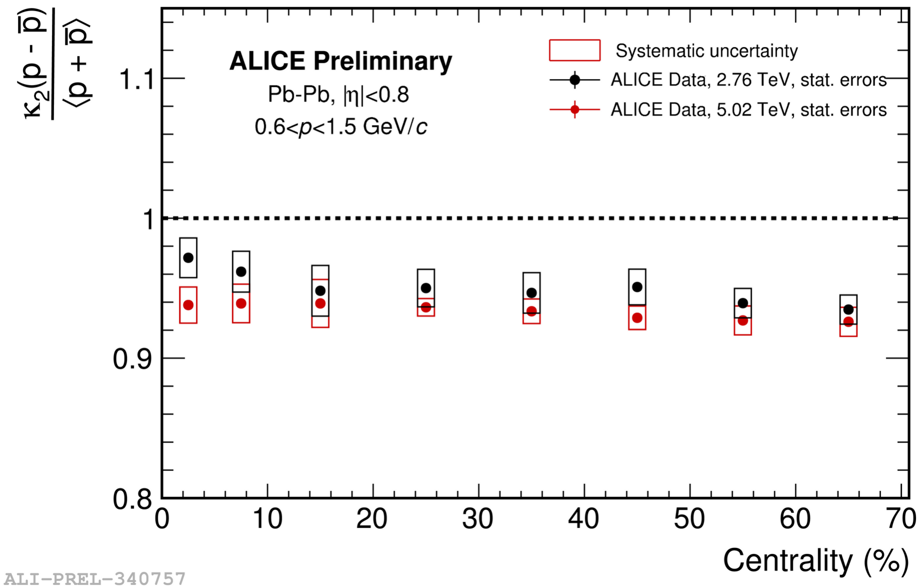
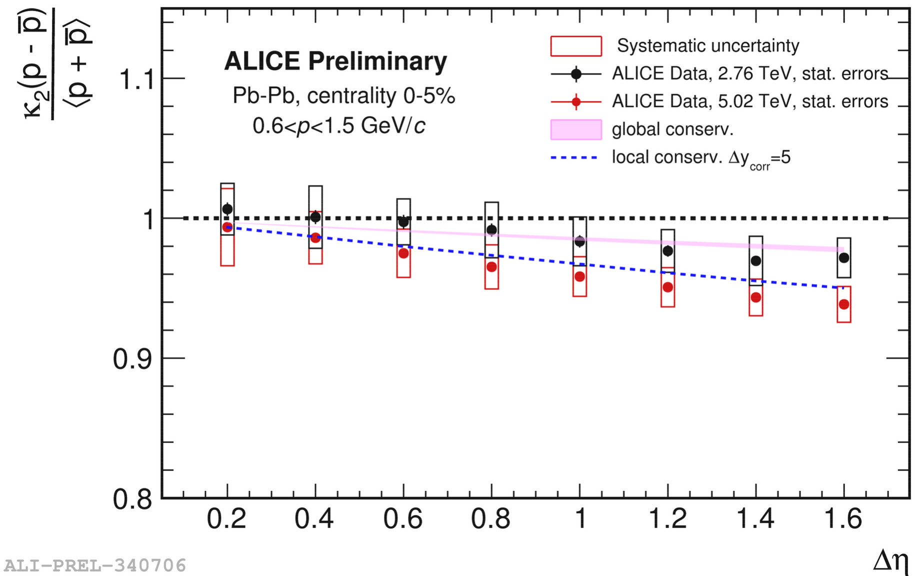


- Short range correlations

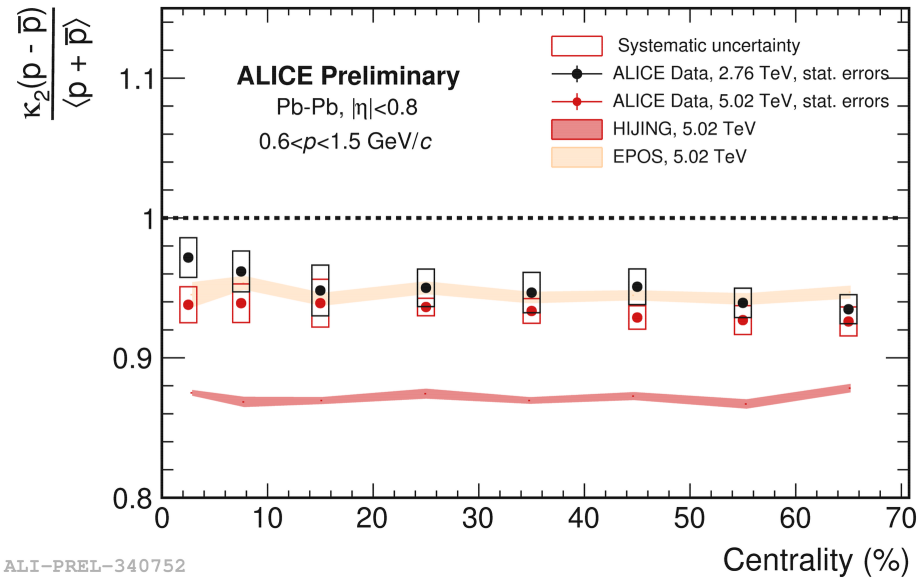
2nd order cumulants

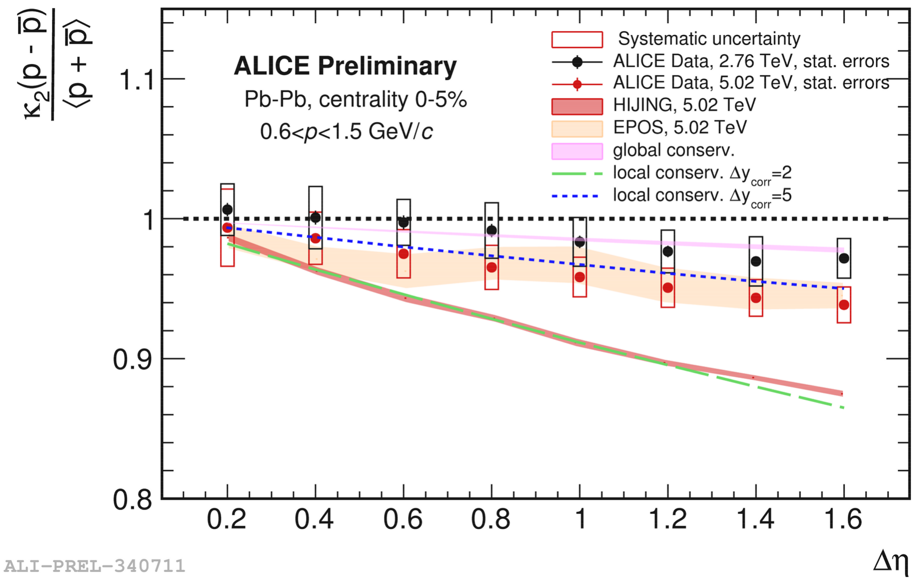




- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations,** $\Delta y = \pm 2.5$ unit **or longer**

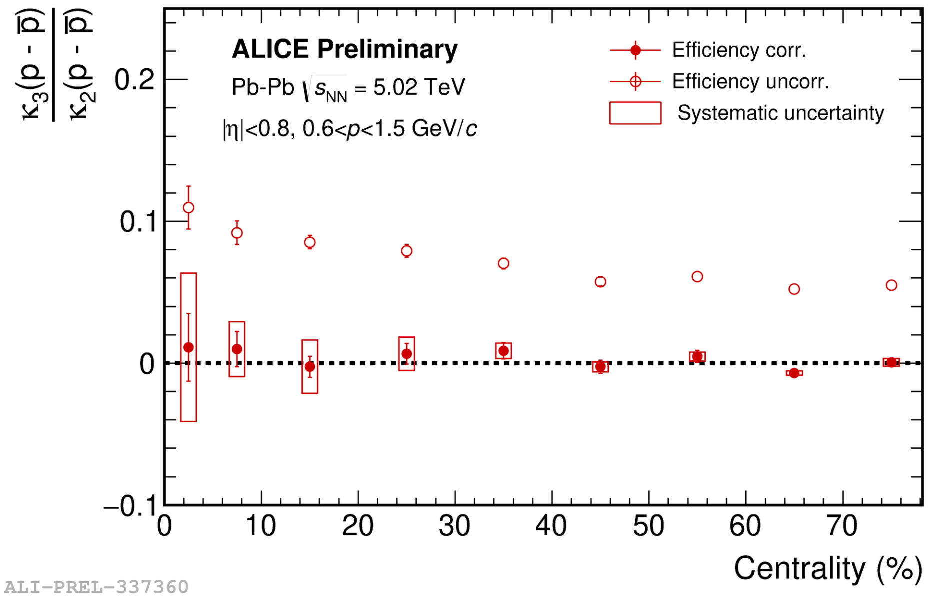
2nd order cumulants

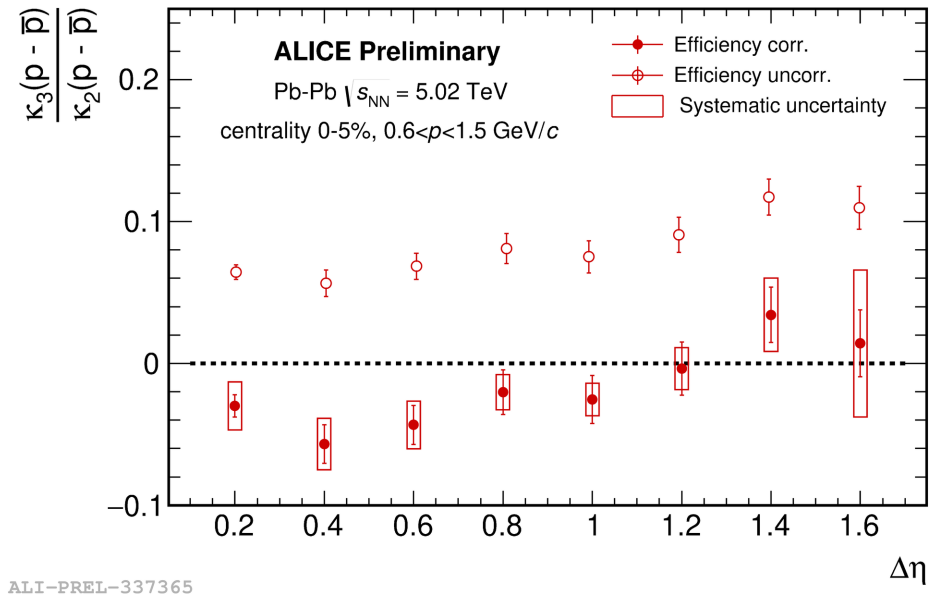




- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations,** $\Delta y = \pm 2.5$ unit **or longer**
- EPOS agrees with ALICE data but HIJING deviates significantly
  - Event generators based on string fragmentation (HIJING) conserve baryon number over $\Delta y = \pm 1$ unit

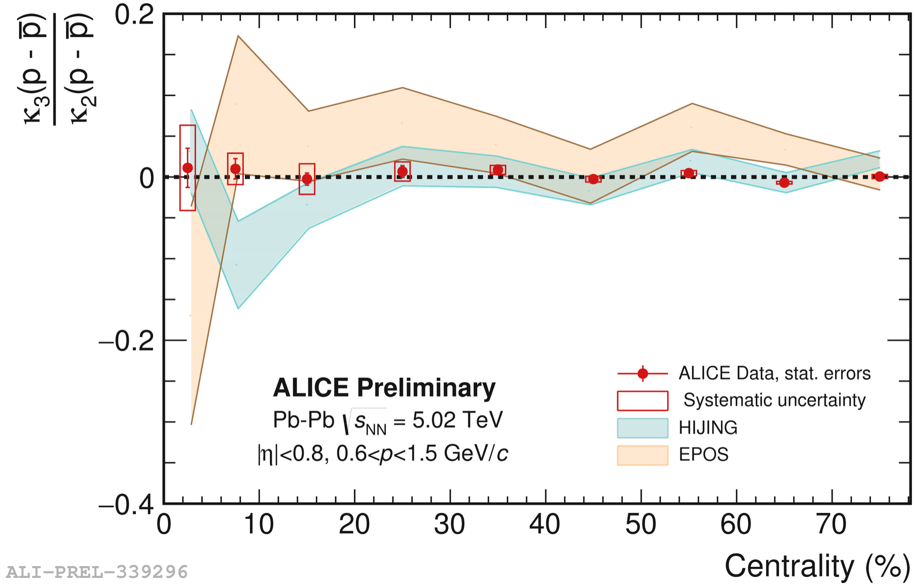
3rd order cumulants

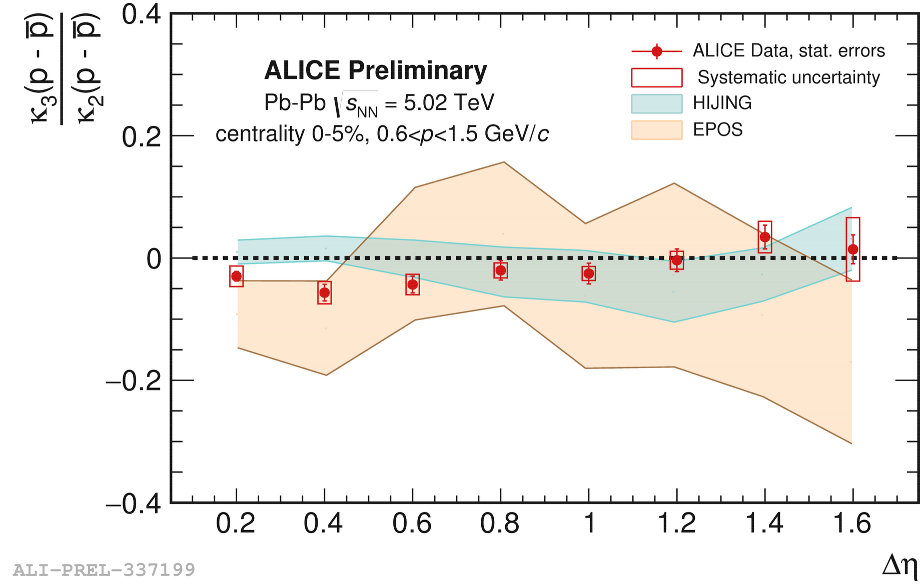




- **Data** agree with Skellam baseline **"0"** as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**

3rd order cumulants

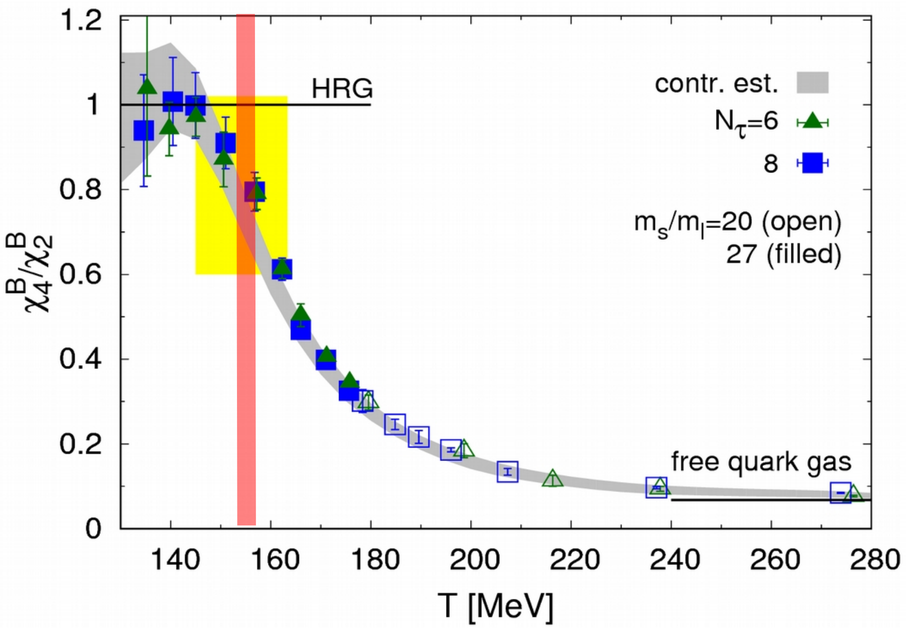




- **Data** agree with Skellam baseline **"0"** as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**
- **EPOS and HIJING in agreement with data** within current statistical errors
  - Both models conserve global charge → net-p within acceptance is ~0

4^{th} order cumulants of net-p

- LQCD shows a **deviation of about 25%** from HRG



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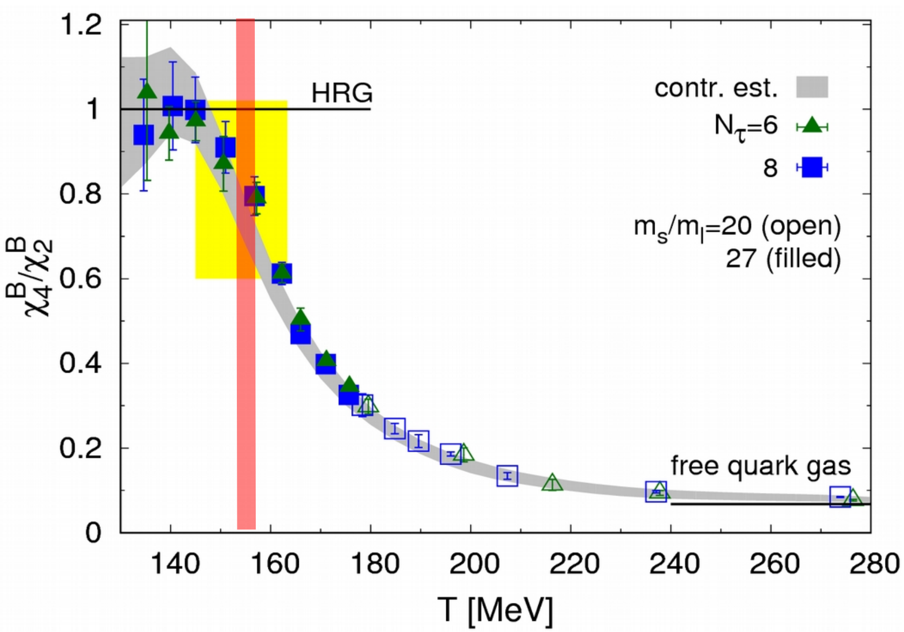
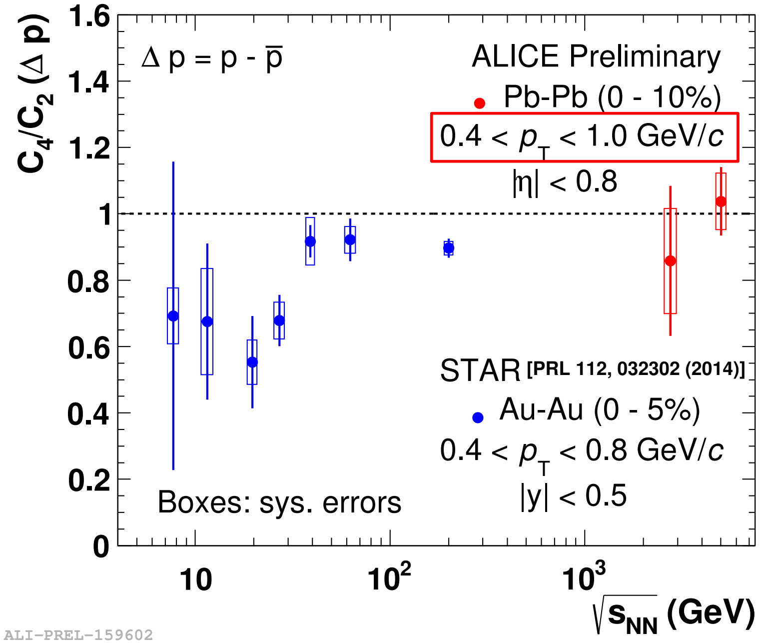
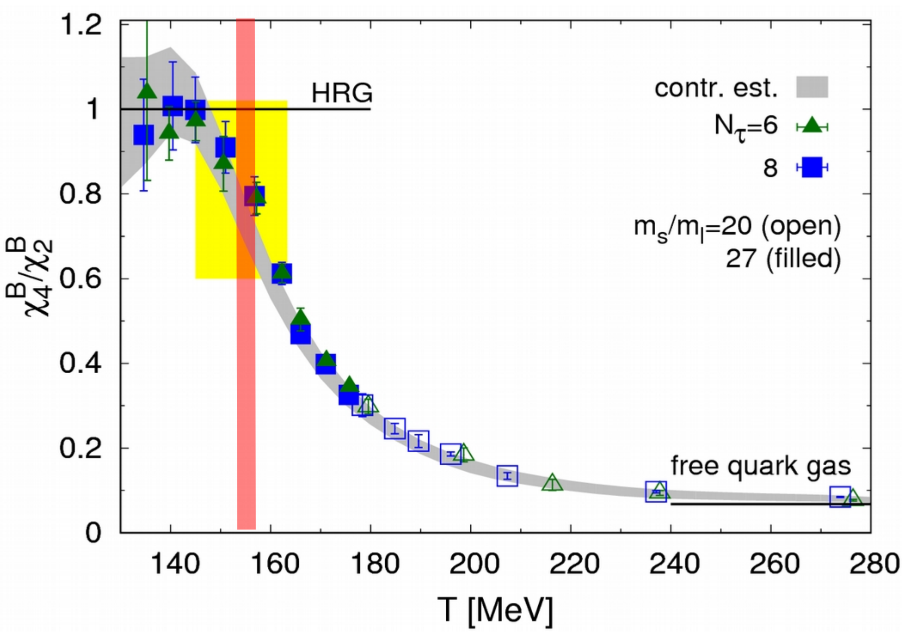
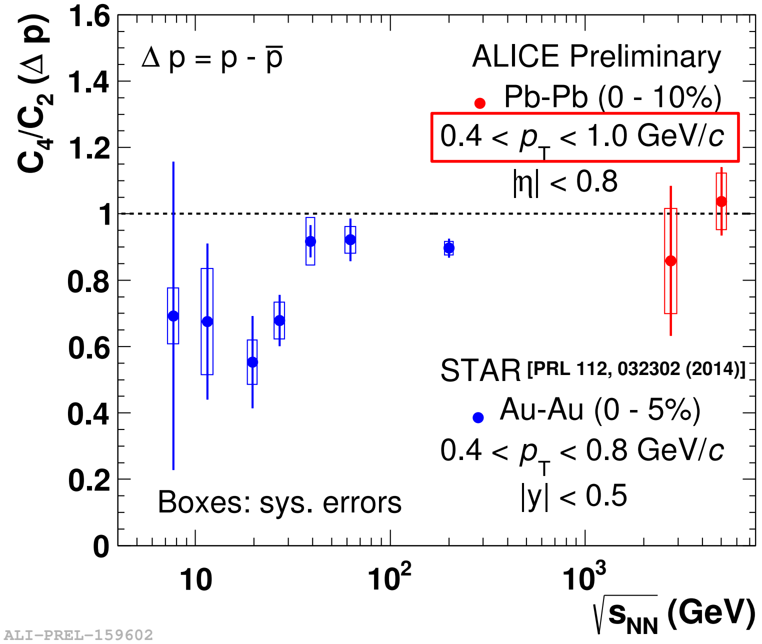




4^{th} order cumulants of net-p

- LQCD shows a **deviation of about 25%** from HRG
- **Preliminary C_4/C_2** agree with Skellam at LHC energies?
  - Small acceptance
  - Low statistics
  - Cut-based approach for PID





Analysis within a larger kinematic acceptance using
Identity Method is in progress

Summary: Current Status

- Σ, υ_{dyn} **etc.** → good to study correlations → **better look more differentially**
- **Net-electric-charge fluctuations:** Challenge are the dominant **resonance contributions**
- **Net-proton fluctuations:**
  - ✓ **1st order:** $T_{fo}^{ALICE} \sim T_{pc}^{LQCD}$
  - ✓ **2nd order:** Deviation from Skellam baseline is due to baryon number conservation
    - ALICE data suggests **long range correlations**
  - ✓ **3rd order:** Agrees with Skellam baseline **"0"** as a function of centrality and pseudorapidity
    - Achieved precision of **better than 5%** is promising for the higher order cumulants
- **Up to 3rd order** ALICE data agree with the LQCD expectations

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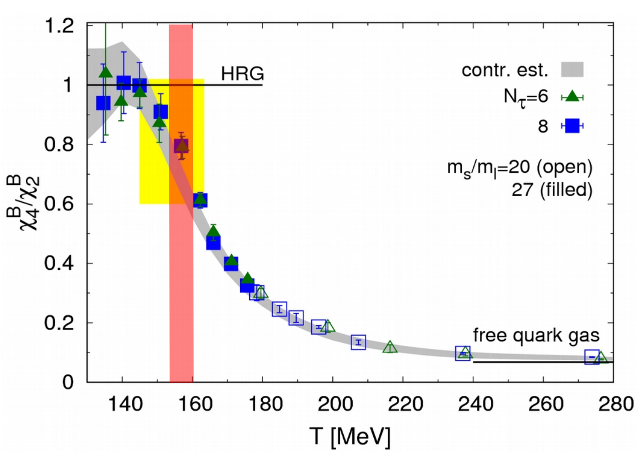
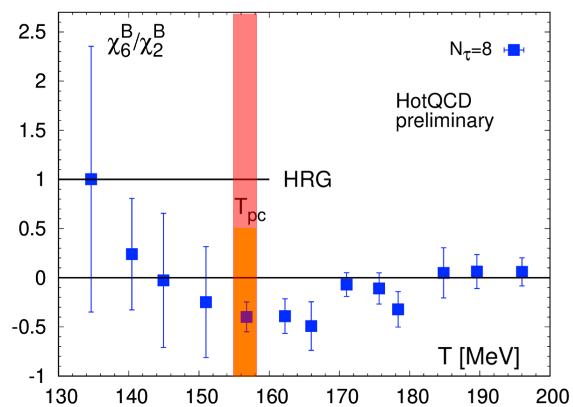
Holy grail: see critical behavior in 6th and higher order cumulants





RUN1: 2nd order (~13M min. bias events)
RUN2: 4th order (~150M central events)
RUN3: **6th …** (>1000M central events)

Outlook: After ALICE upgrade

- **New ITS**: better vertexing
- **Faster TPC**: MWPC → GEMs
- Record minimum-bias Pb-Pb data at 50kHz
  → one to two order of magnitude more events
- 6^{th} order and maybe beyond

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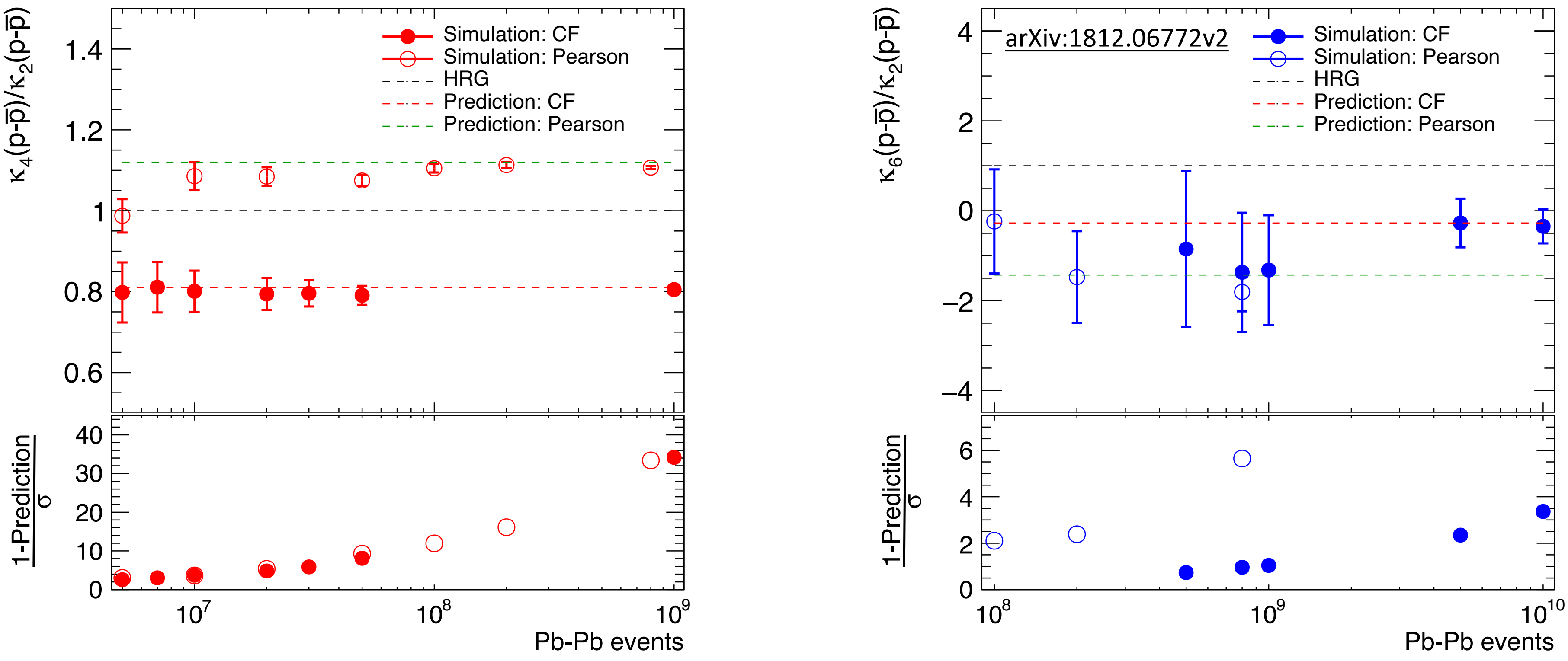


BACKUP

Open Questions

Experiment

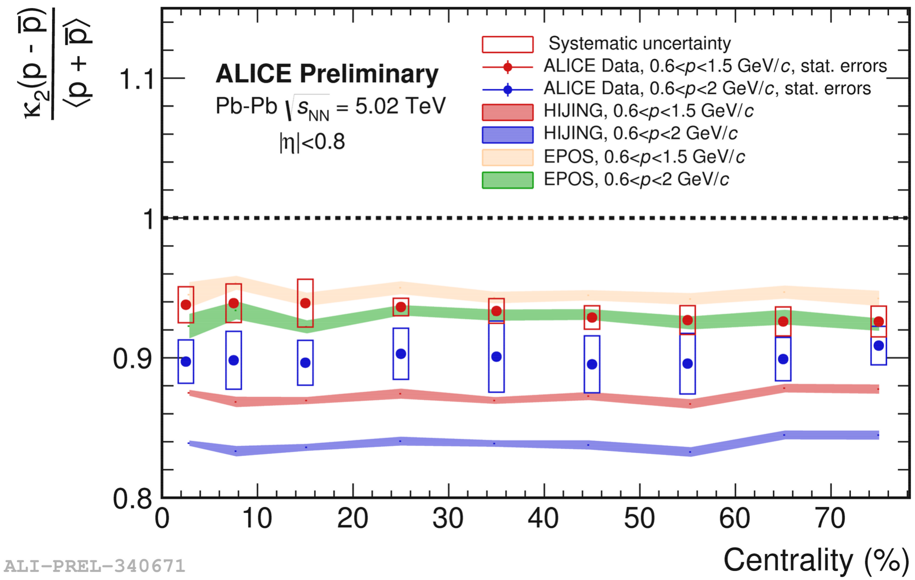
- Efficiency correction
  ⟶ realistic detector simulations
- Volume fluctuations
  ⟶ centrality resolution
- Effect of resonances
- Measurement at low energies
- Systematic uncertainties
- …

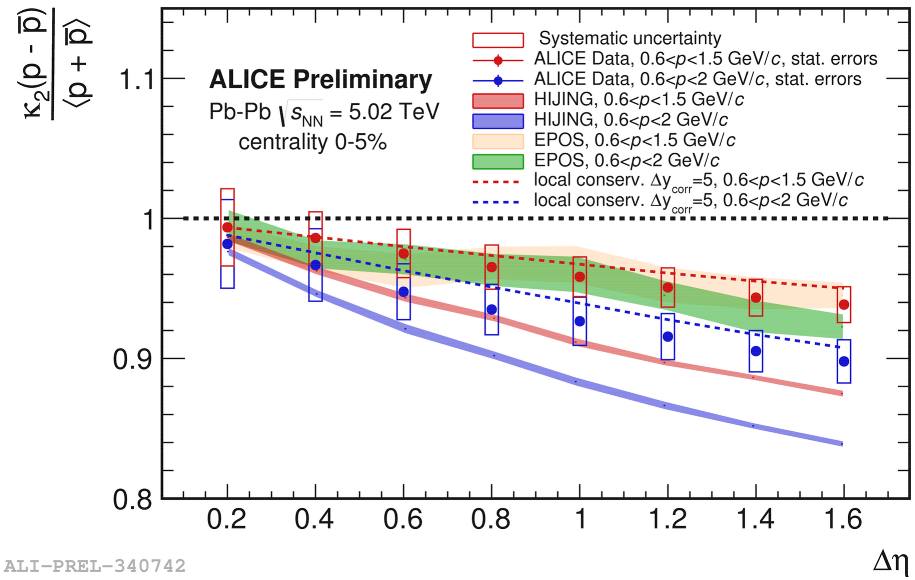
Theory

- Efficiency correction
  ⟶ unfolding or …
- Volume fluctuations
- Effect of resonances
- Measurement at low energies
  ⟶ baryon stopping, deuteron formation …
- Effect of hydrodynamic evolution
- …

- Adam Bzdak et. al., arXiv:1906.00936
- Probing the Phase Structure of Strongly Interacting Matter: Theory and Experiment, https://indico.gsi.de/event/7994/overview

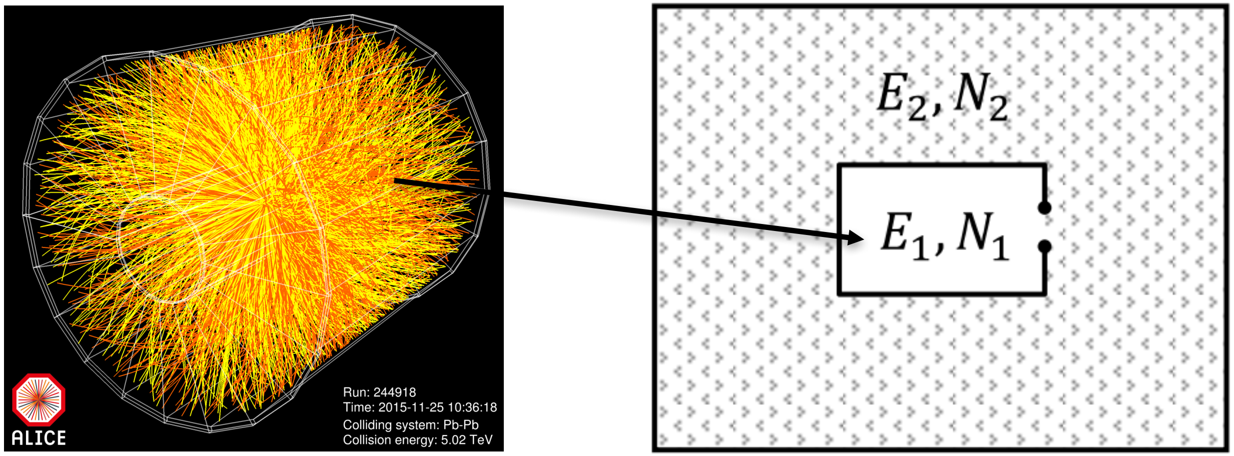
2^{nd} order cumulants of net-p: Acceptance dependence

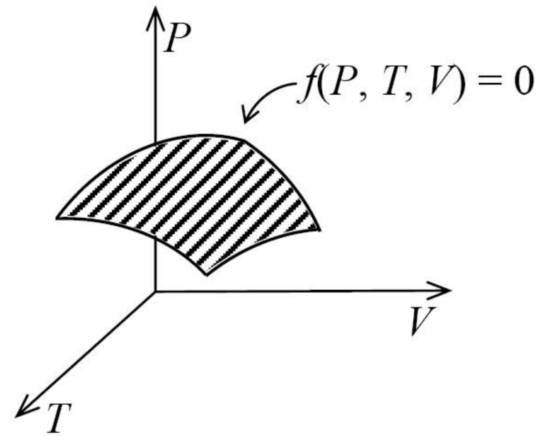




- Consistent with the baryon number conservation picture
  - Increase in fraction of accepted $p, \bar{p}$ -> stronger constraint of fluctuations due to baryon number conservation
- EPOS & HIJING show this drop qualitatively

What kind of a system we are talking about?





<u>Grand canonical ensemble</u> where particles are in a thermal equilibrium

- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate
- EOS can be represented **by a surface** in the state space spanned by P, V and T
- Conservation laws are applied **on average**
- Chemical potential **(μ)**, Volume **(V)** and Temperature (T) are constant
- For a given state E_j and N_j grand canonical partition function

$$Z_{GCE}(T,V,\mu) = \sum_j \exp\left[-\frac{E_j - \mu N_j}{T}\right] \Rightarrow \langle N \rangle = \sum_j N_j p_j = T \frac{\partial \ln Z_{GCE}}{\partial \mu}\bigg|_V$$

Cross Cumulants

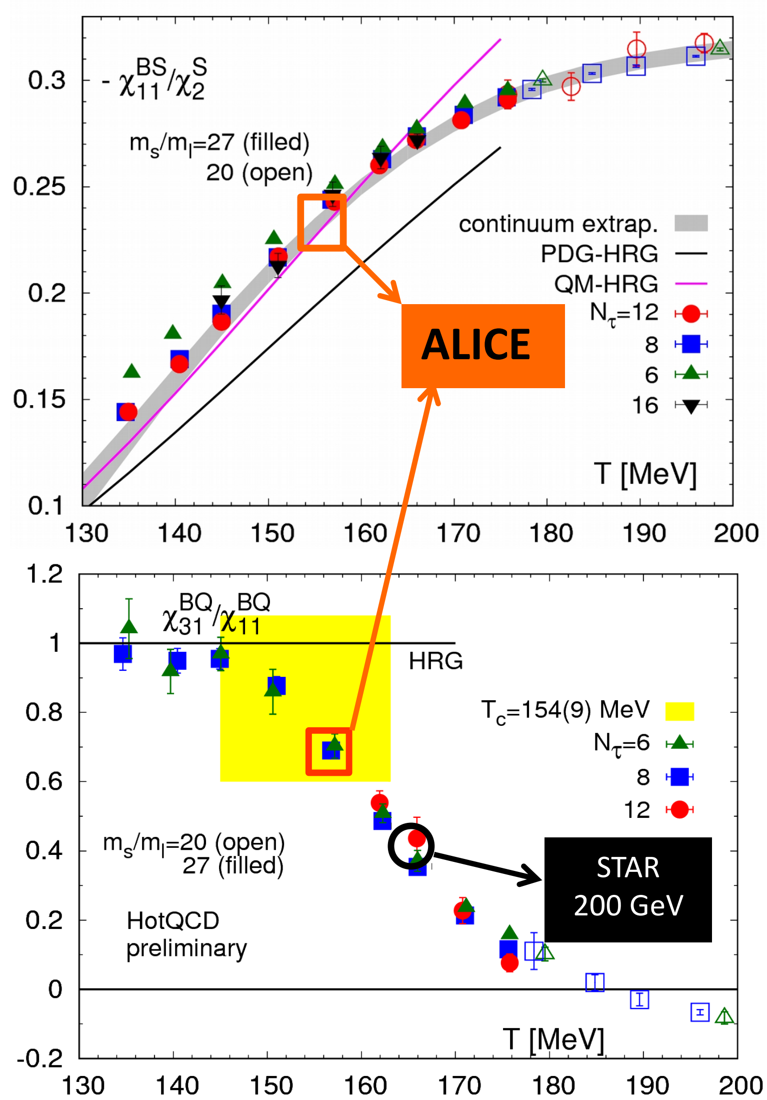
- Taylor expansion of the **QCD** pressure

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Cumulants of **net-charge fluctuations and correlations**

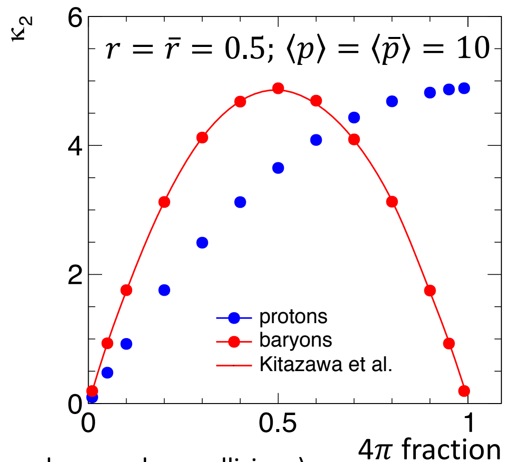
$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0}, \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

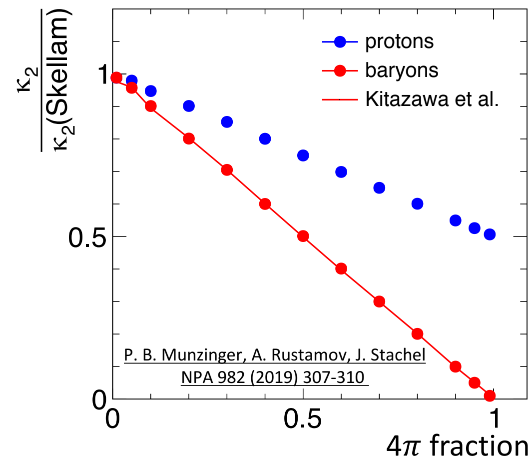


F. Karsch, QM17, arXiv:1706.01620

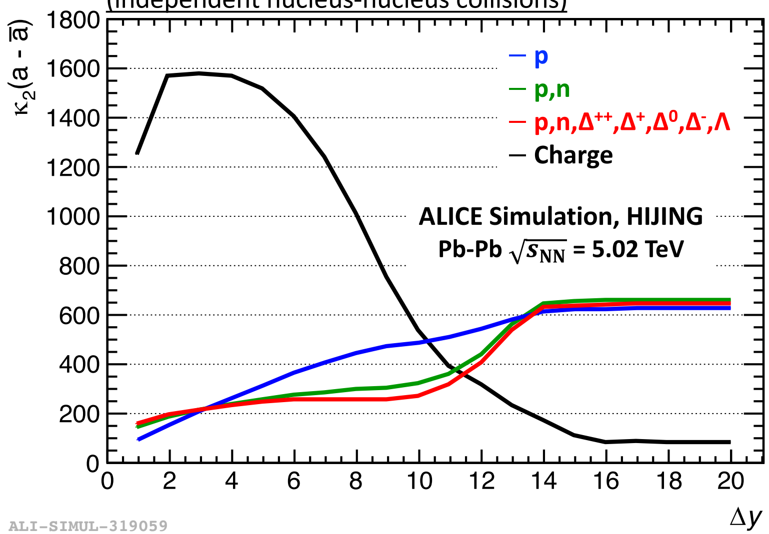
Which acceptance?

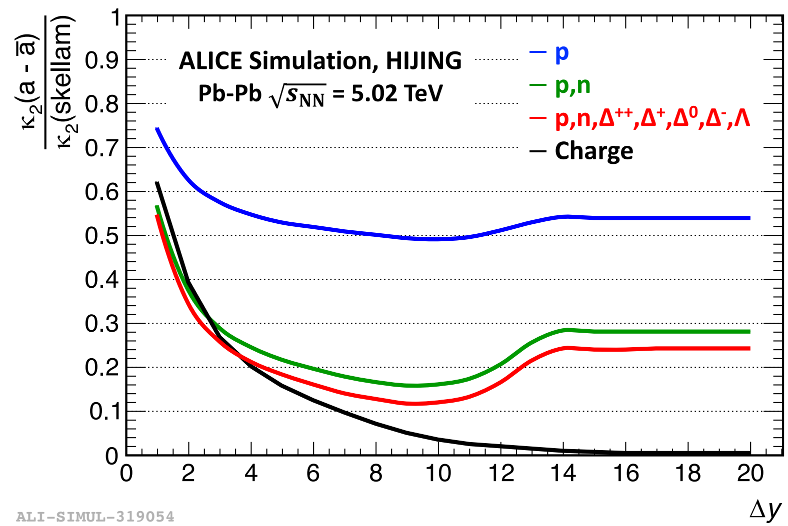
- Due to **isospin randomization,** at $\sqrt{s_{NN}}$ > 10 GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))



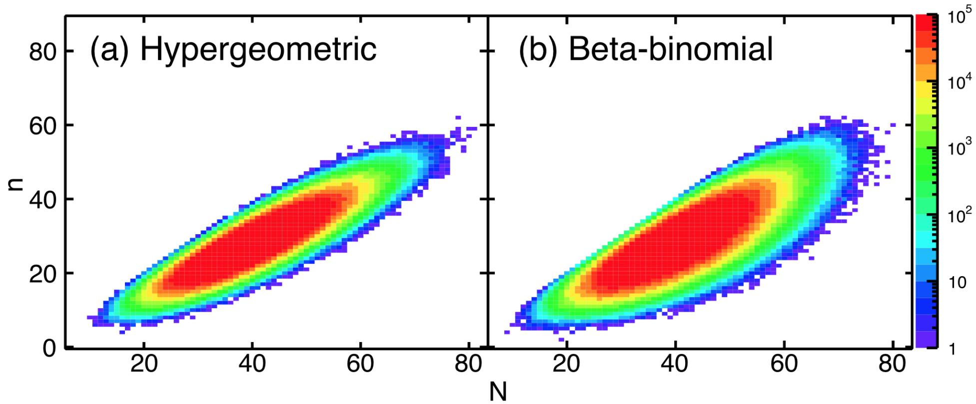


(independent nucleus-nucleus collisions)

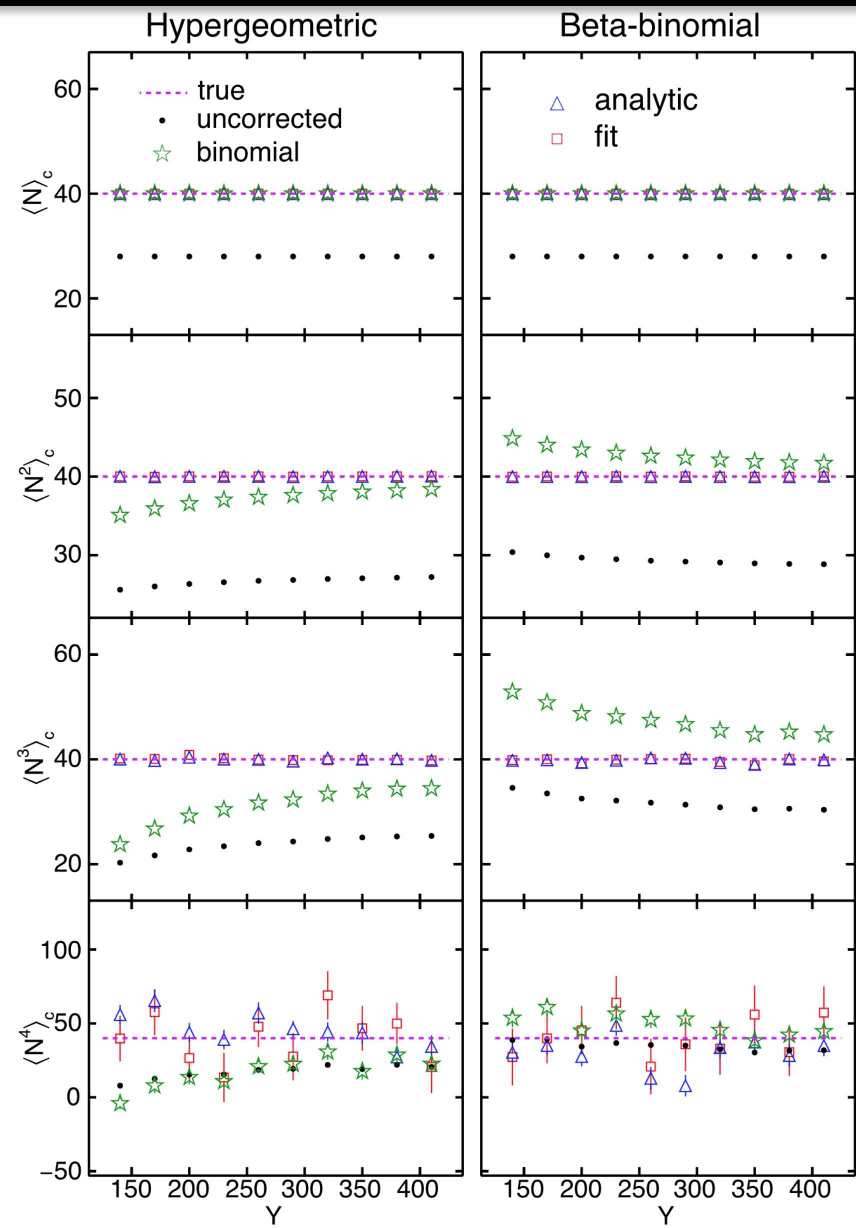




What if efficiency loss is not binomial?



Draw N balls from the urn without returning balls to the urn	In each draw, when one draws a white ball, two white balls are returned to the urn

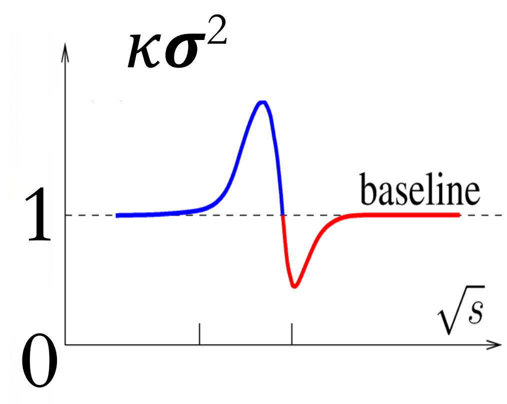


T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 (2018) 10-17
T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)
Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

Expectations for the 3rd and 4th order cumulants

At RHIC:

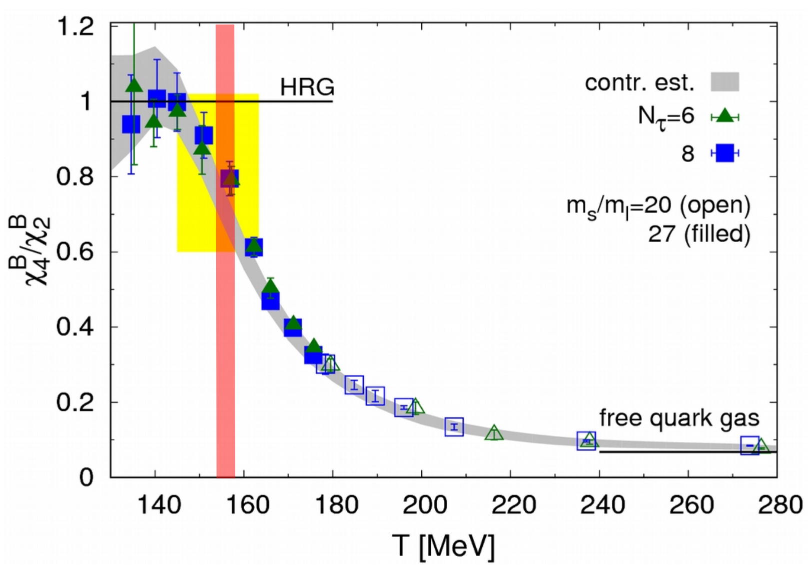
Non-monotonic behavior as a function of energy



[M. Stephanov](#)
[PRL102, 032301 (2009), PRL107, 052301 (2011)](#)

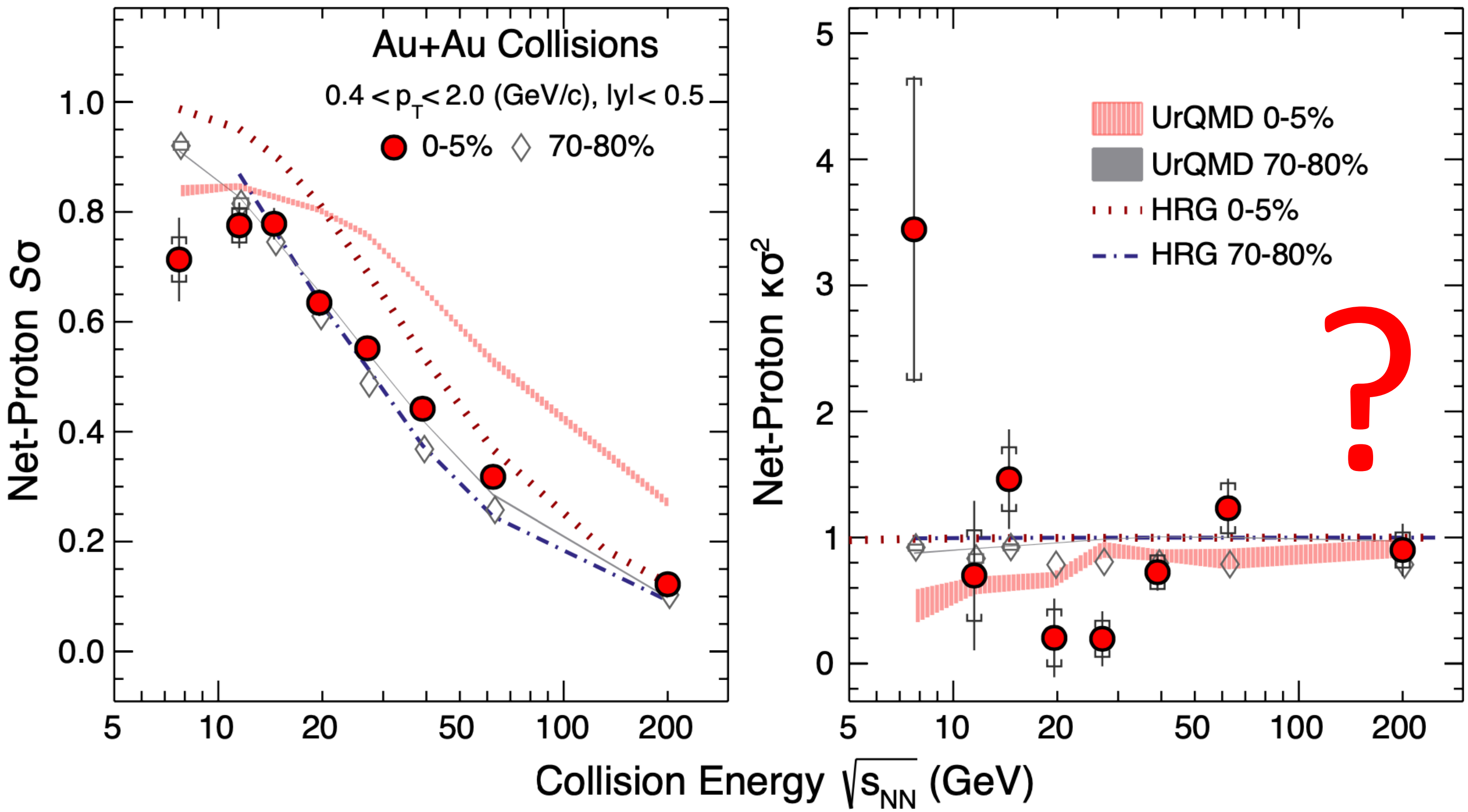
At LHC:

$\sim$ 25% difference between LQCD and HRG at T_{pc}



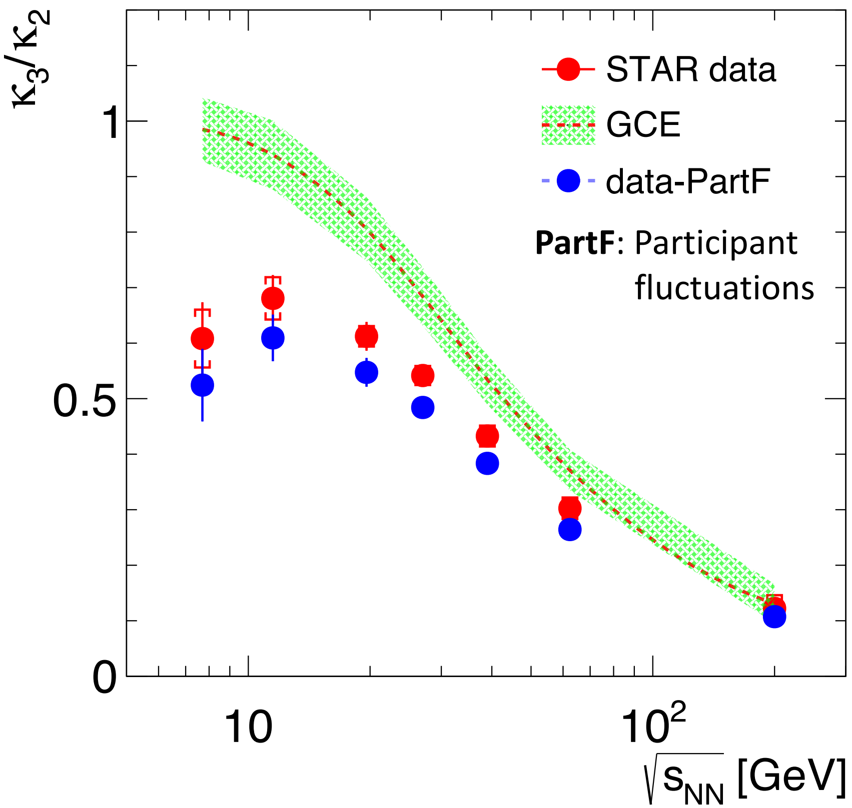
3^{rd} and 4^{th} order cumulants of net-p at RHIC

STAR Collaboration, arXiv:2001.02852

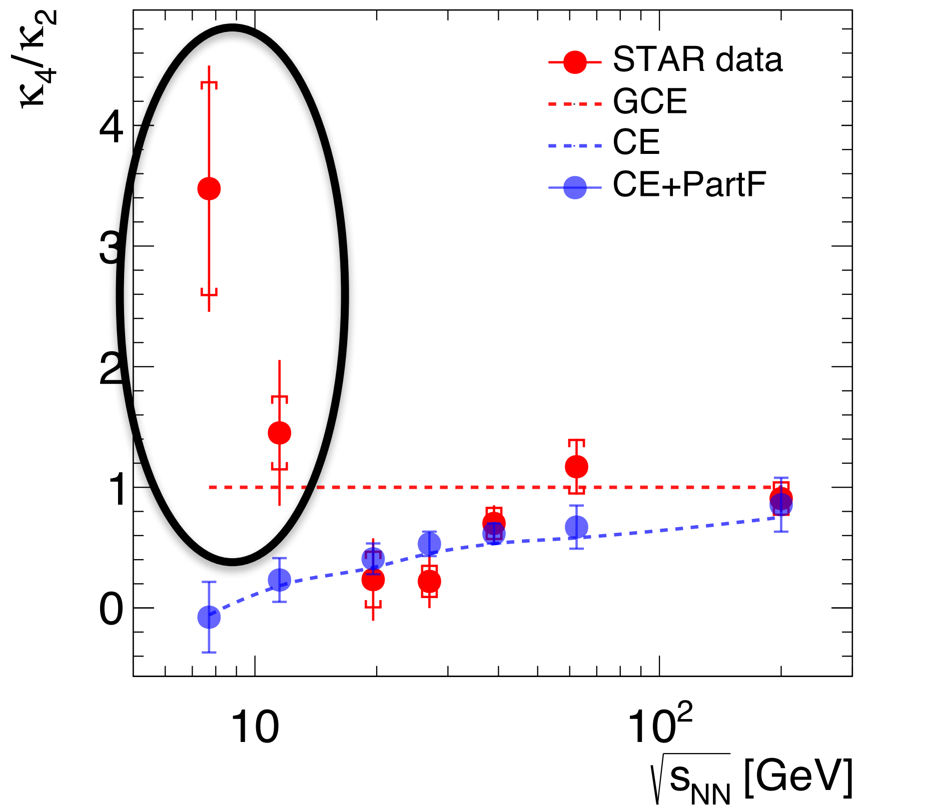


$$\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2 \qquad \frac{\kappa_3}{\kappa_2} = S\sigma$$

Effect of baryon number conservation

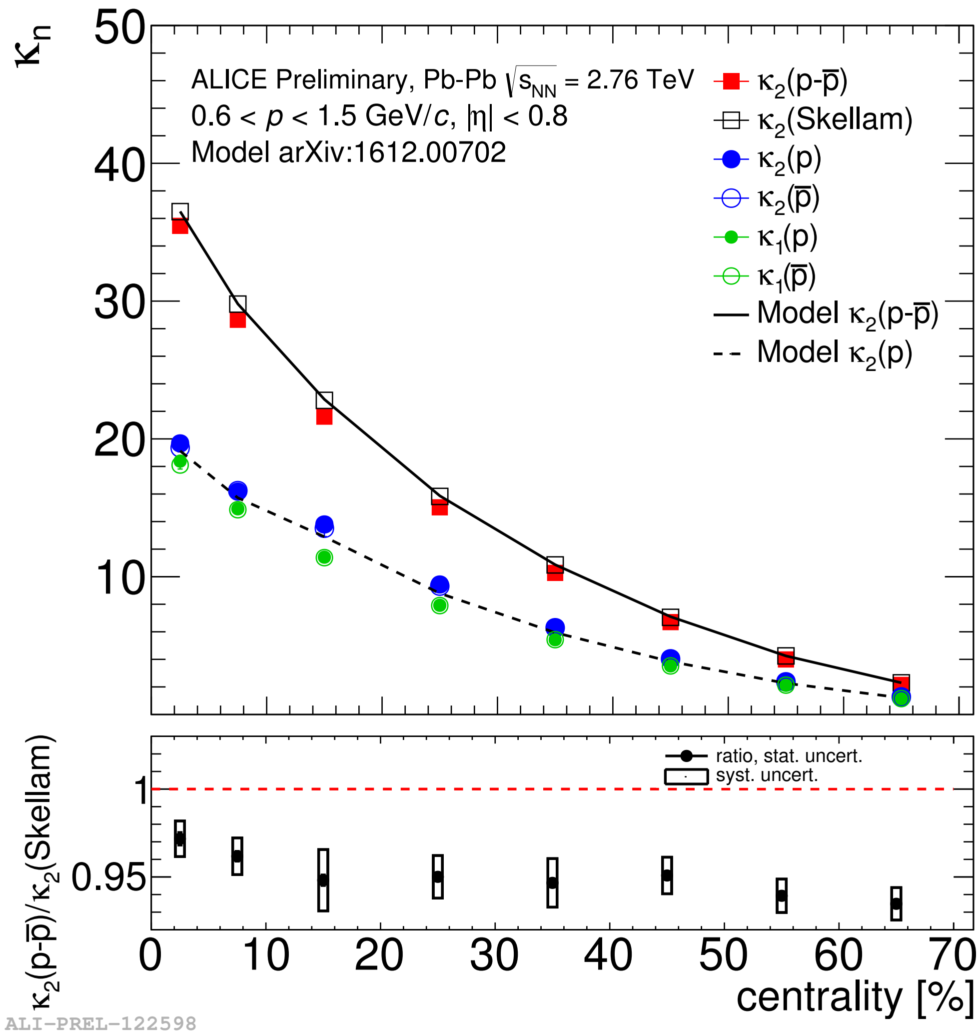


P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310



➢ $\kappa_3/\kappa_2$and κ_4/κ_2 cannot be simultaneously explained for the lowest two energies

“Model” vs ALICE Data



Input to the Model

$\kappa_1(p)$, $\kappa_1(\bar{p})$

centrality selection procedure

Predictions

——— $\kappa_2(p-\bar{p})$

– – – $\kappa_2(p)$

participants

$$\kappa_2(N_B) = \langle N_W \rangle \kappa_2(n_B) + \langle n_B \rangle^2 \kappa_2(N_W)$$

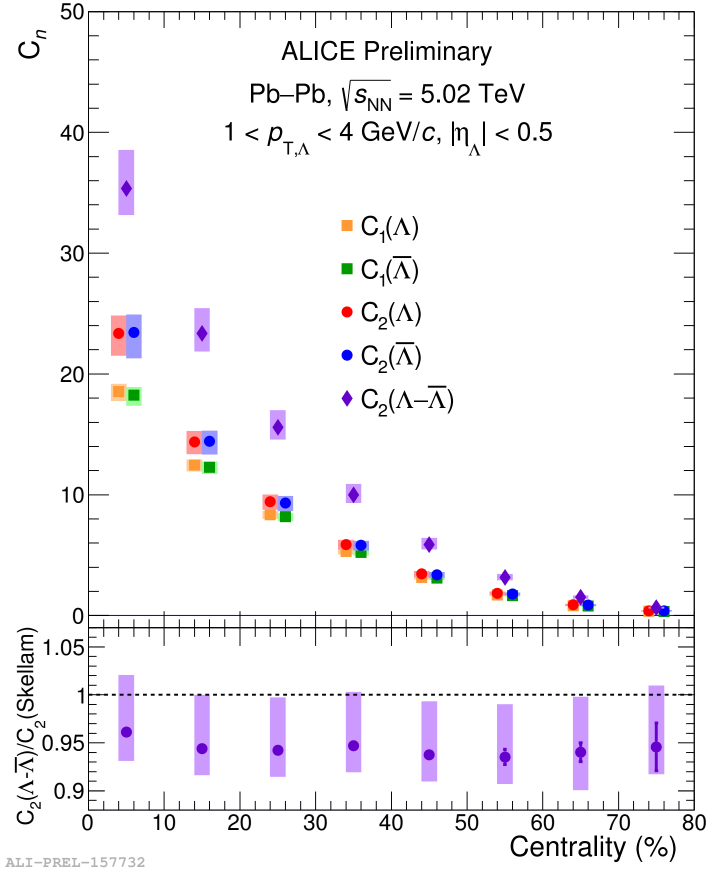
from single participant

Consistent predictions for net-protons, protons and antiprotons

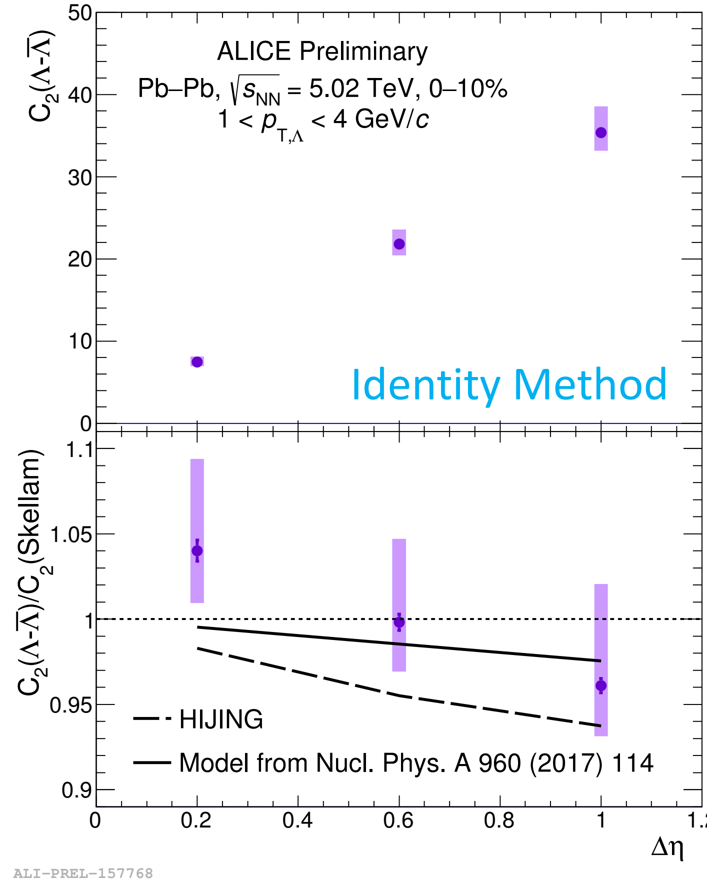
P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

2^{nd} order cumulants of net-Λ at LHC

$$\kappa_2(n-\bar{n}) \equiv C_2(n-\bar{n})$$



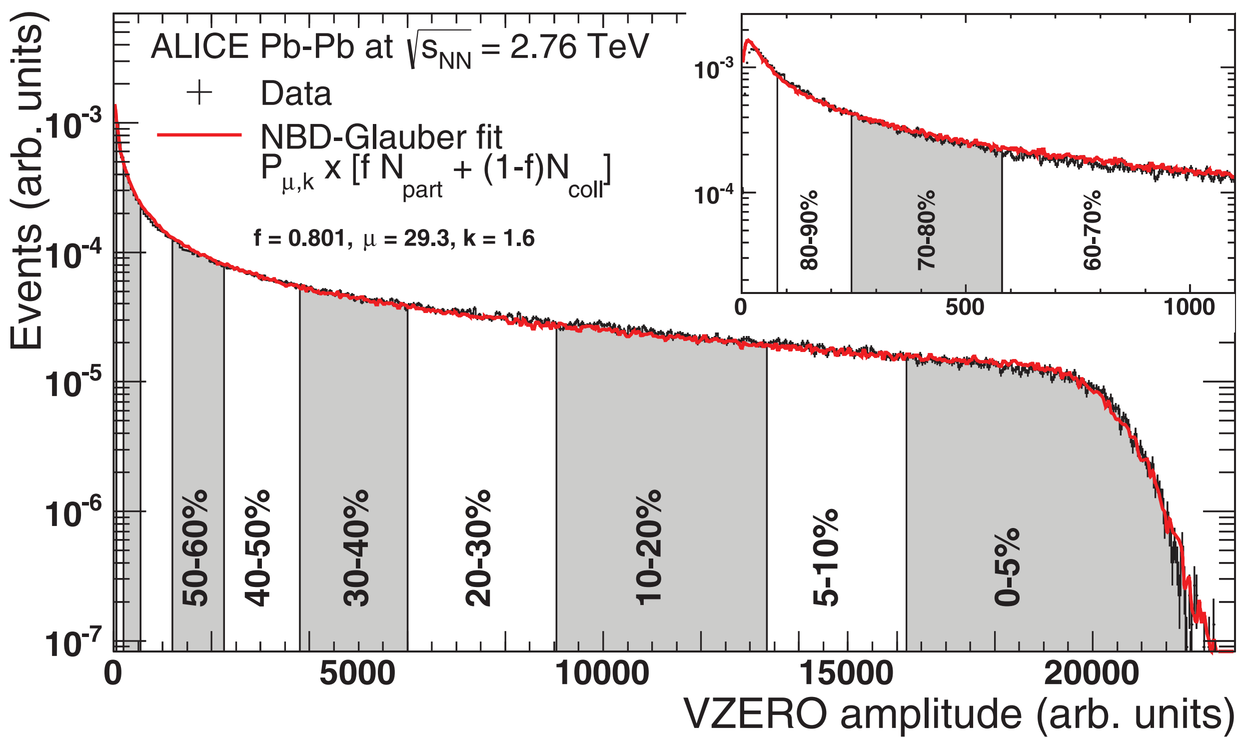
Alice Ohlson, QM2018, NPA 982 (2019) 299

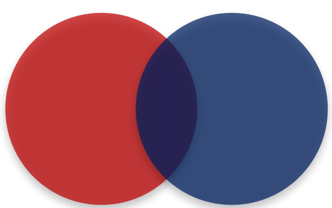


- Similar trend as for **net-p**
- **Better precision** is needed to see the impact of strangeness conservation

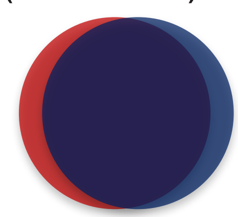
Volume in experiment? ⟶ “Centrality”

ALICE: Phys.Rev. C88 (2013) no.4, 044909



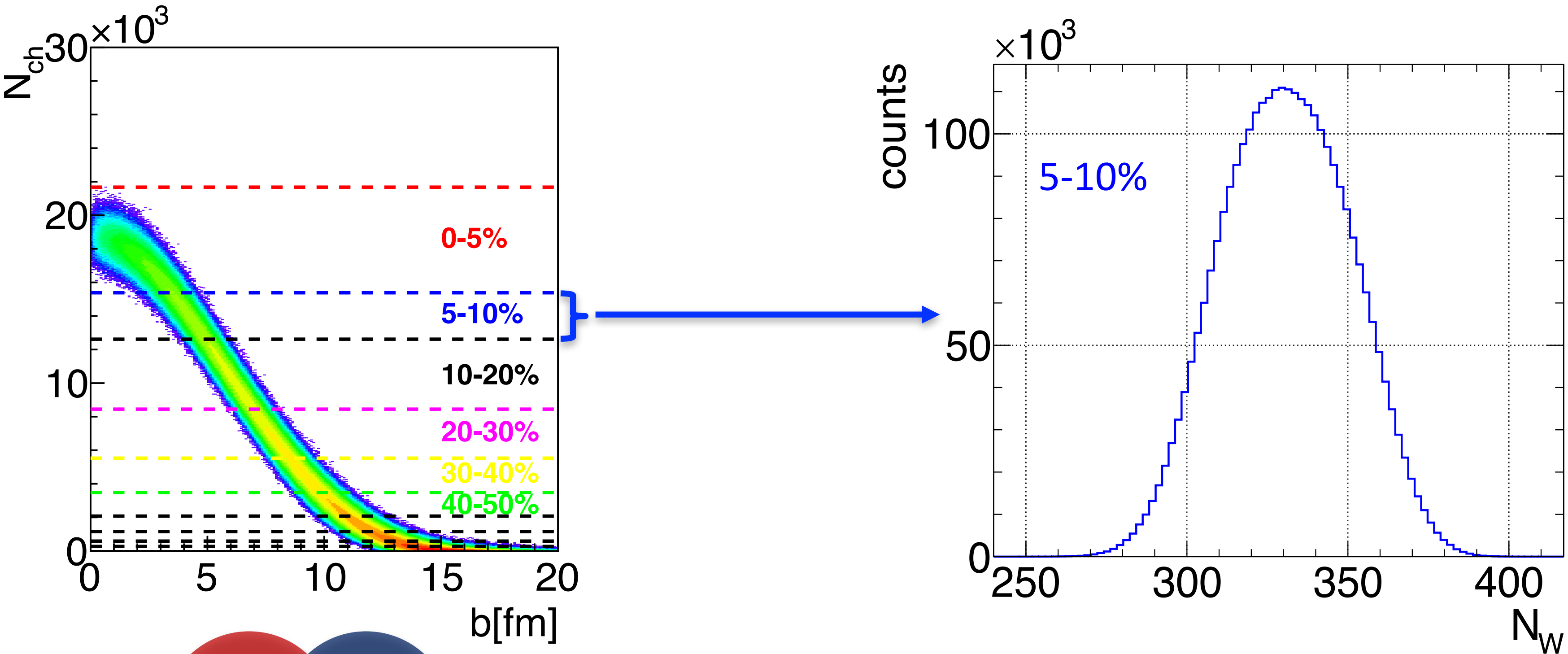


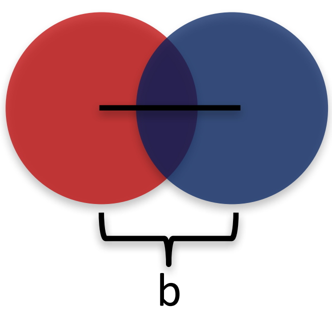
Peripheral collision



Central Collision

Volume Fluctuates



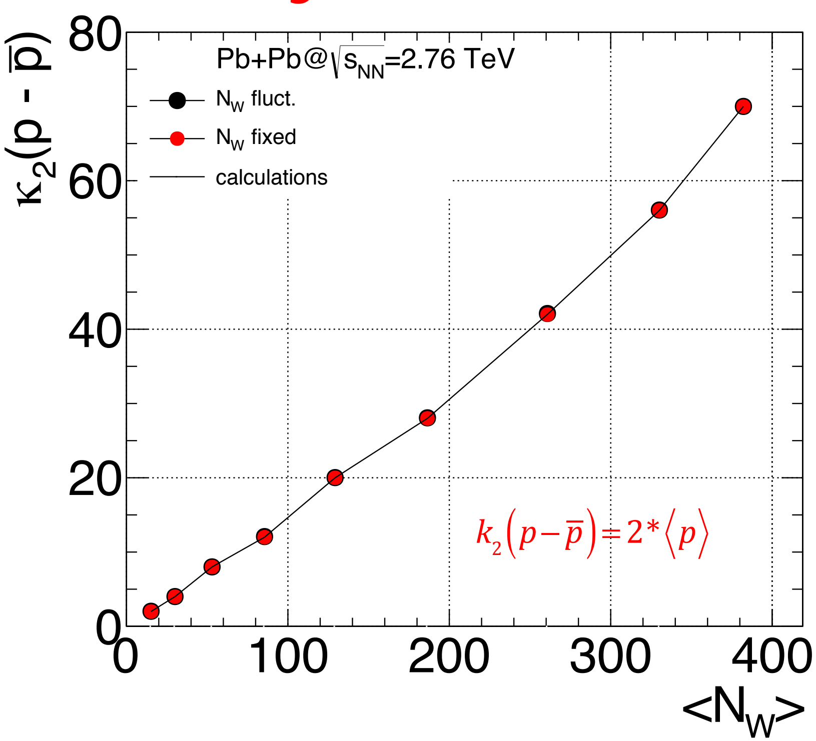


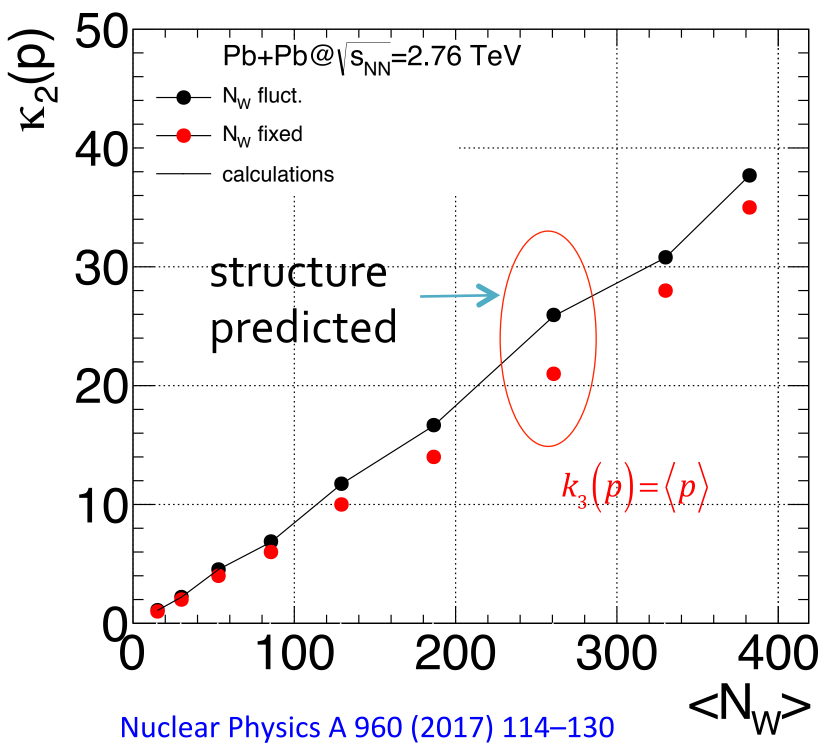
$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \stackrel{?}{=} \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 (2017) 114-130

Volume Fluctuations: 2nd order

$150 * 10^6$ Events





Nuclear Physics A 960 (2017) 114–130

$$k_2(p-\bar{p}) = \langle N_w \rangle k_2(n-\bar{n}) + \langle n-\bar{n} \rangle^2 k_2(N_w)$$

vanishes for ALICE

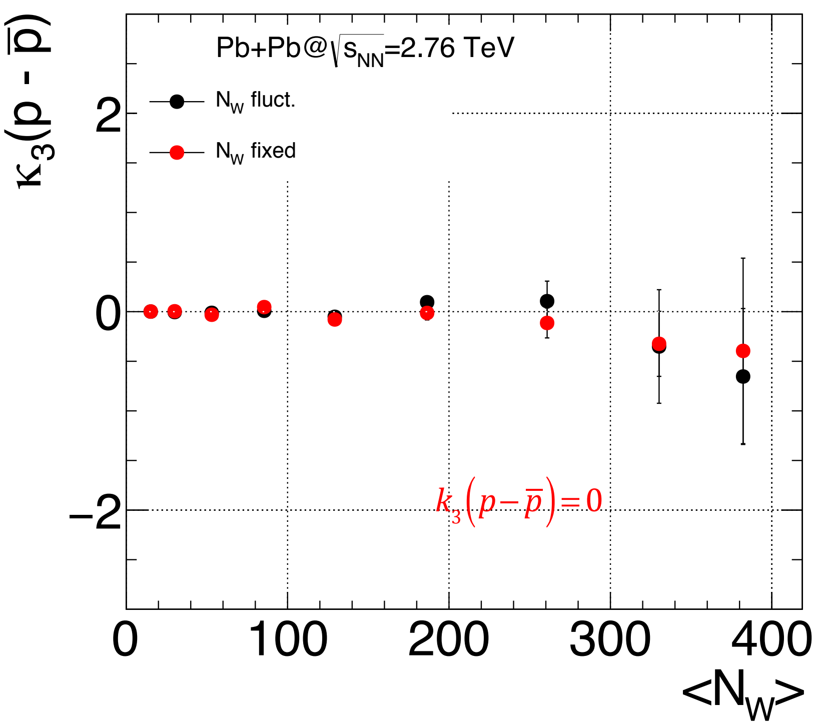
$$k_2(p) = \langle N_w \rangle k_2(n) + \langle n \rangle^2 k_2(N_w)$$

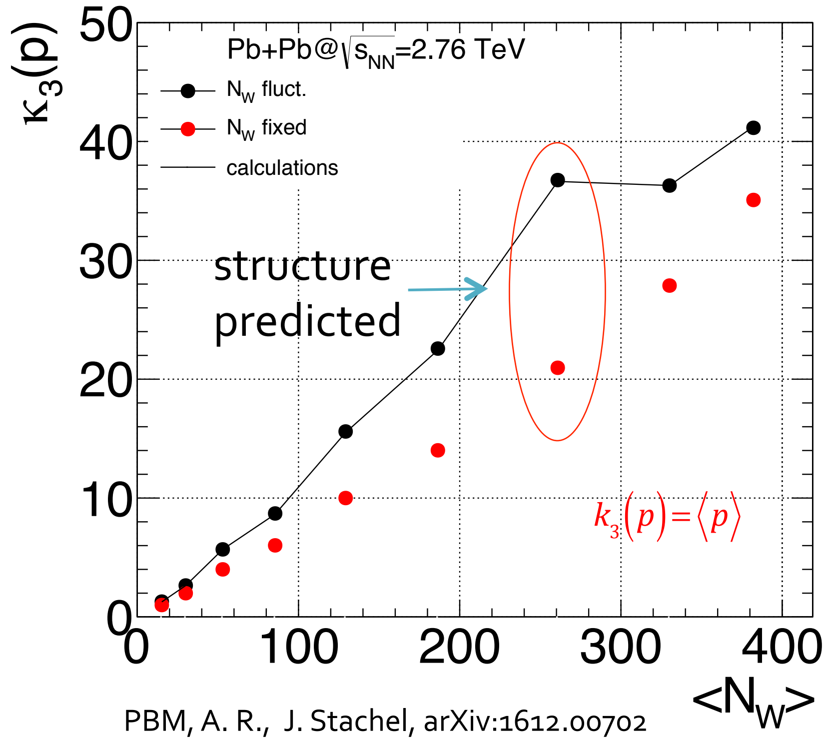
does not vanish

$n, \bar{n}$ from single wounded nucleon

Volume Fluctuations: 3rd order

150*10^6 Events





$k_3(p-\bar{p})=0$

$k_3(p)=\langle p\rangle$

PBM, A. R., J. Stachel, arXiv:1612.00702

$$k_3(p-\bar{p})=\langle N_w\rangle k_3(n-\bar{n})+\langle n-\bar{n}\rangle(\ldots)$$

↓ vanishes for ALICE

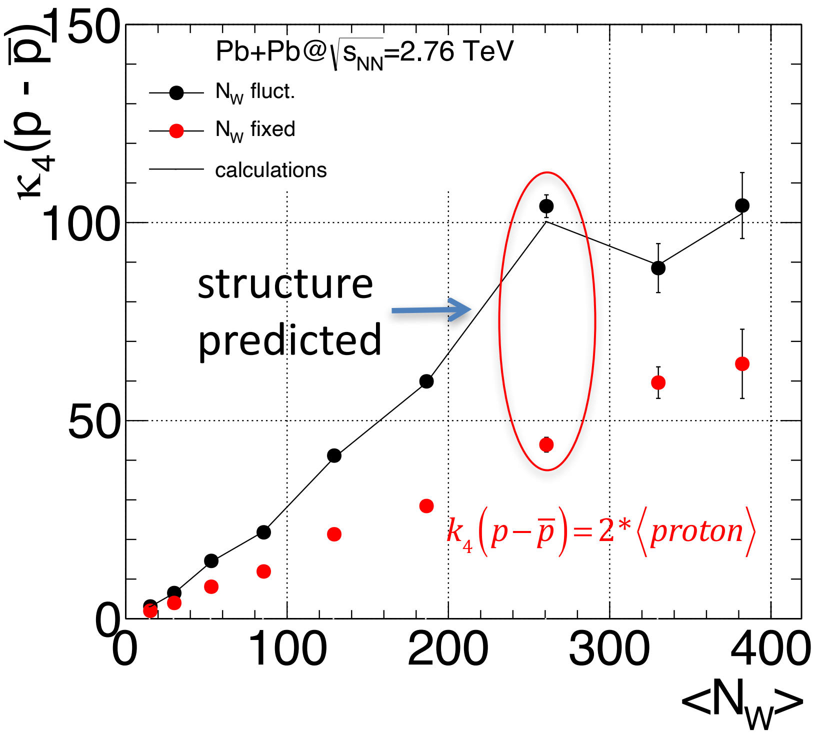
$$k_3(p)=\langle N_w\rangle k_3(n)+\langle n\rangle(\ldots)$$

↓ does not vanish

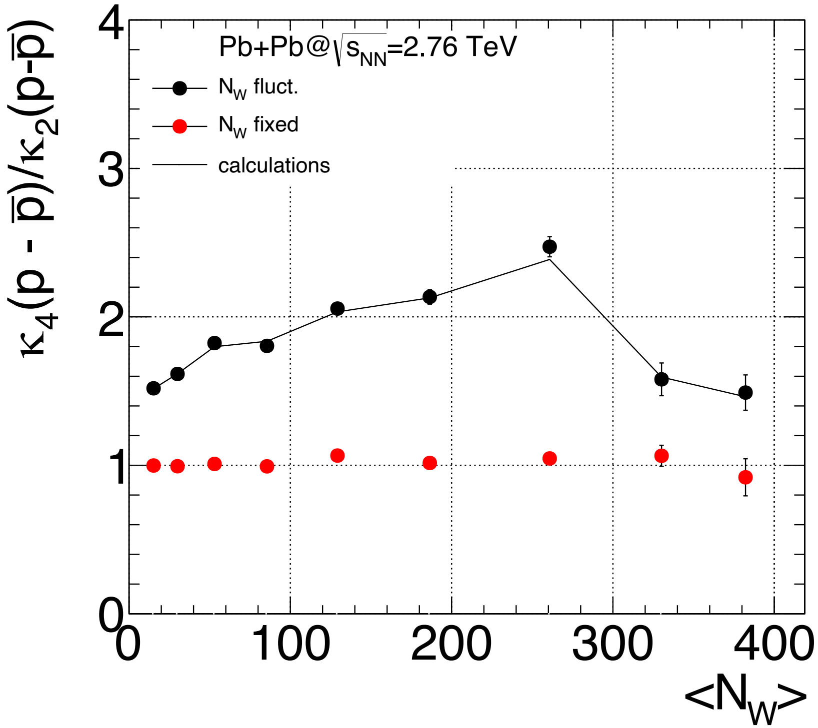
$n,\bar{n}$ from single wounded nucleon

Volume Fluctuations: 4th order

150*10^6 Events



P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

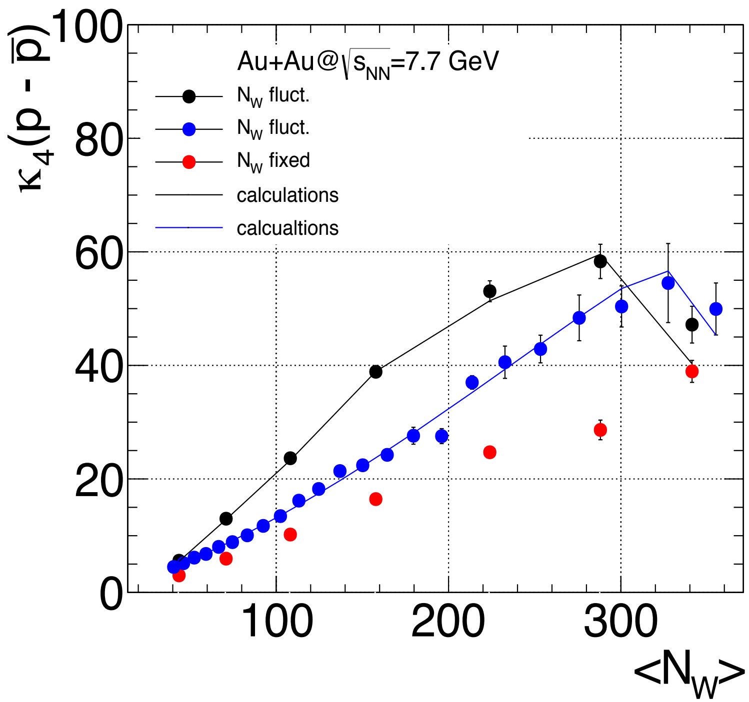


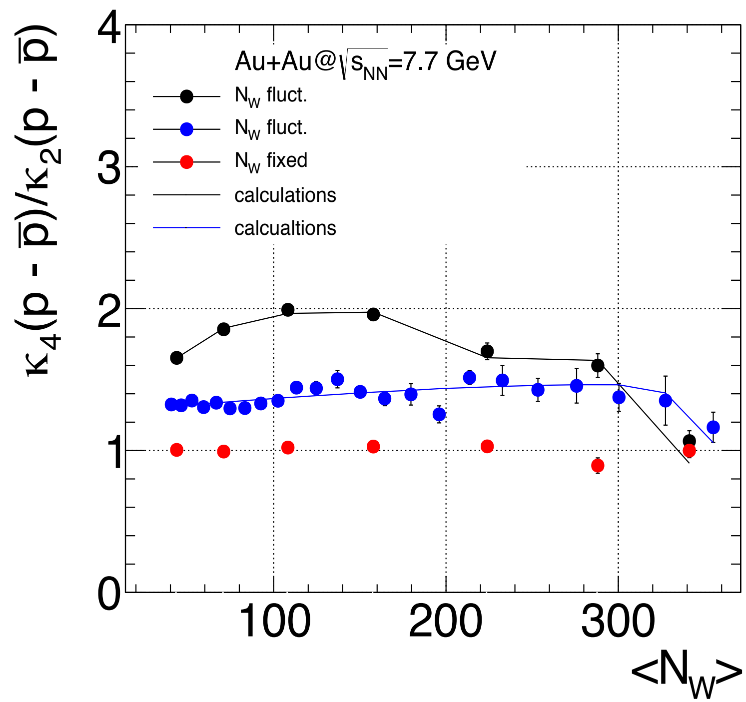
$$k_4(p-\bar{p}) = \langle N_w \rangle k_4(n-\bar{n}) + 3k_2(n-\bar{n})^2 k_2(N_w) + \langle n-\bar{n} \rangle (\ldots)$$

$n, \bar{n}$ → from single wounded nucleon

vanishes for ALICE

Volume Fluctuations at RHIC energies

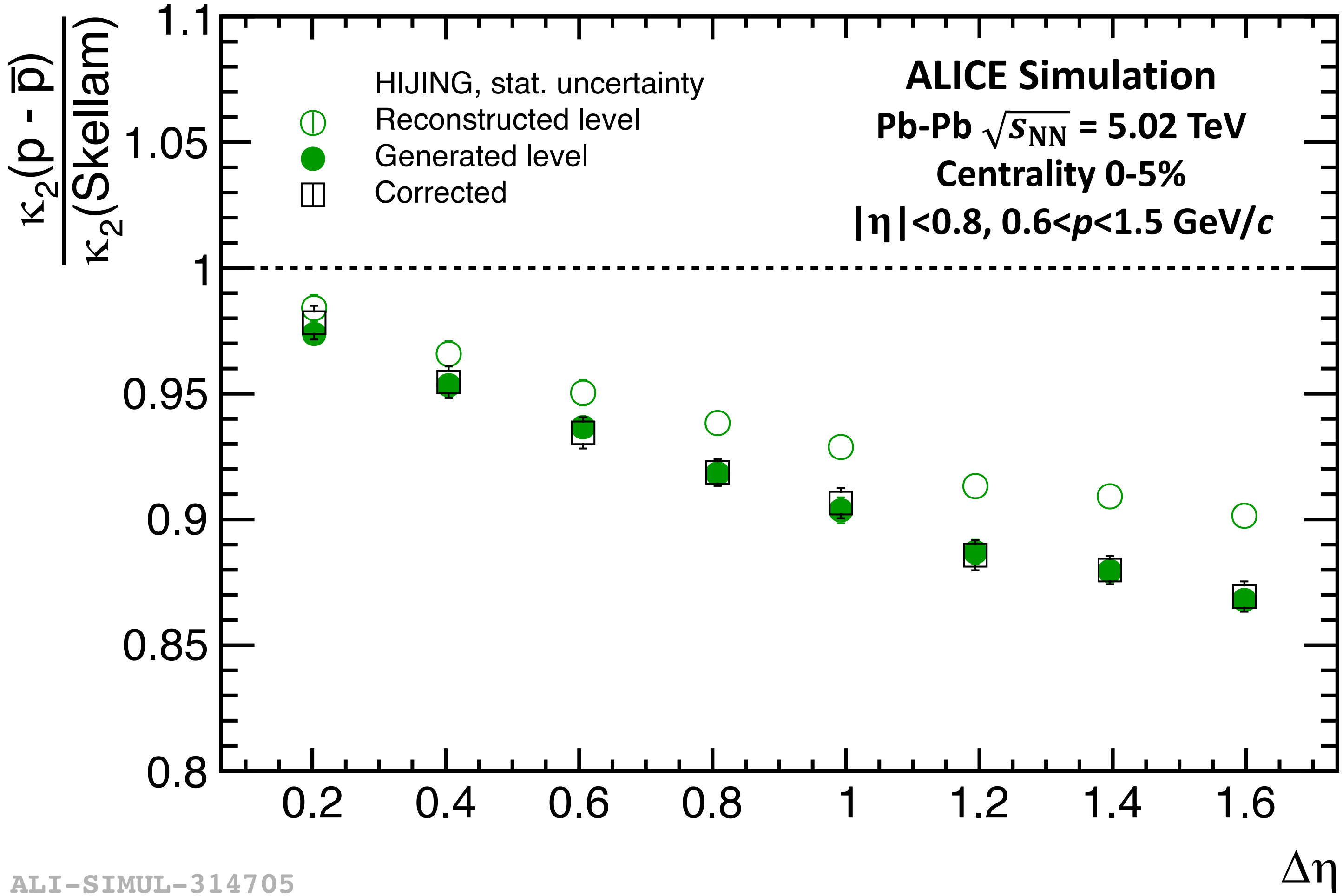




- Participant fluctuations will be present even in the limit of very fine centrality bins
- Incoherent addition of data from intervals with very small centrality bin width will eliminate true dynamical fluctuations.

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nuclear Physics A 960 (2017) 114–130

Efficiency correction: $\kappa_2(p - \bar{p})/\kappa_2(Skellam)$

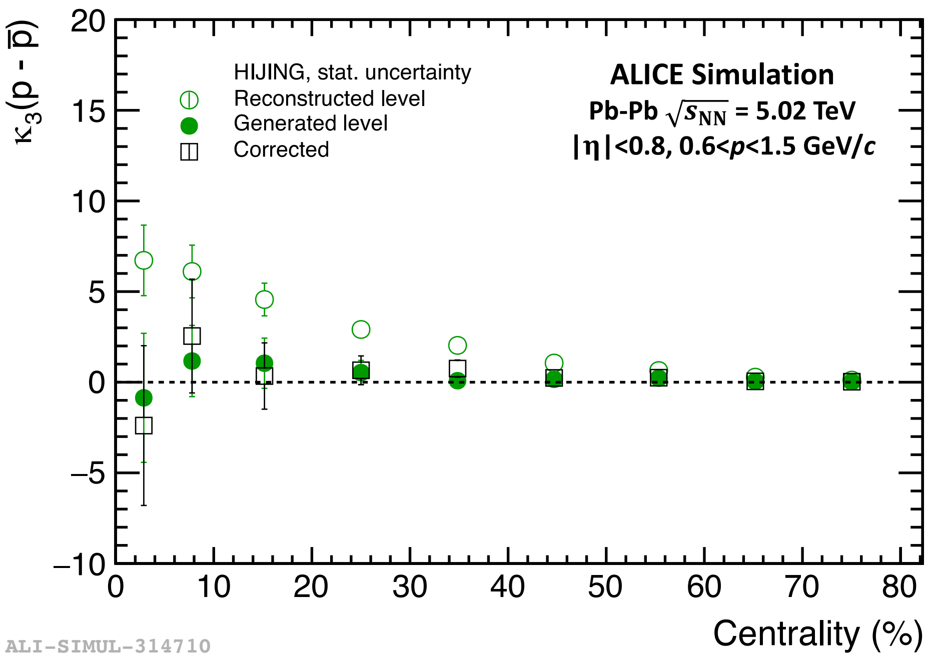


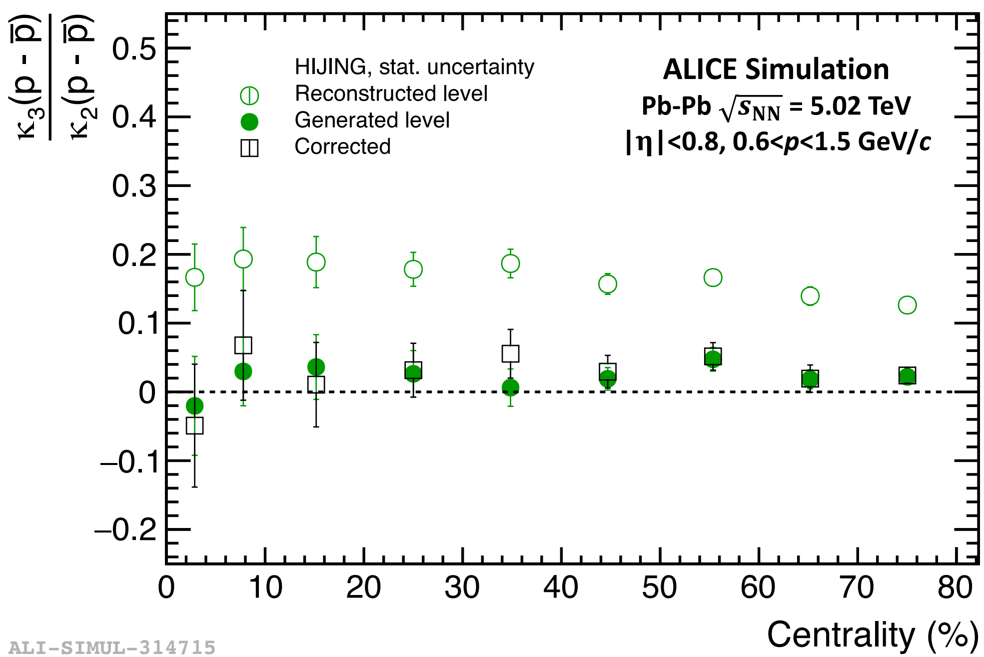
Efficiency correction with binomial assumption:

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

Efficiency correction: $\kappa_3(p-\bar{p})/\kappa_2(p-\bar{p})$





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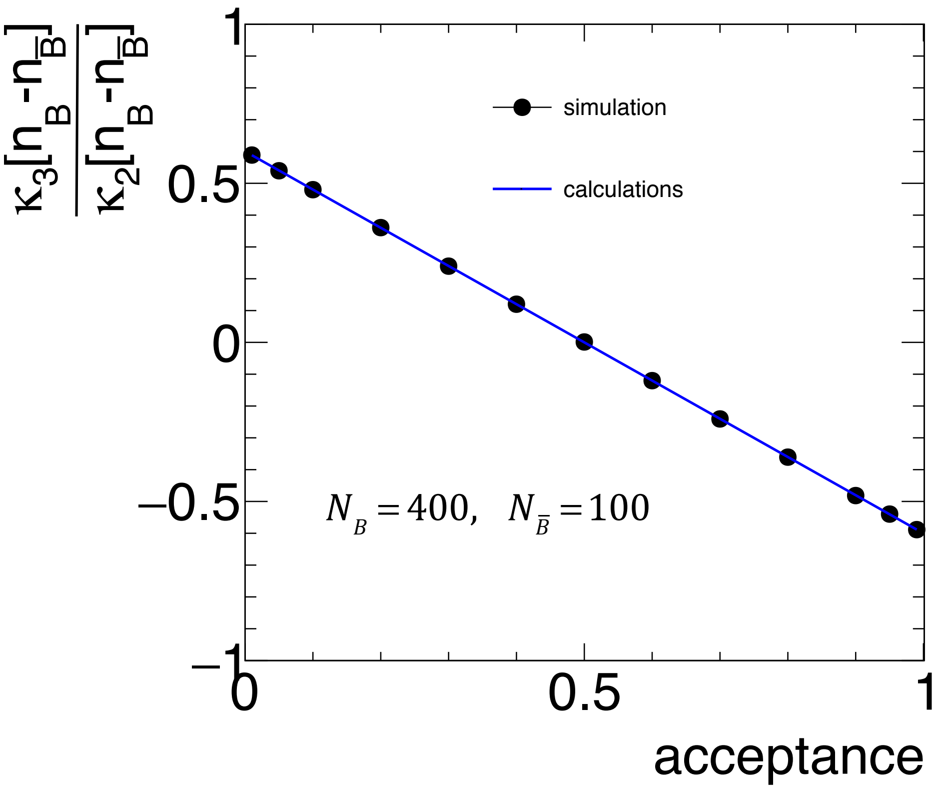
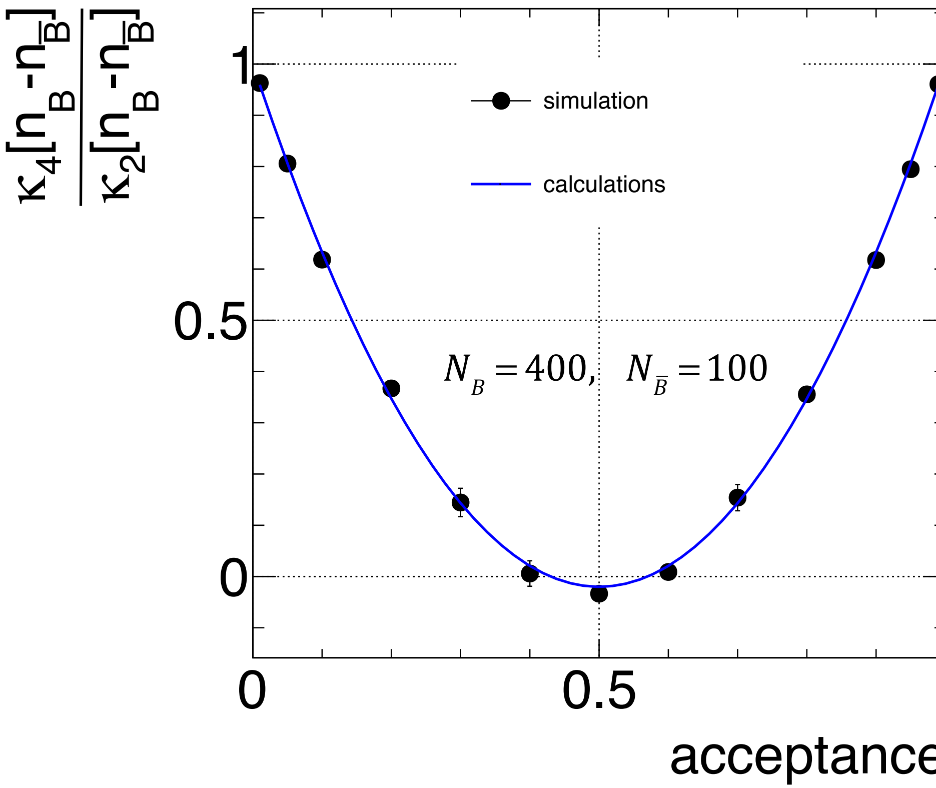
T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

3rd and 4th cumulants

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{B}} \to 0 \rangle} (1-2\alpha)$$





$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}}\left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE}\right]\right)$$

Fluctuation observables

At LHC energies several fluctuation measurements become identical

$$\nu_{dyn} = \frac{\langle N_A(N_A-1)\rangle}{\langle N_A\rangle^2} + \frac{\langle N_B(N_B-1)\rangle}{\langle N_B\rangle^2} - 2\frac{\langle N_A N_B\rangle}{\langle N_A\rangle\langle N_B\rangle}$$

Proper multiplicity scaling

$$\Sigma[a,b] - 1 = \frac{\nu_{\rm dyn}[a,b]}{\frac{1}{\langle N_a\rangle} + \frac{1}{\langle N_b\rangle}}$$

Strongly intensive quantity

M. I. Gorenstein and M. Gazdzicki, Phys. Rev. C 84 (2011)

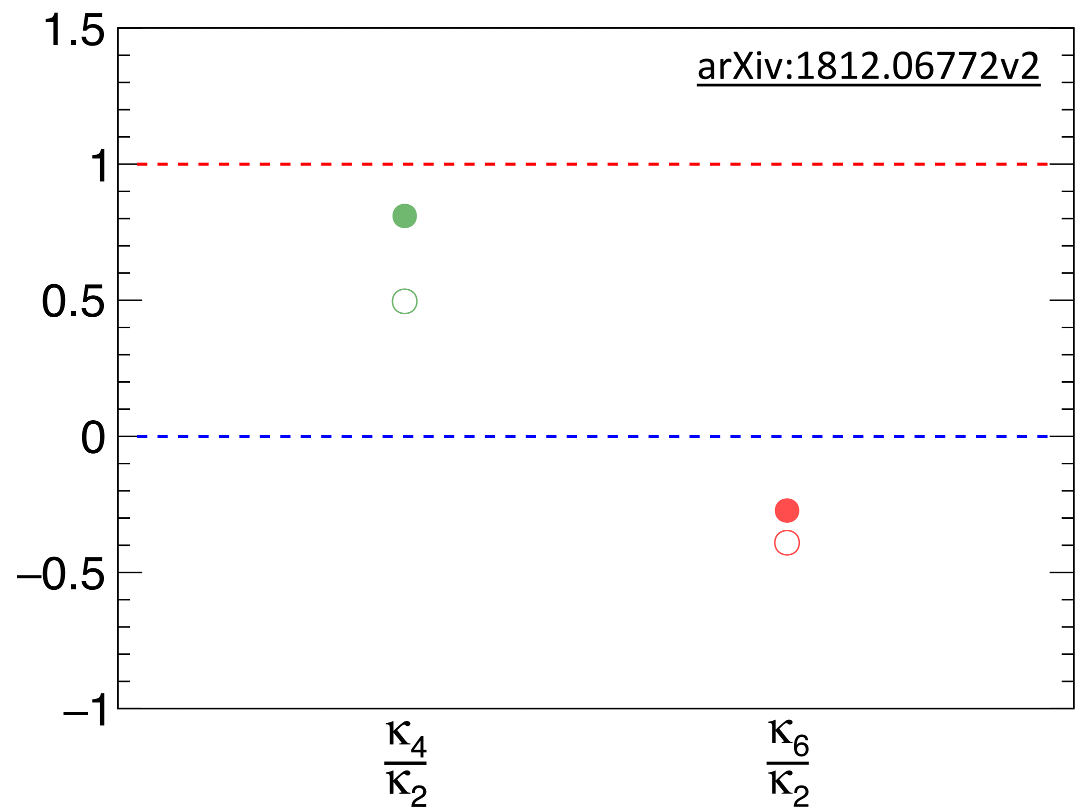
$$\frac{\kappa_2(n_+ - n_-)}{\kappa_2(Skellam)} = 1 + \nu_{dyn}^{Scaled}[n_+, n_-] = \Sigma^{n_+ n_-}$$

Charge conservation

$$\frac{\kappa_2(n_+ - n_-)}{\kappa_2(Skellam)} = 1 - B(Y,Y) = 1 - \frac{\langle n\rangle}{\langle n\rangle_{4\pi}}$$

Integral of balance functions

Good to study correlations → Requires differential analysis



κ4/κ2 and κ6/κ2 as calculated within PQM model (open symbols).
After taking into account contributions from participant nucleon fluctuations
and global baryon number conservation, the deviations from unity decrease (closed symbols).

MC implementation of canonical ensemble

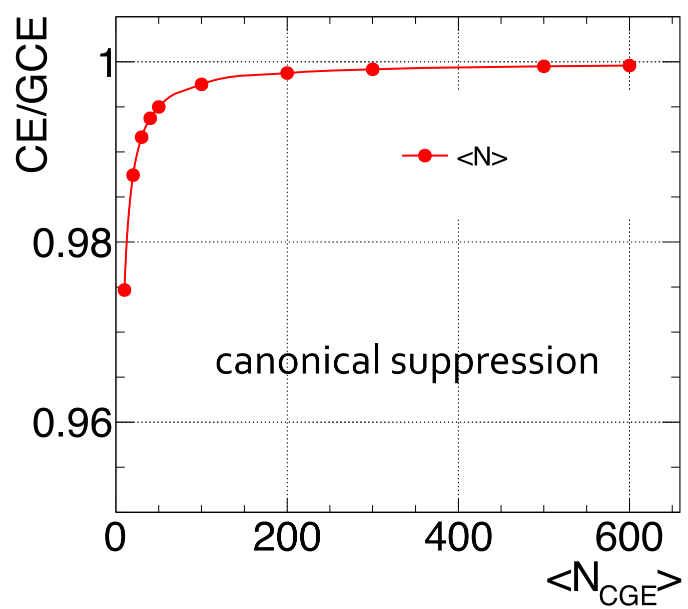
Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh(\mu/T)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta\left(N_B-N_{\bar{B}}-B\right)=I_B(2z)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

$$\left\langle N_{B,\bar{B}}\right\rangle_{GCE}=\lambda_{B,\bar{B}}\frac{\partial \ln Z_{GCE}}{\partial \lambda_{B,\bar{B}}}=e^{\pm\frac{\mu}{T}}z,\quad z=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}$$

$$\left\langle N_{B,\bar{B}}\right\rangle_{CE}=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\frac{I_{B\mp1}\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}{I_B\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}$$



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V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich, V. Skokov, NPA 880 (2012)
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310