Relaxation Time for Strange Quark Spin in Rotating Quark-Gluon Plasma

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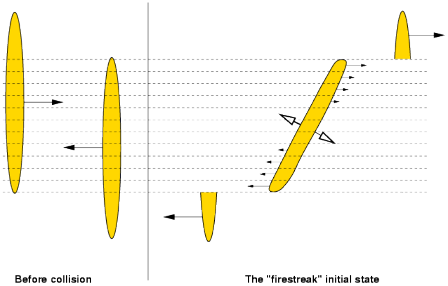
March 5, 2020

Overview

- STAR collaboration at RHIC has observed a predicted polarization of hyperons associated with the angular momentum of non-central heavy ion collisions.
- This is the highest ever (indirectly) measured fluid vorticity.
- How does particle spin couple to vorticity, and what is the relaxation time?
- Estimate relaxation time due to vorticity fluctuations and to strange quark helicity flip in quark-gluon plasma.
- In quark models the spin of the strange quark is transferred to the hyperon at hadronization.

Experimental Setup

Gosset, Kapusta, Westfall, PRC18 (1978)
844 Myers, NPA296 (1978) 177

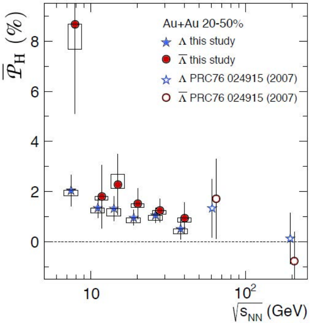


Nuclei colliding at ultra-relativistic energies with nonzero impact parameter have a large initial angular momentum.

Slide courtesy of V. Magas

Λ baryons polarization

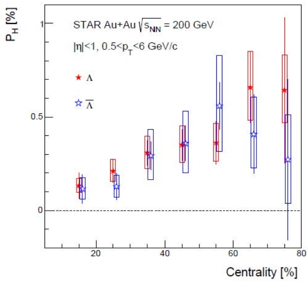
The STAR collaboration, Global Lambda hyperon polarization in nuclear collisions: evidence for the most vortical fluid. Nature volume 548, pages62–65(2017)



- It is natural that particle spin and orbital angular momentum be apportioned according to phase space if equilibration can be attained.
- Decay products of Λ baryons can be measured and used to infer their spin and the vorticity of the fluid.

Λ baryons polarization

J. Adam, et al., Global polarization of Lambda hyperons in Au+Au collisions at $\sqrt{s}$NN=200 GeV. Phys. Rev. C 98 (2018) 14910



- Vorticity should affect particles and anti-particles equally.
- As the impact parameter goes to zero, so does the vorticity and so should the polarization.

Tetrad Formalism

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Vorticity

Rotation around z-axis with frequency ω that may depend on time, but not on position.

$$v_x \equiv -\omega y, \; v_y \equiv \omega x, \; \frac{1}{2}\nabla \times \vec{v} = \vec{\omega}$$

Hamiltonian

$$\begin{aligned} H =& \beta m + \vec{\alpha}\cdot\mathbf{p} - \vec{\omega}\cdot(\mathbf{L}+\mathbf{S}) \\ =& \beta m + \vec{\alpha}\cdot\mathbf{p} - \vec{v}\cdot\mathbf{p} - \vec{\omega}\cdot\mathbf{S}\,. \end{aligned}$$

- When taking the nonrelativistic limit via the Foldy-Wouthuysen procedure the orbital angular momentum term gives rise to the usual Coriolis and centrifugal forces.
  - Energy is lowered when spin is aligned with vorticity.

details

Fluctuations in Vorticity

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Fluctuations in the vorticity tend to restore the polarization to its equilibrium value (fluctuation-dissipation theorem).

Consider a time-independent operator A in the Schrodinger representation.
In the Heisenberg representation the thermal average is

$$\begin{aligned} \langle A_{\rm H}(t)\rangle_0 &= \frac{1}{Z_0}{\rm Tr}\left(e^{-\beta(H_0-\mu N)}A_{\rm H}(t)\right) = \frac{1}{Z_0}{\rm Tr}\left(e^{-\beta(H_0-\mu N)}A_S\right) \\ &= \frac{1}{Z_0}\sum_n e^{-\beta(E_n-\mu N_n)}\langle n|A_{\rm S}|n\rangle = \langle A_{\rm S}\rangle_0 \, . \end{aligned}$$

Now add a time-dependent perturbation whose time average is zero.

$$\begin{aligned} H &= H_0 + V(t) \\ \vec{\omega}(t) &= (\omega_1(t), \omega_2(t), \omega_0 + \omega_3(t)) \end{aligned}$$

Response Theory

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Interaction picture, time evolution operator

$$U_{\mathrm{I}}(t) = 1 + \frac{1}{i}\int_0^t dt' \, V_{\mathrm{I}}(t') + \frac{1}{i^2}\int_0^t dt' \int_0^{t'} dt'' \, V_{\mathrm{I}}(t') V_{\mathrm{I}}(t'') + \cdots .$$

Time evolution of operators

$$\frac{d}{dt}\langle A\rangle_t = i\langle [H_0, A_{\mathrm{S}}]\rangle_t - \int_0^t dt' \langle\langle [V_{\mathrm{I}}(t'), [V_{\mathrm{I}}(t), A_{\mathrm{I}}(t)]]\rangle\rangle_0 .$$

Linear response Second order response

Assumption about fluctuations

$$\overline{\omega_i(t)\omega_j(t')} = \overline{\omega_i^2}\, e^{-|t-t'|/\tau_c}\, \delta_{ij}$$

details

Massive Dirac Particle

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Momentum along z-axis

$$H_0 = m\beta + p\alpha_3 - \frac{1}{2}\omega_0\Sigma_3\,.$$

Normal modes for spin relaxation along z-axis

- $\beta, \alpha_3, i\beta\alpha_3, \Sigma_3$
- Two modes are real and two are complex$\longrightarrow$ oscillation in addition to damping
- parameters: $\omega_0 = 6$ MeV, $\overline{\omega}_\perp^2 = 8$ MeV^2, $\tau_c = 4$ fm/c

details

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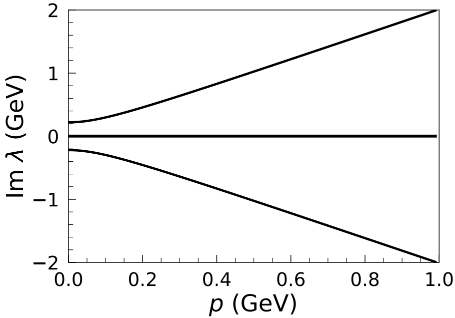

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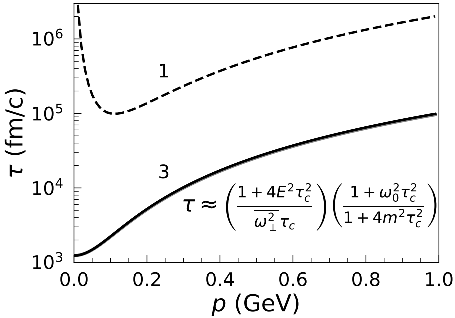

- The largest equilibration time diverges like $1/p^2$ as $p \to 0$, from e-value of $\langle\beta(t)\rangle$
- Transition to $\langle\alpha_3(t)\rangle$ as $p \gtrsim m$
- Three modes with smaller equilibration time scale

Perturbative QCD

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Restoration of the equilibrium polarization due to strange quark scattering via perturbative QCD.

The equilibration time from 2-body scattering can be computed much more simply if Paul suppression and Bose enhancement in the final state are ignored. (Should be a small effect.)

$$\frac{1 - f^{\mathrm{eq}}(E)}{\tau(E)} = \frac{\mathcal{N}\, T}{32(2\pi)^3 E^2} \int \frac{ds}{s} \ln\left(1 \pm e^{-s/4ET}\right)^{\pm 1} \int dt\, |\mathcal{M}(s,t)|^2 \,.$$

For massive s-quark scattering from massless u and d-quarks and gluons we need to apply the hard thermal loop (HTL) approximation.

$$-\frac{(s-m^2)^2}{s} + k_c^2 \leq t \leq -k_c^2\,,\ \ s \geq s_0\,,\ \ \frac{(s_0 - m^2)^2}{s_0} = 2k_c^2\,,$$

Perturbative QCD

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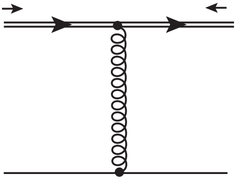
To any finite order in perturbation theory a massless quark cannot change its helicity/chirality in a collision.



$$|\mathcal{M}|^2_{\text{helicity flip}} = -\frac{8}{9}g^4 m^2 \left[\frac{1}{t} + \frac{s}{(s-m^2)^2}\right]$$

Perturbative QCD

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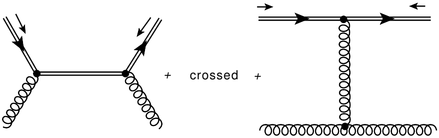

$$|\mathcal{M}|^2_{\text{helicity flip}} = \frac{4g^4 m^2(-t)}{3s_m^4}\left\{4s^2\left[3+\frac{3t}{u_m}+\frac{4t^2}{3u_m^3}\right]+8(m^4-su)^2\left[\frac{3}{t^2}-\frac{3}{u_m t}+\frac{4}{3u_m^2}\right]\right.$$
$$\left.+\frac{7m^4t^2}{3u_m^2}+3\left[\frac{(m^4-su-s_m u_m)}{u_m}-\frac{2(m^4-su)}{t}\right]^2\right\}.$$

$$s_m = s - m^2,\ \ u_m = u - m^2$$

Perturbative QCD

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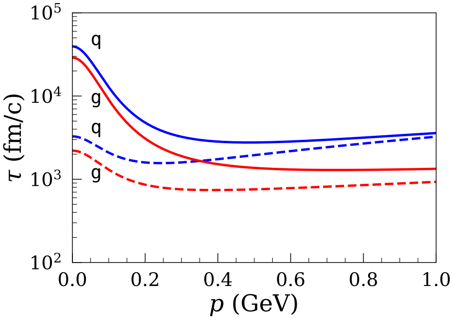

The dashed curves correspond to a temperature of 200 MeV and the solid curves to 400 MeV.

Perturbative QCD

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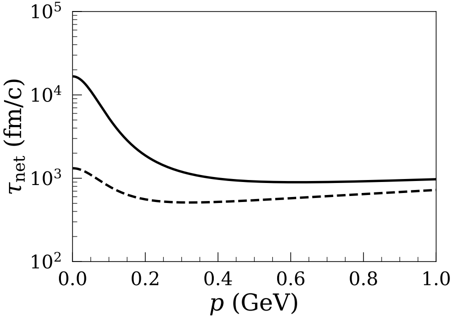

The dashed curves correspond to a temperature of 200 MeV and the solid curves to 400 MeV.

Based on our calculations, and assuming that the strange quarks pass on their polarization to the hyperons, one may be tempted to conclude that the observed polarization is determined at the time the quarks are created and reflect the vorticity of the matter when it was created in the heavy ion collision.

An alternative is that hyperons become polarized at the time of hadronization due to phase space.

Six-quark interaction

M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 44, 1422 (1970).

Kobayashi and Maskawa (1970) – NJL model needs a term which breaks axial U(1) symmetry to solve the problem of the anomalously large η' mass.

$$\mathcal{L}_6 = g_D \left[\det\left(\bar{q}_i(1+\gamma_5)q_j\right) + \det\left(\bar{q}_i(1-\gamma_5)q_j\right)\right]$$

S. Weinberg, Phys. Rev. D 11, 3583 (1975).

Weinberg (1975) – Discusses the axial U(1) problem. Unbroken axial U(1) would imply a light isoscalar pseudoscalar boson which has not observed.

G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D 14, 3432 (1976); Err. 18, 2199 (1978).

't Hooft (1976) – A six-quark interaction of exactly this form arises from instanton physics.

This interaction flips the helicity of the quarks!

Six-quark interaction

Joseph I. Kapusta, Ermal Rrapaj, and Serge Rudaz arXiv:1910.12759

Contribution to elastic scattering

$$\mathcal{L}_{6\to4} = g_D \Big\{ \langle\bar{u}u\rangle \left[\bar{d}(1+\gamma_5)d \times \bar{s}(1+\gamma_5)s + \bar{d}(1-\gamma_5)d \times \bar{s}(1-\gamma_5)s\right] + \langle\bar{d}d\rangle \left[\bar{u}(1+\gamma_5)u \times \bar{s}(1+\gamma_5)s + \bar{u}(1-\gamma_5)u \times \bar{s}(1-\gamma_5)s\right] + \langle\bar{s}s\rangle \left[\bar{u}(1+\gamma_5)u \times \bar{d}(1+\gamma_5)d + \bar{u}(1-\gamma_5)u \times \bar{d}(1-\gamma_5)d\right] \Big\}$$

Contribution to constituent quark masses

$$\mathcal{L}_{6\to2} = 2g_D \left[\langle\bar{u}u\rangle\langle\bar{d}d\rangle\bar{s}s + \langle\bar{u}u\rangle\langle\bar{s}s\rangle\bar{d}d + \langle\bar{d}d\rangle\langle\bar{s}s\rangle\bar{u}u\right]$$

Six-quark Interaction

Joseph I. Kapusta, Ermal Rrapaj, and Serge Rudaz arXiv:1910.12759

Model parameters and temperature dependence of the constituent quark masses and chiral condensates taken from review article Hatsuda and Kunihiro (1994). These parameters provide a very good phenomenological description of hadron properties. Temperature dependence of the axial U(1) symmetry breaking coefficient is highly uncertain.

$$\mathcal{L}_6 = \int_0^\infty d\rho \, d(\rho) \left\{ \prod_f \left[-\frac{\pi^2 \xi}{N_c} \, \bar{q}_f (1+\gamma_5) q_f \, \rho^3 \right] + (\gamma_5 \to -\gamma_5) + \cdots \right\}$$

Instanton density as a function of instanton size.

In the dilute instanton gas approximation the integral is highly divergent.

Instanton screening

D. Diakonov and V. Petrov, Nucl. Phys. B 245, 259 (1984).
T. Schafer and E. V. Shuryak, Rev. Mod. Phys. 70, 323 (1998).

Screening due to instanton repulsion:

$$\exp\left[-\left(\frac{b-4}{2}\right)\frac{\rho^2}{\bar{\rho}^2}\right],\ \ b=\frac{1}{3}(11N_c-2N_f),\ \ \bar{\rho}\approx 0.33\ \mathrm{fm}$$

R. D. Pisarski and L. Yaffe, Phys. Lett. B 97, 110 (1980).
D. J. Gross, R. D. Pisarski and L. Yaffe, Rev. Mod. Phys. 53, 43 (1981).
C. Aragao de Carvalho, Nucl. Phys. B 183, 182 (1981).
V. Baluni, Phys. Lett. B 106, 491, (1981).
E. V. Shuryak, Nucl. Phys. B 203, 140 (1982).
A. A. Abrikosov, Yad. Fiz. 37, 772 (1983).

Screening due to finite temperature and density

$$\exp\left[-\frac{2\pi^2}{g^2}m_{\mathrm{el}}^2\,\rho^2\right],\ \ m_{\mathrm{el}}^2=g^2\left[\left(\frac{N_c}{3}+\frac{N_f}{6}\right)T^2+\frac{1}{2\pi^2}\sum_f\mu_f^2\right]$$

Instanton screening

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Case I: Evaluate screening factor at the average instanton size

$$g_D(T) = g_D(0)\exp\left(-T^2/T_0^2\right), \quad T = 100 \text{ MeV}$$

Case II: Neglect logarithmic corrections

$$g_D(T) \propto \int_0^\infty d\rho\, \rho^{b+3N_f-5} \exp\left[-\left(\frac{b-4}{2}\right)\frac{\rho^2}{\bar{\rho}^2}\right] \exp\left[-\frac{1}{3}(2N_c+N_f)(\pi T)^2\rho^2\right]$$

$$g_D(T) = \frac{g_D(0)}{(1+1.2\pi^2\bar{\rho}^2T^2)^7}, \quad N_c = N_f = 3$$

Results

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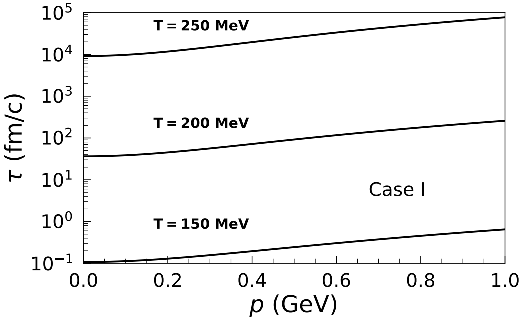

Results

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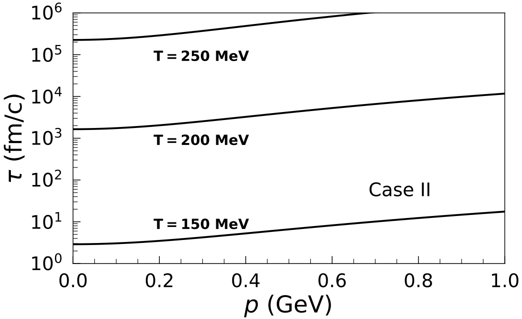

Results

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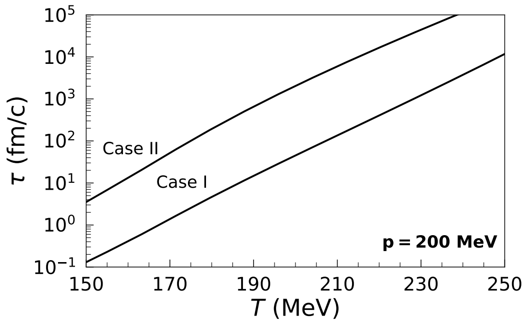

Summary

We found that vorticity fluctuations and helicity flip of strange quarks through perturbative QCD processes result in equilibration times far too long to be relevant.

We then studied the NJL model with the inclusion of the six-quark Kobayashi-Maskawa-'t Hooft interaction term which breaks axial U(1) symmetry, and found that constituent strange quarks are likely to reach spin equilibrium at temperatures below around 170 MeV, just prior to hadronization.

Thank you

Massless Dirac Particle

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Momentum along z-axis

$$H_0 = p\alpha_3 - \tfrac{1}{2}\omega_0\Sigma_3 \,.$$

Normal modes for spin relaxation along z-axis

$$\tau_\pm = \frac{1 + (2p \pm \omega_0)^2 \tau_c^2}{\left(\overline{\omega_1^2} + \overline{\omega_2^2}\right)\tau_c}$$

Hamiltonian from Spin Connection

$$i\partial_0\psi = H\psi$$
$$i\hat{\gamma}^\mu(x)D_\mu\psi = m\psi$$
$$D_\mu\psi = (\partial_\mu + \Gamma_\mu - ieA_\mu)\,\psi$$
$$\Gamma_\mu = -\tfrac{1}{2}\omega_{\mu ab}S^{ab}$$
$$S^{ab} = \frac{i}{2}\sigma^{ab}, \quad \sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$$
$$\omega_\mu{}^a{}_b = e_\nu{}^a e^\lambda{}_b \Gamma^\nu_{\mu\lambda} - e^\lambda{}_b\partial_\mu e_\lambda{}^a$$
$$\hat{\gamma}^\mu(x) = e^\mu{}_a(x)\gamma^a$$
$$e_\mu{}^a(x) = \begin{pmatrix} 1 & v_x & v_y & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$v_x \equiv -\omega y, \;\; v_y \equiv \omega x$$

back

Response Theory

$$\frac{d}{dt}\langle A\rangle_t = \frac{d}{dt}\mathrm{Tr}\left(\rho(t)A_{\mathrm{S}}\right)$$

$$= \frac{d}{dt}\mathrm{Tr}\left(\rho_0 U^\dagger(t)A_{\mathrm{S}}\,U(t)\right)$$

$$\frac{d}{dt}\left[\rho_0 U^\dagger(t)A_{\mathrm{S}}\,U(t)\right] = \frac{d}{dt}\left[\rho_0 U_{\mathrm{I}}^\dagger(t)A_{\mathrm{I}}(t)\,U_{\mathrm{I}}(t)\right]$$

$$= \rho_0\left[U_{\mathrm{I}}^\dagger\frac{dA_{\mathrm{I}}}{dt}U_{\mathrm{I}} + \frac{dU_{\mathrm{I}}^\dagger}{dt}A_{\mathrm{I}}U_{\mathrm{I}} + U_{\mathrm{I}}^\dagger A_{\mathrm{I}}\frac{dU_{\mathrm{I}}}{dt}\right]$$

$$\mathrm{Tr}\left(\rho_0 U_{\mathrm{I}}^\dagger(t)\frac{dA_{\mathrm{I}}}{dt}U_{\mathrm{I}}(t)\right) = i\mathrm{Tr}\left(\rho_0 U_{\mathrm{I}}^\dagger(t)[H_0, A_{\mathrm{I}}(t)]U_{\mathrm{I}}(t)\right)$$

$$= i\mathrm{Tr}\left(\rho_{\mathrm{I}}(t)[H_0, A_{\mathrm{I}}(t)]\right)$$

$$= i\mathrm{Tr}\left(\rho(t)[H_0, A_{\mathrm{S}}]\right) = i\langle[H_0, A_{\mathrm{S}}]\rangle_t$$

$$\mathrm{Tr}\left(\rho_0\left[\frac{dU_{\mathrm{I}}^\dagger}{dt}A_{\mathrm{I}}U_{\mathrm{I}} + U_{\mathrm{I}}^\dagger A_{\mathrm{I}}\frac{dU_{\mathrm{I}}}{dt}\right]\right) = i\rho_0\left[V_{\mathrm{I}}(t), A_{\mathrm{I}}(t)\right] - \rho_0\int_0^t dt'[V_{\mathrm{I}}(t'), [V_{\mathrm{I}}(t), A_{\mathrm{I}}(t)]]$$

back

Massive Dirac Particle

$$\frac{d}{dt}\begin{pmatrix}\langle\Sigma_3(t)\rangle\\ \langle\alpha_3(t)\rangle\\ \langle i\beta(t)\alpha_3(t)\rangle\\ \langle\beta(t)\rangle\end{pmatrix}=\begin{pmatrix}-h_0 & -h_2 & h_1 & 0\\ -h_2 & -h_0 & 2m & -h_3\\ h_1 & -2m & -h_0 & (2p+h_4)\\ 0 & 0 & -2p & 0\end{pmatrix}\begin{pmatrix}\langle\Sigma_3(t)\rangle\\ \langle\alpha_3(t)\rangle\\ \langle i\beta(t)\alpha_3(t)\rangle\\ \langle\beta(t)\rangle\end{pmatrix}$$

$$h_0=\left[\frac{p^2}{2E^2}\left(T_-+T_+\right)+\frac{m^2}{E^2}T_0\right]\overline{\omega_\perp^2}$$

$$h_1=\frac{\tau_c mp}{2E^2}\left[2\omega_0 T_0+(2E-\omega_0)T_- -(2E+\omega_0)T_+\right]\overline{\omega_\perp^2}$$

$$h_2=\frac{p}{2E}\left(T_- - T_+\right)\overline{\omega_\perp^2}$$

$$h_3=\frac{mp}{2E^2}\left(T_- + T_+ - 2T_0\right)\overline{\omega_\perp^2}$$

$$h_4=\frac{\tau_c p}{2E}\left[(2E-\omega_0)T_- +(2E+\omega_0)T_+\right]\overline{\omega_\perp^2}$$

$$\overline{\omega_\perp^2}=\overline{\omega_1^2}+\overline{\omega_2^2}$$

$$T_\pm=\frac{\tau_c}{1+(2E\pm\omega_0)^2\tau_c^2}$$

$$T_0=\frac{\tau_c}{1+\omega_0^2\tau_c^2}$$