Off-diagonal correlators of conserved charges from lattice QCD, and how to relate them to experiment*

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March 3, 2020

The 36th Winter Workshop on Nuclear Dynamics, Puerto Vallarta, Mexico

*Based on: **R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506** (2020)

Collaborators:

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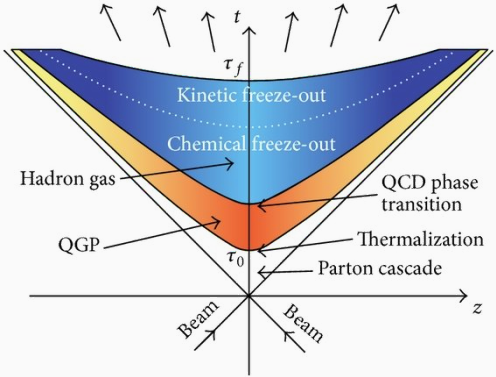
Introduction - Freeze-out

The stages of a heavy-ion collision (HIC)

- **Thermalization**: after a short time τ_0 the system thermalizes to a QGP (if the energy density is sufficient)
- **Hadronization**: when the system reaches T_C, hadrons are formed
- **Chemical freeze-out**: all inelastic collision cease and chemical composition is fixed (**yields, fluctuations**)
- **Kinetic freeze-out**: elastic collisions cease and spectra are fixed → free streaming to the detectors



Hui Wang's PhD thesis [Wang:2012jua]

Introduction: Fluctuations of conserved charges

- **Theory**
  Fluctuations are defined as the susceptibilities of the QCD pressure:

$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \frac{\partial^{i+j+k} P\left(T,\mu_B,\mu_Q,\mu_S\right)/T^4}{\partial\left(\mu_B/T\right)^i \partial\left(\mu_Q/T\right)^j \partial\left(\mu_S/T\right)^k}$$

- **Experiment**
  We can measure the moments/cumulants of net-particle distributions:

mean:	$M = \chi_1$	variance:	$\sigma^2 = \chi_2$
skewness:	$S = \chi_3/\left(\chi_2\right)^{3/2}$	kurtosis:	$\kappa = \chi_4/\left(\chi_2\right)^2$

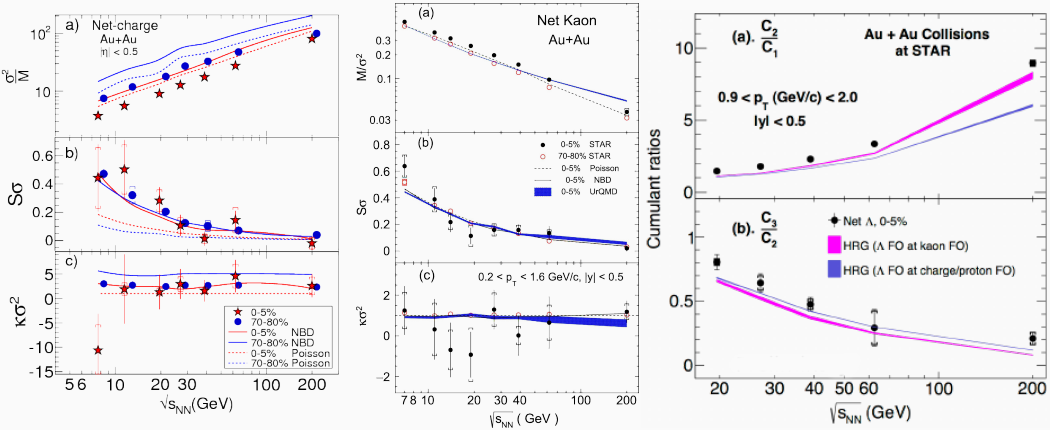
Both in theory and experiment, volume-independent ratios are often used:

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma = \chi_3/\chi_2$$

$$S\sigma^3/M = \chi_3/\chi_1 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

Introduction: Fluctuations of conserved charges - Experiment

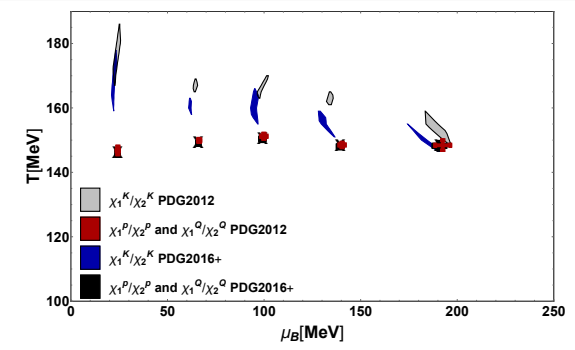
Event-by-event net-particle distributions allow to measure different cumulants (and ratios thereof):

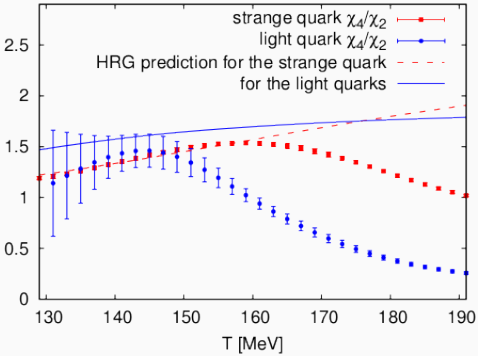


STAR: Phys. Rev.Lett. 113 92301 (2014); Phys. Lett. B 785 551 (2018); arXiv: 2001.06419 [nucl-ex]

Introduction: Fluctuations of conserved charges - Theory

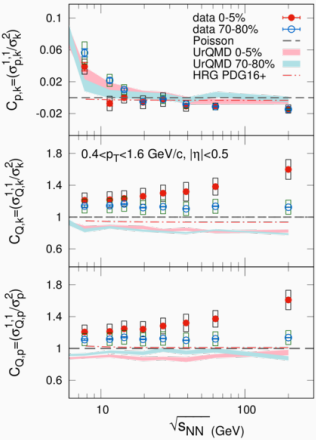
- Fluctuations have been largely utilized to study the freeze-out in HIC
- Lattice QCD calculations provide information on the relevant degrees of freedom near the QCD transition





R. Bellwied, PP *et al.*, Phys.Rev.C 99 034912 (2019); P. Alba, PP *et al.*, arXiv: 2002.12395 [hep-ph]; R. Bellwied *et al.*, Phys.Rev.Lett. 111 202302 (2013)

Introduction: Off-diagonal fluctuations of conserved charges



STAR: Phys.Rev.C 100 014902 (2019)

- The measurable species in HIC are only a handful. How much do they tell us about the **correlation between conserved charges**?
- Historically, the proxies for B, Q and S have been p, p, π, K and K themselves $\rightarrow$ **what about off-diagonal correlators?**

In this talk:

- **Main contributions to χ_{ijk}^{BQS}?**
- **Find a way (proxy) to compare *directly* lattice and experiment**
- **Provide results from lattice QCD at finite (real) μ_B**

Off-diagonal correlators: hadron resonance gas model

Can we compare lattice to experiment? What are the obstacles?

A number of effects cannot be taken into account on the lattice

- Resonance decays feed-down into lighter hadrons
- Kinematic cuts on particle measurements
- Correlations of $BQS \neq$ correlations of hadrons
- Many particles just cannot be detected!

$\longrightarrow$ **Analysis based on the hadron resonance gas (HRG) model where we can include and study these effects**

Off-diagonal correlators: hadron resonance gas model

Simple formulation, ideal gas of *all* hadronic resonances[1]. The pressure reads:

$$\frac{P\left(T,\mu_B,\mu_Q,\mu_S\right)}{T^4} = \sum_R \frac{\left(-1\right)^{B_R+1} d_R}{2\pi^2 T^3} \int_0^\infty dp\, p^2 \log\left[1+\left(-1\right)^{B_R+1} \exp\left(-\sqrt{p^2+m_R^2}/T + \mu_R/T\right)\right]$$

with:

$$\mu_R = \mu_B B_R + \mu_Q Q_R + \mu_S S_R$$

Susceptibilities in the HRG simply read:

$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \sum_R B_R^i\, Q_R^j\, S_R^k\, I_{i+j+k}^R(T,\mu_B,\mu_Q,\mu_S)$$

where:

$$I_{i+j+k}^R(T,\mu_B,\mu_Q,\mu_S) = \frac{\partial^{i+j+k} P_R/T^4}{\partial\left(\mu_R/T\right)^{i+j+k}}$$

[1] we use the list PDG2016+ from **P. Alba, PP *et al.*, Phys.Rev.D 96 034517 (2017)**

Off-diagonal correlators: hadron resonance gas model

We have the advantage we can include experimental effects:

- Include **resonance decay** feed-down, considering (strongly) stable hadrons:

$$\sum_R B_R\, Q_R\, S_R \longrightarrow \sum_{\mathrm{i}\in\mathrm{stable}} \sum_R P_{R\to i}\, B_i\, Q_i\, S_i$$

  where $P_{R\to i} = \mathrm{BR}_{R\to i}\, n_i^R$ is the average number of particles i produced by a particle R

- Include **acceptance cuts** on the kinematics of measured particles:

$$p_T^{\mathrm{m}} \leq p_T \leq p_T^{\mathrm{M}} \qquad\qquad |y| < y^* \quad (\text{or } |\eta| < \eta^*)$$

- Impose **strangeness neutrality** at finite chemical potential:

$$\langle n_S \rangle = 0 \qquad\qquad \langle n_Q \rangle = 0.4\, \langle n_B \rangle$$

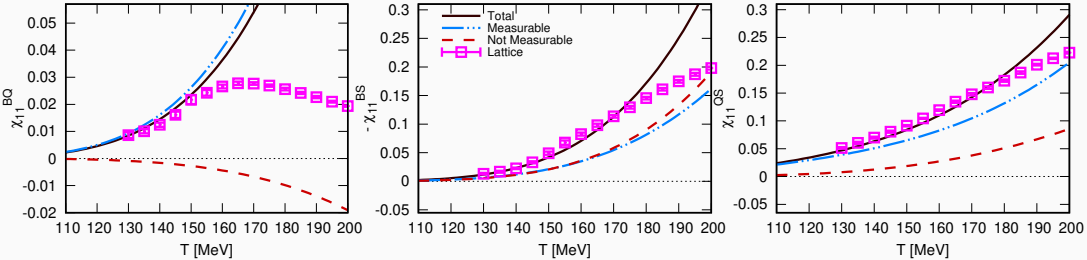
- The species that are stable <u>under strong interactions</u>, AND are **measurable**

$$\boxed{\pi^\pm,\ K^\pm,\ p\,(\overline{p})\,,\ \Lambda(\overline{\Lambda}),\ \Xi^-(\overline{\Xi}^+),\ \Omega^-(\overline{\Omega}^+)}$$

$\longrightarrow$ we inevitably lose a good chunk of conserved charges!

Off-diagonal correlators: measurable vs non-measurable

- Thanks to the **separation between observable and non-observables species**, one can pinpoint what can be measured and what cannot of χ_{ijk}^{BQS}

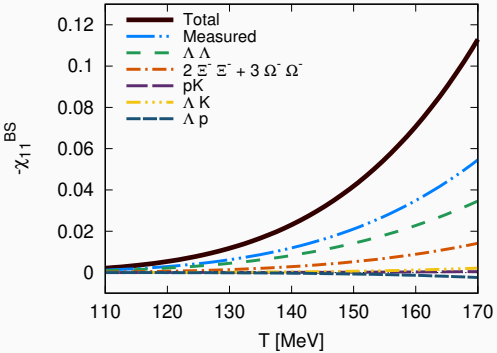


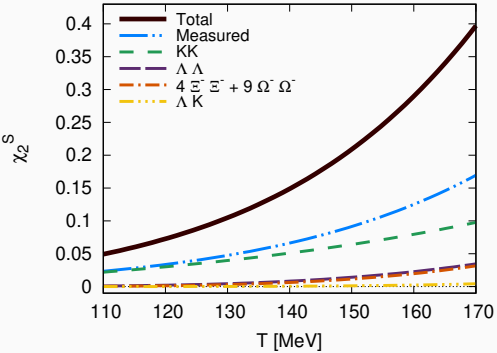
- For the **proton- and kaon-dominated** χ_{BQ} **and** χ_{QS}, a large part of the full correlator is carried by measurable particles
- χ_{BS} is less transparent, and requires careful analysis of its contributions

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Measured contributions: a breakdown of χ_{11}^{BS} and χ_2^S

- Each 2-particle correlation can be isolated in the HRG model
- In light of studying the ratio χ_{11}^{BS}/χ_2^S, we consider χ_{11}^{BS} and χ_2^S





- Different-particle correlations are negligible throughout, while the contribution from multi-strange baryons is sizable

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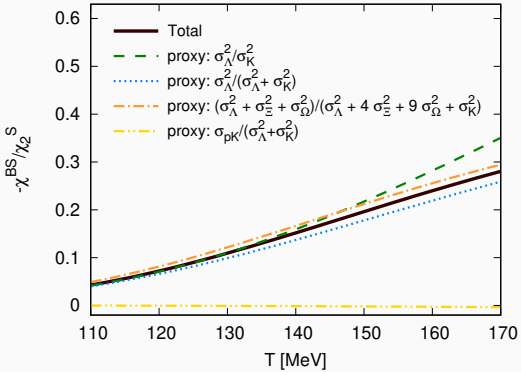
Hadronic proxies

Constructing a proxy not a trivial task: consider main contributions to numerator and denominator



- Good proxy for χ_{11}^{BS}/χ_2^S: $\widetilde{C}_{BS,SS}^{\Lambda,\Lambda K} = \sigma_\Lambda^2/(\sigma_K^2+\sigma_\Lambda^2)$

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Hadronic proxies

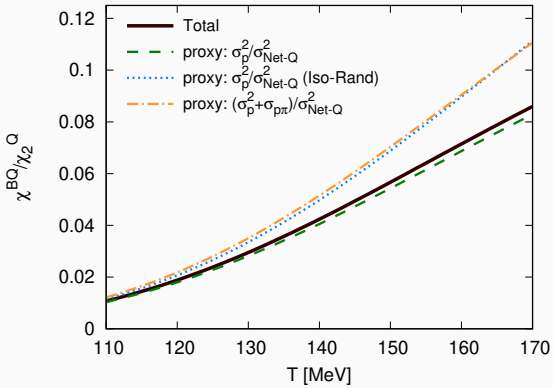
- The case of χ_{11}^{BQ}: a perfect proxy when GCE is considered

$$\sigma_p^2/\sigma_{\mathrm{Net-Q}}^2$$

- Consider isospin randomization from reactions of the like (**Kitazawa, Asakawa, Phys.Rev. C85 021901 (2012), Phys.Rev. C86 024904 (2012)**):

$$p + \pi^0 \longleftrightarrow \Delta^+ \longleftrightarrow n + \pi^+$$

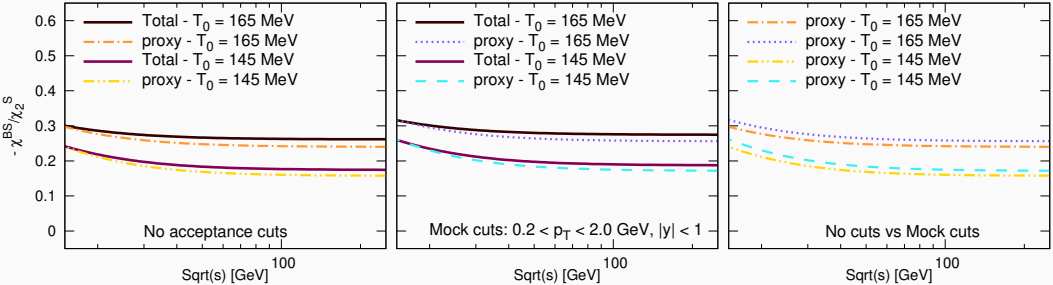
- Huge impact on measured net-p and net-π distributions makes the proxy unusable



- Even if not for BQ, we have a good proxy for BS and QS. Are we done?
- **No!** We need to check impact of:
  - Cuts on the kinematics of measured particles
  - Non-zero baryon chemical potential

Hadronic proxies: finite μ_B and kinematic cuts

- Consider our proxy along parametrized freeze-out lines with different $T(\mu_B = 0)$
- We look the ratio χ_{11}^{BS}/χ_2^S, in the case:
  - With no acceptance cuts
  - With "mock" cuts: $0.2 \leq p_T \leq 2.0\,\text{GeV}$, $|y| \leq 1.0$



- The proxy works well at finite μ_B, and **the effect of cuts is minimal!**

Note: when taking these ratios, **the same cuts** were applied to all species involved

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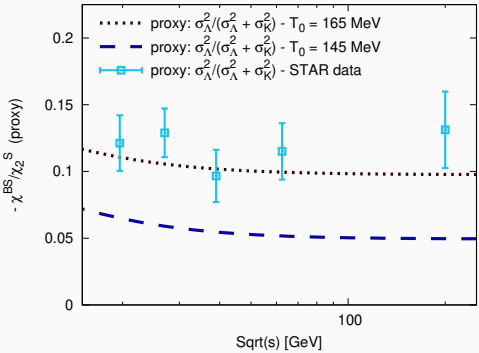
Hadronic proxies: a final comparison

- Compare to STAR data with the same cuts as in the experiment:

$$\Lambda: \quad 0.9 < p_T < 2.0\,\text{GeV} \quad |y| < 0.5$$
$$\text{K}: \quad 0.4 < p_T < 1.6\,\text{GeV} \quad |y| < 0.5$$

- A comparison along the same freeze-out lines as before shows a preferred $T(\mu_B = 0) \sim 165\,\text{MeV}$
- **Note:** a factor ~ 3 separates the case with same and different cuts! (see previous slide)



Crucial to have same cuts if comparing with lattice results

STAR: Phys. Lett. B 785, 551 (2018); arXiv: 2001.06419 [nucl-ex]
R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Hadronic proxies: wrap up

- Net-proton and net-kaon dominate χ_{11}^{BQ} and χ_{11}^{QS}, while χ_{11}^{BS} is less obvious
- **Main contributions** to χ_{11}^{BS} come from **net-Λ and multi-strange baryons**, and d**ifferent-particle correlations are negligible**
- We could not find a good proxy for $\chi_{11}^{BQ}/\chi_{2}^{Q}$ due to isospin randomization, however...
- ...**We found a good proxy** for the ratio $\chi_{11}^{BS}/\chi_{2}^{S}$ in:

$$\widetilde{C}_{BS,SS}^{\Lambda,\Lambda K} = \sigma_{\Lambda}^{2}/(\sigma_{K}^{2} + \sigma_{\Lambda}^{2})$$

- We found **its dependence on kinematic cuts to be small**
- A comparison of HRG to experiment hints at high chemical FO temperature

$\longrightarrow$ **We want to calculate this ratio on the lattice**

Off-diagonal correlators from lattice QCD at $\mu_B = 0$

- On the lattice we have access to quark number susceptibilities, then we can use the relationship between chemical potentials to connect to B, Q, S fluctuations:

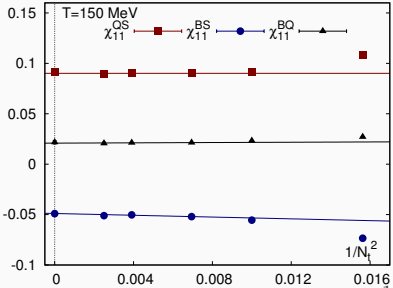
$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \qquad \chi_{11}^{BQ} = \frac{1}{9}\left[\chi_2^u - \chi_2^s - \chi_{11}^{us} + \chi_{11}^{ud}\right]$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \qquad \chi_{11}^{BS} = -\frac{1}{3}\left[\chi_2^s + 2\chi_{11}^{us}\right]$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \qquad \chi_{11}^{QS} = \frac{1}{3}\left[\chi_2^s - \chi_{11}^{us}\right]$$

- We extract results from a system of $N_f = 2+1+1$ flavors ($m_u = m_d \neq m_s \neq m_c$) with physical quark masses

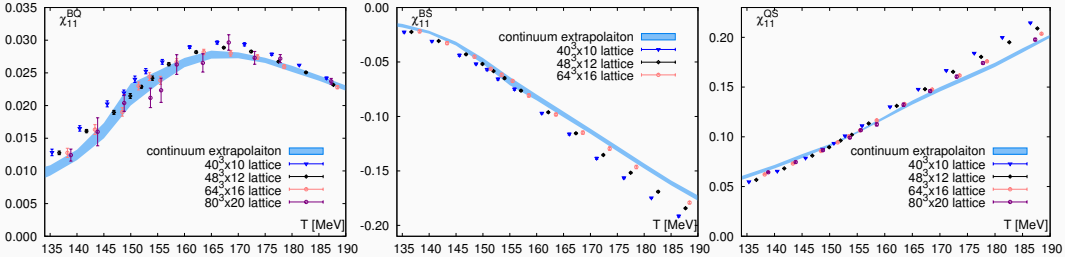
- We perform a continuum extrapolation from $40^3 \times 10$, $48^3 \times 12$, $64^3 \times 16$ and $80^3 \times 20$ lattices



R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Off-diagonal correlators from lattice QCD at $\mu_B = 0$

- Continuum extrapolated results in the range $T = 130 - 190\,\mathrm{MeV}$



Remember: due to the sign problem, lattice QCD cannot access real μ_B *directly*

- Extrapolate to real μ_B: we consider two methods
  - **Taylor method**: based on higher order derivatives at $\mu_B = 0$
  - **Sectors method**: based on decomposition of QCD pressure
- **Both methods** make use of simulations at imaginary chemical potential

Off-diagonal correlators: extrapolation to real μ_B

- **Taylor method:** expand around $\mu_B = 0$

$$\left.\frac{\chi_{11}^{BS}}{\chi_2^S}\right|_{\mu_B/T} = \frac{\chi_{11}^{BS}}{\chi_2^S} + \frac{\hat{\mu}_B^2}{2}\frac{\chi_{11}^{BS,(NLO)}\chi_2^S - \chi_2^{S,(NLO)}\chi_{11}^{BS}}{(\chi_2^S)^2} + \mathcal{O}(\hat{\mu}_B^4)$$

with

$$\begin{aligned}
\chi_{11}^{BS,(NLO)} &= \chi_{13}^{BS}s_1^2 + 2\chi_{22}^{BS}s_1 + \chi_{31}^{BS} \\
\chi_2^{S,(NLO)} &= \chi_{22}^{BS} + 2\chi_{13}^{BS}s_1 + \chi_4^S s_1^2 \\
s_1 &= -\chi_{11}^{BS}/\chi_2^S
\end{aligned}$$

NOTE:

- all derivatives on the RHS are taken at $\mu_B = \mu_S = 0$, but
- they are obtained from ensembles at imaginary chemical potential (same as in **S. Borsányi *et al.* JHEP 1810 205 (2018)**)

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Off-diagonal correlators: extrapolation to real μ_B

- **Sectors method:** We can separate particles in families by quantum numbers:

$$P(T,\, \hat{\mu}_B,\, \hat{\mu}_Q,\, \hat{\mu}_S) = \sum_{i,j,k} P_{ijk}^{BQS}(T)\, \cosh(i\,\hat{\mu}_B + j\,\hat{\mu}_Q - k\,\hat{\mu}_S)$$

If we consider only contributions from known hadrons (leading order):

$$\begin{aligned} P_{\rm LO}(T,\, \hat{\mu}_B,\, \hat{\mu}_Q,\, \hat{\mu}_S) &= P_{00}^{BS} + P_{10}^{BS}\cosh(\,\hat{\mu}_B) + P_{01}^{BS}\cosh(\,\hat{\mu}_S) + P_{11}^{BS}\cosh(\,\hat{\mu}_B - \,\hat{\mu}_S) + \\ &+ P_{12}^{BS}\cosh(\,\hat{\mu}_B - 2\,\hat{\mu}_S) + P_{13}^{BS}\cosh(\,\hat{\mu}_B - 3\,\hat{\mu}_S) \end{aligned}$$

which correspond to (for simplicity, we have dropped the dependence on μ_Q):

$$\begin{aligned} P_{00}^{BS} &\longmapsto \pi, \eta, f, \rho, ... & P_{01}^{BS} &\longmapsto K, ... \\ P_{10}^{BS} &\longmapsto p, \Delta, ... & P_{11}^{BS} &\longmapsto \Lambda, \Sigma, ... \\ P_{12}^{BS} &\longmapsto \Xi, ... & P_{13}^{BS} &\longmapsto \Omega, ... \end{aligned}$$

Note: contributions from interactions also fall into these sectors (e.g., K-Λ counted in P_{12}^{BS})

Off-diagonal correlators: extrapolation to real μ_B

When working with imaginary chemical potential:

$$P(T,\, \hat{\mu}_B,\, \hat{\mu}_Q,\, \hat{\mu}_S) = \sum_{i,k} P_{ik}^{BQS}(T)\, \cos(i\, \hat{\mu}_B^I - k\, \hat{\mu}_S^I)$$

the partial pressures become Fourier coefficients.

- In previous work[2] we used this decomposition to calculate the partial pressures P_{ik}^{BS} and study the hadron spectrum
- We fixed $\mu_B = 0, \mu_S = i\mu_S^I$ and calculated:

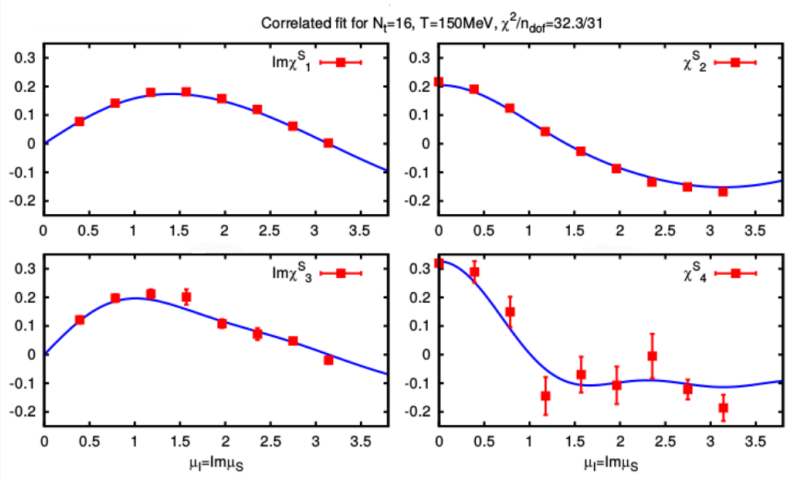
$$\begin{aligned}
\mathrm{Im}\chi_1^B &= -P_{11}^{BS}\sin(\mu_I) - P_{12}^{BS}\sin(2\mu_I) - P_{13}^{BS}\sin(3\mu_I) \\
\chi_2^B &= P_{10}^{BS} + P_{11}^{BS}\cos(\mu_I) + P_{12}^{BS}\cos(2\mu_I) + P_{13}^{BS}\cos(3\mu_I) \\
\mathrm{Im}\chi_1^S &= \left(P_{01}^{BS} + P_{11}^{BS}\right)\sin(\mu_I) + 2P_{12}^{BS}\sin(2\mu_I) + 3P_{13}^{BS}\sin(3\mu_I) \\
\chi_2^S &= \left(P_{01}^{BS} + P_{11}^{BS}\right)\cos(\mu_I) + 4P_{12}^{BS}\cos(2\mu_I) + 9P_{13}^{BS}\cos(3\mu_I)
\end{aligned}$$

and so on...

[2] **P. Alba, PP *et al.*, Phys.Rev.D 96 034517 (2017)**

Off-diagonal correlators: extrapolation to real μ_B

A combined fit of data at different μ_S^I gave the partial pressures



P. Alba, PP *et al.*, Phys.Rev.D 96 034517 (2017)

Off-diagonal correlators: extrapolation to real μ_B

In this work, we added a NLO:

$$\begin{aligned} P_{\rm NLO}(T,\,\hat{\mu}_B,\,\hat{\mu}_Q,\,\hat{\mu}_S) &= P_{02}^{BS}\cosh(2\,\hat{\mu}_S) + P_{1,-1}^{BS}\cosh(\,\hat{\mu}_B + \,\hat{\mu}_S) + \\ &+ P_{21}^{BS}\cosh(2\,\hat{\mu}_B - \,\hat{\mu}_S) + P_{22}^{BS}\cosh(2\,\hat{\mu}_B - 2\,\hat{\mu}_S) + \cdots \end{aligned}$$

where we included:

$$P_{02}^{BS} \longmapsto K\text{-}K, \text{tetraquark(e.g., } \bar{\text{u}}\text{s}\bar{\text{d}}\text{s)}, \ldots \qquad P_{1,-1}^{BS} \longmapsto \text{pentaquark(e.g., uudd}\bar{\text{s}}\text{)}, \ldots$$

$$P_{21}^{BS} \longmapsto p\text{-}\Lambda, \text{exaquark(e.g., uududs)}, \ldots \qquad P_{22}^{BS} \longmapsto p\text{-}\Xi, \text{exaquark(e.g., uuduss)}, \ldots$$

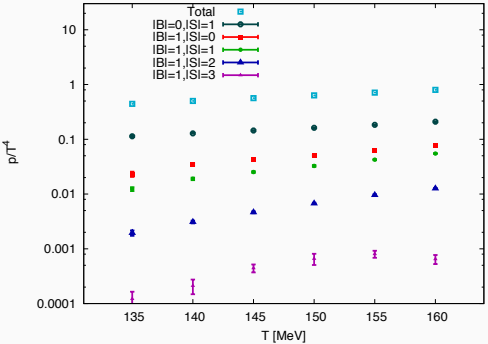
Note:

- the definition of what is NLO is somewhat ambiguous. We tried adding further sectors like P_{23}^{BS}, but it did not help improve the fit

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Off-diagonal correlators: extrapolation to real μ_B

- (Left) Continuum-extrapolated results for HRG model sector decomposition
- (Right) results on a $48^3 \times 12$ lattice for the NLO sectors



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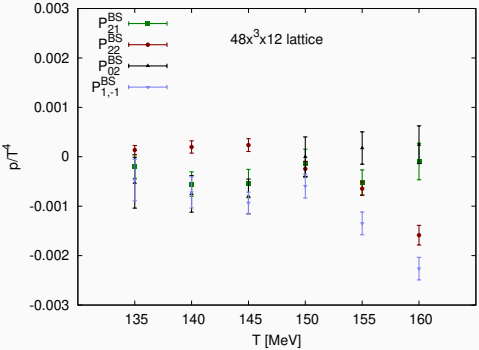
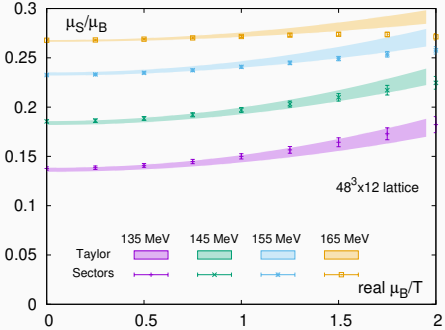


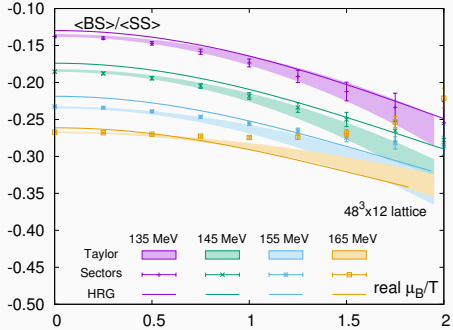
NLO

P. Alba, PP *et al.*, Phys.Rev.D 96 034517 (2017), R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Off-diagonal correlators: extrapolation to real μ_B

- We compare the ratio χ_{11}^{BS}/χ_2^S and the ratio μ_S/μ_B which realizes strangeness neutrality





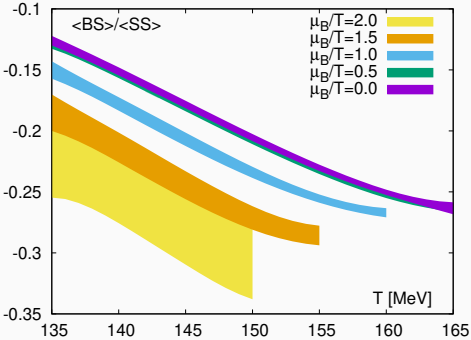
- We compare the two methods at finite (real) chemical potential on a $48^3 \times 12$ lattice
- Good agreement for up to $T \simeq 155\,\mathrm{MeV}$ (at high T the sectors method is bound to fail anyway)

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Off-diagonal correlators: extrapolation to real μ_B

Finally:

- As a final result, we perform a continuum extrapolation of the sectors method in the regime where the two methods agree
- We provide results for $\mu_B/T = 0 - 2.0$
- Uncertainties grow quickly with the chemical potential, but can be systematically improved



- This result can be compared to experimental results when they will become available
- Nonetheless, uncertainties at larger μ_B need to be improved for a proper comparison

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Conclusions

- Off-diagonal correlators offer yet another chance to study freeze-out in heavy-ion collisions
- **Main contributions to χ_{11}^{BS} come from net-Λ and multi-strange baryons**
- A very good proxy for the ratio χ_{11}^{BS}/χ_2^S was found to be

$$\widetilde{C}_{BS,SS}^{\Lambda,\Lambda K} = \sigma_\Lambda^2/(\sigma_K^2 + \sigma_\Lambda^2)$$

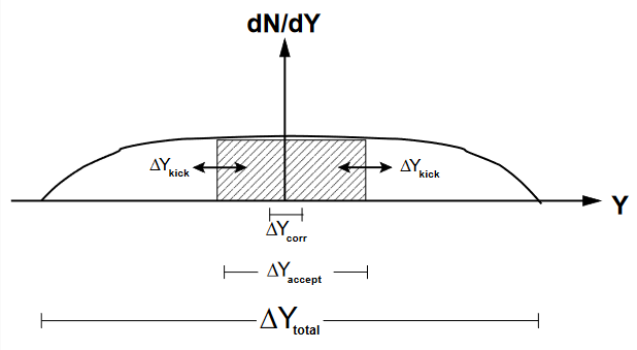
- From out continuum-extrapolated results at $\mu_B = 0$ and imaginary μ_B, we could **extrapolate to real chemical potential with two different methods**
- The two methods showed agreement in a fairly sizable range of T and μ_B/T, and in this range **we continuum-extrapolated the sectors method result**
- Although improvement is necessary at larger μ_B, these result could be already be compared to experiment

BACKUP

Fluctuations of conserved charges

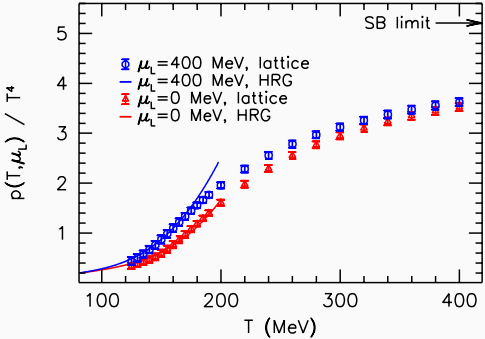
How can CONSERVED CHARGES fluctuate?

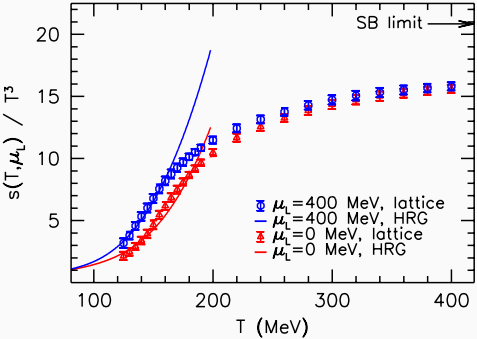
- If we could measure ALL particles in a collision, they would not
- If we look at a small enough subsystem, fluctuations occur and become meaningful



Hadron resonance gas model and lattice QCD

Many thermodynamic observables show an excellent agreement between the HRG model and lattice data in the hadronic phase

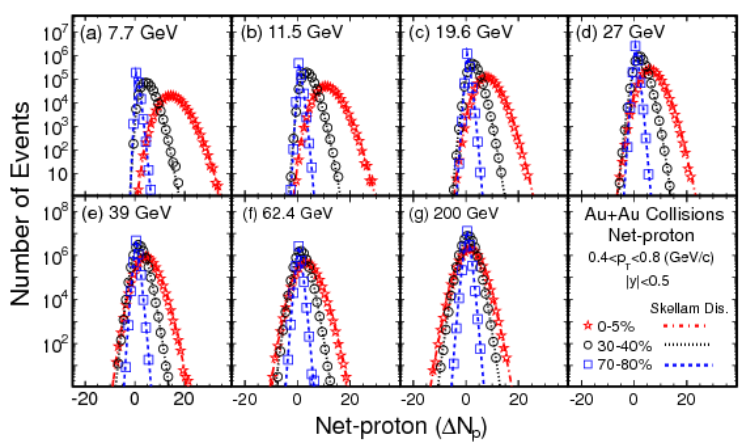




S. Borsanyi *et al.* JHEP 1208 053, (2012)

Fluctuations: event-by-event distributions

Due to limited acceptance and efficiency of detectors, conserved charges are conserved only on average. Possible to **measure the moments of the distribution**



STAR COllaboration: Phys.Rev.Lett. 112 032302 (2014)

Off-diagonal correlators: hadron resonance gas model

The species that are stable <u>under strong interactions</u>:

- π^0, π^+, π^-
- $K^+, K^-, K^0, \overline{K}^0$
- $p, \overline{p}, n, \overline{n}$
- $\Lambda, \overline{\Lambda}, \Sigma^+, \overline{\Sigma}^-, \Sigma^-, \overline{\Sigma}^+$
- $\Xi^-, \overline{\Xi}^+, \Xi^0, \overline{\Xi}^0$
- $\Omega^-, \overline{\Omega}^+$

Measurable

$\Longrightarrow$

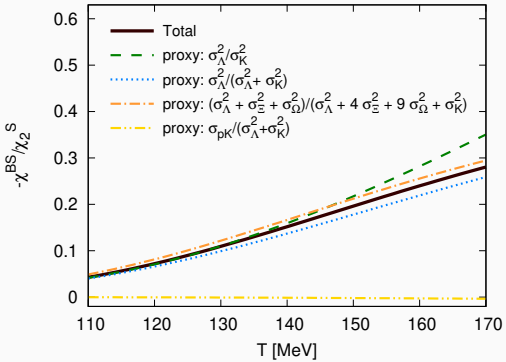
- π^+, π^-
- K^+, K^-
- $p, \overline{p}$
- $\Lambda, \overline{\Lambda}$
- $\Xi^-, \overline{\Xi}^+$
- $\Omega^-, \overline{\Omega}^+$

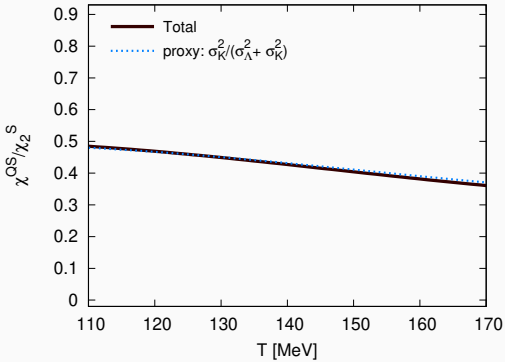
Define the net-particle number $\widetilde{A} = A - \overline{A}$. Then, we have:

- net-B: $\left(\widetilde{\mathbf{p}} + \widetilde{n} + \widetilde{\mathbf{\Lambda}} + \widetilde{\Sigma}^+ + \widetilde{\Sigma}^- + \widetilde{\mathbf{\Xi}}^- + \widetilde{\Xi}^0 + \widetilde{\mathbf{\Omega}}^-\right)$
- net-Q: $\left(\widetilde{\boldsymbol{\pi}}^+ + \widetilde{\mathbf{K}}^+ + \widetilde{\mathbf{p}} + \widetilde{\Sigma}^+ - \widetilde{\Sigma}^- - \widetilde{\mathbf{\Xi}}^- - \widetilde{\mathbf{\Omega}}^-\right)$
- net-S: $\left(\widetilde{\mathbf{K}}^+ + \widetilde{K}^0 - \widetilde{\mathbf{\Lambda}} - \widetilde{\Sigma}^+ - \widetilde{\Sigma}^- - 2\,\widetilde{\mathbf{\Xi}}^- - 2\,\widetilde{\Xi}^0 - 3\,\widetilde{\mathbf{\Omega}}^-\right)$

Hadronic proxies

Constructing a proxy not a trivial task: main contributions to numerator and denominator





- Good proxy for χ_{11}^{BS}/χ_2^S: $\widetilde{C}_{BS,SS}^{\Lambda,\Lambda K} = \sigma_\Lambda^2/(\sigma_K^2+\sigma_\Lambda^2)$
- Get χ_{11}^{QS}/χ_2^S for free: $\widetilde{C}_{QS,SS}^{K,\Lambda K} = \frac{1}{2}\sigma_K^2/(\sigma_\Lambda^2+\sigma_K^2)$

R. Bellwied, PP *et al.*, Phys.Rev.D 101 034506 (2020)

Strangeness neutrality and charge-baryon number ratio

- **Strangeness neutrality:** to meet experimental conditions, we have

$$\langle n_S \rangle = 0 \qquad\qquad \langle n_Q \rangle = \frac{Z}{A} \langle n_B \rangle$$

  where Z/A is the charge-to-baryon-number ratio for colliding nuclei

- This forces a relationship between chemical potentials:

$$\begin{aligned}
\frac{\mu_Q}{\mu_B} &= \frac{Z/A\left(\chi_2^B \chi_2^S - \chi_{11}^{BS}\chi_{11}^{BS}\right) - \left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)}{\left(\chi_2^Q \chi_2^S - \chi_{11}^{QS}\chi_{11}^{QS}\right) - Z/A\left(\chi_{11}^{BQ}\chi_2^S - \chi_{11}^{BS}\chi_{11}^{QS}\right)} \\
\frac{\mu_S}{\mu_B} &= -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\,\frac{\mu_Q}{\mu_B}
\end{aligned}$$

- In the HRG analysis, we strictly impose these conditions; in the lattice analysis, we use $Z/A = 0.5$ for simplicity (this results in $\mu_Q = 0$)

Off-diagonal correlators: extrapolation to real μ_B

Sector method: Take the HRG model in Boltzmann approximation:

$$\frac{P}{T^4} = \sum_R (-1)^{B_R+1} \frac{d_R}{2\pi^2 T^3} \int dk\, k^2 \log\left(1 + (-1)^{B_R+1} \exp\left[-\left(\epsilon_R - \mu_R\right)/T\right]\right) \simeq$$

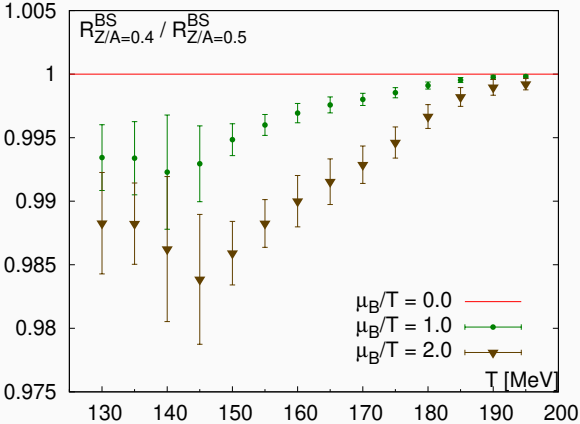
$$\simeq \sum_R \frac{d_R}{2\pi^2 T^3} \int dk\, k^2 e^{-(\epsilon_R - \mu_R)/T} \simeq \sum_R e^{\mu_R/T} \frac{d_R}{2\pi^2 T^3} \int dk\, k^2 e^{-\epsilon_R/T} = \sum_R e^{\mu_R/T} I_R(T)$$

where:

- the chemical potential $\mu_R = B_R \mu_B + Q_R \mu_Q + S_R \mu_S$
- the integral $I_R(T)$ does not depend on μ_R, and is the same for particle-antiparticle
- every particle-antiparticle pair leads to a term $\sim \cosh(\mu_R)$
- all particles with same quantum numbers (B, Q, S) have the same μ_R

Correction for non-zero μ_Q

We also checked for the correction induced by $Z/A = 0.5$ on a $48^3 \times 12$ lattice:



$\longrightarrow$ Well within the error bars in the whole range

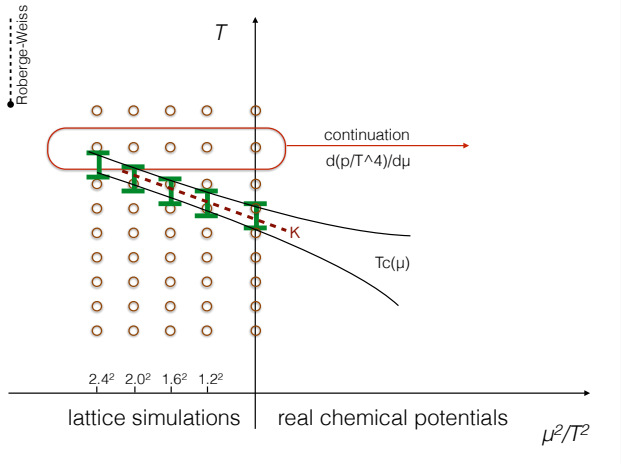
The sign problem

The QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\ e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U\ \det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U) e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

Analytic continuation



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- ...