

# Exploring the partonic phase at finite chemical potential in and out-of equilibrium

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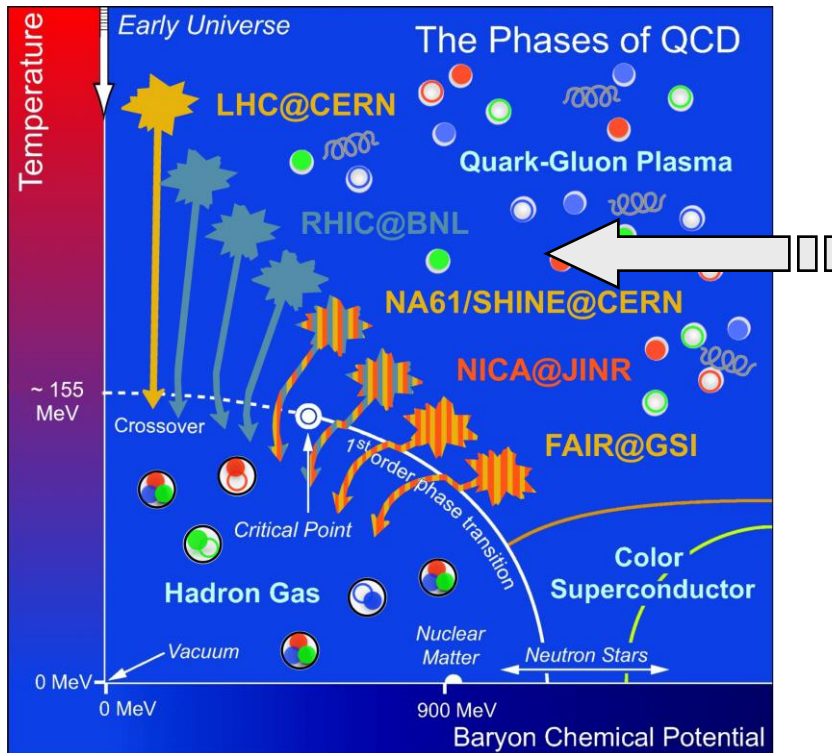
**Pierre Moreau, Olga Soloveva, Lucia Oliva,  
Taesoo Song, Wolfgang Cassing**



**The 36th Winter Workshop on Nuclear Dynamics  
Puerto Vallarta, Mexico, March 1-7, 2020**



# The ,holy grail' of heavy-ion physics:



## The phase diagram of QCD

- Study of the **phase transition** from hadronic to partonic matter – **Quark-Gluon-Plasma**
- Search for possible **critical point**
- Search for signatures of **chiral symmetry restoration**
- Study of the **in-medium properties** of hadrons at high baryon density and temperature

**Our goal:** to study the properties of strongly interacting matter created in heavy-ion collisions on a **microscopic basis**

**Theory:** QCD + many body theory + microscopic transport theory

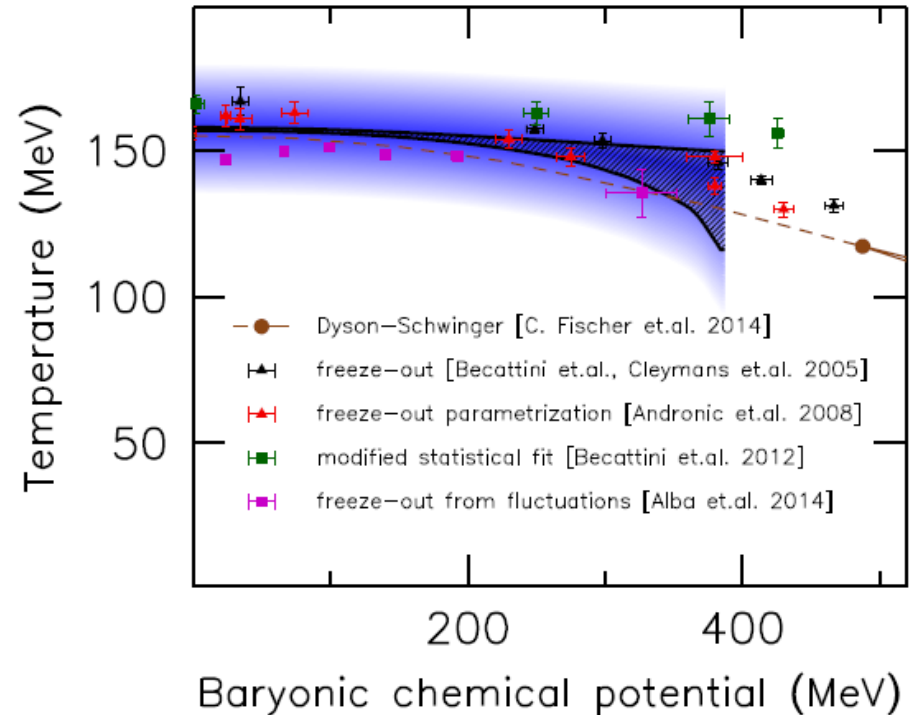
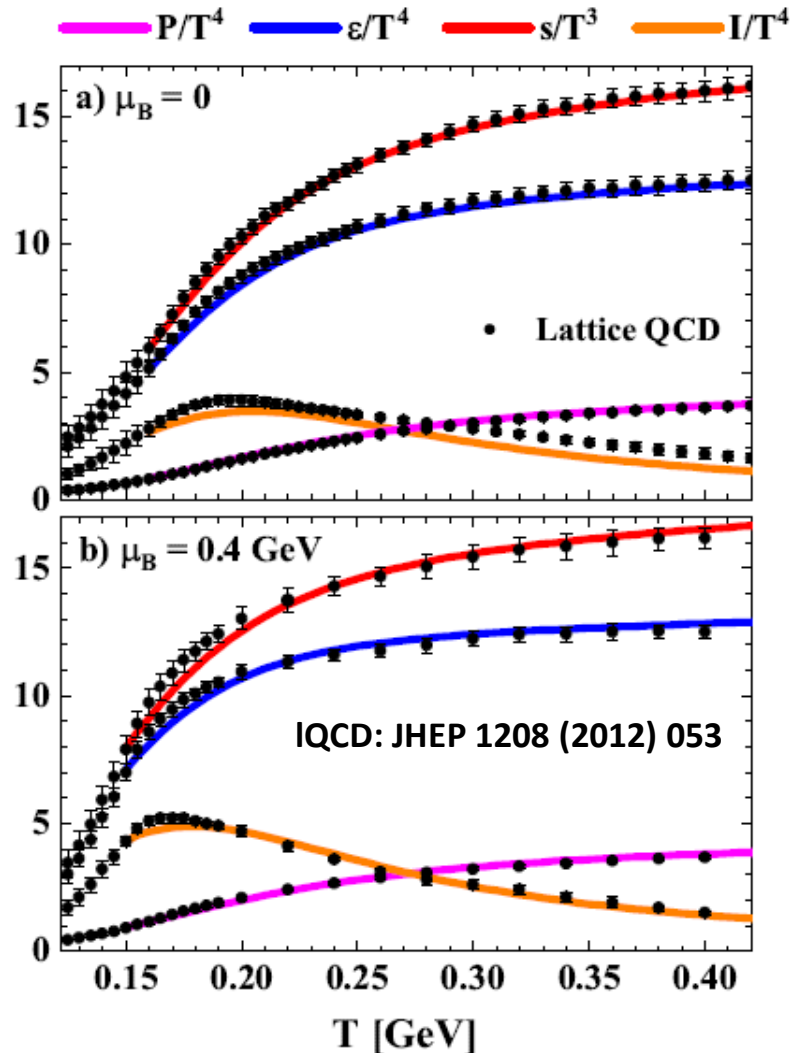
**Realization:** dynamical transport approach → **PHSD**



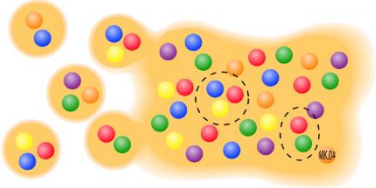
# Theory: lattice QCD data for $\mu_B = 0$ and finite $\mu_B > 0$

□ Deconfinement phase transition from hadron gas to QGP with increasing T and  $\mu_B$

IQCD: J. Guenther et al., Nucl. Phys. A 967 (2017) 720



→ Lattice QCD results: up to  $\mu_B < 400 \text{ MeV}$ :  
Crossover: hadron gas → QGP



# Degrees-of-freedom of QGP

For the microscopic transport description of the system one **needs to know all degrees of freedom** as well as their properties and interactions!

❖ IQCD gives QGP EoS at finite  $\mu_B$



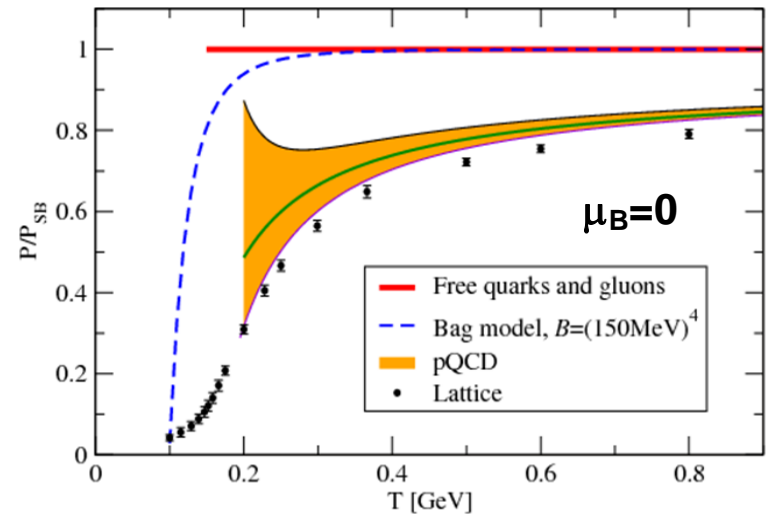
**! need to be interpreted in terms of degrees-of-freedom**

**pQCD:**

- weakly interacting system
- massless quarks and gluons

How to learn about the degrees-of-freedom of QGP from HIC?

- ➔ microscopic transport approaches
- ➔ comparison to HIC experiments

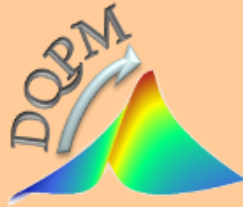


Non-perturbative QCD ← pQCD

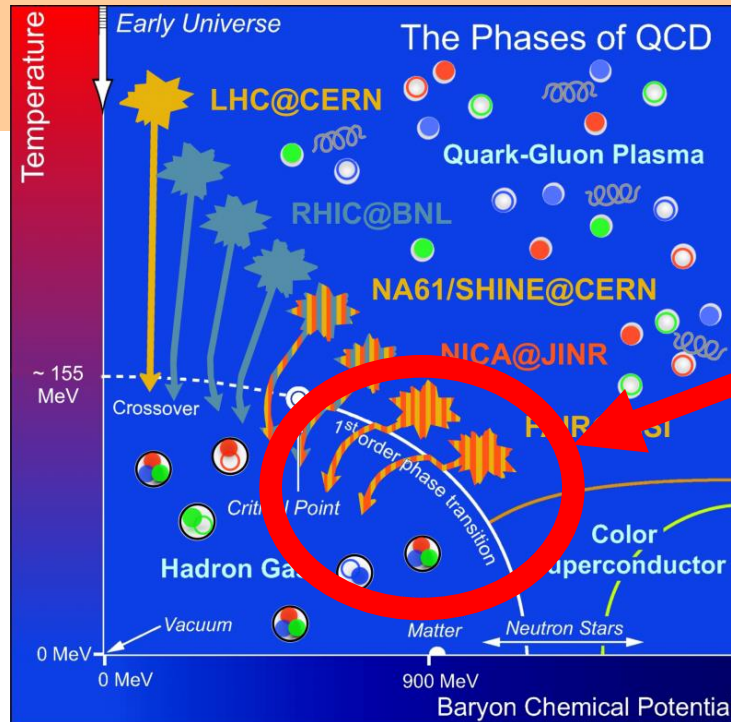
**Thermal QCD**

= QCD at high parton densities:

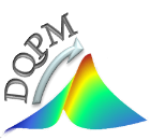
- strongly interacting system
- massive quarks and gluons
- ➔ quasiparticles
- = effective degrees-of-freedom



# DQPM ( $T, \mu_q$ )



**finite  $\mu_q$**



# Dynamical QuasiParticle Model (DQPM)

**DQPM** describes **QCD** properties in terms of **,resummed‘ single-particle Green’s functions** (propagators  $G^R$ ) – in the sense of a two-particle irreducible (2PI) approach:

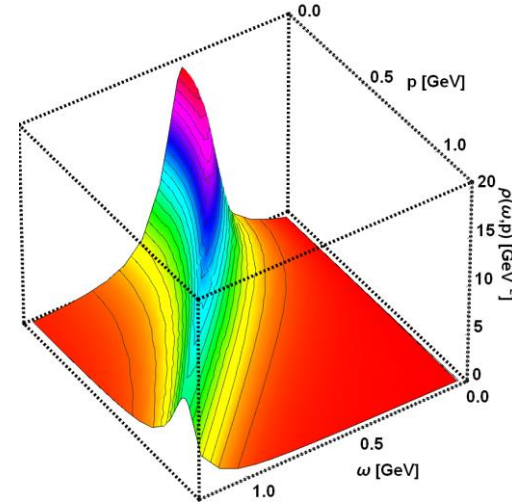
- **Degrees-of-freedom: interacting quasiparticles - quarks and gluons** with Lorentzian spectral functions:

$$\rho_j(\omega, \mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j} \left( \frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2} \right)$$

$$\equiv \frac{4\omega\gamma_j}{(\omega^2 - \mathbf{p}^2 - M_j^2)^2 + 4\gamma_j^2\omega^2}$$

$$\rho = -2 \text{Im } G^R$$

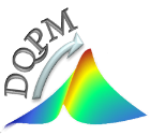
$$\tilde{E}_j^2(\mathbf{p}) = \mathbf{p}^2 + M_j^2 - \gamma_j^2$$



- **Resummed properties of the quasiparticles** are specified by scalar **complex self-energies**:

<p>gluon propagator: <math>\Delta^{-1} = P^2 - \Pi</math>    &amp;    quark propagator <math>S_q^{-1} = P^2 - \Sigma_q</math>  gluon self-energy: <math>\Pi = M_g^2 - i2\gamma_g\omega</math>    &amp;    quark self-energy: <math>\Sigma_q = M_q^2 - i2\gamma_q\omega</math></p>
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- **Real part of the self-energy: thermal mass** ( $M_g, M_q$ )
- **Imaginary part of the self-energy: interaction width** of partons ( $\gamma_g, \gamma_q$ )



# Parton properties

- Modeling of the quark/gluon **masses** and **widths** (inspired by HTL calculations)

## Masses:

$$M_{q(\bar{q})}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} g^2(T, \mu_B) \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

$$M_g^2(T, \mu_B) = \frac{g^2(T, \mu_B)}{6} \left( \left( N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

## Widths:

$$\gamma_{q(\bar{q})}(T, \mu_B) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

$$\gamma_g(T, \mu_B) = \frac{1}{3} N_c \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

→ **DQPM :**

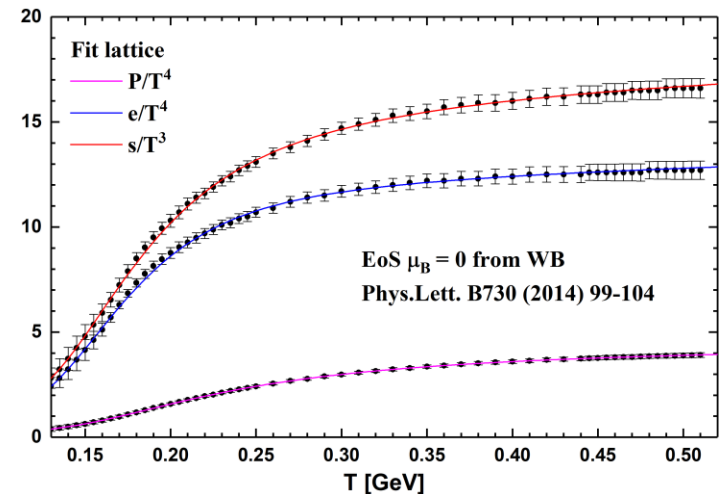
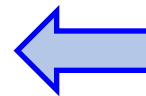
only **one parameter** ( $c = 14.4$ )  
+  $(T, \mu_B)$ - dependent **coupling constant** has to be determined from lattice results

- **Coupling:** input: IQCD **entropy density** as a function of temperature for  $\mu_B$

→ Fit to lattice data at  $\mu_B=0$  with

$$g^2(s/s_{SB}) = d \left( (s/s_{SB})^e - 1 \right)^f$$

$$s_{SB}^{QCD} = 19/9 \pi^2 T^3$$



# DQPM at finite $(T, \mu_q)$ : scaling hypothesis

- Scaling hypothesis for the effective temperature  $T^*$  for  $N_f = N_c = 3$

$$\mu_u = \mu_d = \mu_s = \mu_q$$

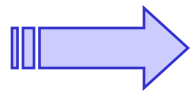
$$T^{*2} = T^2 + \frac{\mu_q^2}{\pi^2}$$

- Coupling:

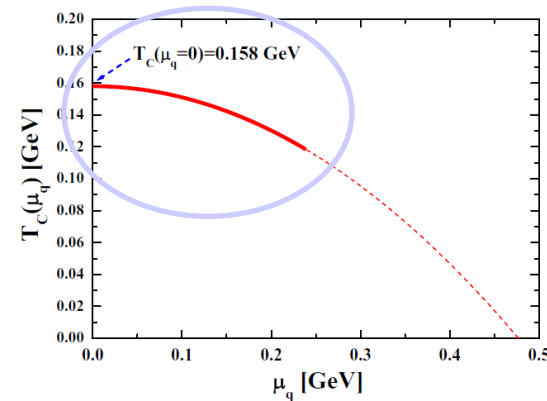
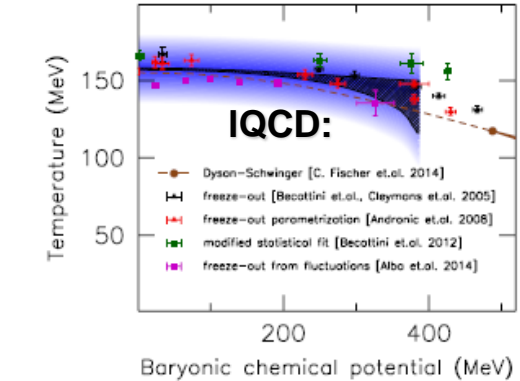
$$g(T/T_c(\mu=0)) \longrightarrow g(T^*/T_c(\mu))$$

- Critical temperature  $T_c(\mu_q)$ :

obtained by assuming a constant energy density  $\varepsilon$  for the system at  $T=T_c(\mu_q)$ , where  $\varepsilon$  at  $T_c(\mu_q=0)=156$  GeV is fixed by IQCD at  $\mu_q=0$



$$\frac{T_c(\mu_q)}{T_c(\mu_q=0)} = \sqrt{1 - \alpha \mu_q^2} \approx 1 - \alpha/2 \mu_q^2 + \dots$$



$$\alpha \approx 8.79 \text{ GeV}^{-2}$$

! Consistent with lattice QCD:

IQCD: C. Bonati et al., PRC90 (2014) 114025

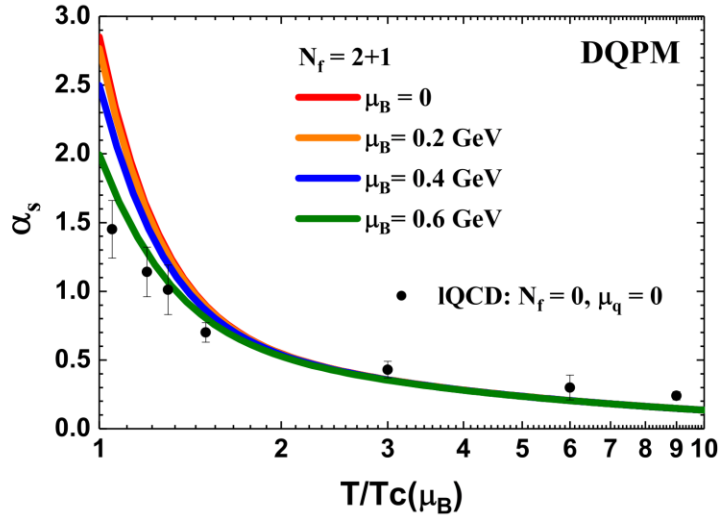
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left( \frac{\mu_B}{T_c} \right)^2 + \dots$$

$$\text{IQCD } \kappa = 0.013(2) \longleftrightarrow \kappa_{DQPM} \approx 0.0122$$

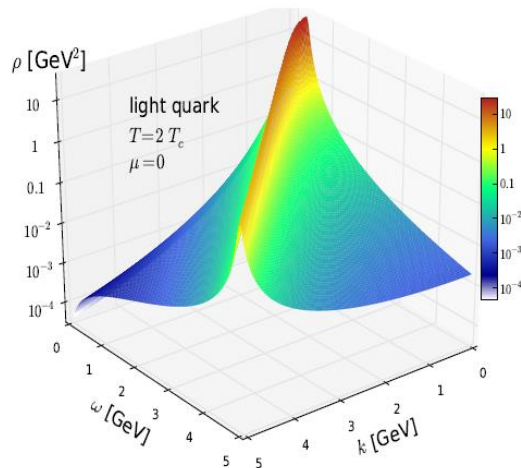
H. Berrehrah et al, PRC 93 (2016) 044914,  
Int.J.Mod.Phys. E25 (2016) 1642003,

# DQPM: parton properties

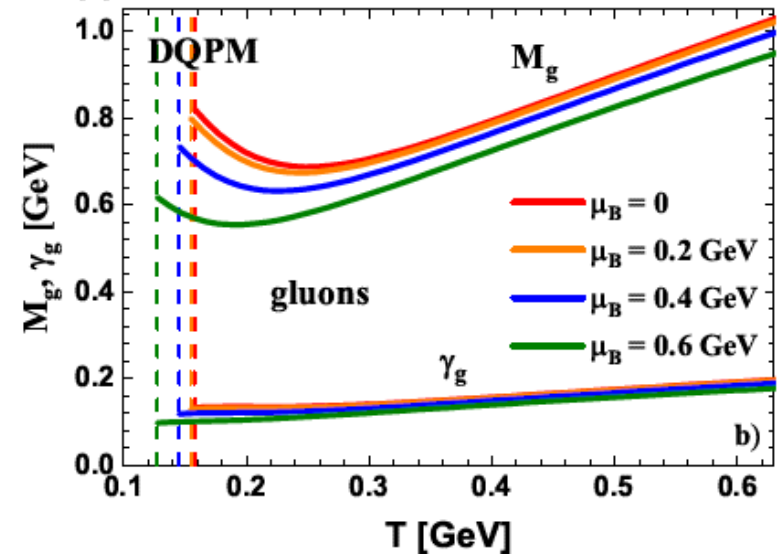
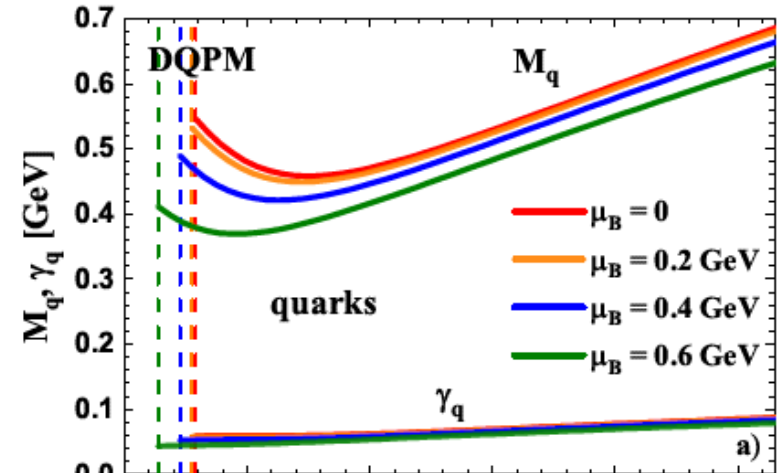
## □ Coupling as a function of $(T, \mu_B)$

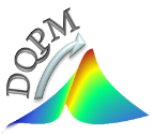


## → Lorentzian spectral function:



## □ Masses and widths as a function of $(T, \mu_B)$





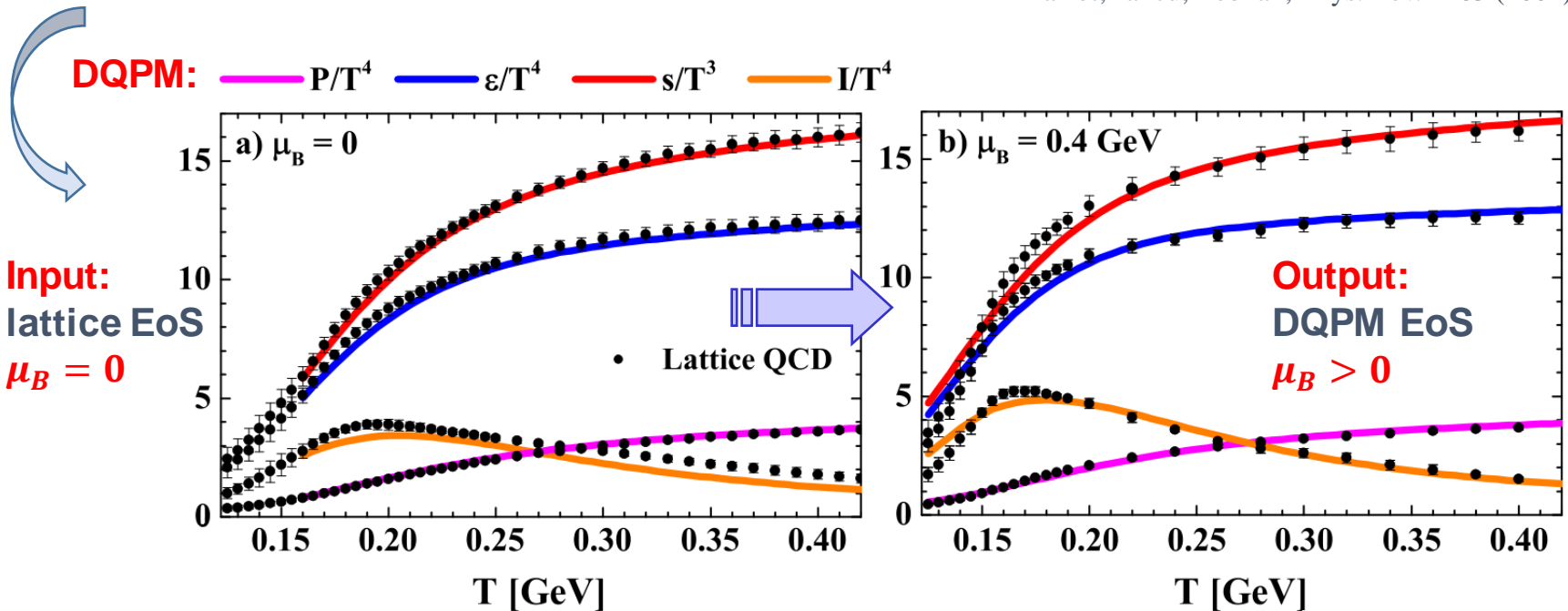
# DQPM thermodynamics at finite $(T, \mu_q)$

□ Entropy and baryon density in the quasiparticle limit (G. Baym 1998):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ d_g \frac{\partial n_B}{\partial T} (\text{Im}(\ln -\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta) \right. \\ \left. + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

Blaizot, Iancu, Rebhan, Phys. Rev. D 63 (2001) 065003



# DQPM: Mean-field potential for quasiparticles

Space-like part of energy-momentum tensor  $T_{\mu\nu}$  defines the **potential energy density**:

$$V_p(T, \mu_q) = T_{g-}^{00}(T, \mu_q) + T_{q-}^{00}(T, \mu_q) + T_{\bar{q}-}^{00}(T, \mu_q)$$

**space-like gluons**      **space-like quarks+antiquarks**

→ **mean-field scalar potential (1PI)** for quarks and gluons ( $U_q, U_g$ ) vs **scalar density**  $\rho_s$ :

$$U_s(\rho_s) = \frac{dV_p(\rho_s)}{d\rho_s}$$

$$U_q = U_s, \quad U_g \sim 2U_s$$

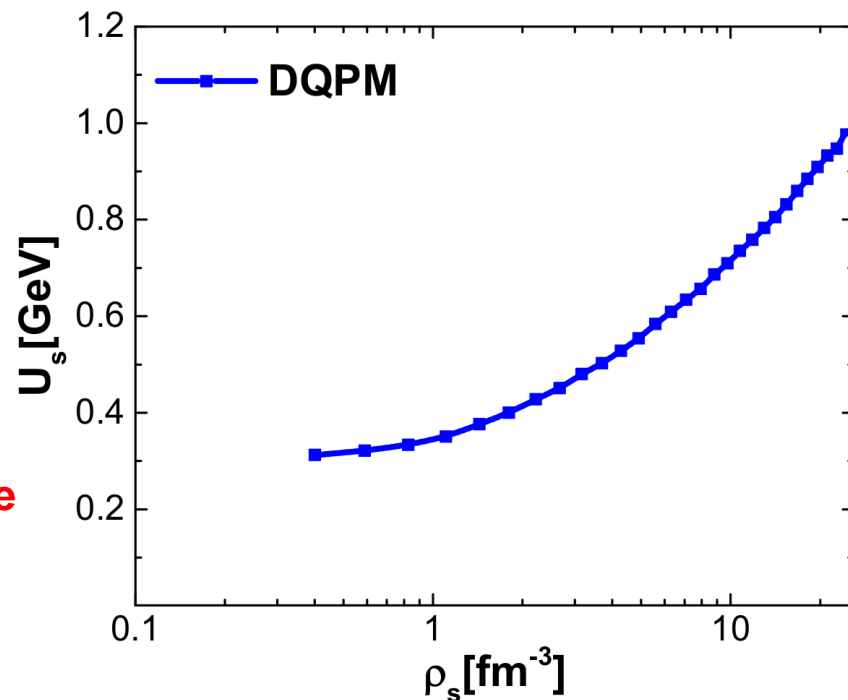
□ **Quasiparticle potentials ( $U_q, U_g$ ) are repulsive**

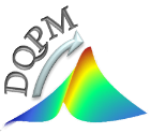
→ **the force** acting on a quasiparticle  $j$ :

$$F \sim M_j/E_j \nabla U_s(x) = M_j/E_j \frac{dU_s}{d\rho_s} \nabla \rho_s(x)$$

$$j = g, q, \bar{q}$$

→ **accelerates particles**





# Partonic interactions: matrix elements

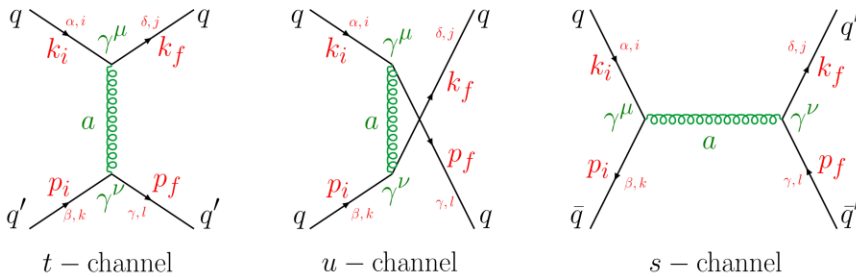
DQPM partonic cross sections → **leading order diagrams**

□ **Propagators** for massive bosons and fermions:

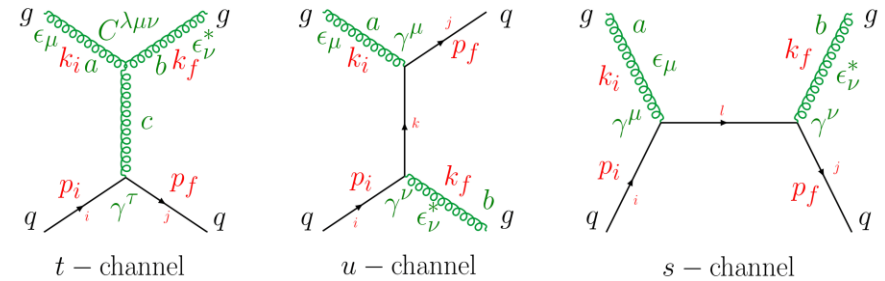
$$\frac{\mu, a \quad \nu, b}{q} = -i\delta_{ab} \frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

$$\begin{matrix} i & & j \\ \longrightarrow & & \longrightarrow \\ q & & \end{matrix} = i\delta_{ij} \frac{q\!\!\!/ + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

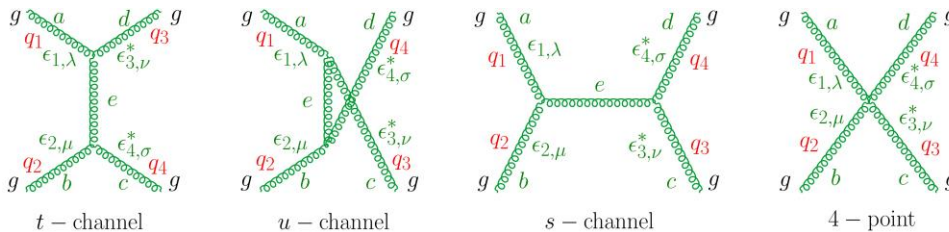
**qq' → qq' scattering**



**gg → gg scattering**



**gg → gg scattering**

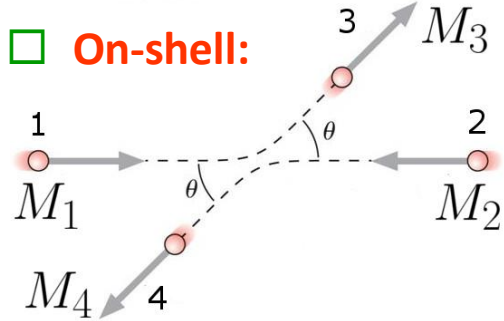


H. Berrehrh et al, PRC 93 (2016) 044914,  
Int.J.Mod.Phys. E25 (2016) 1642003,

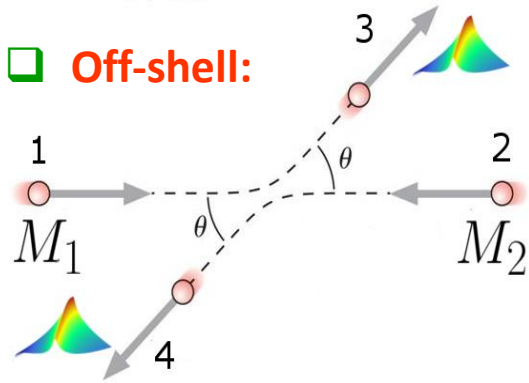


P. Moreau et al., PRC100 (2019) 014911

# Differential cross sections

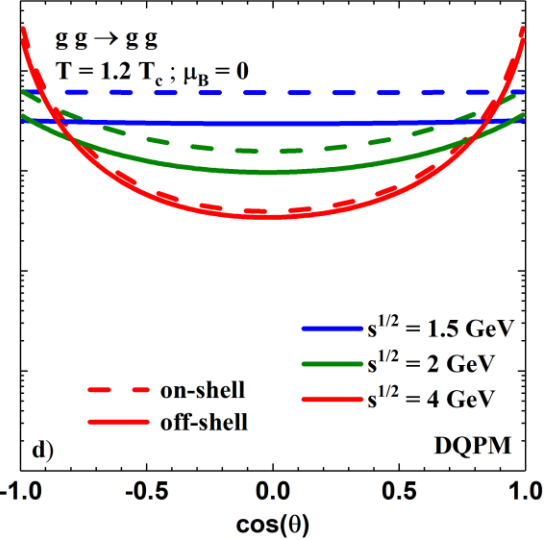
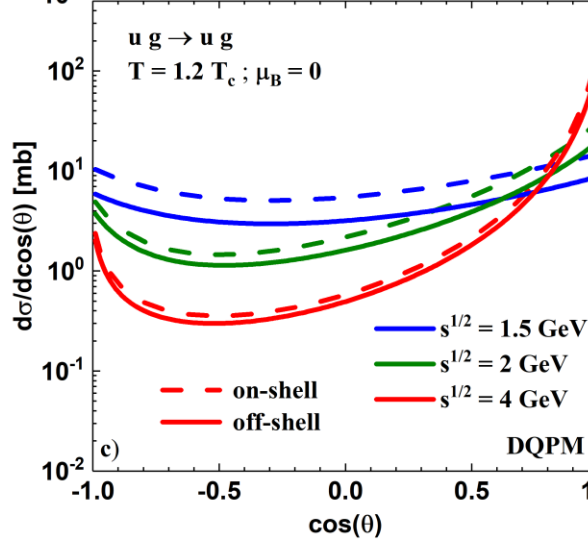
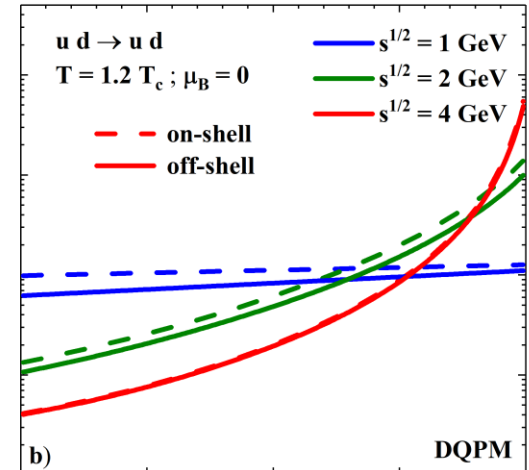
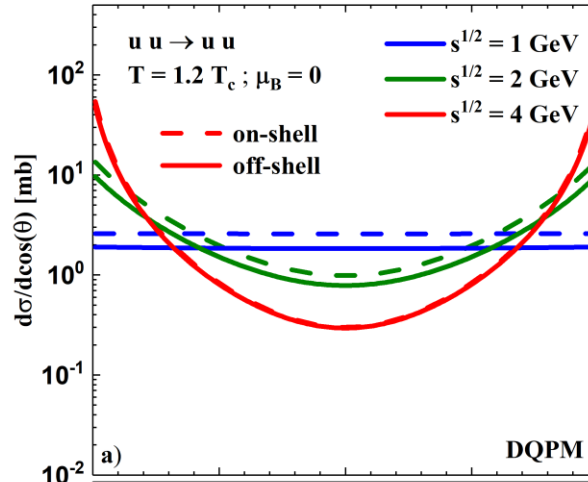


**Initial masses:** pole masses  
**Final masses:** pole masses



**Initial masses:** pole masses  
**Final masses:** integrated over spectral functions

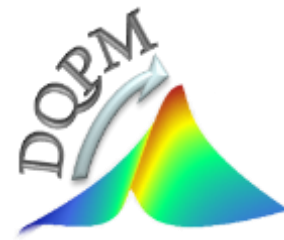
- At lower  $s$ : off-shell  $\sigma <$  on-shell  $\sigma$  since  $\omega_3 + \omega_4 < \sqrt{s}$



# QGP in-equilibrium

DQPM ( $T, \mu_q$ ):

transport properties at finite ( $T, \mu_q$ )

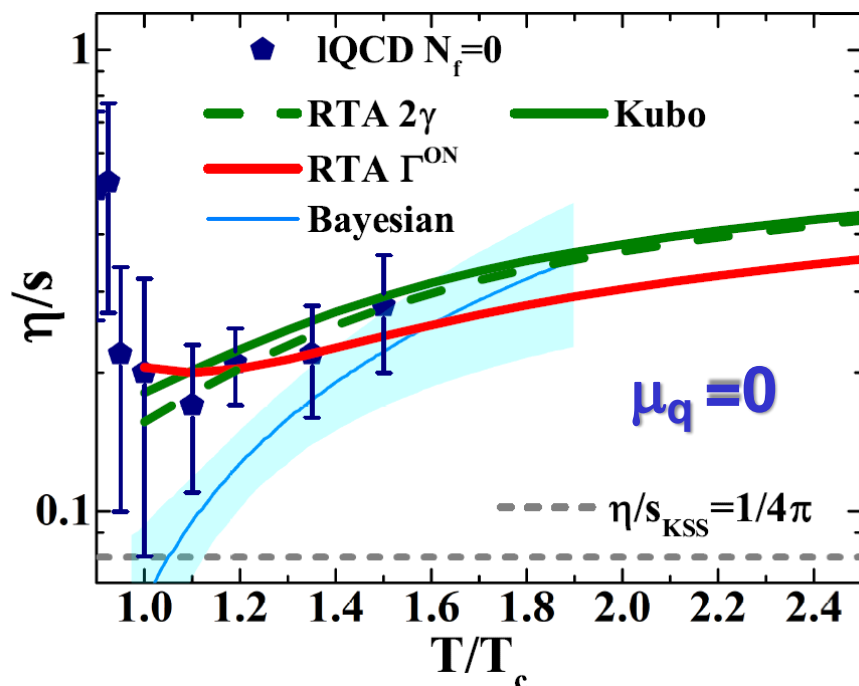


# Transport coefficients: shear viscosity

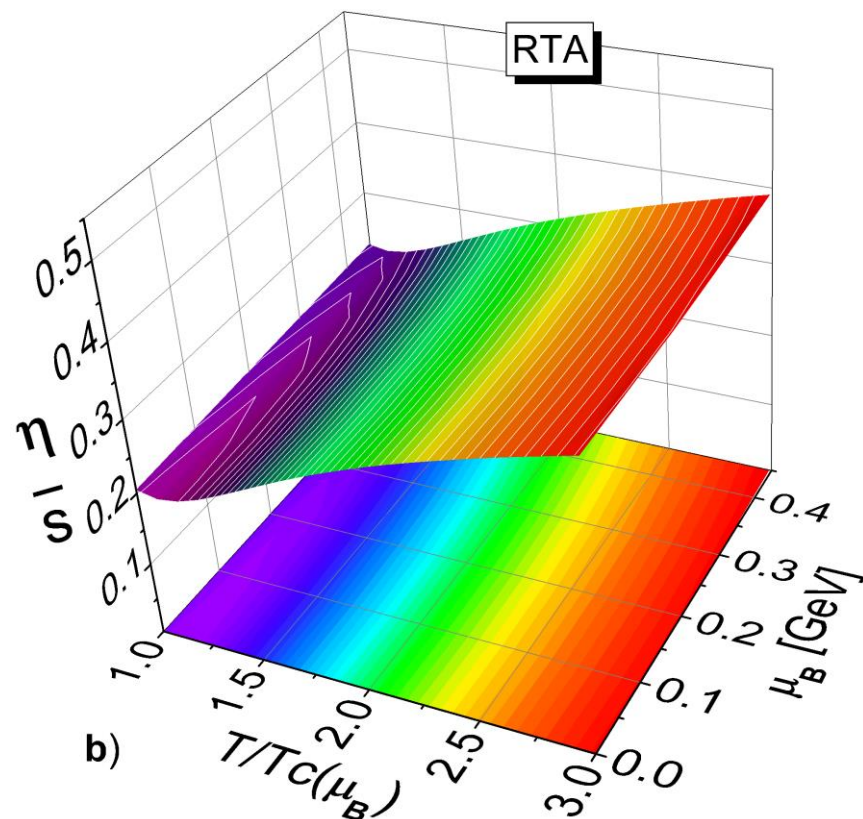
## Shear viscosity $\eta/s$ at finite $T$ , $\mu_q=0$

- **DQPM**: Relaxation Time Approximation (RTA) and Kubo formalism

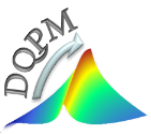
Hydro: Bayesian analysis, S. Bass et al., NPA967 (2017) 67



## Shear viscosity $\eta/s$ at finite $(T, \mu_q)$



- Very **weak dependence** of **shear** viscosity on  $\mu_B$

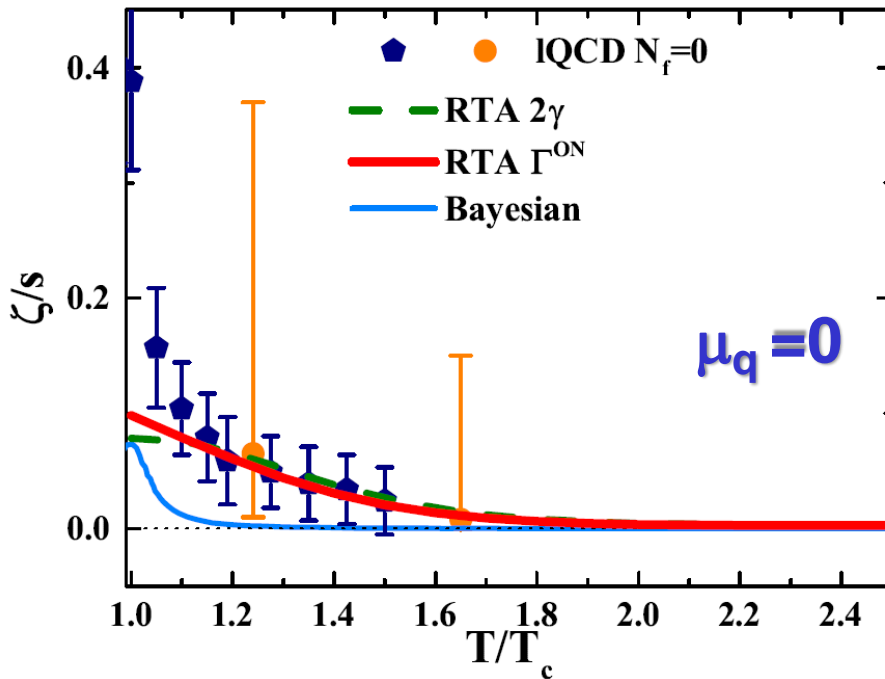


# Transport coefficients: bulk viscosity

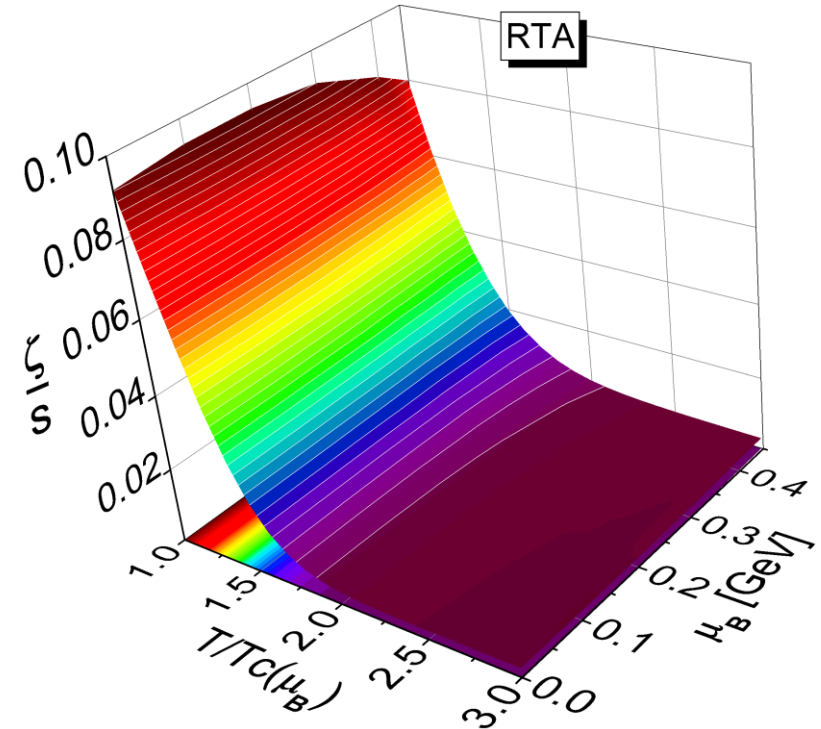
## Bulk viscosity $\zeta/s$ at finite $T$ , $\mu_q=0$

- **DQPM**: Relaxation Time Approximation (RTA) and Kubo formalism

Hydro: Bayesian analysis, S. Bass et al., NPA967 (2017) 67

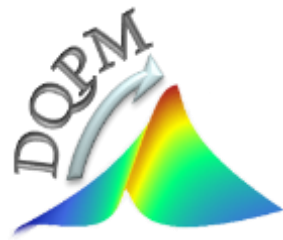


## Bulk viscosity $\zeta/s$ at finite $(T, \mu_q)$



- Very **weak dependence** of **bulk** viscosity on  $\mu_B$

**QGP:**  
**in-equilibrium → off-equilibrium**



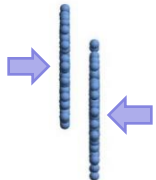


# Parton-Hadron-String-Dynamics (PHSD)

**PHSD** is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

**Dynamics:** based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A collision



Initial A+A collisions :

$N+N \rightarrow$  **string formation**  $\rightarrow$  decay to pre-hadrons + leading hadrons

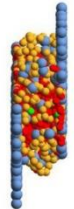
Formation of QGP stage if local  $\varepsilon > \varepsilon_{\text{critical}}$  :

dissolution of **pre-hadrons**  $\rightarrow$  partons

Partonic phase - QGP:

QGP is described by the **Dynamical QuasiParticle Model (DQPM)** matched to reproduce **lattice QCD EoS** for finite  $T$  and  $\mu_B$  (crossover)

Partonic phase



- **Degrees-of-freedom:** strongly interacting quasiparticles: **massive quarks and gluons ( $g, q, q_{\text{bar}}$ )** with sizeable collisional widths in a self-generated mean-field potential

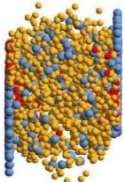
- **Interactions:** (quasi-)elastic and inelastic collisions of partons

**Hadronization** to colorless **off-shell mesons and baryons:**

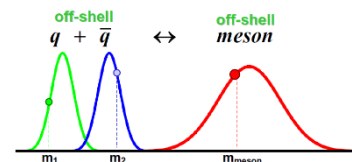
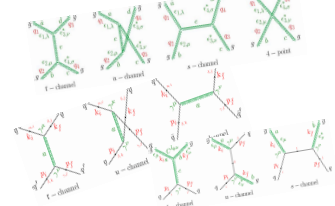
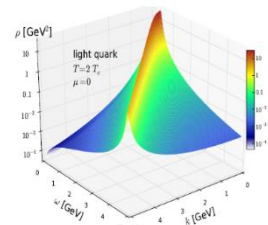
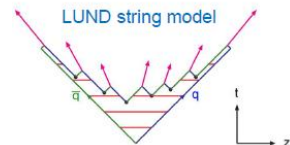
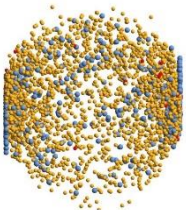
Strict 4-momentum and quantum number conservation

**Hadronic phase:** hadron-hadron interactions – **off-shell HSD**

Hadronization



Hadronic phase





# Extraction of $(T, \mu_B)$ in PHSD

□ For each cell in PHSD :

In order to extract  $(T, \mu_B)$  use IQCD relations (up to 4<sup>th</sup> order) - Taylor series :

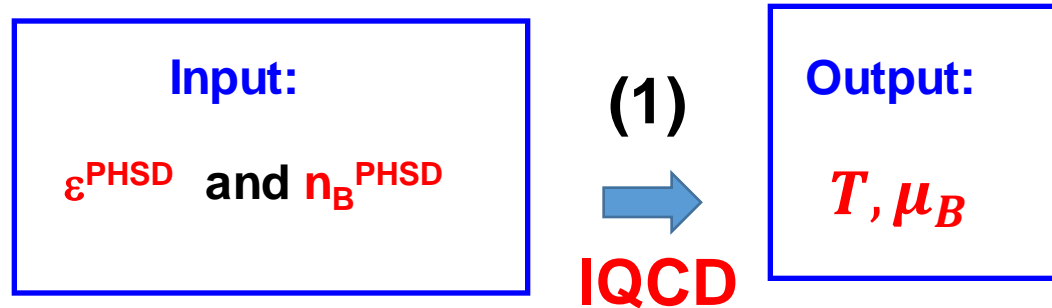
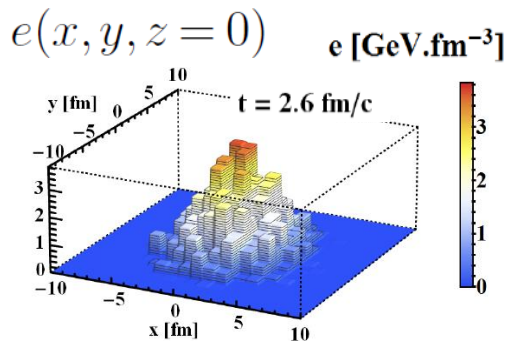
(1)  
IQCD

$$\left\{ \begin{array}{l} \frac{n_B}{T^3} \approx \chi_2^B(T) \left( \frac{\mu_B}{T} \right) + \dots \\ \Delta\epsilon/T^4 \approx \frac{1}{2} \left( T \frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T) \right) \left( \frac{\mu_B}{T} \right)^2 + \dots \end{array} \right.$$

\* Use baryon number susceptibilities  $\chi_n$  from IQCD

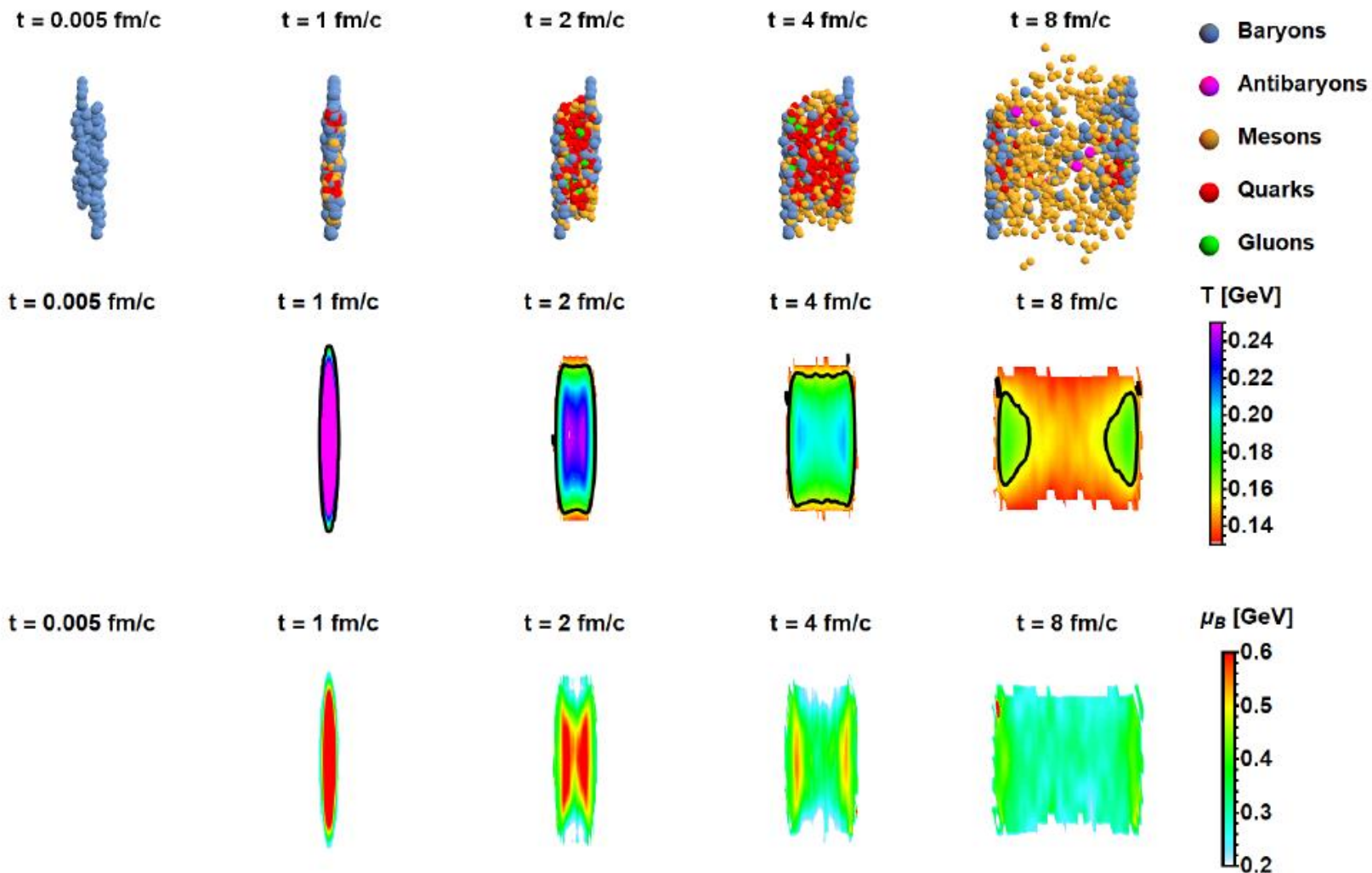
➔ obtain  $(T, \mu_B)$  by solving the system of coupled equations using  $\epsilon^{\text{PHSD}}$  and  $n_B^{\text{PHSD}}$

\* Done by the Newton-Raphson method

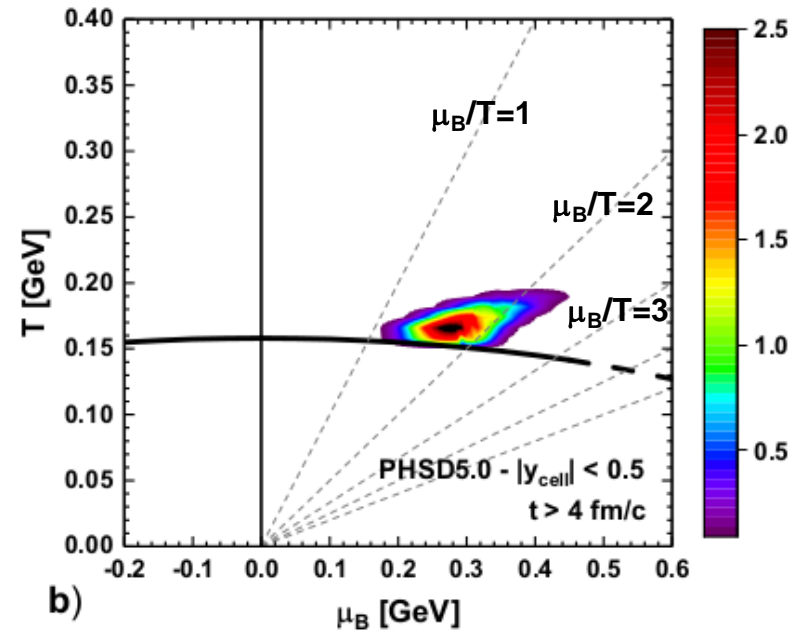
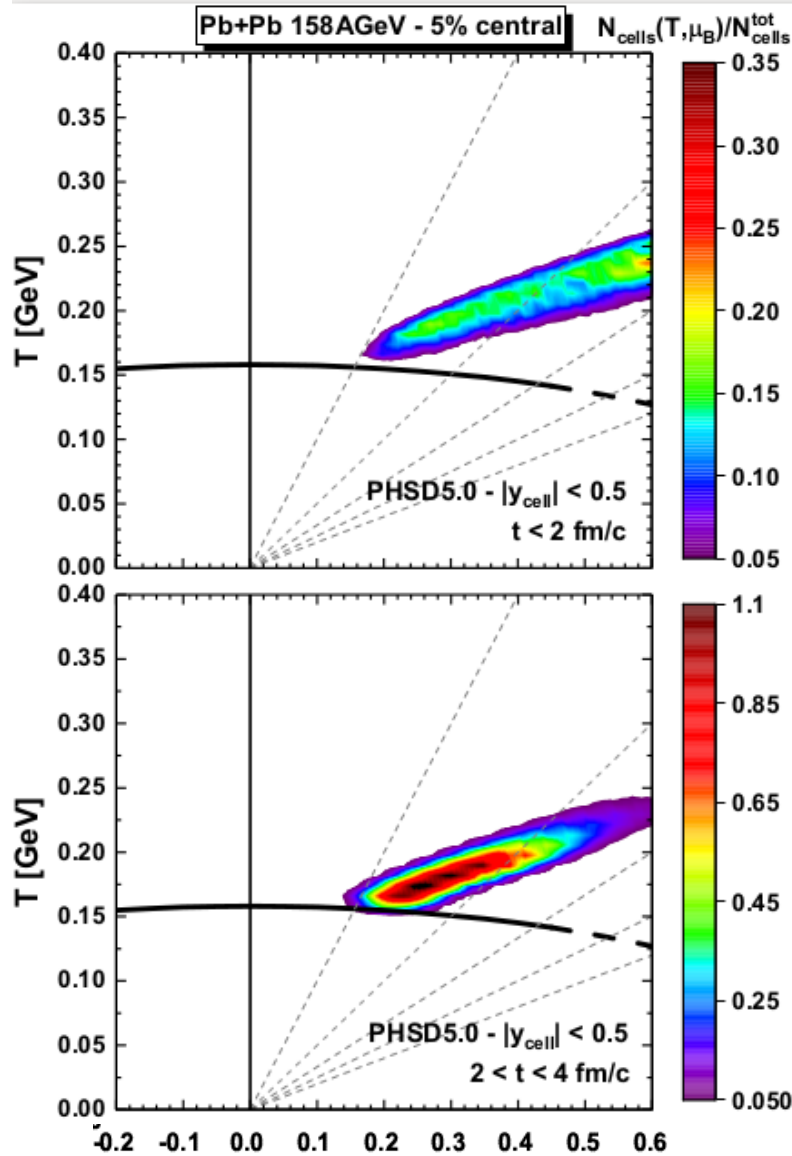


# Illustration for a HIC ( $\sqrt{s_{NN}} = 19.6 \text{ GeV}$ )

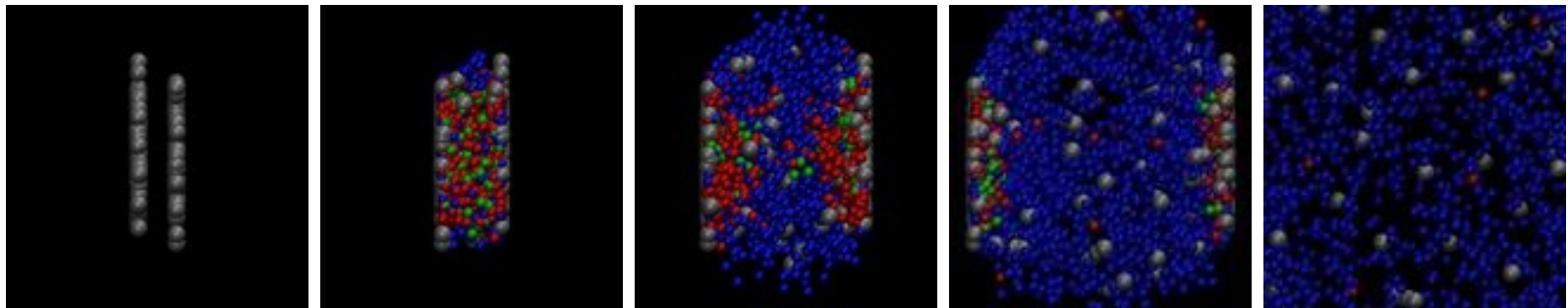
Au + Au  $\sqrt{s_{NN}} = 19.6 \text{ GeV} - b = 2 \text{ fm} - \text{Section view}$



# Illustration for a HIC ( $\sqrt{s_{NN}} = 17$ GeV)



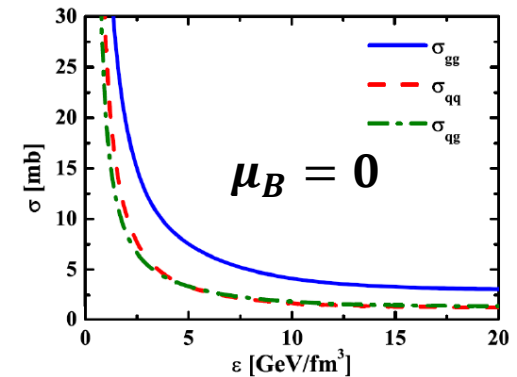
Traces of the QGP at finite  $\mu_q$  in  
observables  
in high energy heavy-ion collisions



➤ Comparison between three different results:

1) PHSD 4.0 : only  $\sigma(T)$  and  $\rho(T)$

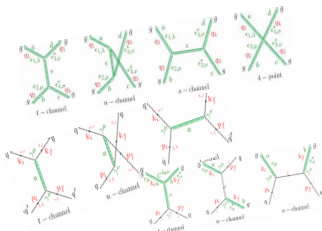
$\sigma(T)$  – parton interaction cross sections  
 $\rho(T)$  – spectral function of partons  
 → (masses and widths)



2) PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \underline{\mu_B = 0})$  and  $\rho(T, \mu_B = 0)$

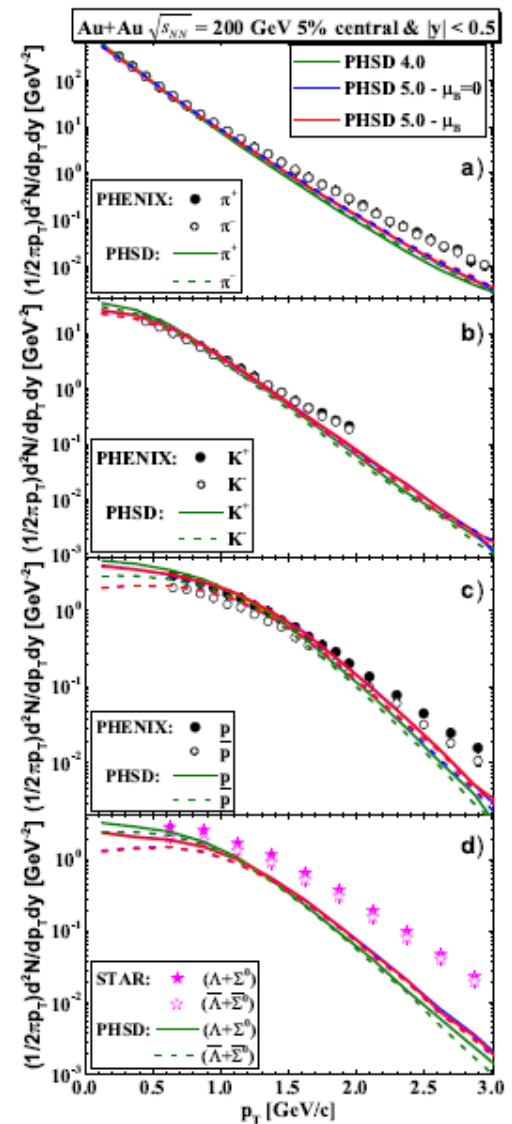
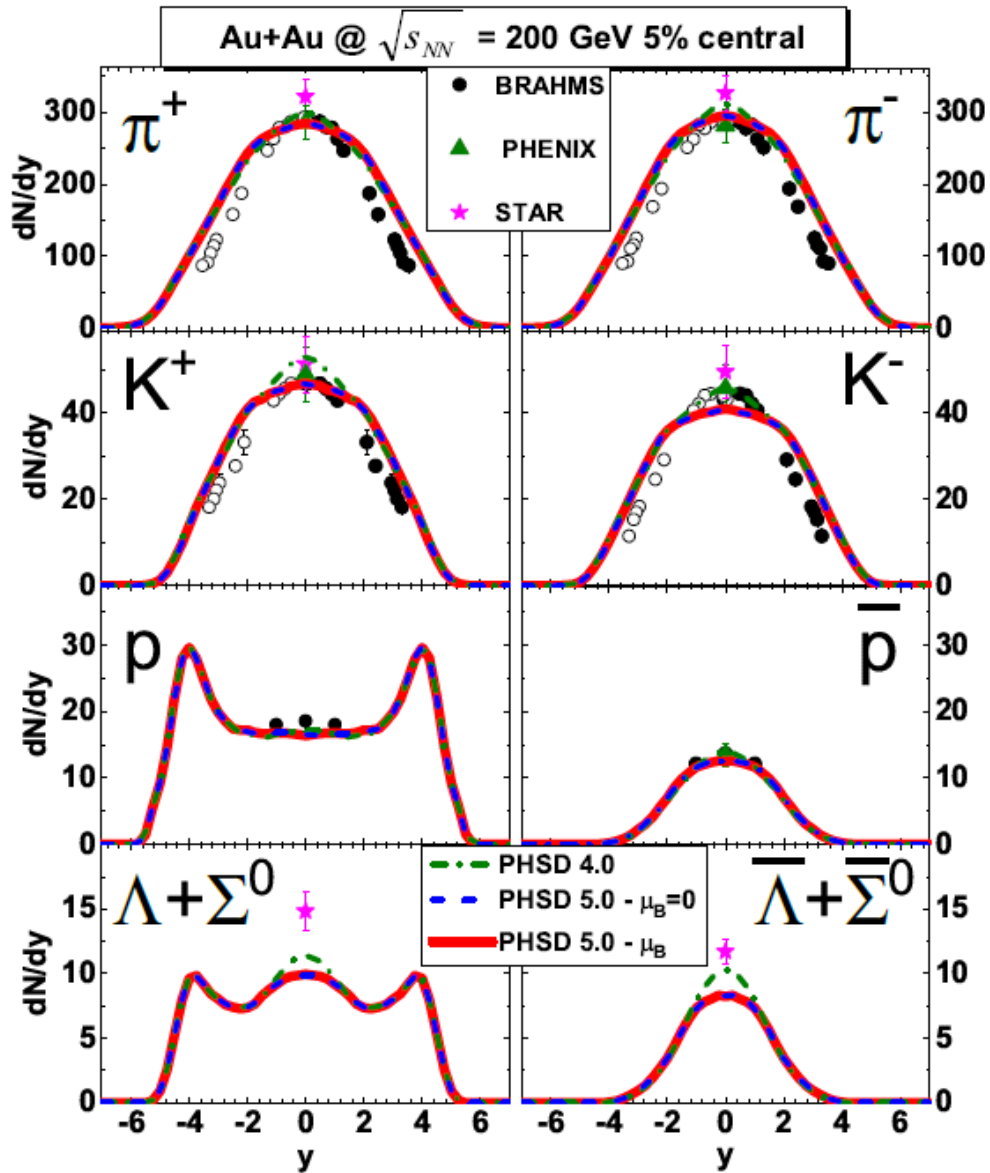
In v.5.0: + angular dependence of diff. partonic cross sections

3) PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$  and  $\rho(T, \mu_B)$





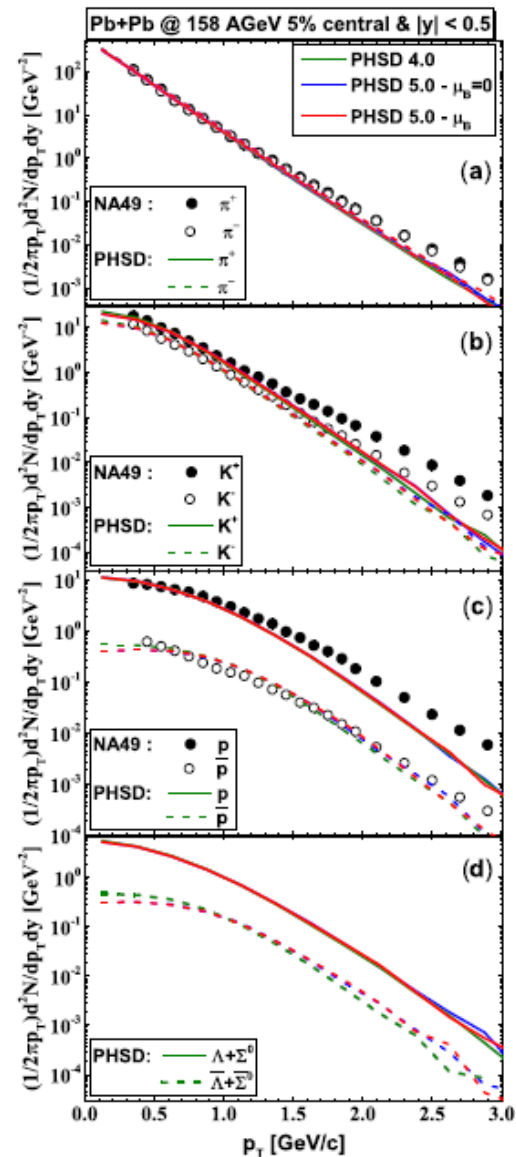
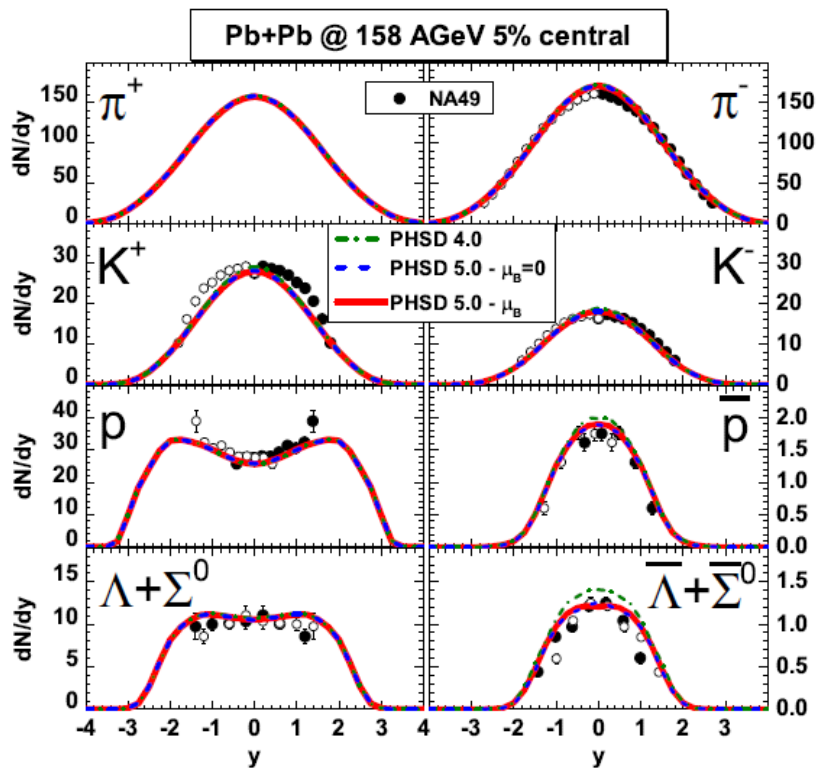
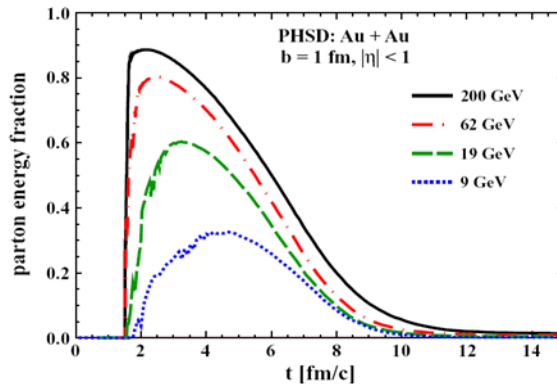
# Results for HICs ( $\sqrt{s_{NN}} = 200$ GeV)





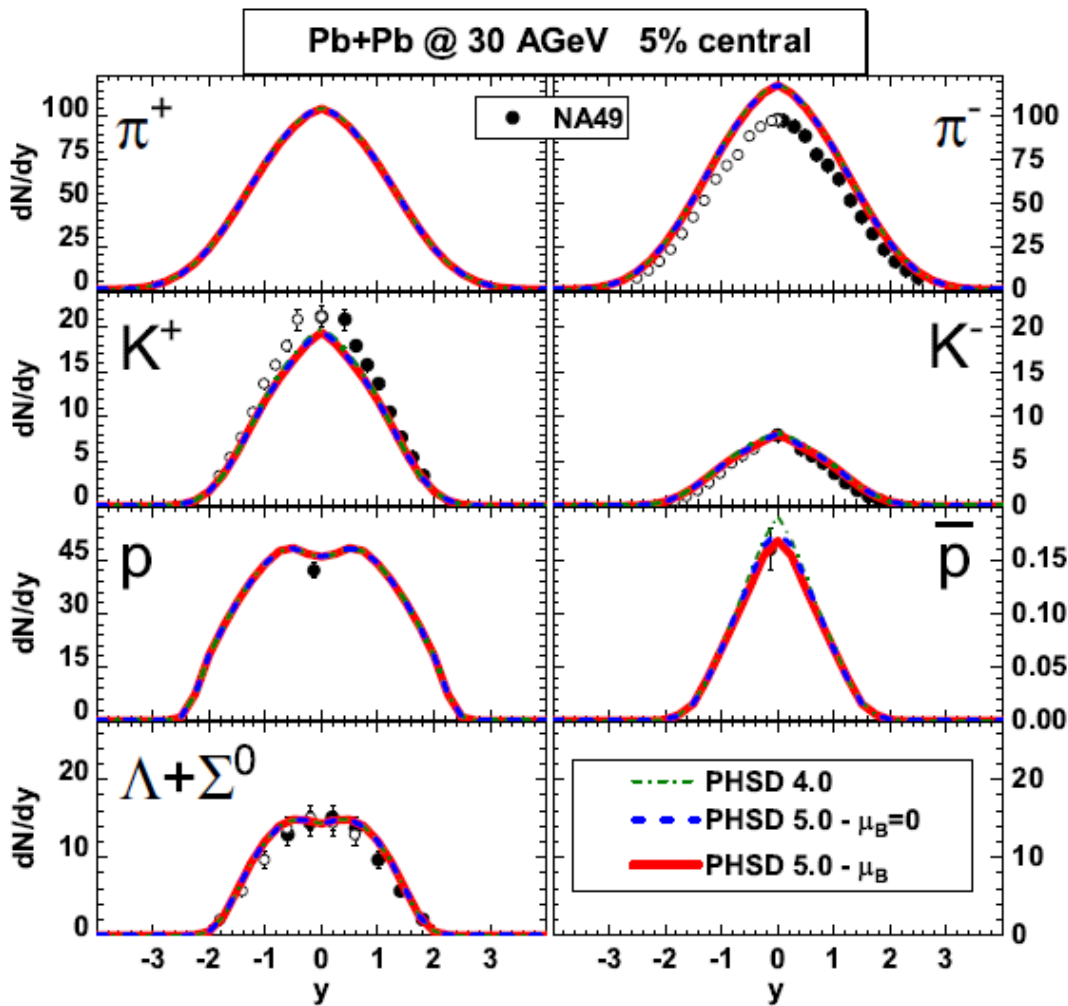
# Results for HICs ( $\sqrt{s_{NN}} = 17$ GeV)

High- $\mu_B$  regions are probed at **low  $\sqrt{s_{NN}}$  or high rapidity regions**  
 But, **QGP fraction is small** at low  $\sqrt{s_{NN}}$





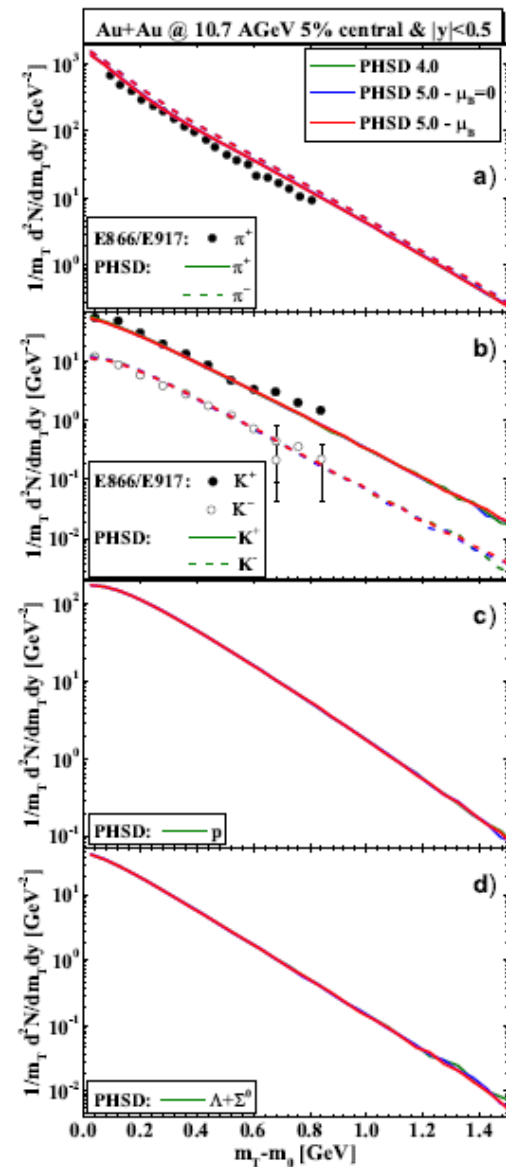
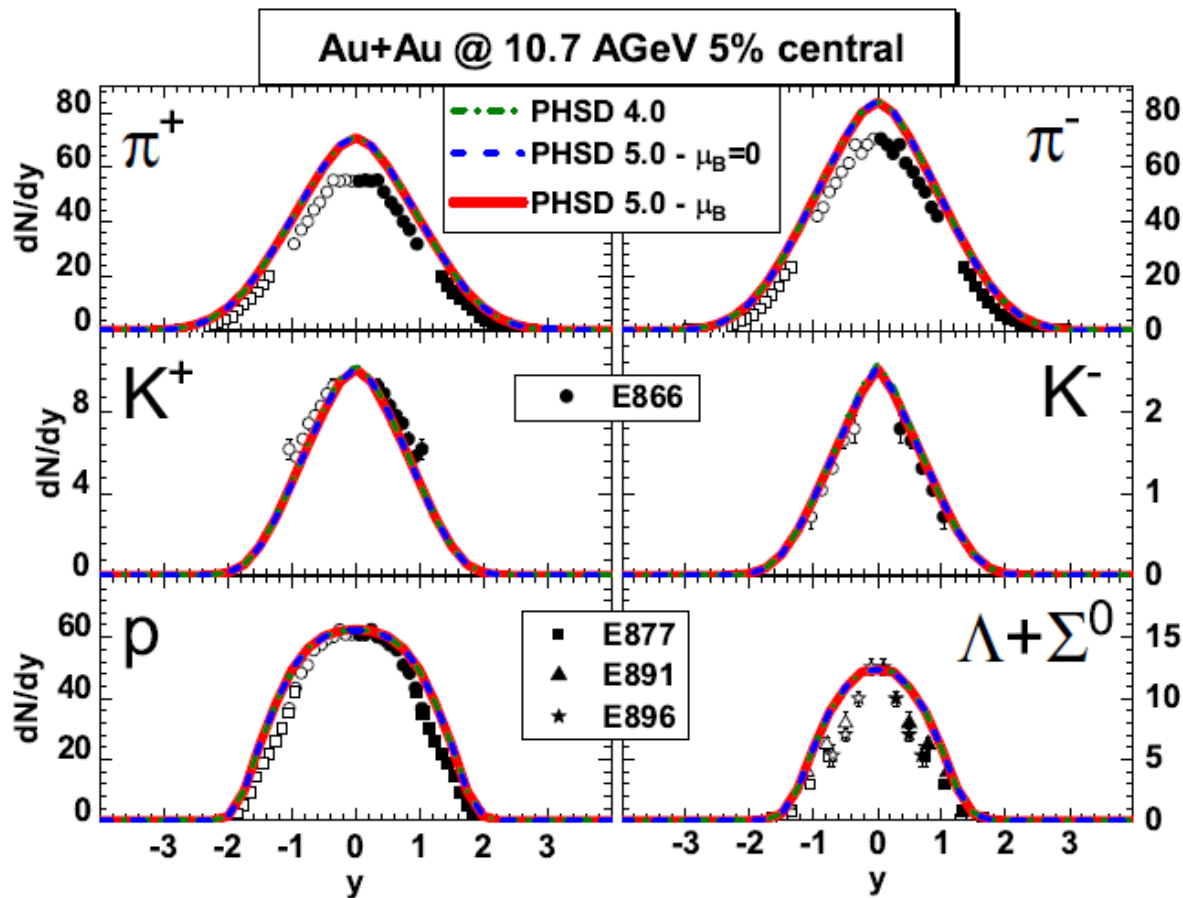
# Results for HICs ( $\sqrt{s_{NN}} = 7.6 \text{ GeV}$ )



➤ very weak dependence of 'bulk' observables on  $\mu_B$

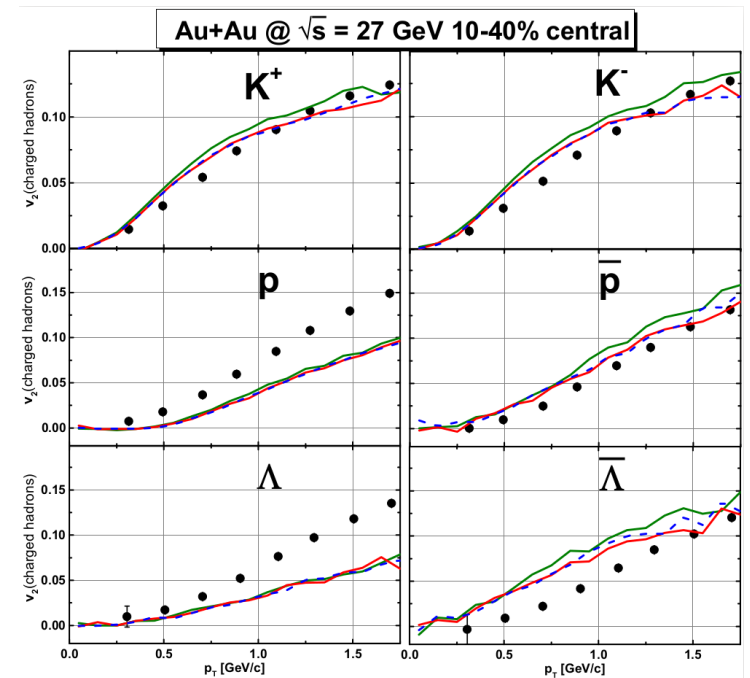
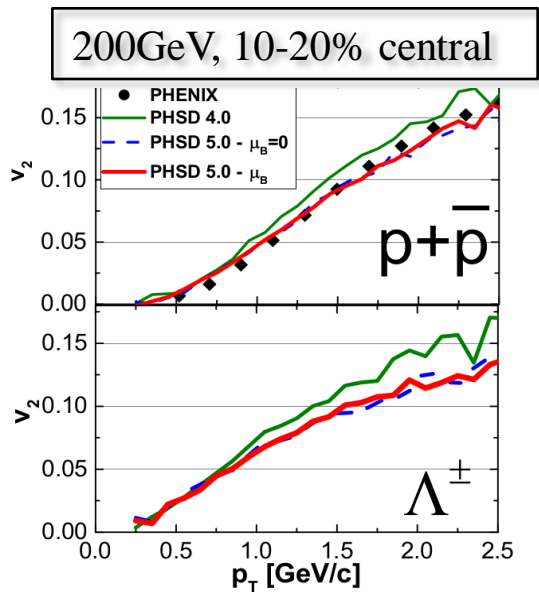
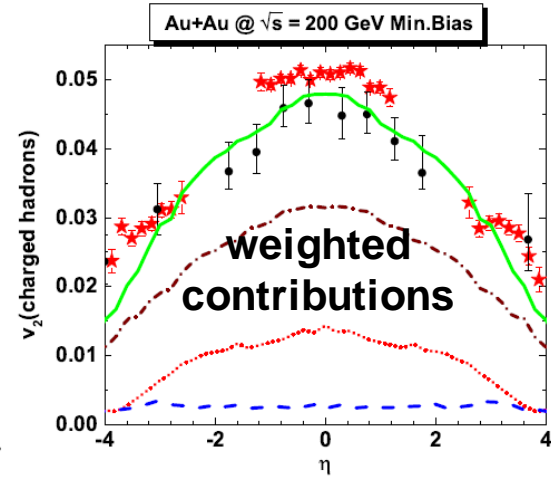
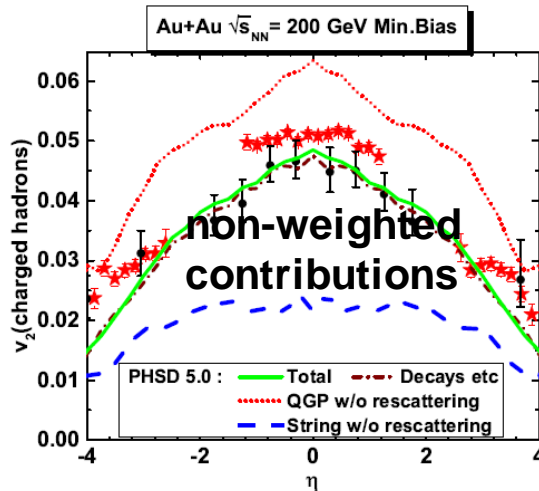
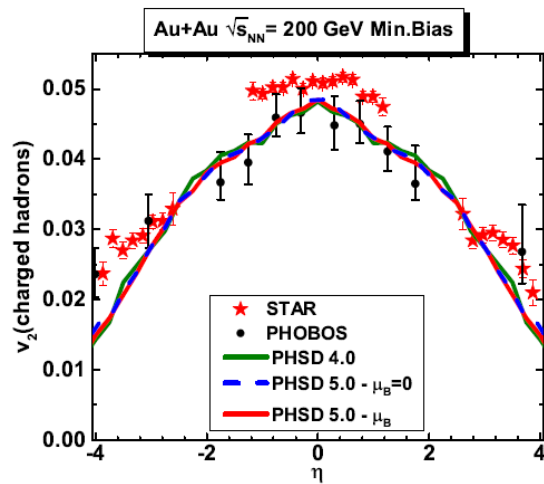


# Results for HICs ( $\sqrt{s_{NN}} = 4.86$ GeV)



➤ very weak dependence of 'bulk' observables on  $\mu_B$

# Elliptic flow $v_2$ ( $\sqrt{s_{NN}} = 200 \text{ GeV vs } 27 \text{ GeV}$ )

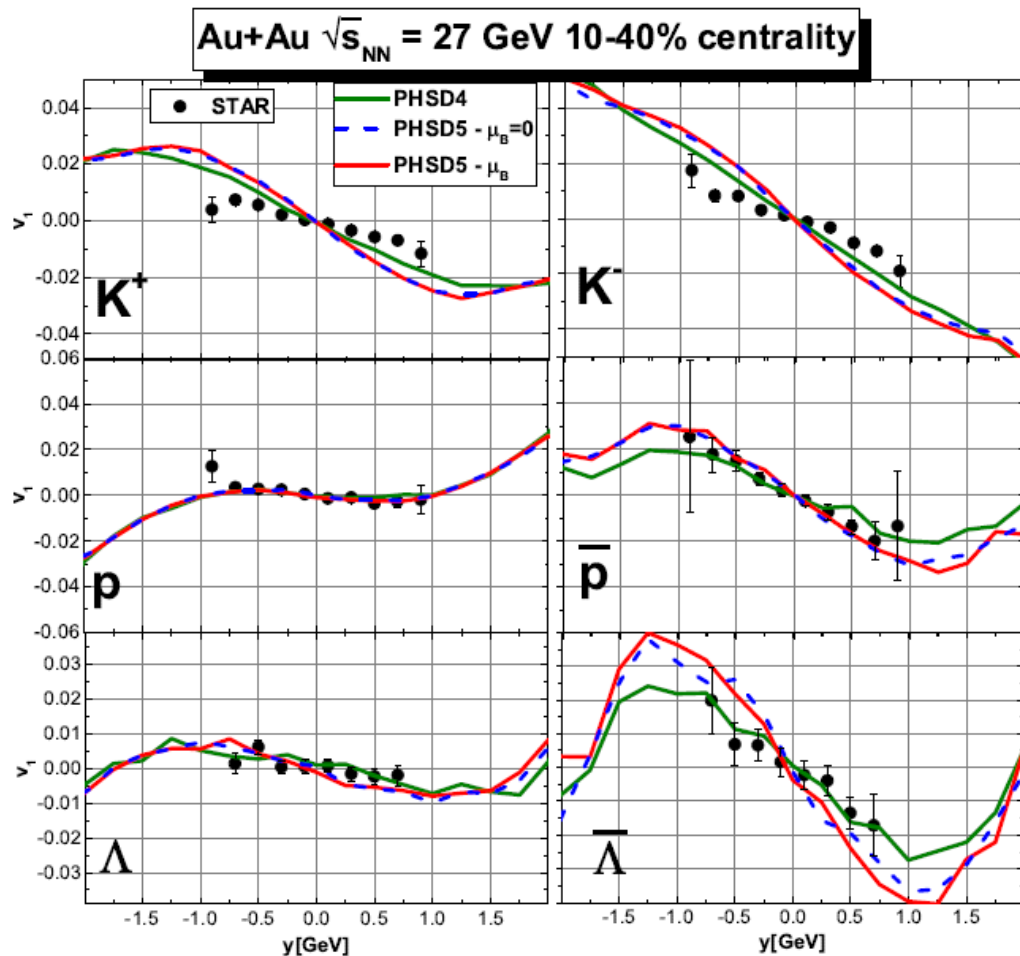


➔ Small effect of  $\mu_B$  dependence on  $v_2$



# Results for $v_1$ for HICs ( $\sqrt{s_{NN}} = 27$ GeV)

$v_1$



## $v_1, v_2$ analysis:

- weak dependence of  $v_1, v_2$  on  $\mu_B$
- small influence on  $v_1, v_2$  of explicit  $\sqrt{s}$ -dependence of total partonic cross sections  $\sigma$  + angular dependence of  $d\sigma/d\cos\theta$  due to the relatively small QGP volume
- strong flavor dependence of  $v_1, v_2$

O. Soloveva et al., arXiv:2001.07951, MDPI Particles 2020, 3, 178



# Summary / Outlook

- ❑  $(T, \mu_B)$ -dependent partonic cross sections and masses/widths of quarks and gluons have been implemented in PHSD
- ❑ High- $\mu_B$  region is probed at low bombarding energies or high rapidity regions
- ❑ But, QGP fraction is small at low bombarding energies:
  - ➔ no effects of  $(T, \mu_B)$ -dependent partonic cross sections and masses/widths seen in 'bulk' observables –  $dN/dy$ ,  $p_T$ -spectra
- ❑ Flow harmonics  $v_1, v_2$  show :  
visible sensitivity to the explicit  $\sqrt{s}$ -dependence of total partonic cross sections  $\sigma$  + angular dependence of  $d\sigma/d\cos\theta$ , however, weak dependence on  $\mu_B$
- ❑ Outlook:
  - More precise EoS at large  $\mu_B$
  - Possible 1<sup>st</sup> order phase transition at even larger  $\mu_B$ ?!

High- $\mu_B$  region of QCD phase diagram ➔ challenge for FAIR, NICA, BES RHIC