Far-From-Equilibrium Hydrodynamics and the Beam Energy Scan

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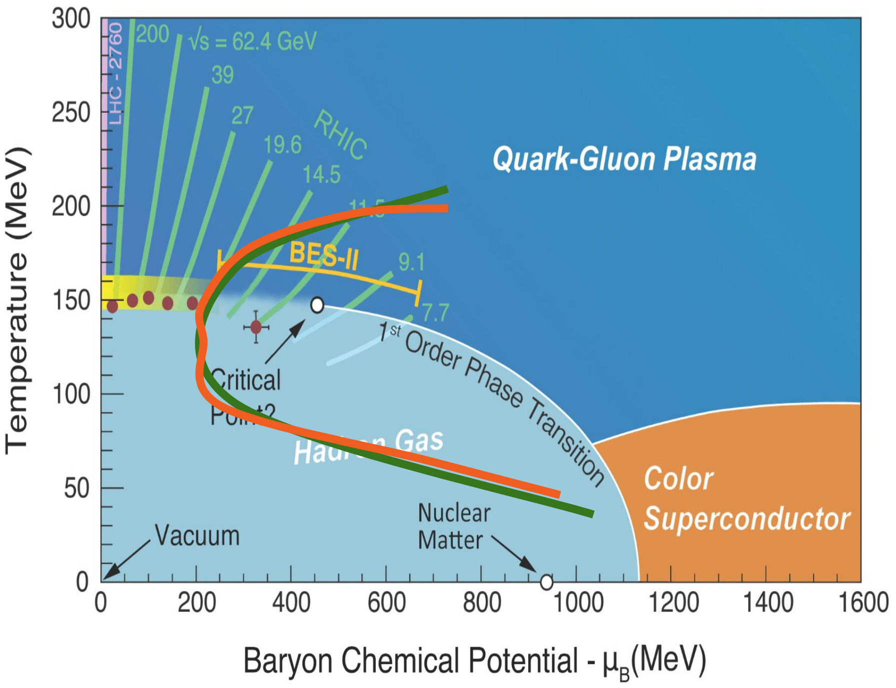
BES Physics and T-Mu Trajectories

Simple Equilibrium Picture:

- System expands isentropically
- *reversible process*
  - Ideal Hydrodynamics
  - Trajectories follow isentropes

Cannot Be The Case:

- Process is clearly *irreversible*
- Entropy *must* be being produced
- ***Viscosity*** important contribution to hydrodynamic evolution



Question: *How do non-equilibrium dynamics influence trajectories?*

What Does Out Of Equilibrium Mean?

Well, entropy should certainly increase..

Self-Consistent Method: ***Phenomenological*** Israel-Stewart (IS) Equations

Postulate: $S^\mu = S^\mu_{eq} - u^\mu \beta_\pi \pi_{\mu\nu}\pi^{\mu\nu} - u^\mu \beta_\Pi \Pi^2$

W. Israel, J.M. Stewart, Annals of Physics, 1979

Shear-Stress Viscous Effects

Bulk Viscous Effects

$$\partial_\mu S^\mu \geq 0 \rightarrow$$

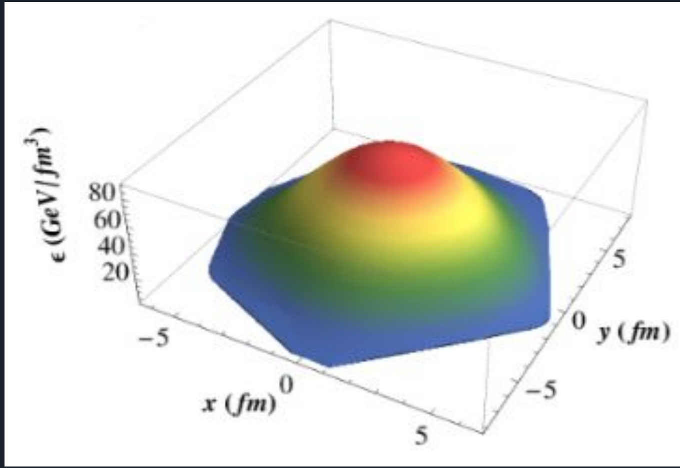
$$\tau_\pi D\pi_{\mu\nu} = -\pi_{\mu\nu} + 2\eta\sigma_{\mu\nu} - \eta T \pi_{\mu\nu} \nabla_\alpha(\beta_\pi u^\alpha)$$

$$\tau_\Pi D\Pi = -\Pi - \zeta\theta - \frac{\zeta T}{2}\Pi\nabla_\alpha(\beta_\Pi u^\alpha)$$

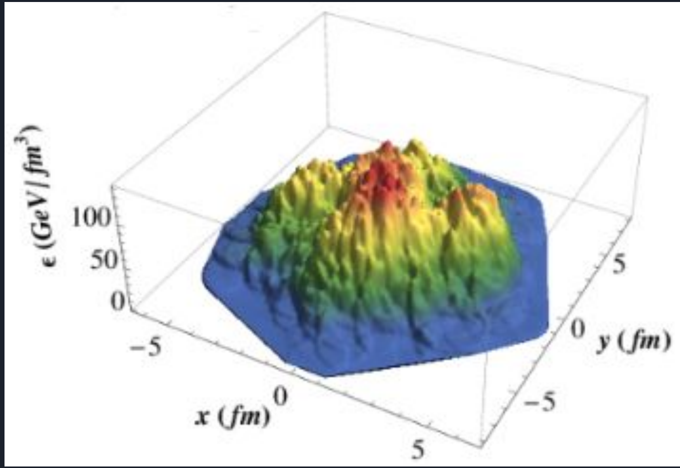
Viscous effects are *dynamical*, describe relaxation to equilibrium

Quantifying Initial Viscous Effects

Close to Equilibrium Initial State



Fluctuations and Large Gradients



J. Noronha-Hostler,
J .Noronha,
M. Gyulassy
Phys. Rev. C 93, 024909 (2016)

Far-From-Equilibrium initial state
is possible given quantum fluctuations

How to explain robustness of hydrodynamic predictions?

Attractors in Hydrodynamics

<u>Working Definition:</u> There exists an attractor if, after some finite amount of time, solutions converge on to a *non-trivial*, *universal* curve

Some Previous Study:

Attractors in relativistic, viscous, hydrodynamics, **Conformal EoS**, no bulk

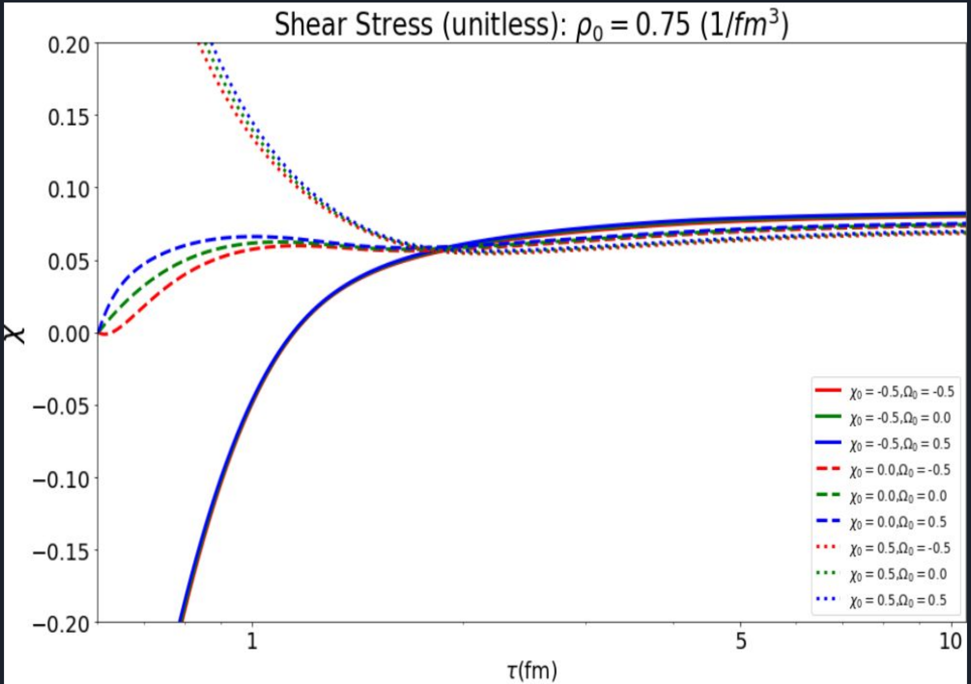
Gabriel S. Denicol and Jorge Noronha, Phys. Rev. D 97, 056021

Attractors in kinetic theory,
Non- Conformal EoS

Romatschke, P. J. High Energ. Phys. (2017) 2017: 79

Attractors in relativistic, viscous, hydrodynamics, **Non-Conformal EoS**, small bulk, finite baryon densities

TD and Jacquelyn Noronha-Hostler, QM 19 Poster



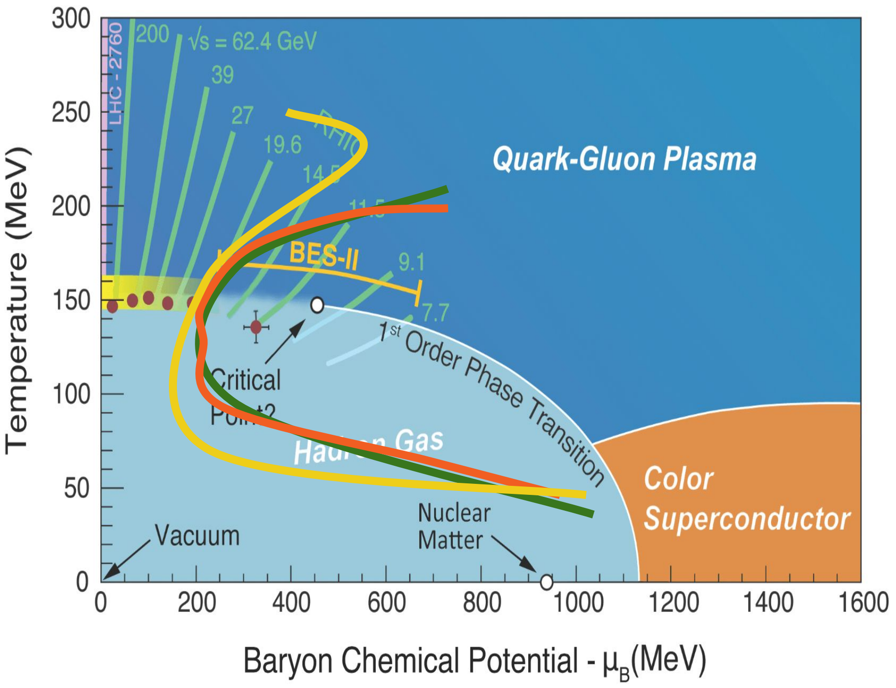
An Evolving Hydrodynamic Picture

Close to Equilibrium:

- System expands hydrodynamically with a viscosity, gradients not too large
  - Assumption under which IS derived
  - Did we get out more than we put in?

Far From Equilibrium Initial State:

- No reason to believe system should be initially close to equilibrium
- Existence of attractors offers unique way to make sense of the success of hydro



Question: *How do far-from-equilibrium dynamics influence trajectories?*

A Tale of Two Hydros: Israel-Stewart vs DNMR

G. S. Denicol, H. Niemi, E. Molnár, and D. H. Rischke, Phys. Rev. D 85, 114047

Historical Note:
Israel and Stewart were interested in cosmological scale hydro, could throw out terms that *should* be interesting for us

Truncate at second order in 4-momentum moments of distribution

→Kinetic Theory
→"2nd order"
→Causal relaxation to equilibrium (*in linear regime*)
→Entropy production?

Truncate at second order in power counting scheme

Contemporary Note:
DNMR is commonly implemented in the field (MUSIC, v-USPHydro)

$$\nabla_\alpha(\beta u^\alpha) = \theta\beta + \dot{\beta}$$

Cosmological Assumption

$\dot{\beta} \ll \theta\beta$ Not True For Heavy Ions!

Toy Model: *Bjorken Symmetric Flow*

1. Boost Invariance
2. Cylindrical Symmetry
3. Trival Flow

$$u_\mu = (1, 0, 0, 0)$$

$$g_{\mu\nu} = diag(1, -1, -1, -\tau^2)$$

$$\nabla_\mu u^\mu = \frac{1}{\tau}$$

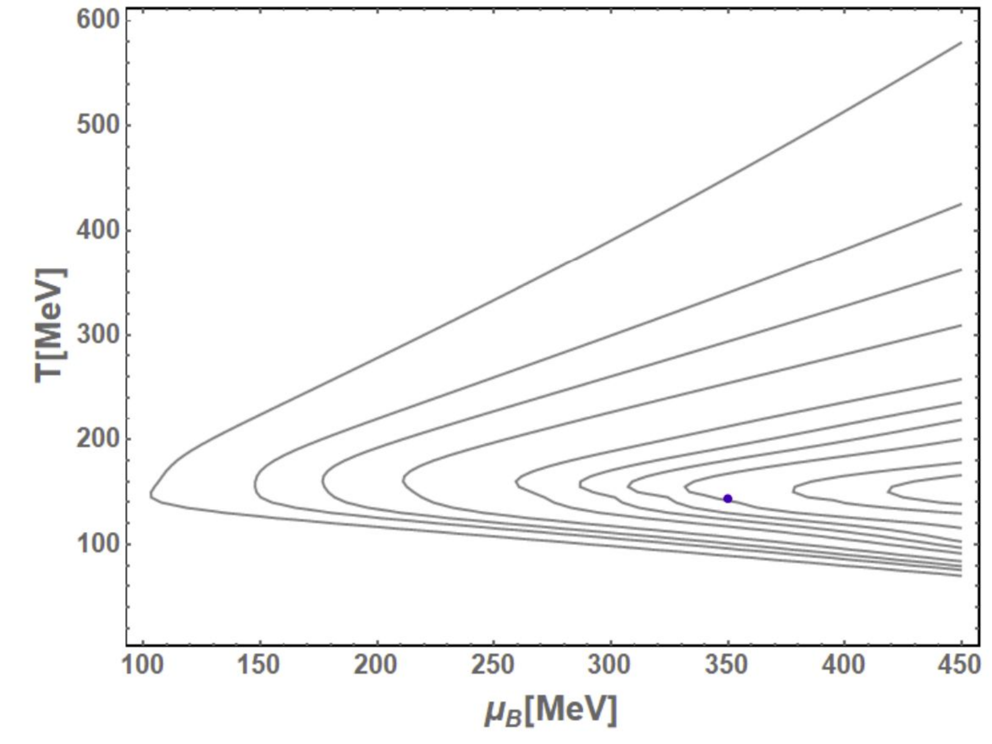
Three Cases To Compare

DNMR Israel-Stewart with $\dot{\beta}$ Israel-Stewart w/o $\dot{\beta}$

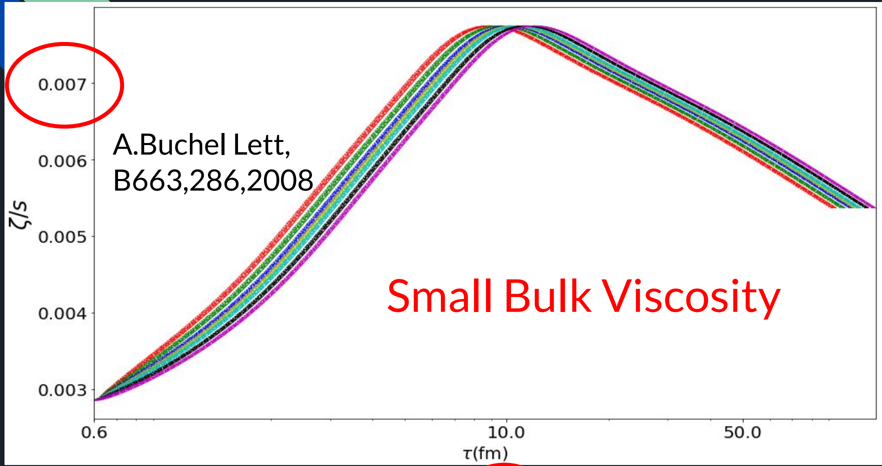
Equation of State

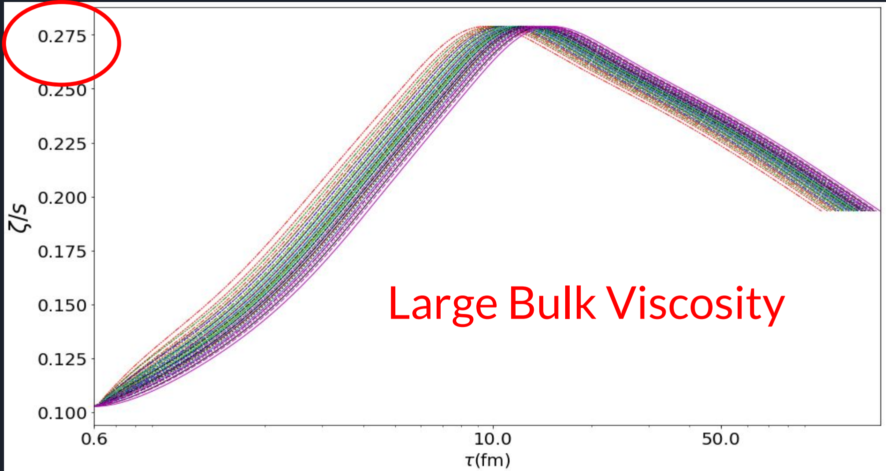
Parotto et al,
arXiv:1805.05249v1

- How do hydro paths deviate from isentropes?
  - *Where does out of equilibrium take you?*
- The point of BES is to probe the QCD EoS
  - *Need to use most realistic and up to date EoS with critical point*



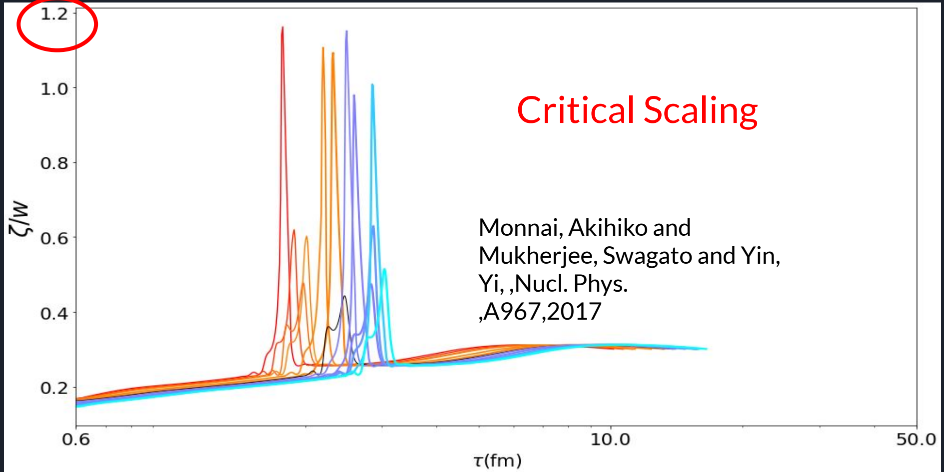
Transport Coefficients: bulk viscosity





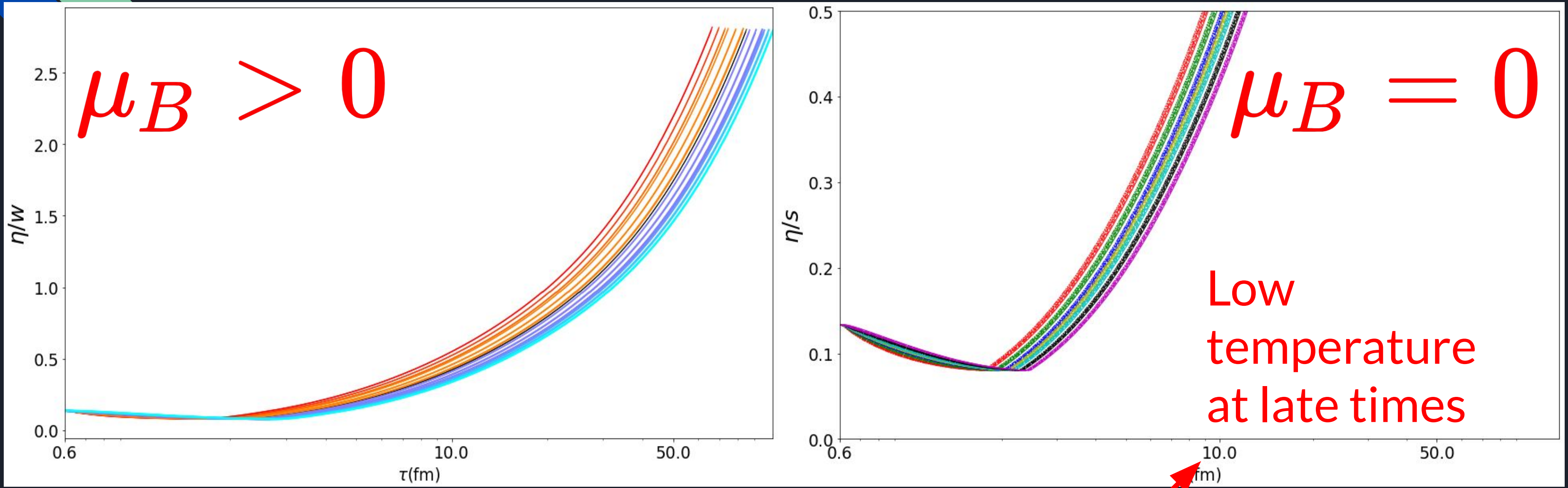
Different ways to parametrize bulk viscosity

$$\frac{\zeta}{w} = \frac{\zeta}{w}(\mu_B, T)$$



Different initial conditions will also yield different time evolution

Transport Coefficients: shear viscosity



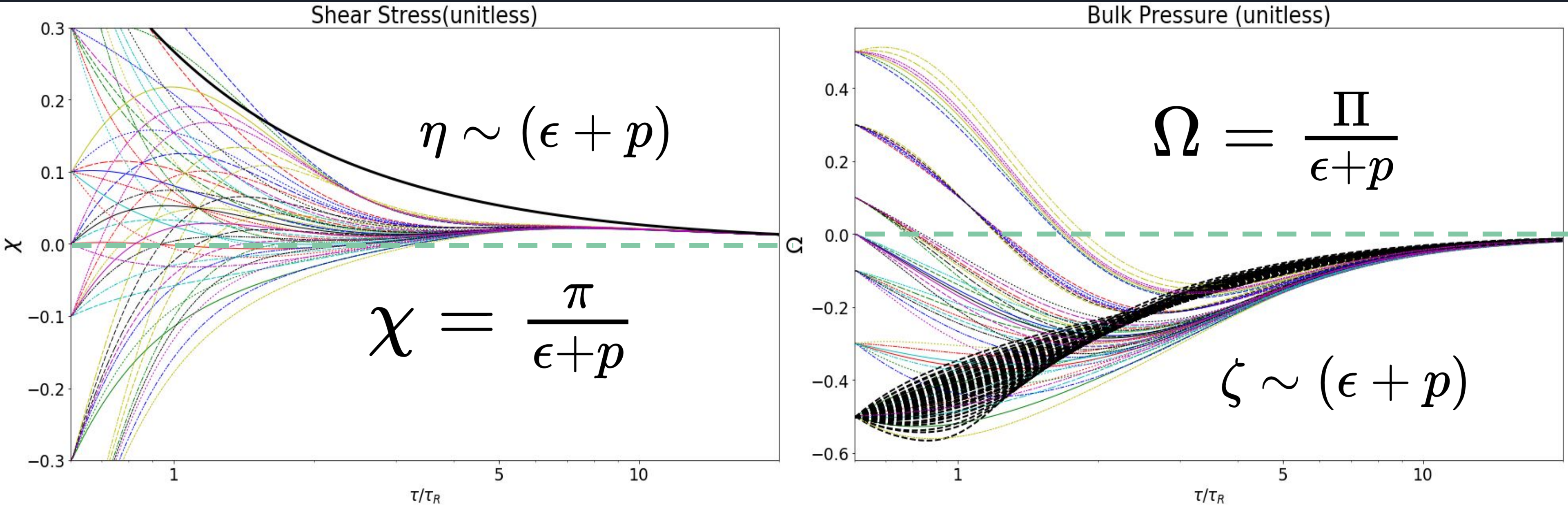
- ➢ Same shear viscosity
- ➢ Different initial conditions
- ➢ Work done with REU student Emma McLaughlin

$$\frac{\eta}{w} = \frac{\eta}{w}(\mu_B, T)$$

Rises at low temperature

Warm up!

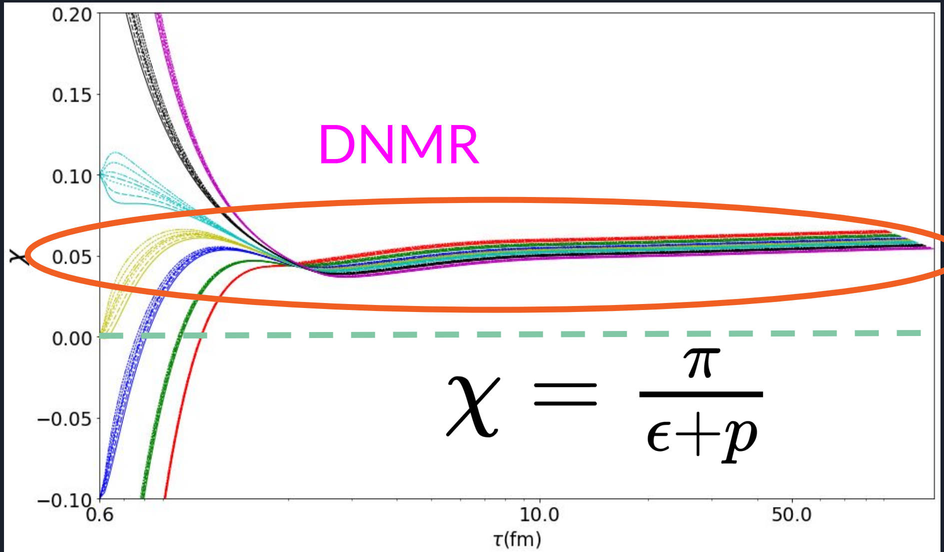
DNMR: Unphysical Case 'RTA'

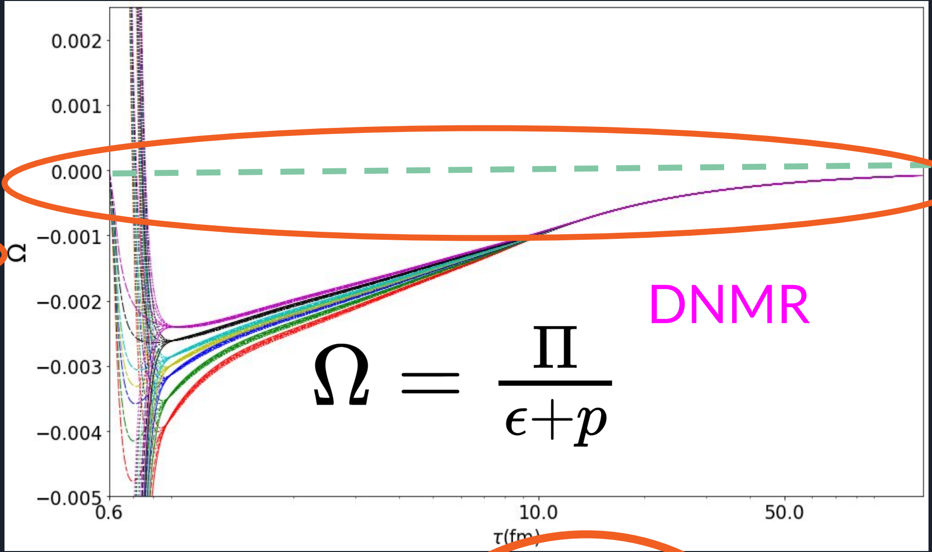


- EoS: Alba et al, Phys.Rev. C98 (2018) no.3, 034909
- Converges before Navier-Stokes
- System eventually reaches equilibrium

Well defined attractors with constant relaxation time

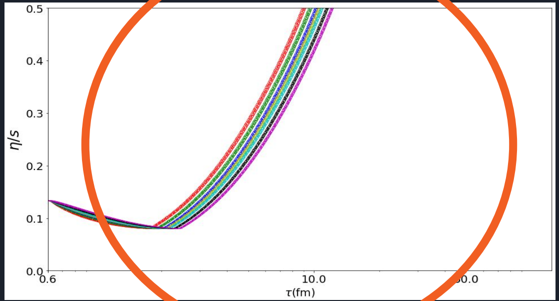
DNMR: Non-conformal, Small $\zeta > 0,\ \mu_B = 0$



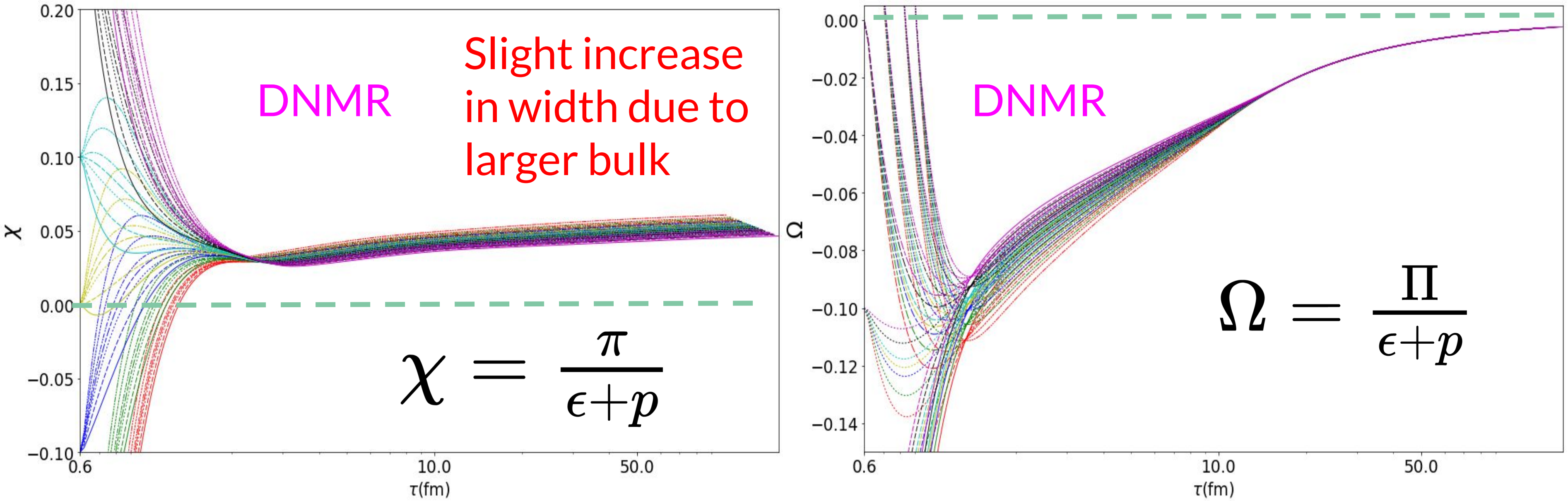


- Bulk still shows very clear attractor behavior
- Quasi-Attractor behavior in shear

➢ **System never reaches equilibrium**
  - Shear viscosity rises at low temperatures



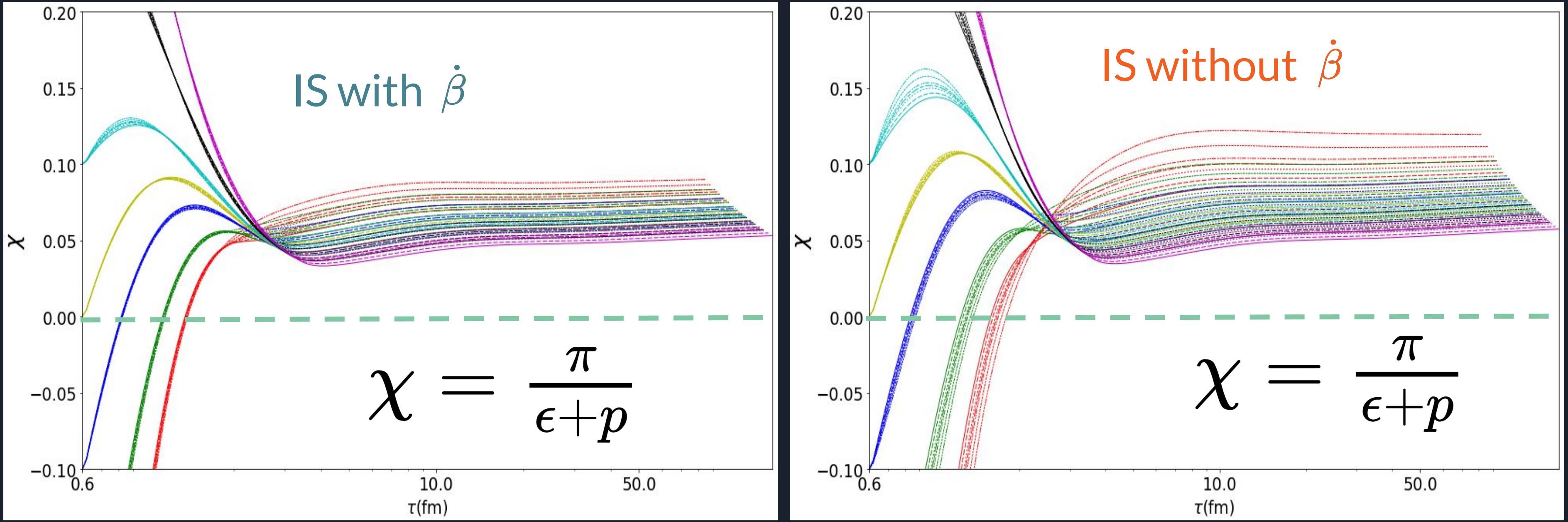
DNMR: Large Bulk Viscosity Affected *Shear* Attractor Width



- Interplay of time scales and strength of viscosities
- When is the strength more important than peak/minimum timing and vice versa

Center Manifold?

Israel-Stewart: Shear More Sensitive to Initial State



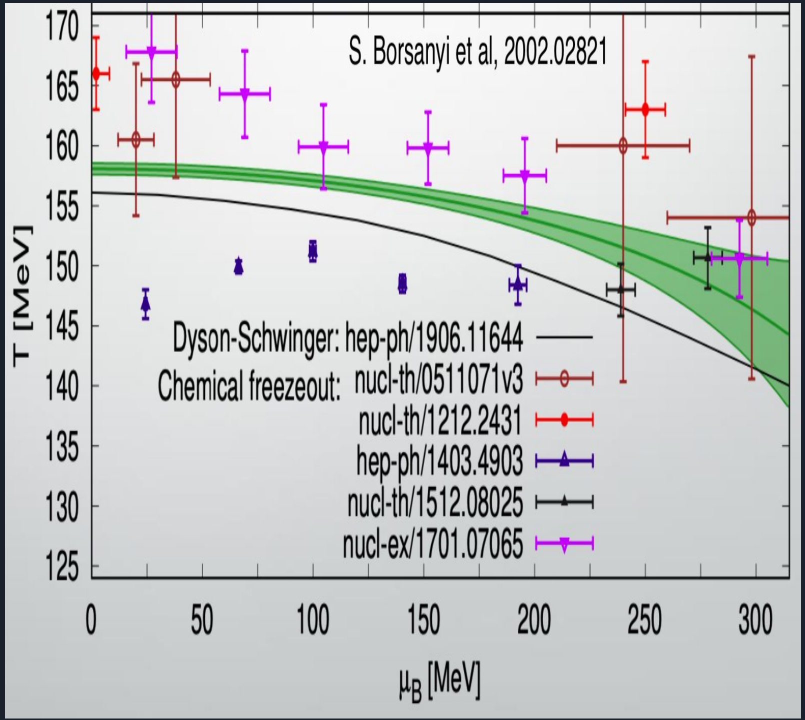
- Why do the Beta-Dot contribution generates a smaller final state width?
  - Bulk attractor persists in both cases
  - Does DNMR incorporate all of the physics of this term?
  - Might one need to include higher order terms if these derivatives are large?

Connecting Experiment and Data At Finite Baryon Density

Particle ratios inferred from Lattice QCD fluctuation calculations for given (T, μ_b)

Particle yields measured, (T, μ_b) of freeze out inferred

Can a (T, μ_b) trajectory be uniquely inferred from a freeze out point?

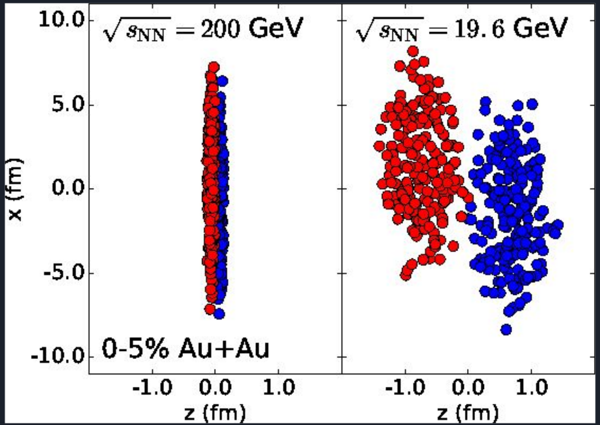


Talk From Claudia Ratti (3/3/20)

Initial Viscous Effects and Baryon Current in HIC

Another open question!

At the BES :
'Baryon Stopping' in lower energy collisions
→Initialize Baryon Current

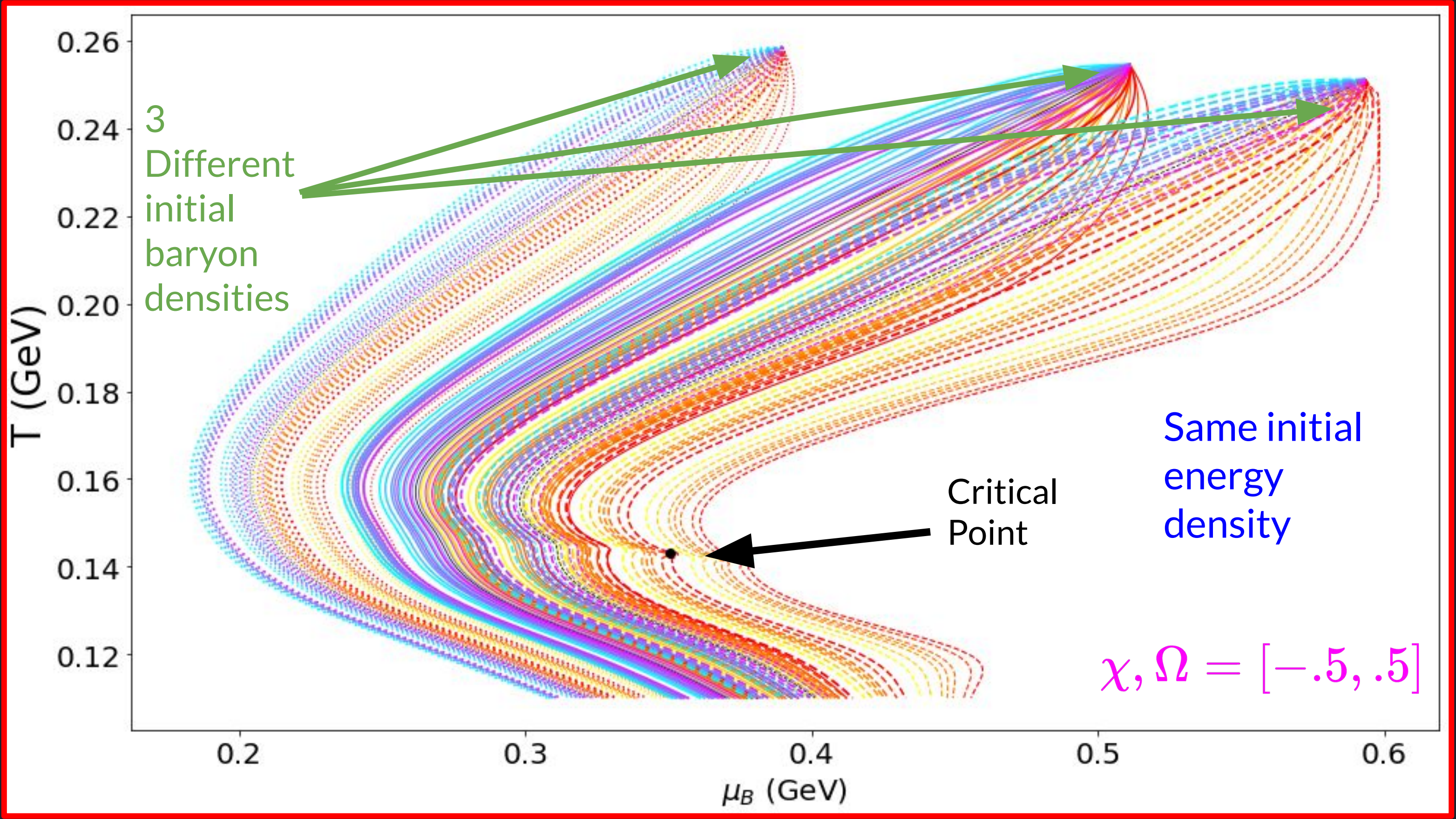


Much less attention has been given to initializing full $T^{\mu\nu}$

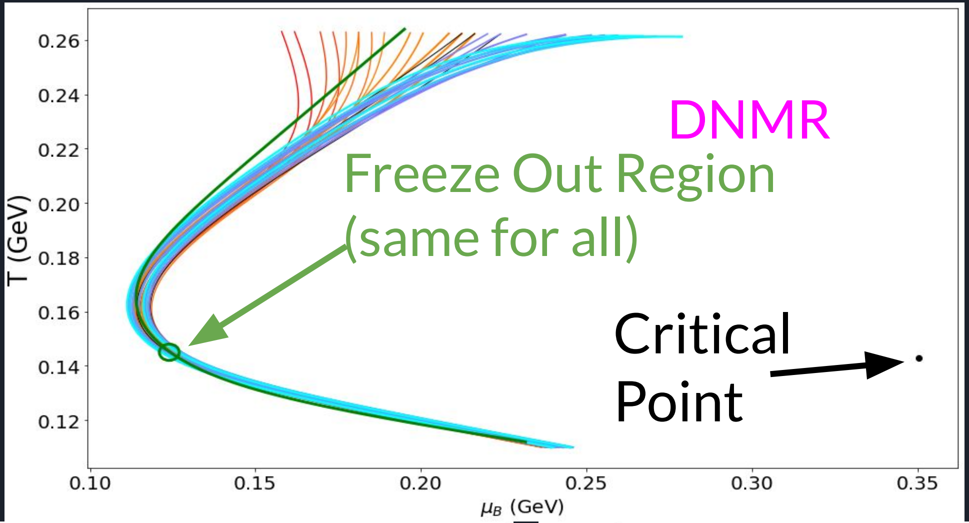
Some modelling has been done to implement this C. Shen, B. Schenke Phys Rev C. **97** 024907

More realistic Hydro equations of motion are needed for data comparisons
G. Denicol, et Al Phys. Rev. C 98, 034916

Out of Equilibrium Effects Matter



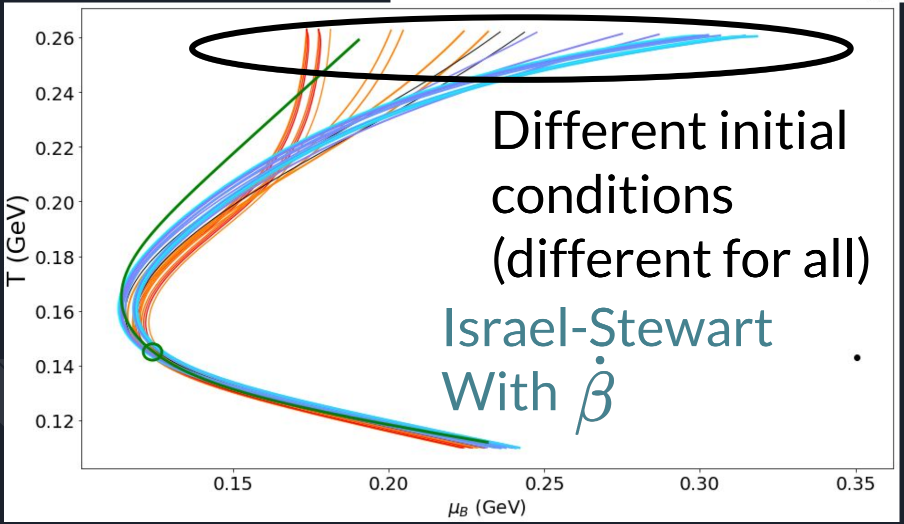
$\sqrt{s_{NN}} = 27 GeV$

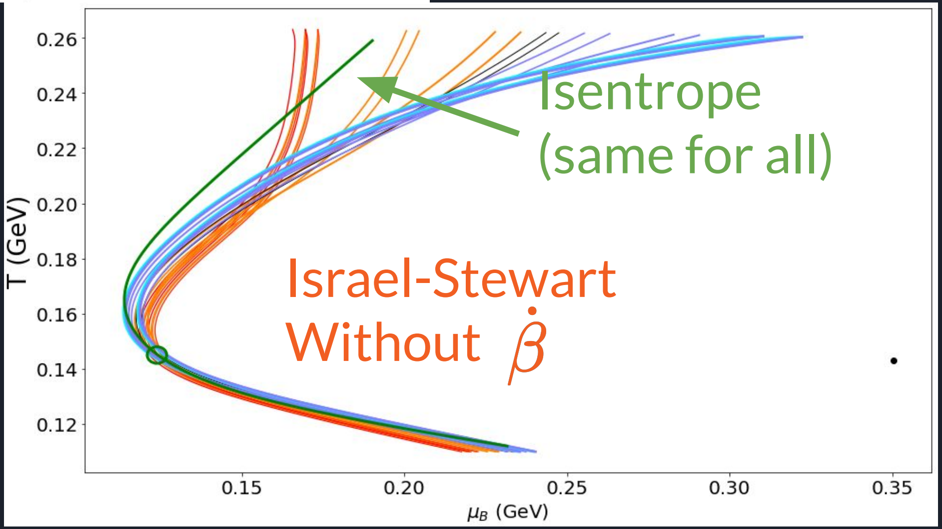


Large bulk viscosity

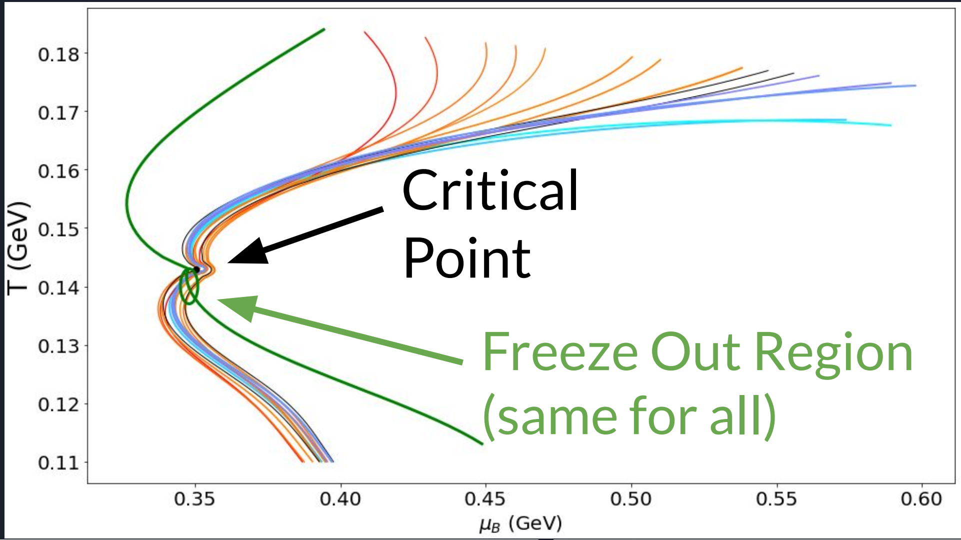
$\mu_{Bc} = 350\ MeV$

$T_c = 143\ MeV$





Critical Point effects on trajectories



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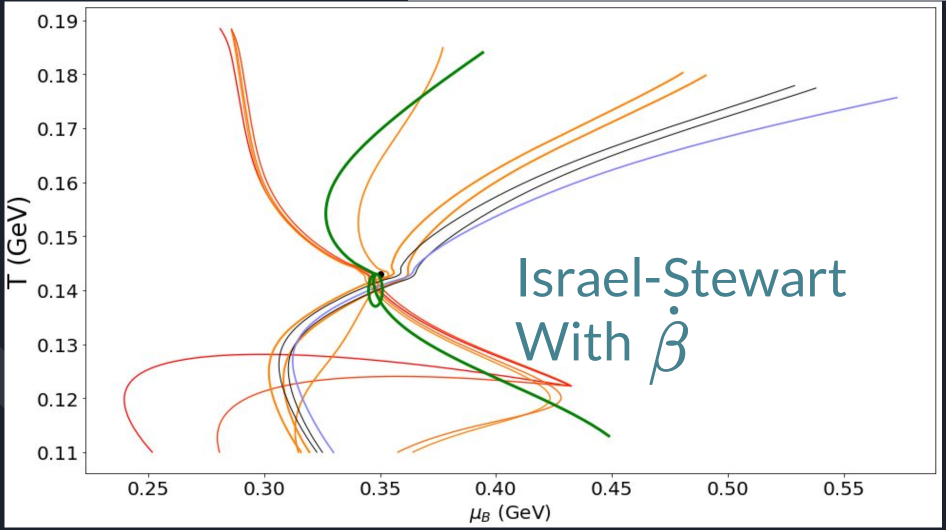
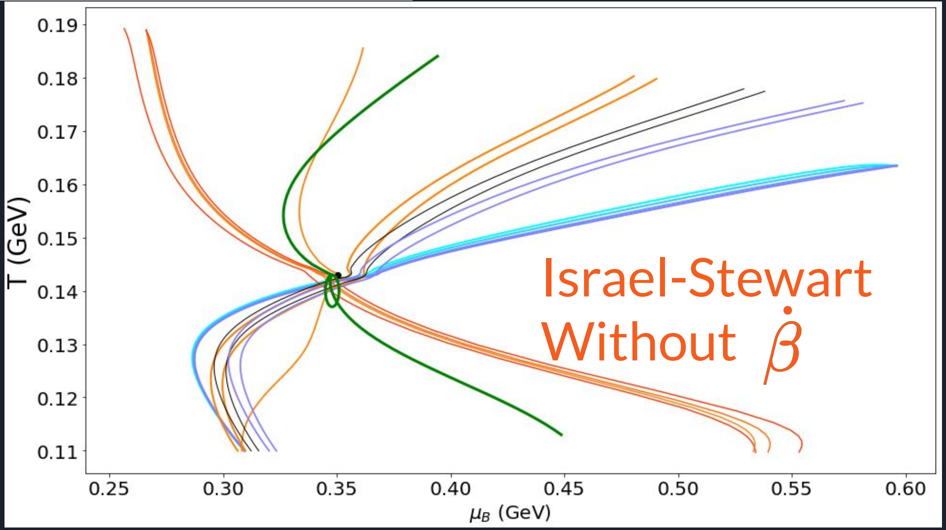
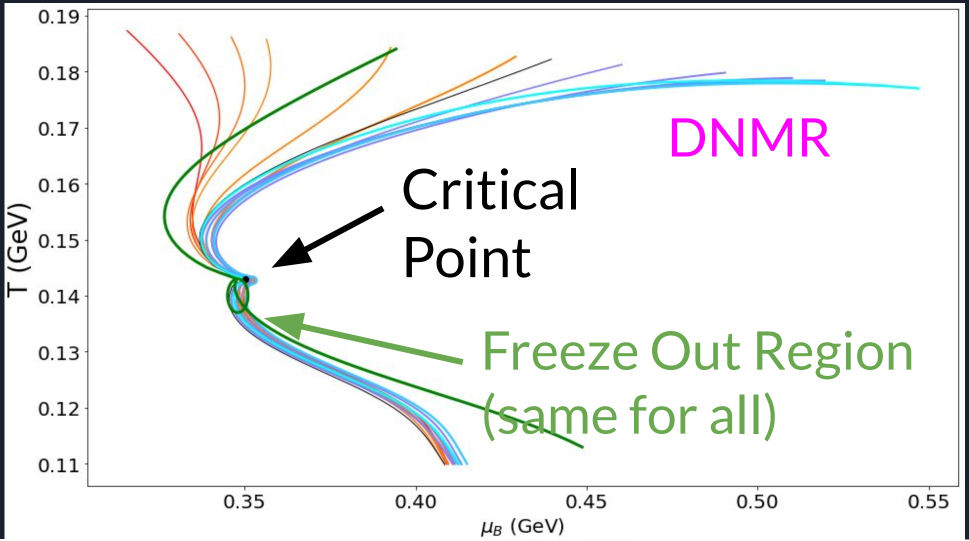




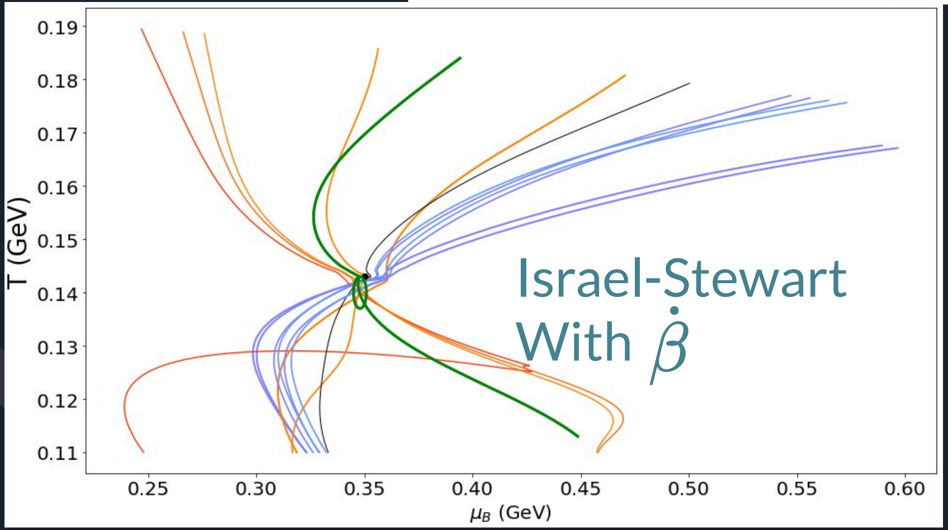
Critical Point effects on trajectories

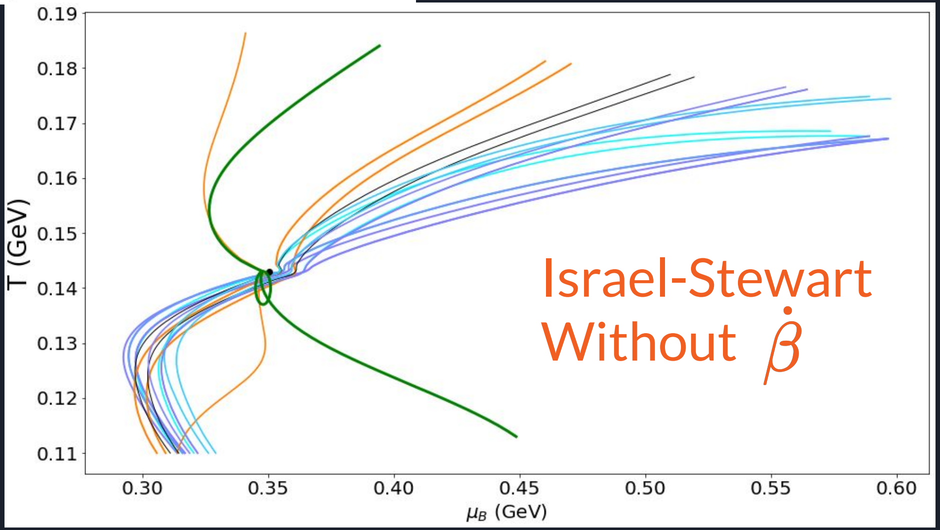


Critically scaled bulk viscosity

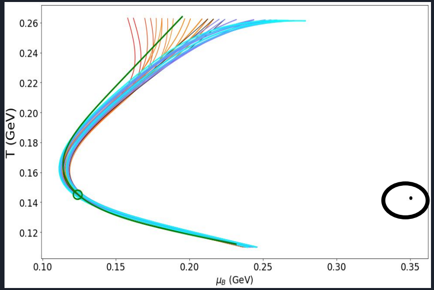
$$\mu_{Bc} = 350\ MeV$$

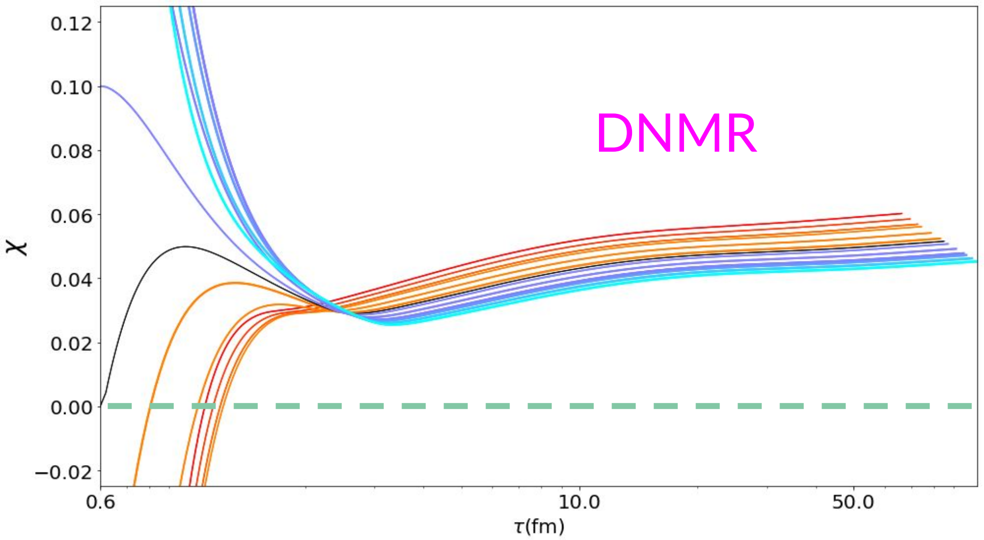
$$T_c = 143\ MeV$$





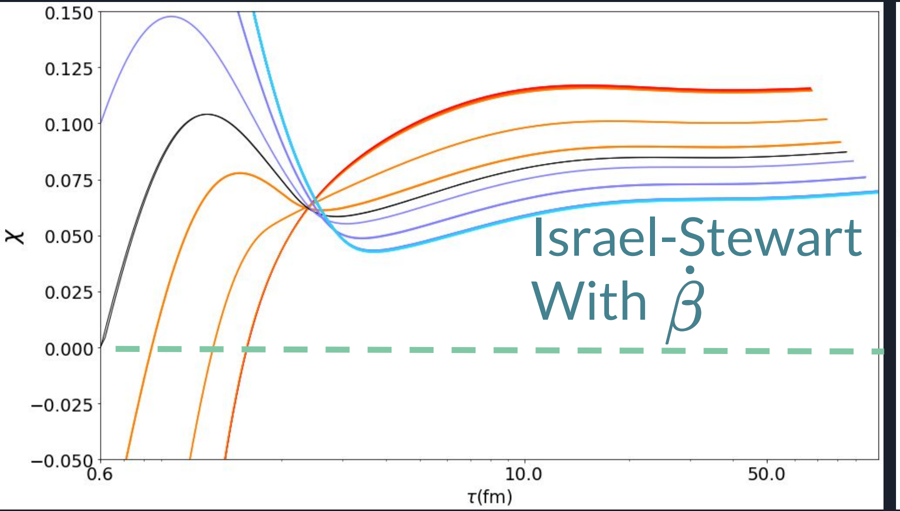
Far from CP path

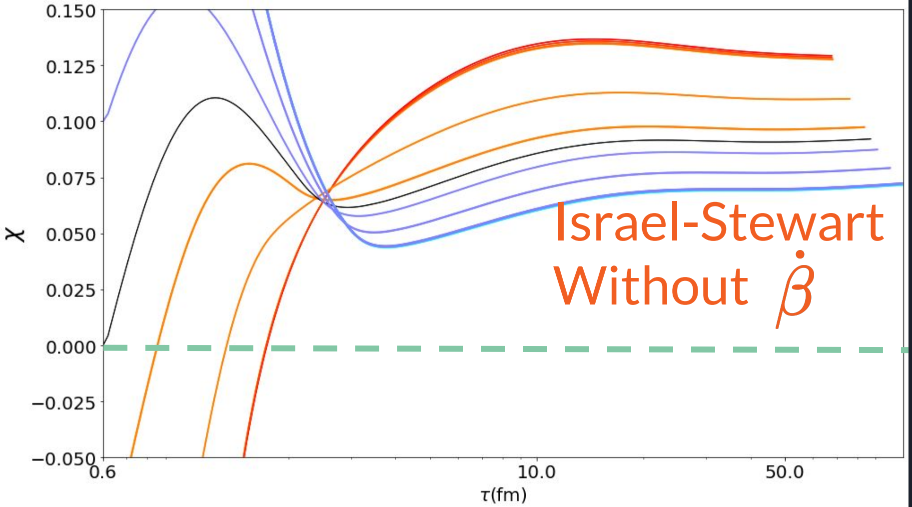




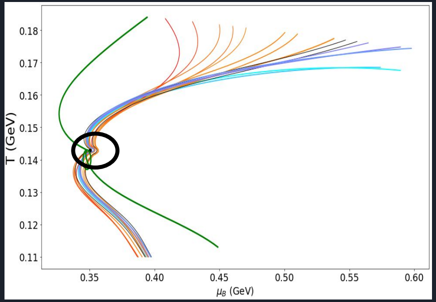
Large bulk viscosity

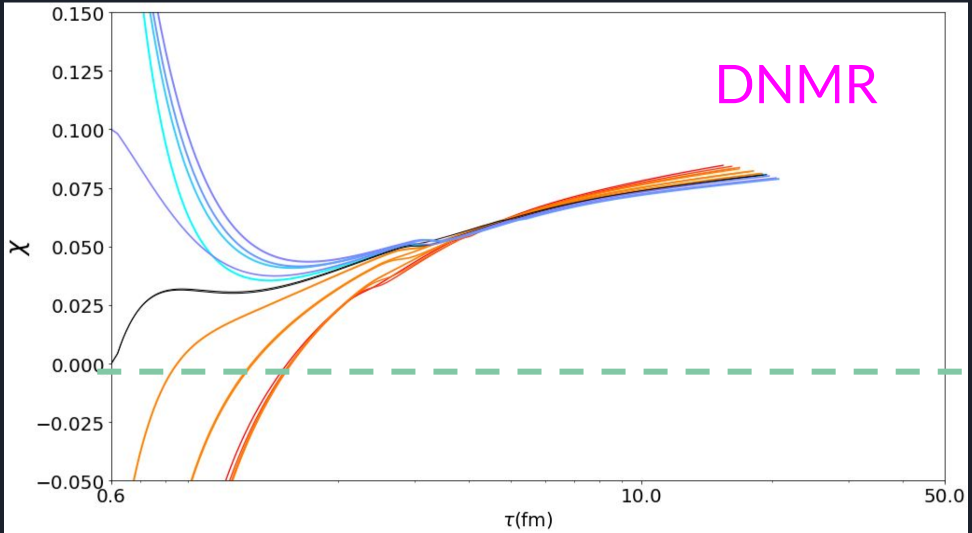
Attractor behavior clearly less well defined





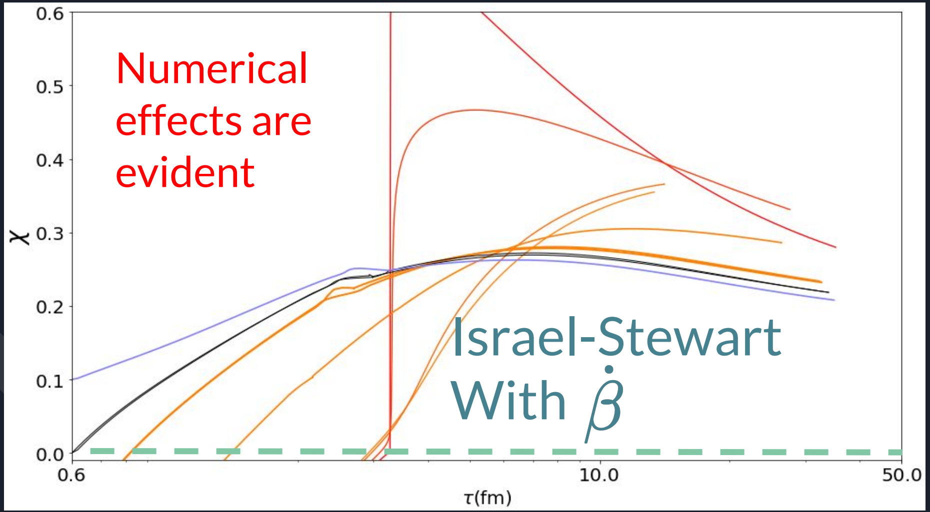
Close to CP path

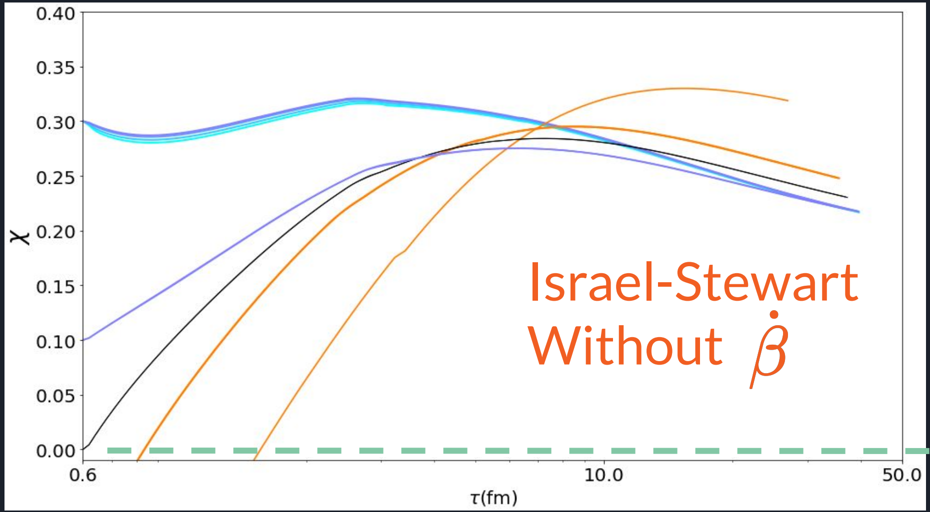




Large bulk viscosity

Are we still comfortable with attractor and equilibrium interpretations?





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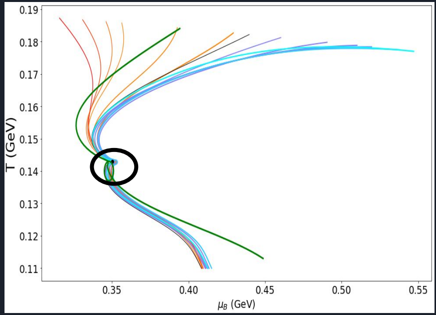

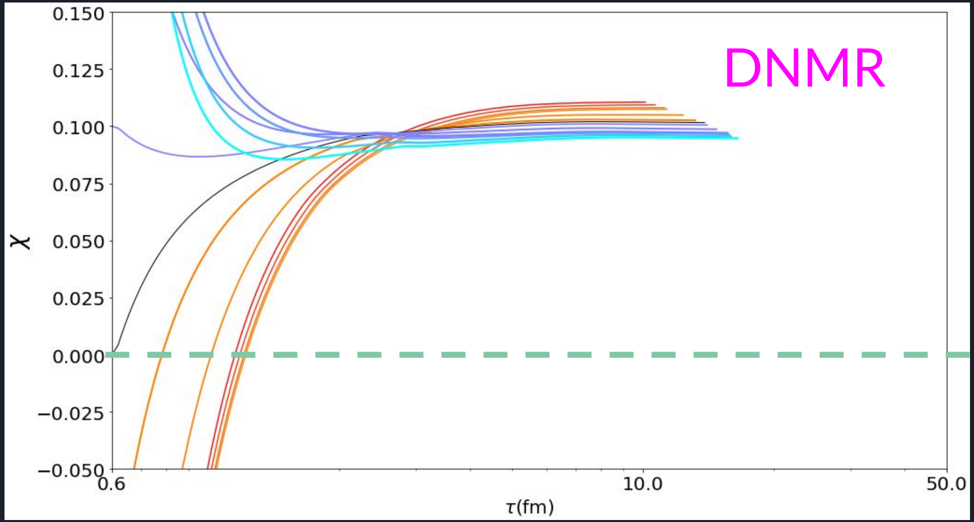

Critically scaled bulk viscosity

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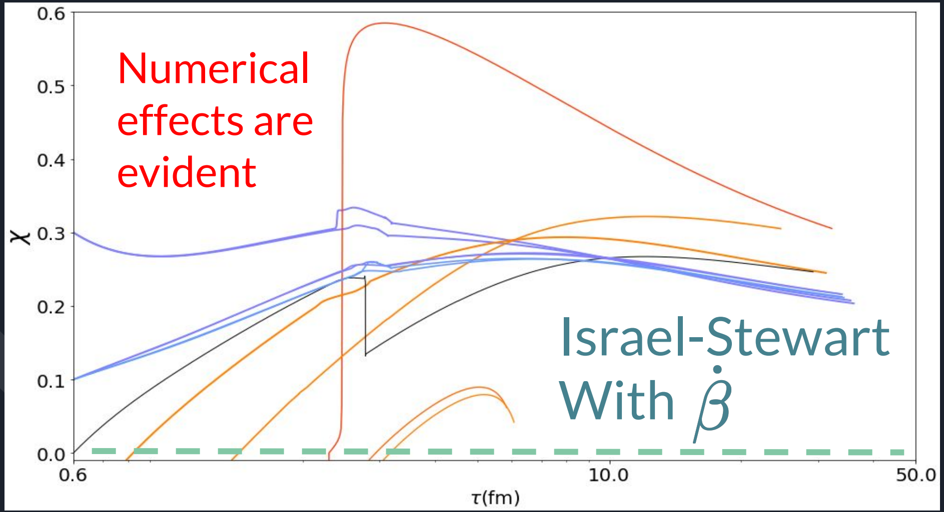
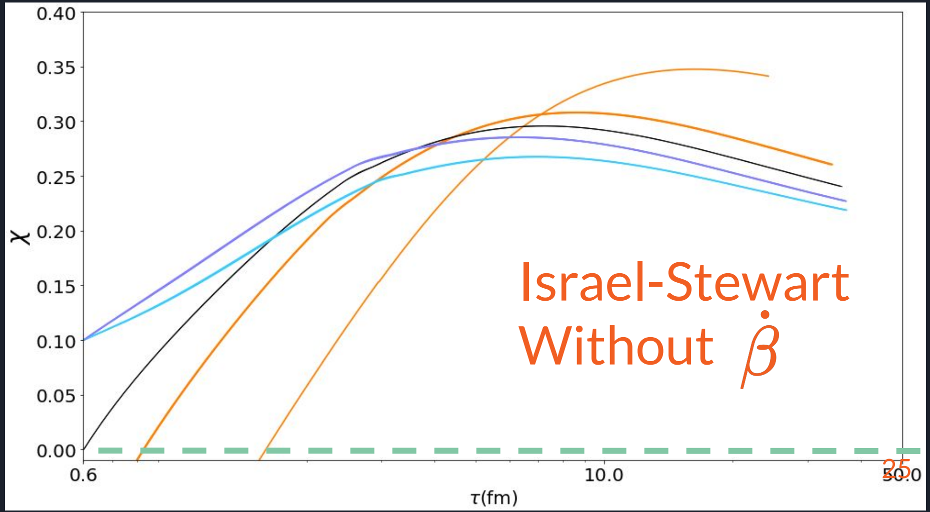




Outlook

- Attractors haven't saved us yet
  - There are clearly cases when one can break them
  - To what extent depends largely on the EoS and transport coefficients
- We should begin to think carefully about far from equilibrium effects on our interpretation of theory and data
- Ongoing work: Putting Constraints on Viscous Effects
  - Physical conditions on energy-momentum tensor
  - Entropy Production in IS like hydrodynamics

Backup Slides

Toy Model: Bjorken Symmetric Flow

Boost Invariance

$$\tau \equiv \sqrt{t^2 - z^2}$$

$$u_z = \frac{z}{\tau}, \xi = arctanh(u_z)$$

$$u_\xi = -u_t \frac{\sinh \xi}{\tau} + u_z \frac{\cosh \xi}{\tau} = 0$$

Cylindrical Symmetry

$$u_x = u_y = 0$$

So it must be that:

$$u_\mu = (1, 0, 0, 0)$$

Radial Expansion

$$\nabla_\mu u^\mu = \partial_\mu u^\mu + \Gamma^\mu_{\mu\nu} u^\nu$$

$$\Gamma^\xi_{\xi\tau} = \tau^{-1}, \Gamma^\tau_{\xi\xi} = \tau$$

$$\Rightarrow \nabla_\mu u^\mu = \frac{1}{\tau}$$

$$\tau_\pi \dot{\pi} = -\pi + \frac{4\eta}{3\tau} - \frac{1}{\tau}\left(\left(\frac{4}{3} + \lambda\right)\pi + \frac{2}{3}\lambda_{\pi\Pi}\Pi\right)$$

$$\tau_\Pi \dot{\Pi} = -\Pi - \frac{\zeta}{\tau} - \frac{1}{\tau}\left(\delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi\right)$$

$$\tau_\pi \dot{\pi} = -\pi - \frac{4\eta}{3\tau} - \frac{\pi}{2\tau} - \dot{\beta}_\pi$$

$$\tau_\Pi \dot{\Pi} = -\Pi - \frac{\zeta}{\tau} - \frac{\Pi}{2\tau} - \dot{\beta}_\Pi$$

$$\dot{\rho} = \frac{\rho_0}{\tau}$$

Valid for both given symmetries

Transport coefficients

$$\frac{\zeta}{\tau_\Pi} = 15\left(\frac{1}{3} - c_s^2\right)^2(\epsilon + p)$$

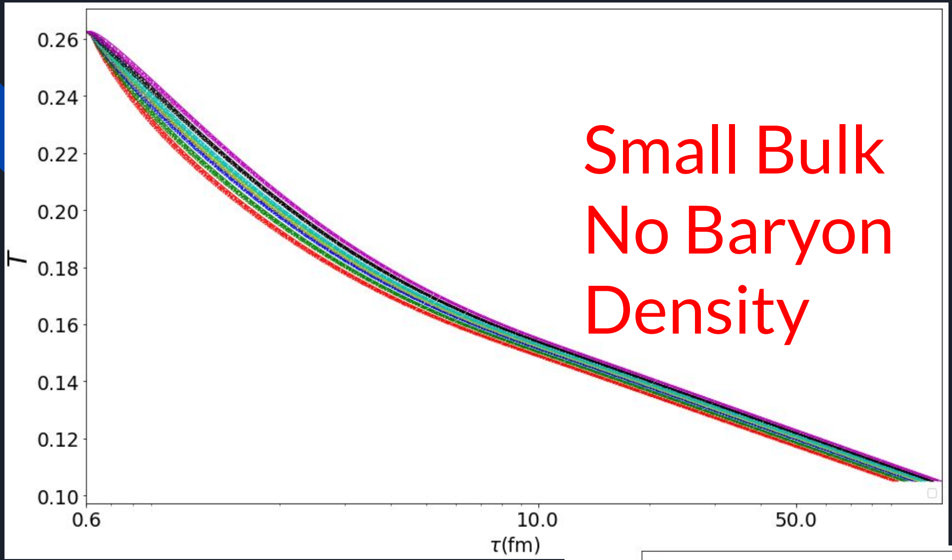
$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}$$

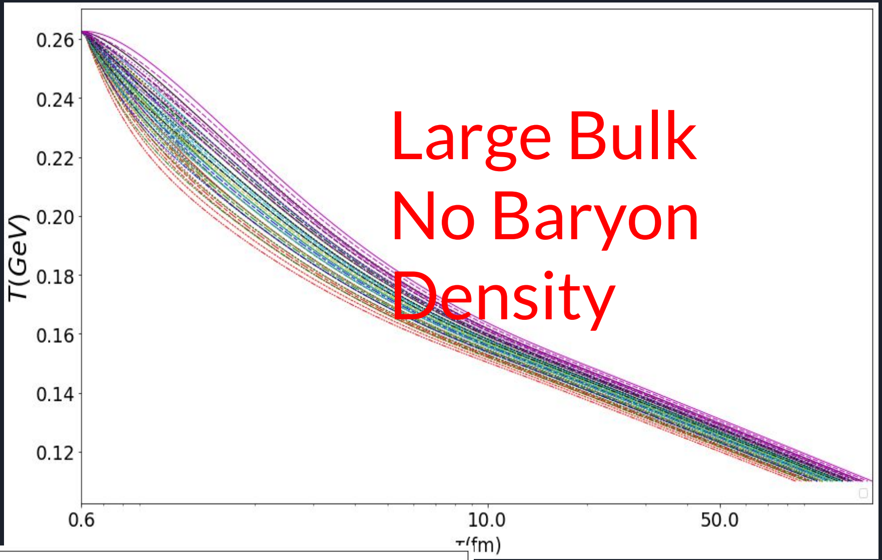
$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)$$

$$\frac{\eta}{\tau_\pi} = \frac{\epsilon+p}{5}$$

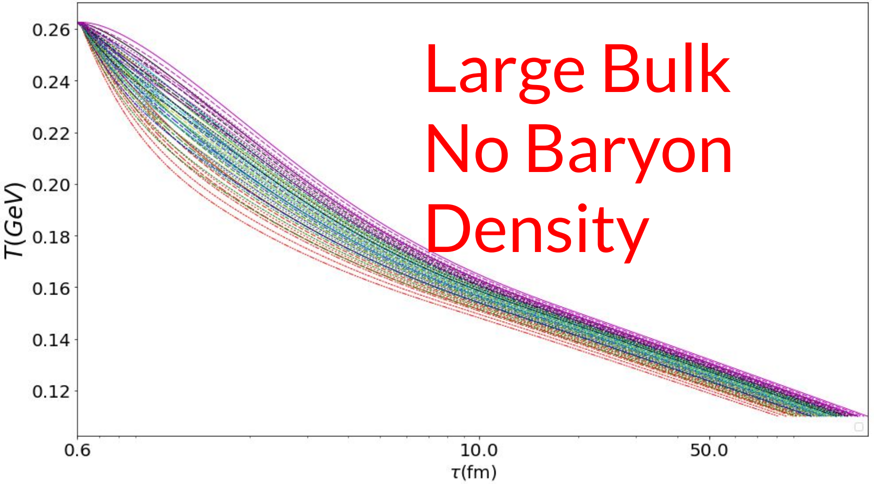
$$\frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}$$

$$\frac{\lambda}{\tau_\pi} = \frac{10}{7}, \quad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}$$





Beta Dot



No Beta Dot