From small to large x:

toward a unified description of high energy collisions

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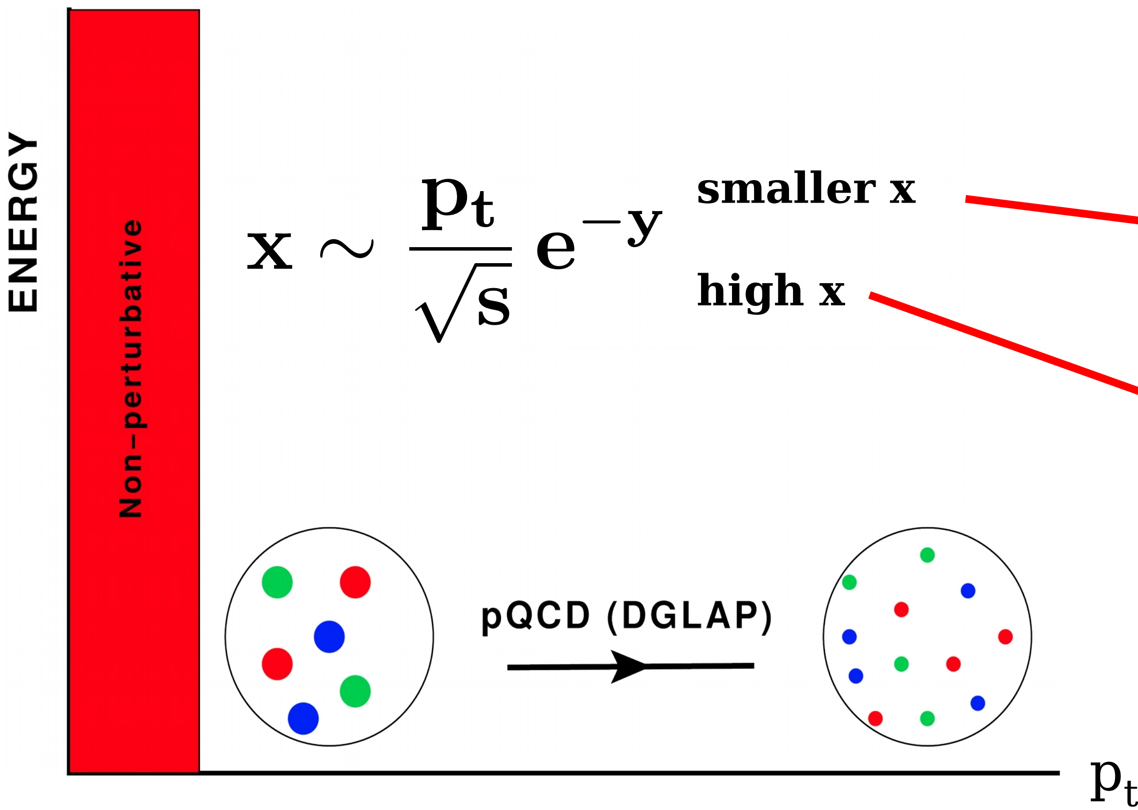
pQCD: the standard paradigm

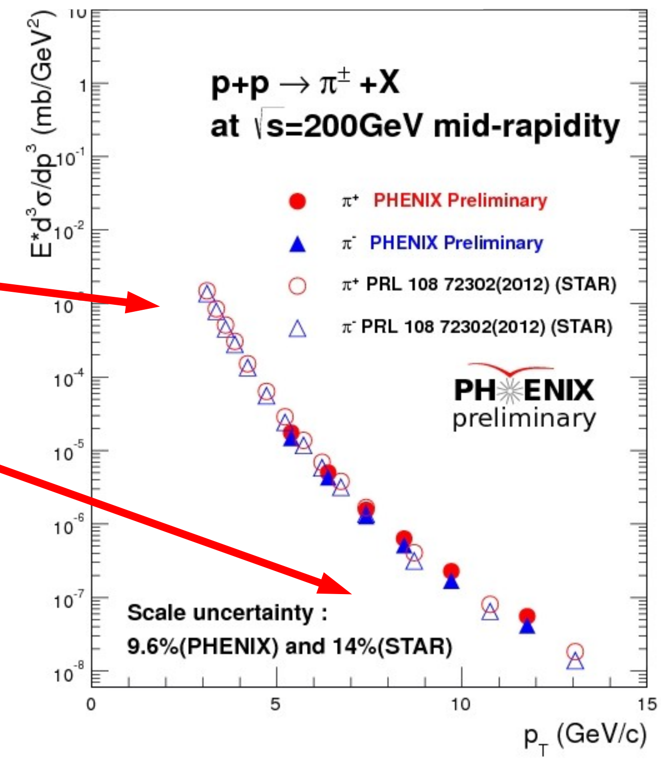
$$\mathbf{E}\,\frac{\mathbf{d}\sigma}{\mathbf{d^3p}} \sim \mathbf{f_1(x, p_t^2)} \otimes \mathbf{f_2(x, p_t^2)} \otimes \frac{\mathbf{d}\sigma}{\mathbf{dt}} \otimes \mathbf{D(z, p_t^2)} + \cdots\cdots$$

$$\mathbf{x} \sim \frac{\mathbf{p_t}}{\sqrt{\mathbf{s}}}\,\mathbf{e^{-y}}$$

smaller x

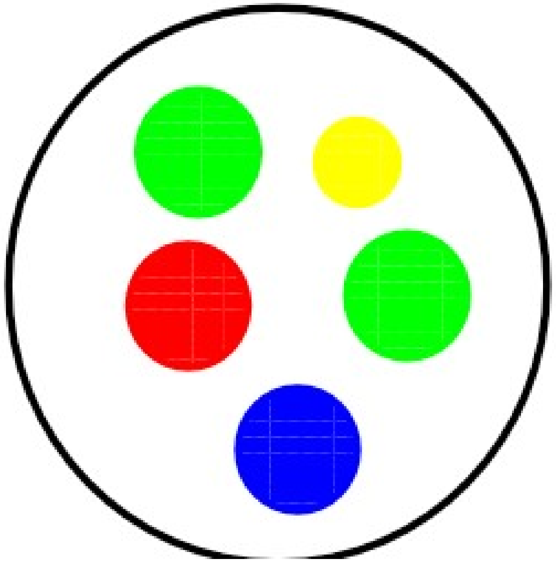
high x





bulk of QCD phenomena happens at low p_t (small x)

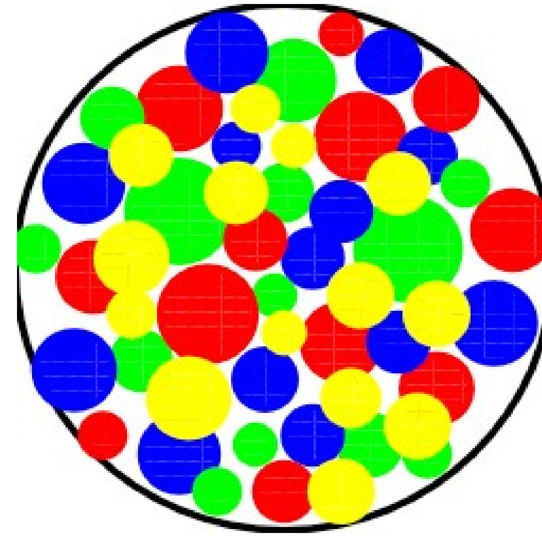
Gluon saturation (Gribov, Levin, Ryskin, early 80's)



$S \to \infty, \;\; Q^2 \;\; fixed$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$

gluon radiation $\mathbf{P_{gg}(x) \sim \frac{1}{x}}$



$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi R^2} \sim 1$$

<u>McLerran-Venugopalan</u> (93)

saturation scale $\mathbf{Q_s^2(x, b_t, A)}$

$\alpha_{\mathbf{s}}(\mathbf{Q_s^2}) \ll \mathbf{1}$

<u>MV: an effective action approach to QCD at high energy</u>

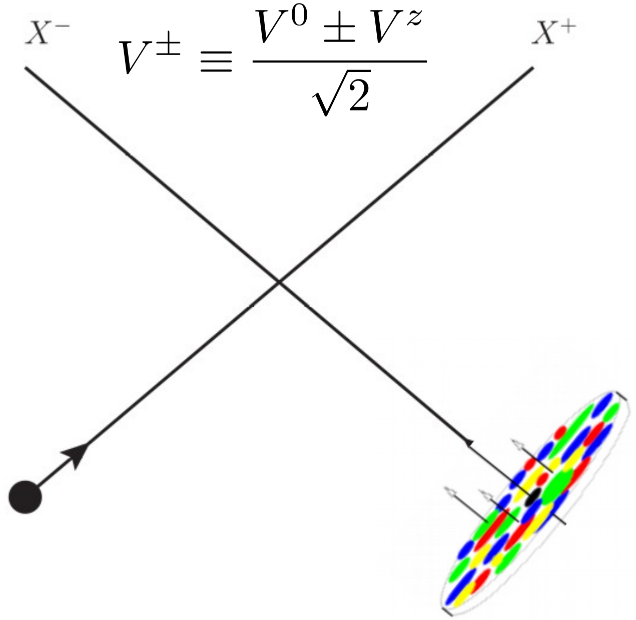
novel and exciting phenomena

universal properties

Scattering at high energy (small x) *(proton-nucleus)*

Eikonal approximation

$$V^{\pm} \equiv \frac{V^0 \pm V^z}{\sqrt{2}}$$



$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$

$$D_\mu \, J^\mu \;=\; D_- \, J^- = 0$$

$$\partial_- J^- \;=\; 0 \qquad \text{(in } A^+ = 0 \text{ gauge)}$$

does not depend on x^-

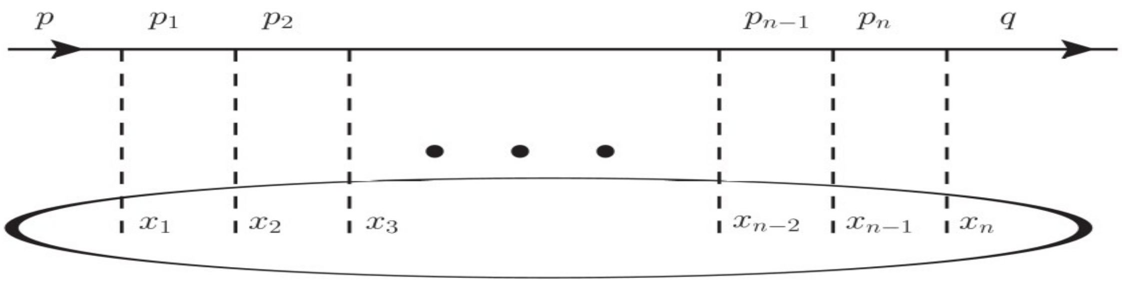
solution to EOM: $$A_a^-(x^+, x_t) \equiv n^- \; S_a(x^+, x_t)$$

with
$$n^\mu \;=\; (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$n^2 \;=\; 2\, n^+ n^- - n_\perp^2 = 0$$

recall (eikonal limit): $$\bar{u}(q)\gamma^\mu u(p) \to \bar{u}(p)\gamma^\mu \, u(p) \sim p^\mu$$
$$\bar{u}(q) \not{A} \, u(p) \to p \cdot A \sim p^+ \, A^-$$

multiple scattering of a quark from background color field $S_a(x^+, x_t)$

$$A_a^-(x^+, x_\perp) \equiv n^- \, S_a(x^+, x_\perp)$$

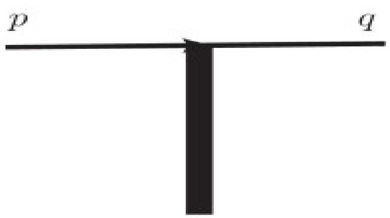


$$\begin{aligned} i\mathcal{M}_n &= 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\ &\Big\{(ig)^n\,(-i)^n (i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \\ &\left[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t)\right] \Big\}\, u(p) \end{aligned}$$

$$i\mathcal{M} = \sum_n i\,\mathcal{M}_n$$

$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\,\left[V(x_t) - 1\right]\, u(p)$$

with $V(x_t) \equiv \hat{P}\, \exp\left\{ig \int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a\right\}$



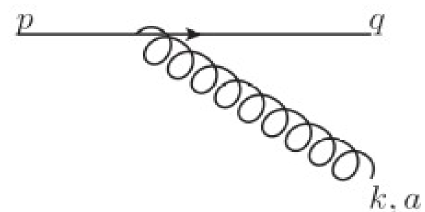
DIS, proton-nucleus collisions involve dipoles

$$< Tr\, V(x_\perp)\, V^\dagger(y_\perp) > \sim e^{-r_t^2\, Q_s^2\, \log 1/r_t^2\, \Lambda^2}$$ **MV model**

scattering from small x modes of the target can cause only a *small angle deflection*

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



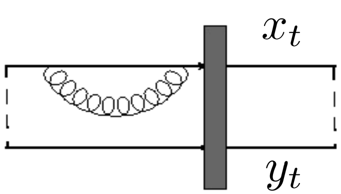
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^\mu_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

x_t, z_t are transverse coordinates of the quark and gluon

virtual corrections:



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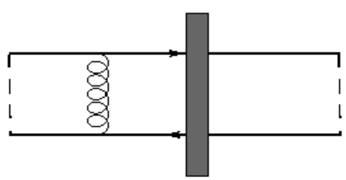
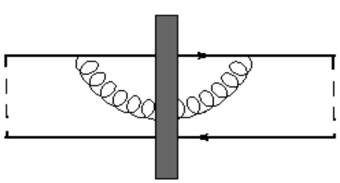


$+ \quad \longrightarrow Tr\,V(x_t)\,V^\dagger(y_t)$ *a dipole*

real corrections:



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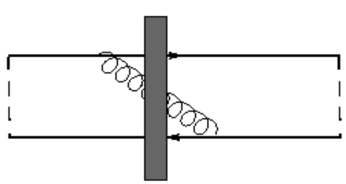


$+ \quad \longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

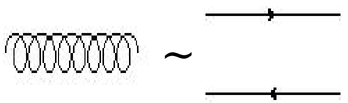
the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c} Tr\,V(x_t)\,V^\dagger(y_t)$$

1-loop correction: BK eq.

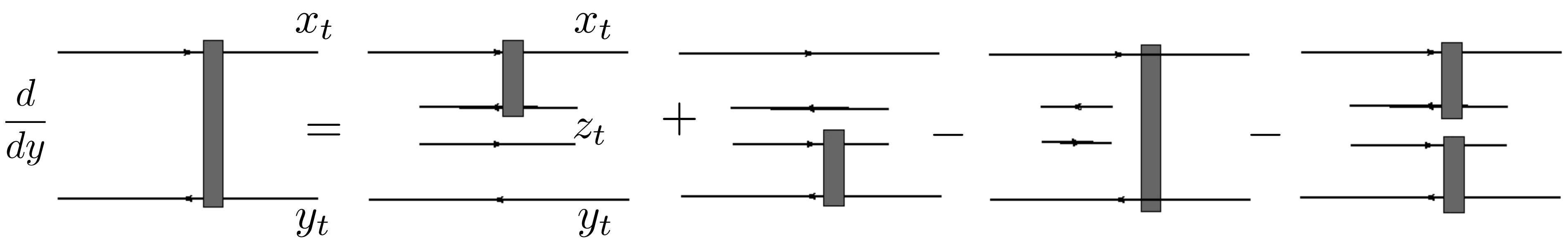
at large N_c

$$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$$



$$\frac{d}{dy} T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t) T(z_t, y_t) \right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \gg p_t^2$$

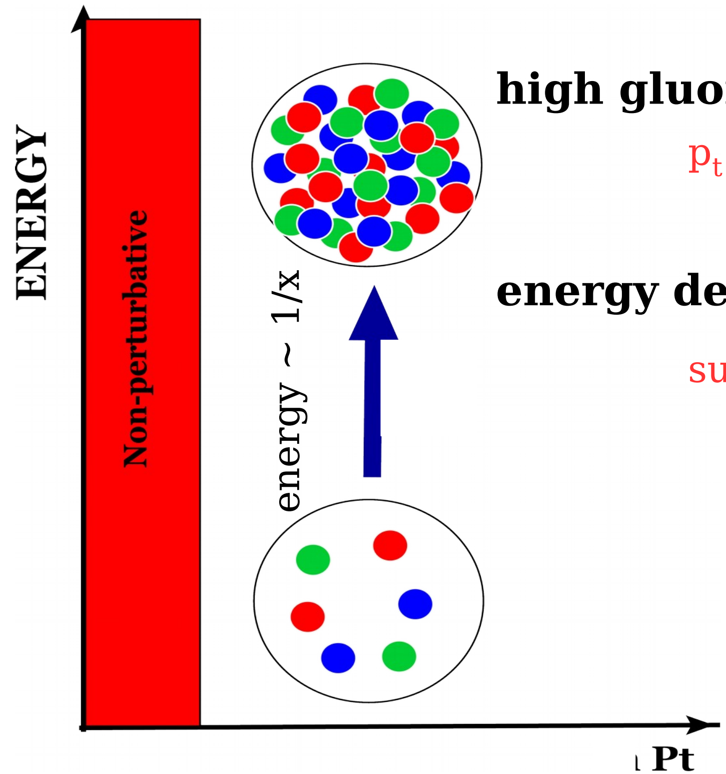
$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t \, dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t \, dy}}$$

nuclear shadowing
suppression of p_t spectra
disappearance of away side peak
....

A hadron/nucleus at high energy: CGC



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Shadowing/Nuclear modification factor
Azimuthal angular correlations (di-jets,...)
Long range rapidity correlations (ridge,...)
Initial conditions for hydro
Thermalization ?

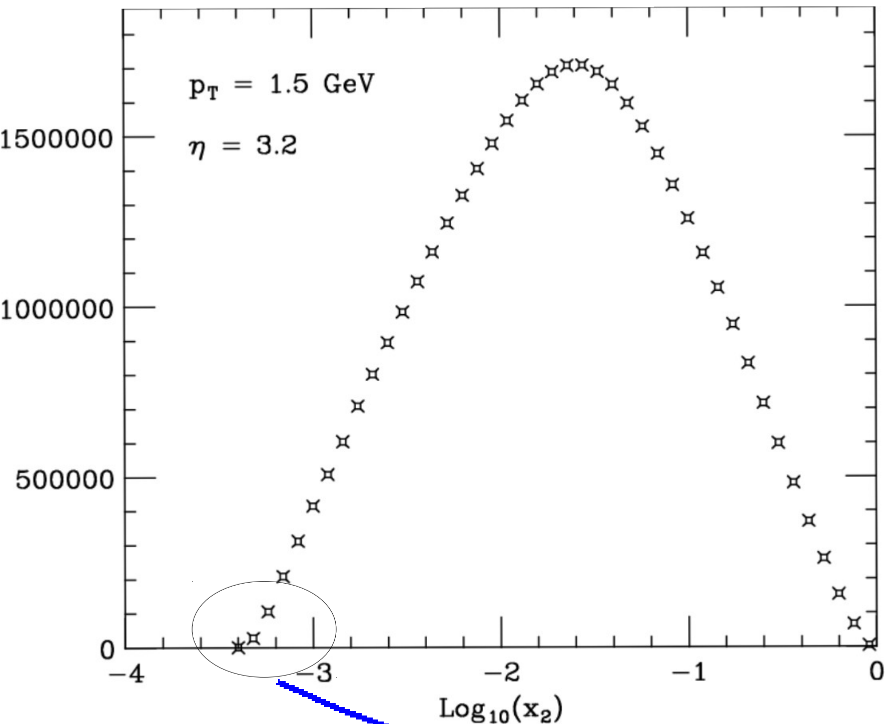
$$\mathbf{x \leq 0.01}$$

NLO: toward precision

Pion production at RHIC: kinematics

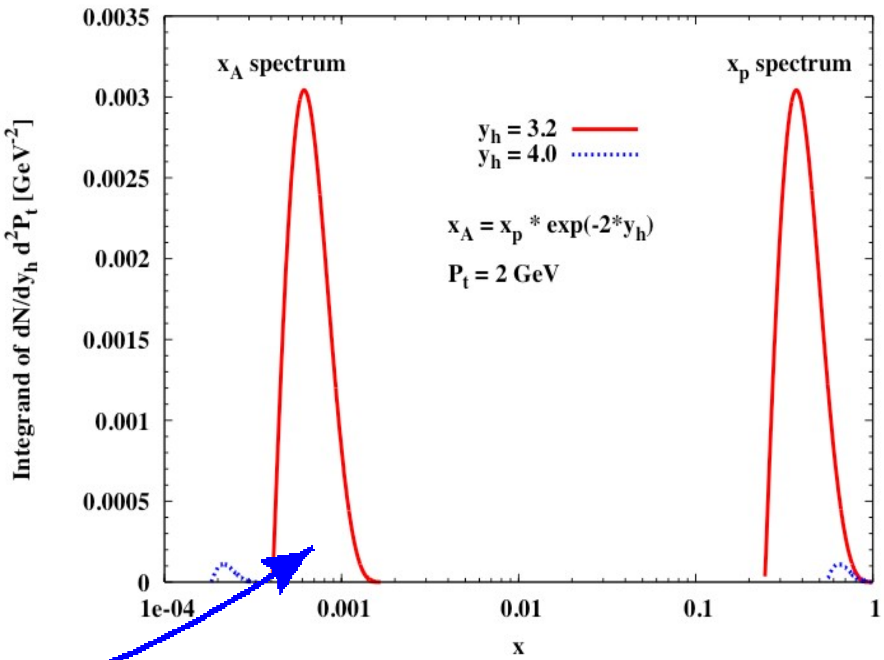
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

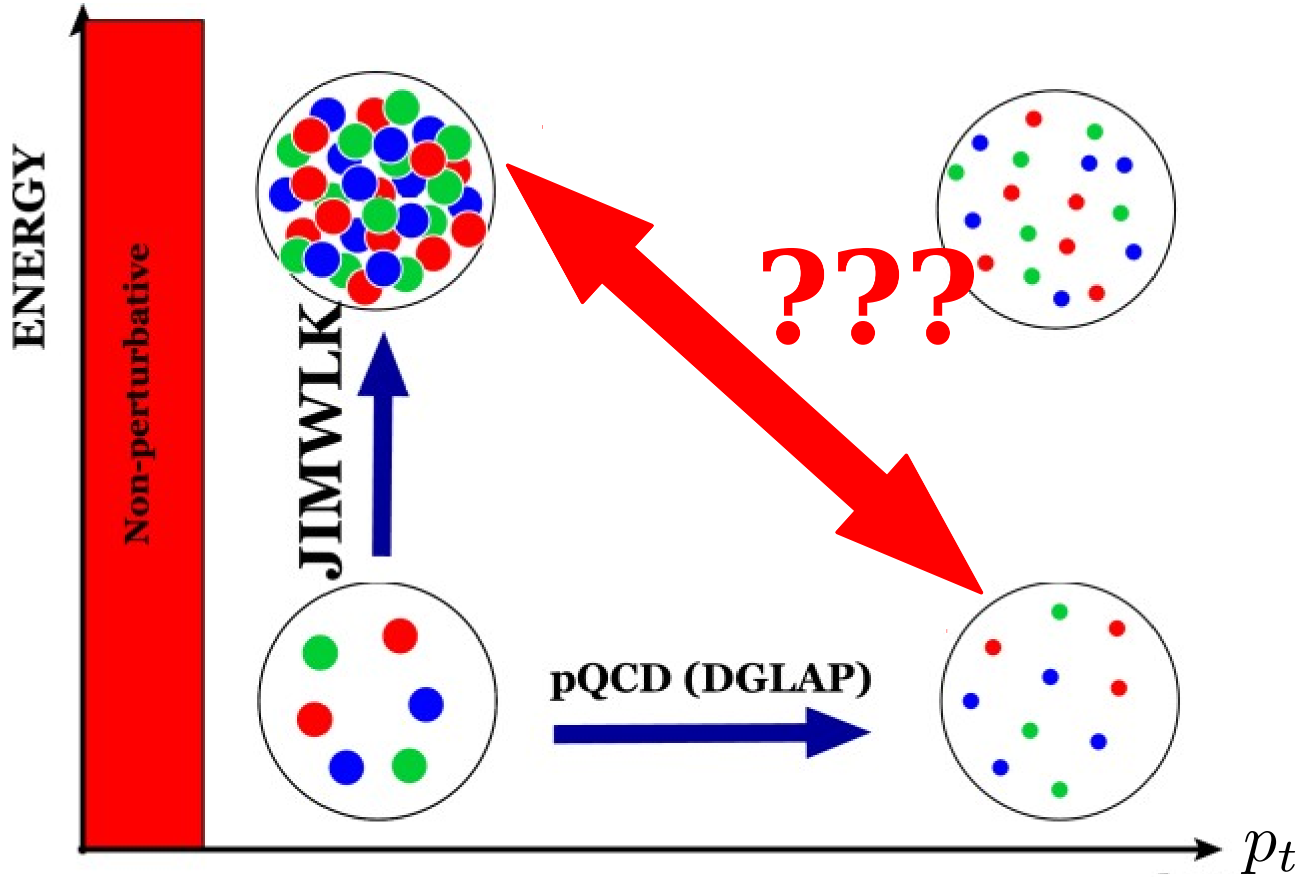
DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x,Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min},Q^2)\cdots$$

this is an extreme approximation with severe consequences!

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

<u>kinematics of saturation: *where is saturation applicable?*</u>

jet physics, high p_t (polar and azimuthal) angular correlations

cold matter energy loss, spin physics, ...

beyond eikonal approximation: tree level

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{S}, p^- = 0, p_\perp = 0)$$

$$A^- : (p^+ \sim 0, \frac{p^-}{P^-} = x \ll 1, p_\perp = 0)$$

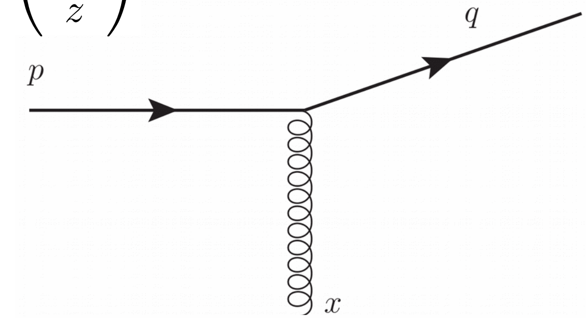
allow hard scattering by including one hard field $A_a^\mu(x^+, x^-, x_t)$ during which large momenta can be exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

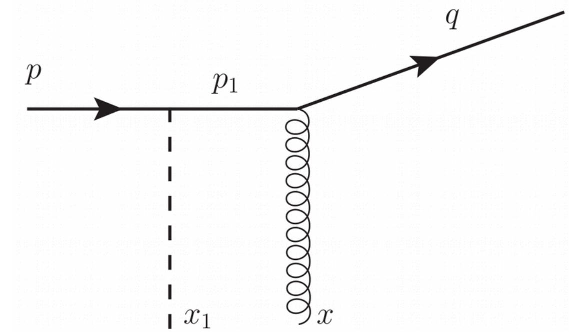
hard scattering: large deflection
scattered quark travels in the new “z” direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

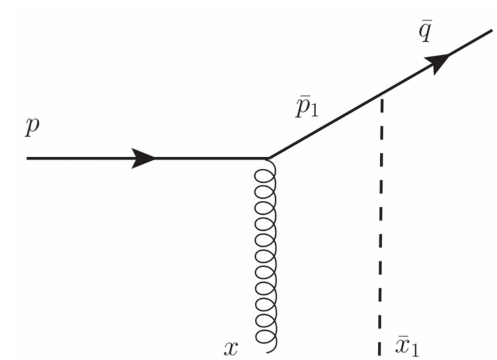
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x) \right] \, u(p)$$



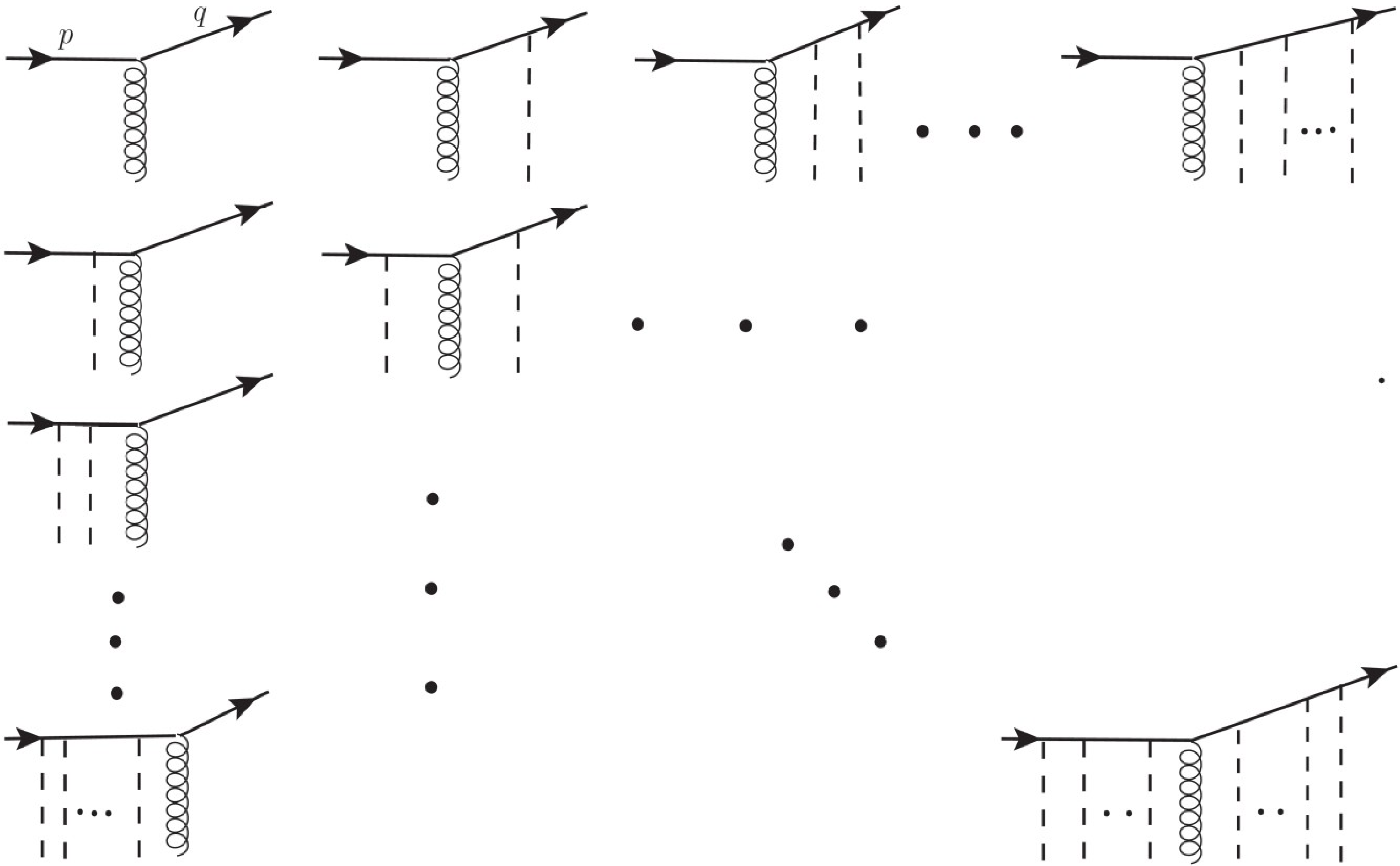
$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \\ & \qquad \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] u(p) \end{aligned}$$



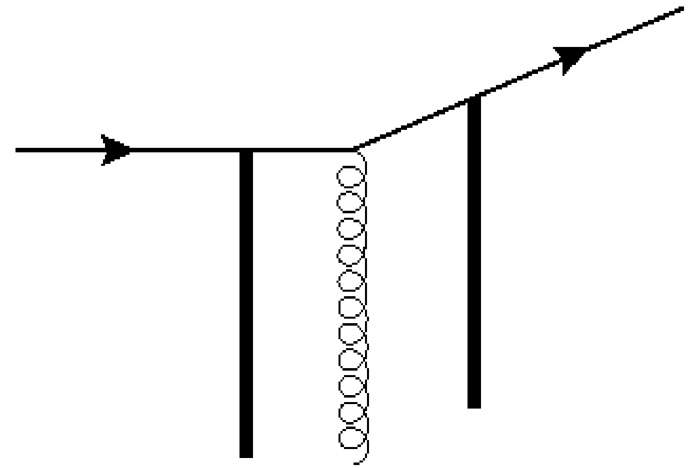
$$\begin{aligned} i\mathcal{M}_2 \quad &= \quad (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \\ & \qquad \bar{u}(\bar{q}) \left[\not{\bar{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x) \right] u(p) \end{aligned}$$



with $\quad \vec{\bar{v}} = \mathcal{O} \, \vec{v}$


p
q

summing all the terms gives:



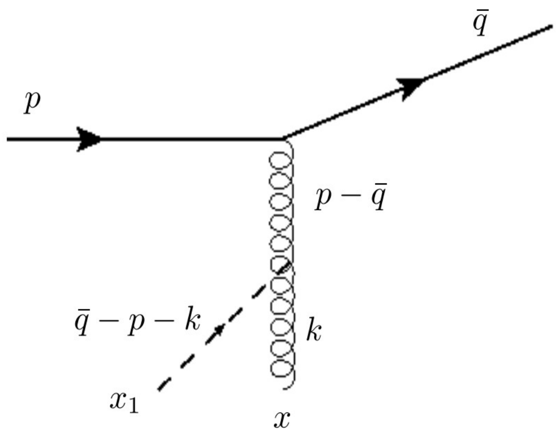
$$
\begin{aligned}
i\mathcal{M}_1 \quad = \quad & \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2} \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t} \\
& \bar{u}(\bar{q}) \left[\overline{V}_{AP}(x^+,\bar{z}_t)\, \not{\bar{n}}\, \frac{\not{\bar{k}}}{2\bar{k}^+}\, \left[ig\not{A}(x)\right]\, \frac{\not{k}}{2k^+}\, \not{n}\, V_{AP}(z_t,x^+) \right] u(p)
\end{aligned}
$$

with

$$
\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\, \exp\left\{ ig \int_{x^+}^{+\infty} d\bar{z}^+\, \bar{S}_a^-(\bar{z}_t,\bar{z}^+)\, t_a \right\}
$$

$$
V_{AP}(z_t,x^+) \equiv \hat{P}\, \exp\left\{ ig \int_{-\infty}^{x^+} dz^+\, S_a^-(z_t,z^+)\, t_a \right\}
$$

interactions of large and small x modes

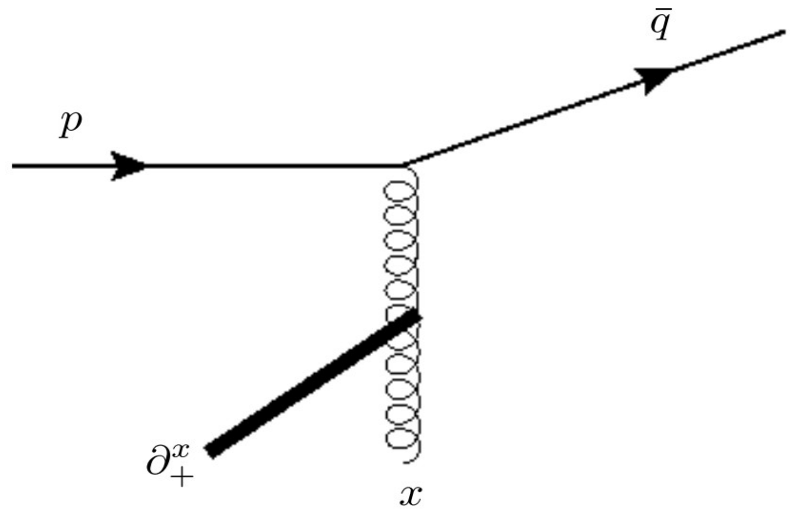


$$
\begin{aligned}
i\mathcal{M} \;=\;& f_{acd} \int \frac{d^4k}{(2\pi)^4}\, d^4x\, d^4x_1\, e^{i(\bar{q}-p-k)x_1}\, e^{ikx} \\
& \bar{u}(\bar{q})\,(ig\,\gamma^\mu\, t^a)\, u(p)\, A^c_\lambda(x)\ \left[i\,g\,S^d(x_1)\right] \\
& \frac{1}{(p-\bar{q})^2 + i\epsilon} \Bigg[-g^\mu_\lambda\, n\cdot(p-\bar{q}-k) \\
& + n^\mu \left[p_\lambda - \bar{q}_\lambda\,(1 - \frac{n\cdot k}{n\cdot(p-\bar{q})}) \right] \Bigg]
\end{aligned}
$$

performing k$^-$ integration sets $x_1^+ = x^+$

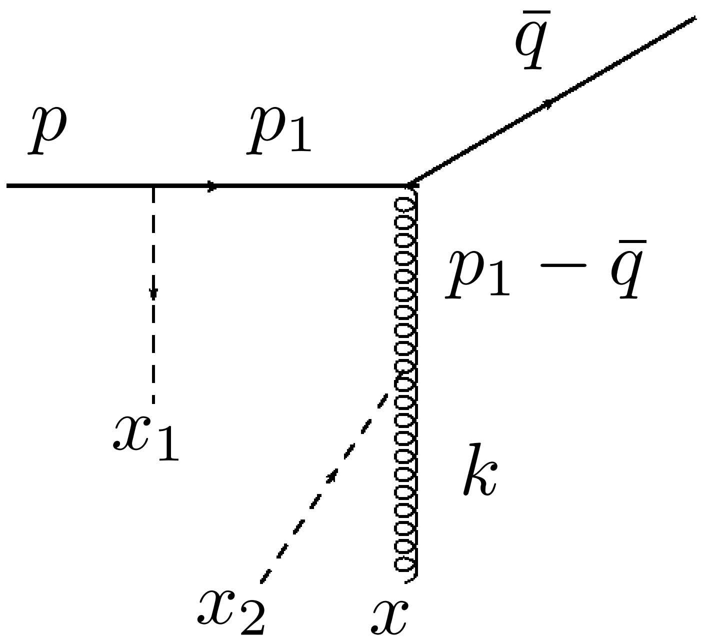
$$
\begin{aligned}
i\mathcal{M} \;=\;& 2f_{acd} \int d^4x\, e^{i(\bar{q}-p)x} \\
& \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_c(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p)\ \left[i\,g\,S^d(x^+, x_t)\right]
\end{aligned}
$$

all re-scatterings of hard gluon can be re-summed



$$i\mathcal{M}_2 \quad = \quad \frac{2\,i}{(p-\bar{q})^2}\int d^4x\, e^{i(\bar{q}-p)x}\,\bar{u}(\bar{q})\left[(ig\,t^a)\,\left[\partial_{x^+}\,U^{\dagger}_{AP}(x_t,x^+)\right]^{ab}\right.$$
$$\left.\left[n\cdot(p-\bar{q})\not{A}_b(x)-(p-\bar{q})\cdot A_b(x)\,\not{n}\right]\right]u(p)$$

with $$U_{AP}(x_t,x^+)\equiv \hat{P}\,exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z^+,x_t)\,T_a\right\}$$



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\ [(p_1 - \bar{q})^2 + i\epsilon]}$$

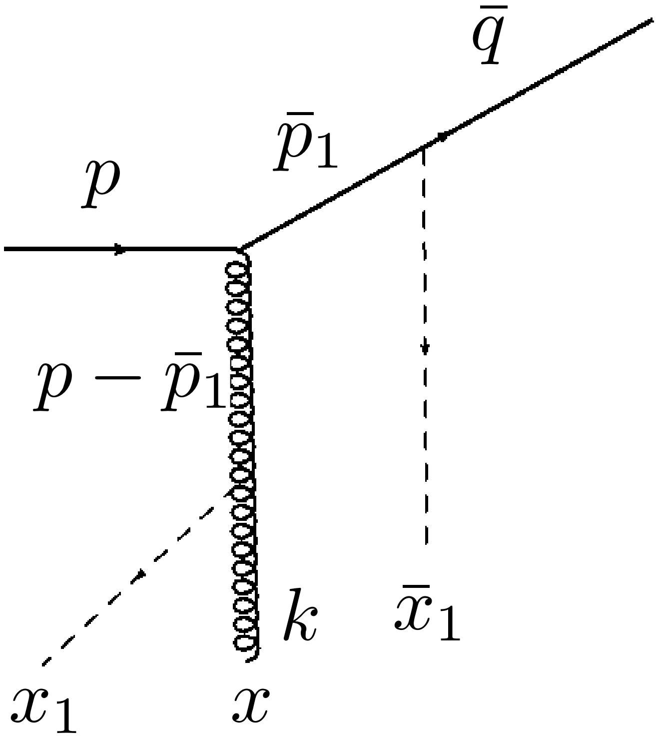
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\ [(p_1-\bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

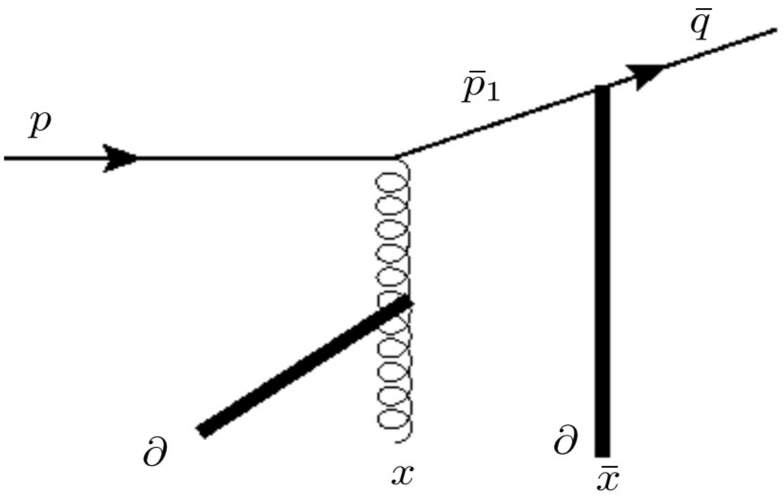
$\theta(x^+ - \bar{x}_1^+)$ and $\theta(\bar{x}_1^+ - x^+)$

ignoring the phases the contribution of the two poles add!

path ordering is lost!

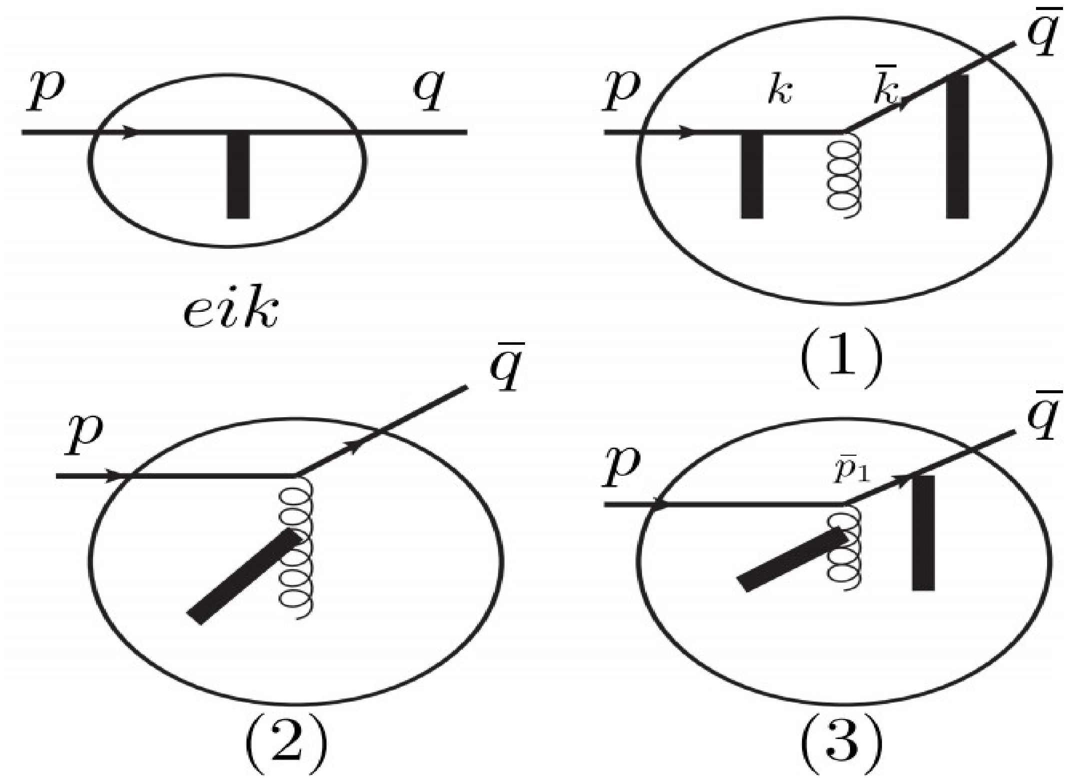
however further rescatterings are still path-ordered before/after $\mathbf{x_1^+}$, $\mathbf{\bar{x}_1^+}$

Re-scatterings of hard gluon and final state quark re-sum to



$$
\begin{aligned}
i\mathcal{M}_3 \;=\; & -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+ \,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+ - p^+)x^-}\; e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\; e^{-i(\bar{q}_t - \bar{p}_{1t})\cdot\bar{x}_t} \\
& \bar{u}(\bar{q}) \Bigg[\left[\partial_{\bar{x}^+}\, \overline{V}_{AP}(\bar{x}^+, \bar{x}_t)\right]\, \not{n}\, \not{\bar{p}}_1\, (igt^a)\, \left[\partial_{x^+}\, U^\dagger_{AP}(x_t, x^+)\right]^{ab} \\
& \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]} \Bigg]\, u(p)
\end{aligned}
$$

full amplitude: $\mathbf{i}\mathcal{M} = \mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}$



small x limit:

$$\begin{aligned} A^\mu(x) &\rightarrow n^- S(x^+, x_t) \\ n\cdot\bar{q} &\rightarrow n\cdot p \end{aligned}$$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

cross section: $|\mathbf{i}\mathcal{M}|^{\mathbf{2}} = |\mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}|^{\mathbf{2}}$

small x limit: $i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$

spinor helicity formalism: light-front spinors

spin asymmetries (intermediate p_t)

$$|i\mathcal{M}_2^+|^2 \sim g^2 \frac{q^+}{p^+}\frac{1}{q_\perp^4}\int d^4x\, d^4y\, e^{i(q^+-p^+)(x^- - y^-)}\, e^{-i(q_t-p_t)\cdot(x_t-y_t)}$$

$$\Big\{\Big[(p^+ - q^+)^2\, q_\perp^2\, A_\perp^b(x)\cdot A_\perp^c(y) + 4\, p^+ q^+\, q_\perp\cdot A_\perp^b(x)\, q_\perp\cdot A_\perp^c(y)\Big]$$

$$+\, \mathbf{i}\, \epsilon^{ij}\, [(p^+)^2 - (q^+)^2]\Big[q_i\, A_j^b(x)\, q_\perp\cdot A_\perp^c(y) - q_i\, A_j^c(y)\, q_\perp\cdot A_\perp^b(x)\Big]\Big\}$$

$$\big[\partial_{y^+} U_{AP}\big]^{ca}\big[\partial_{x^+} U_{AP}^\dagger\big]^{ab}$$

$|i\mathcal{M}_2^-|^2 = \left(|i\mathcal{M}_2^+|^2\right)^\star \longrightarrow \mathbf{d}\sigma^{++} - \mathbf{d}\sigma^{--} \neq \mathbf{0}$ this is zero in eikonal limit

azimuthal asymmetries

rapidity loss, ...

SUMMARY

CGC is a systematic approach to high energy collisions

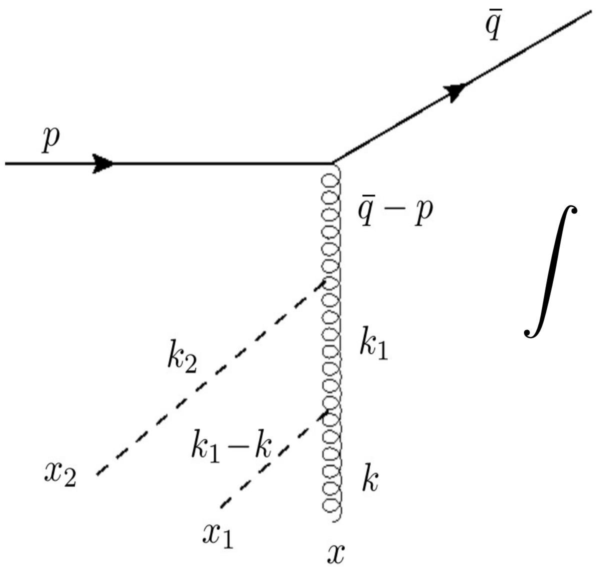
- ***strong hints from RHIC, LHC,...***
- ***connections to TMD,...***
- ***toward precision: NLO,...***

CGC breaks down at large x (high p_t)

- ***a significant portion of EIC phase space is at large x***
  - ***transition from large x to small x physics***

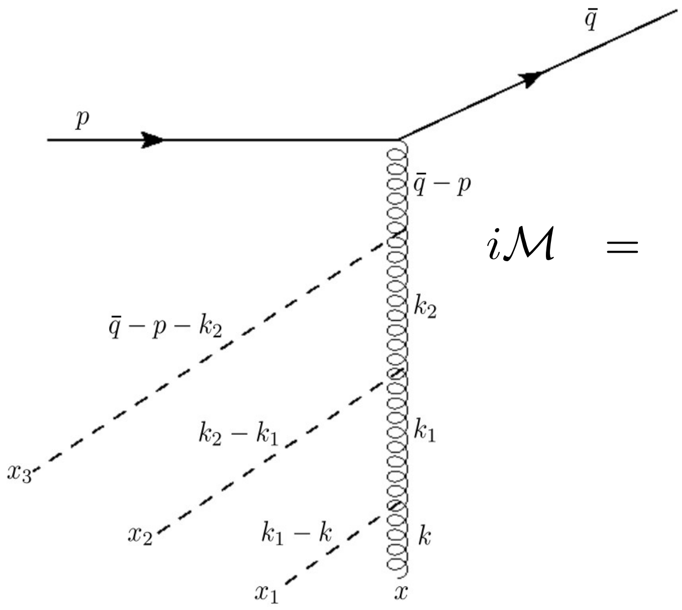
Toward a unified formalism:

- ***particle production in both small and large x (p_t) kinematics***
- ***spin, azimuthal asymmetries in intermediate p_t region***
- ***jet energy loss***
- ***one-loop correction to cross section: from JIMWLK to DGLAP ?***



$$\int \frac{dk_1^-}{(2\pi)} \frac{e^{ik_1^-(x^+-x_2^+)}}{2(\bar{q}^+-p^+)\left[k_1^- - \frac{k_{1t}^2 - i\epsilon}{2(\bar{q}^+-p^+)}\right]} \sim \theta(x^+-x_2^+)$$

$$\begin{aligned} i\mathcal{M} \quad = \quad & 2\, f_{abc}\, f_{cde} \int d^4x\, dx_2^+\, \theta(x^+-x_2^+)\, e^{i(\bar{q}^+-p^+)x^- - i(\bar{q}_t-p_t)\cdot x_t} \\ & \bar{u}(\bar{q})\, \frac{\left[\not{n}\,(p-\bar{q})\cdot A_e(x) - \not{A}_c(x)\, n\cdot(p-\bar{q})\right]}{(p-\bar{q})^2}\, (ig\, t^a)\, u(p) \\ & \left[i\, g\, S_d(x^+, x_t)\right]\ \left[i\, g\, S_b(x_2^+, x_t)\right] \end{aligned}$$



$$
\begin{aligned}
i\mathcal{M} \quad = \quad & \frac{2(i)^2}{(\bar{q}-p)^2}\, f^{abc} f^{cde} f^{egf} \int d^4x\, dx_2^+ dx_3^+\, \theta(x^+ - x_2^+)\, \theta(x_2^+ - x_3^+) \\
& \bar{u}(\bar{q})\, (ig\, t^a)\, \Big[n \cdot (p-\bar{q}) \not{A}_f(x) - (p-\bar{q}) \cdot A_f(x) \not{n} \Big]\, u(p) \\
& \big[i\, g\, S_g(x^+, x_t) \big]\; \big[i\, g\, S_d(x_2^+, x_t) \big]\; \big[i\, g\, S_b(x_3^+, x_t) \big] \\
& e^{i(\bar{q}^+ - p^+)x^- - i(\bar{q}_t - p_t)\cdot x_t}
\end{aligned}
$$

recall

$$
\begin{aligned}
\partial_{x^+} \left[U^\dagger_{AP}(x_t, x^+) \right]^{ab} \quad = \quad & (if^{bca})\, [igS_c(x^+, x_t)] \\
+ \quad & (if^{bce})\, (if^{eda}) \int dx_1^+ \theta(x^+ - x_1^+) [[igS_c(x^+, x_t)]\, [igS_d(x_1^+, x_t)] \\
+ \quad & (if^{bch})\, (if^{gdf})\, (if^{fea}) \int dx_1^+\, dx_2^+\, \theta(x^+ - x_1^+)\, \theta(x_1^+ - x_2^+) \\
& [[igS_c(x^+, x_t)]\, [igS_d(x_1^+, x_t)]\, [[igS_c(x_2^+, x_t)] + \cdots\cdots
\end{aligned}
$$

Particle production in high energy collisions

pQCD and collinear factorization at high p_t

precision physics

breaks down at low p_t (small x)

CGC at low p_t

toward precision physics

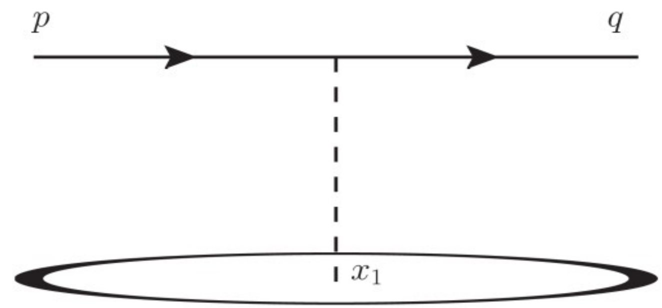
breaks down at large x (high p_t)

to firmly establish CGC, need a unified formalism

CGC at low x (low p_t)

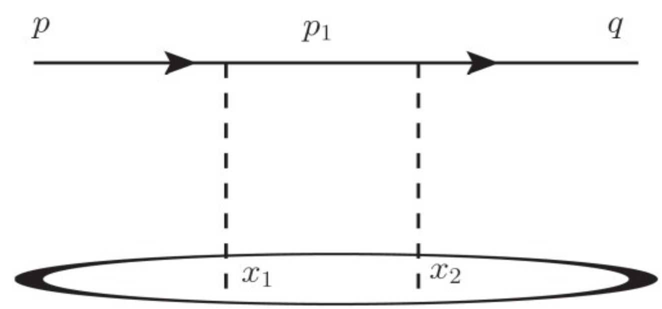
leading twist pQCD (DGLAP) at large x (high p_t)

$$
\begin{aligned}
i\mathcal{M}_1 &= (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\ \left[\not{n}\, S(x_1)\right]\ u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}} \\
&\qquad \bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)
\end{aligned}
$$



$$
A_a^-(x^+, x_\perp) \equiv n^-\, S_a(x^+, x_\perp)
$$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\int d^4x_1\, d^4x_2\, \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2} \\
&\qquad \bar{u}(q)\ \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\ u(p)
\end{aligned}
$$



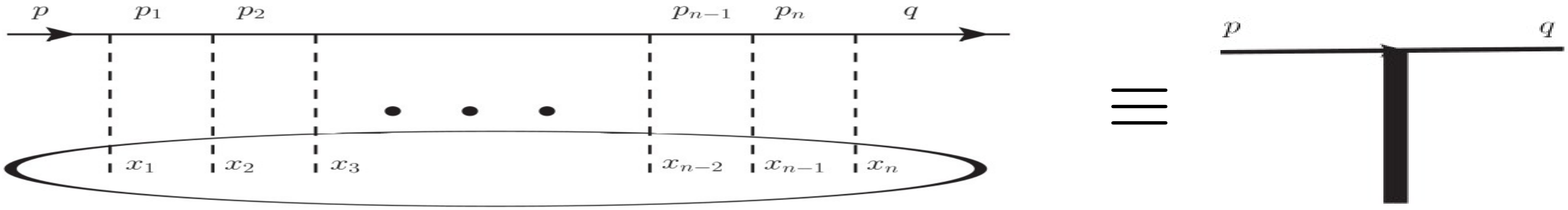
$$
\int \frac{dp_1^-}{(2\pi)}\frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\,\theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}
$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\,\frac{\not{p}_1}{2n\cdot p}\,\not{n} = \not{n}$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\,(-i)(i)\,2\pi\delta(p^+ - q^+)\, \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+)\, \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}} \\
&\qquad \bar{u}(q)\ \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\ u(p)
\end{aligned}
$$

CGC: tree level (eikonal approximation)



$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\bar{u}(q)\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\,[V(x_t)-1]\,u(p)$$

with $V(x_t) \equiv \hat{P}\exp\left\{ig\int_{-\infty}^{+\infty} dx^+ S_a^-(x^+, x_t)t_a\right\}$

Dipole: DIS, proton-nucleus collisions

$$< Tr\, V(x_\perp)\, V^\dagger(y_\perp) >$$

scattering from small x gluons of the target can cause only a *small angle deflection*

$$S_a^-(k^+ \sim 0, k^-/\sqrt{s} \ll 1, k_t \sim Q_s)$$

toward precision: NLO evolution

beyond eikonal approximation: tree level

large x partons of target: can cause a large-angle deflection of the quark (high q_t)

$$A_a^\mu(x^+, x^-, x_\perp)$$

include eikonal multiple scatterings before and after the hard scattering