Magnetic effects of QCD parameters from finite energy sum rules

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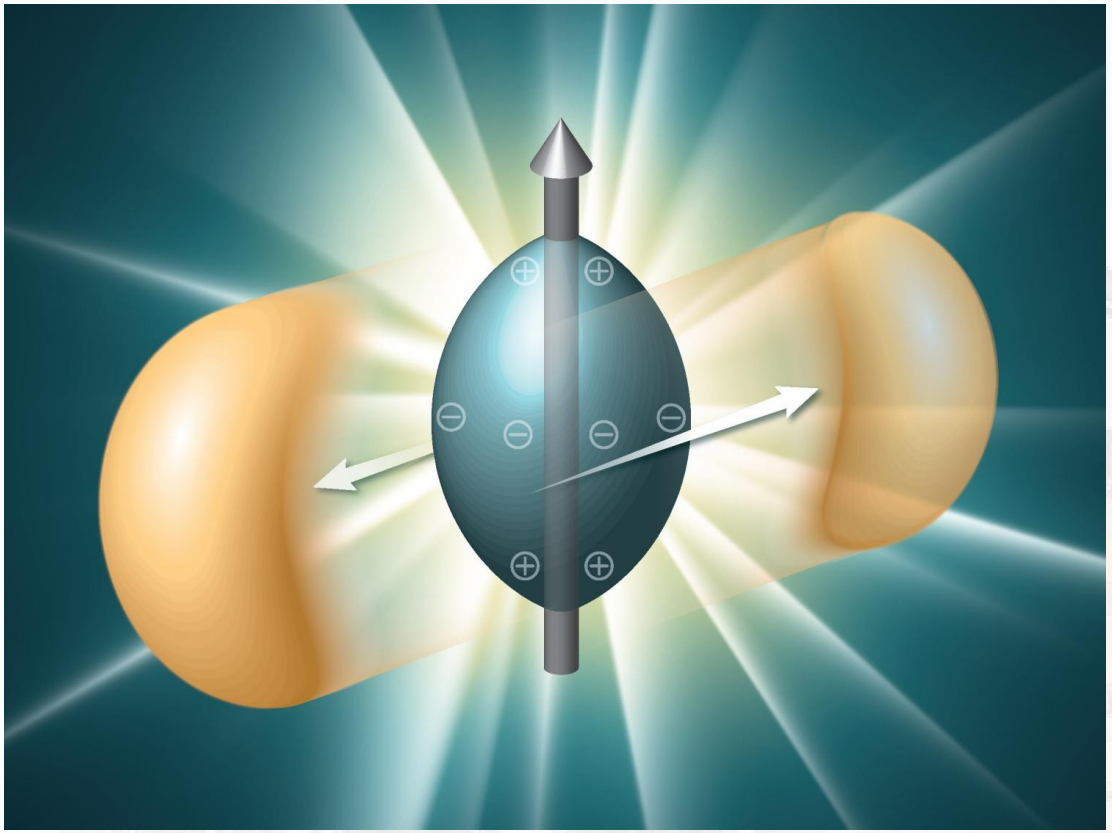
Motivation

Intense magnetic fields

- Heavy ion experiments

$$B \sim 10^{19} - 10^{20} \text{ Gauss}$$

$$0.1 - 1 \text{ GeV}^2$$



- Magnetars

$$B \sim 10^{18} - 10^{19} \text{ Gauss}$$

$$0.01 - 0.1 \text{ GeV}^2$$

Commonly used Models for QCD and Hadronic physics

- Perturbative QCD → do not include non-perturbative effects
- Lattice QCD → sign problem
- Schwinger Dyson equation → extremely complicated
- Nambu Jona Lasinio → cutoff dependent
- Sigma models → good for low temperature and chemical potential
- Functional renormalization group → recently studied at finite B
- **QCD sum rules** → studied and forgotten at finite B since late 90's (Borel sum rules)

Outline

- Introduction to Finite Energy Sum Rules (FESR)
- FESR at vacuum
- FESR at finite magnetic field
- Results
- About finite temperature and chemical potential

Introduction to finite energy sum rules (FESR)

Spectral function

Two current correlator

$$\Pi_{\mu\nu}(x-y) = i\langle 0|TJ_\mu(x)J_\nu^\dagger(y)|0\rangle$$

Fourier transformation

$$\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$$

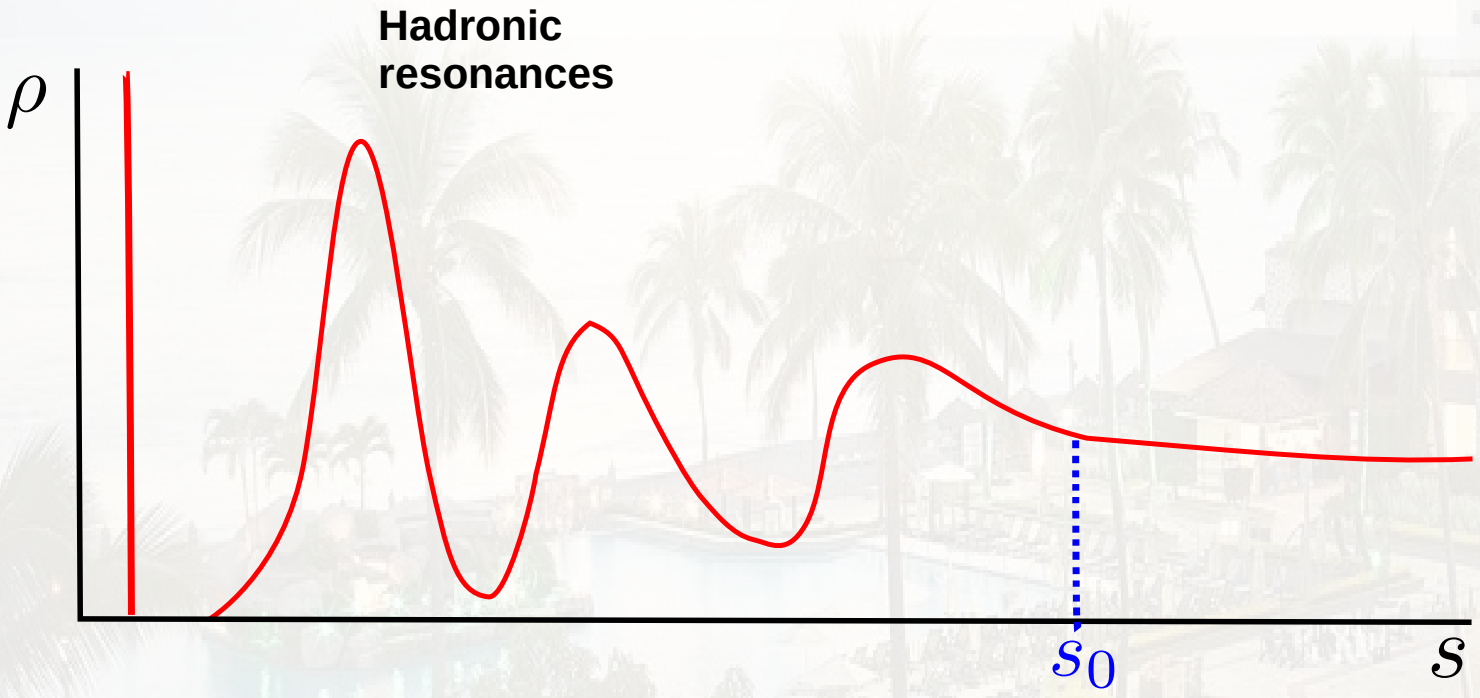
Spectral function: imaginary part of the correlator

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$$

hadronic continuum threshold s_0

Vacuum: $s_0 \sim 1\ \mathrm{GeV}^2$ ➡ Nonperturbative QCD

Finite temperature $s_0 \to 0$ ➡ Deconfinement order parmeter



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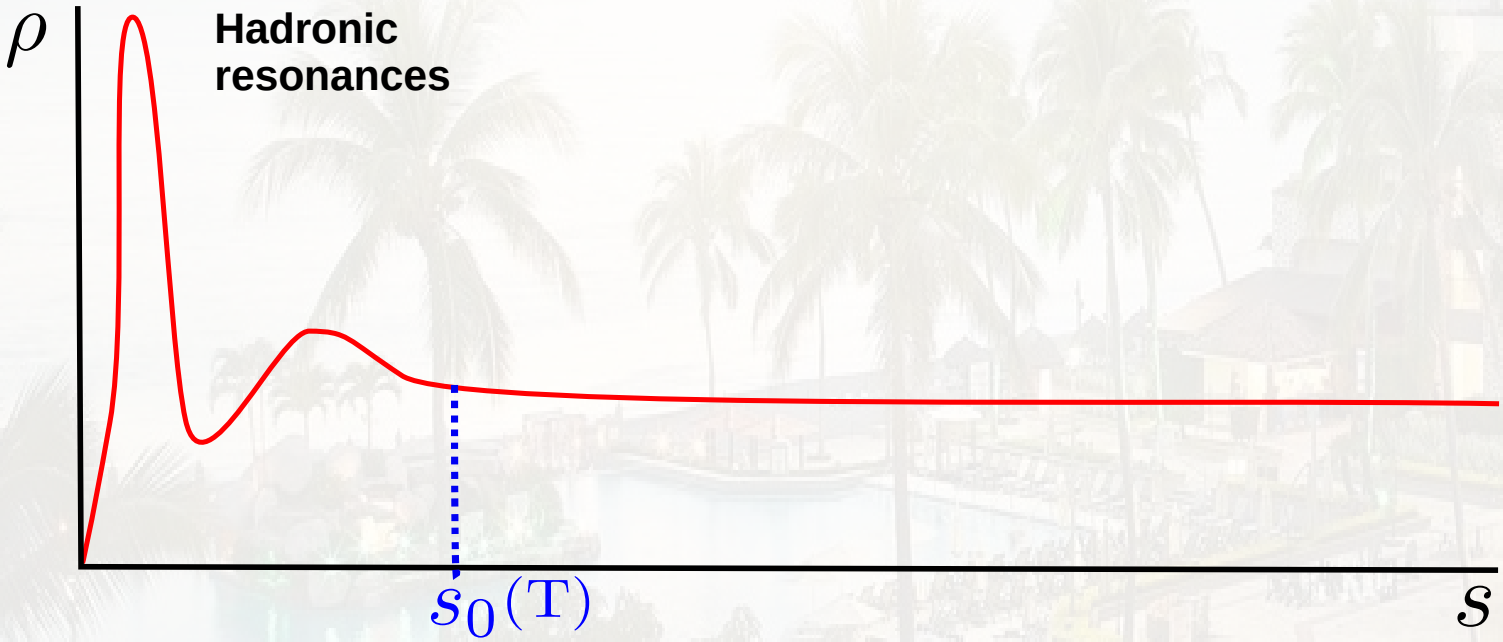
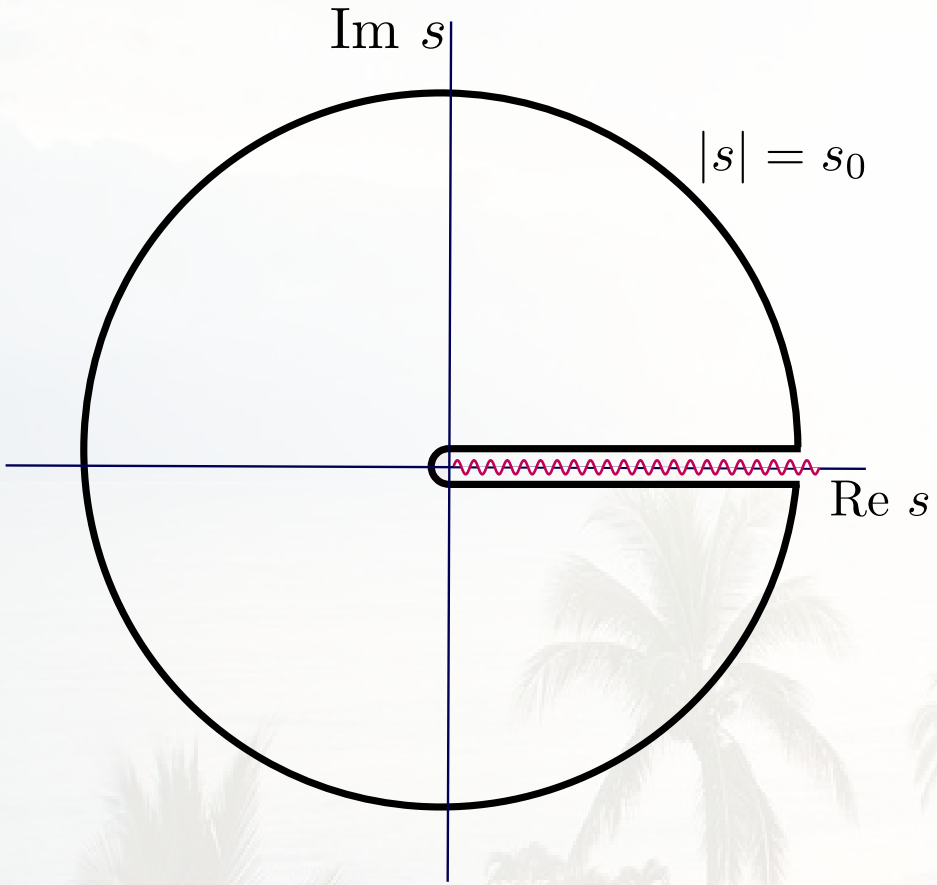


Quark-hadron duality

$$\Pi^{\mathrm{Had}} \leftrightarrow \Pi^{\mathrm{QCD}}$$



Cauchy's theorem

Had

$$\frac{1}{\pi}\int_0^{s_0} ds\, p(s)\, \mathrm{Im}\Pi(s+i\epsilon) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, p(s)\, \Pi(s)$$

QCD

FESR ⇒ $p(s) = s^{N-1}$

Operator product expansion (OPE)

$$\Pi^{\rm QCD}(s) = \sum_{n=0}^{\infty} C_{2n}(s,\mu)\frac{\langle O_{2n}(\mu)\rangle}{s^n}$$

C_{2n} Wilson parameters (dimensionless)

$\langle O_{2n}\rangle$ Condensates (dimension *2n*)

$$\begin{aligned}
\langle O_0\rangle &= 1 \\
\langle O_2\rangle &= 0 \\
\langle O_4\rangle &\sim \alpha_s\langle G^a_{\mu\nu}G^{a\mu\nu}\rangle,\ m_q\langle\bar{q}q\rangle \\
\langle O_6\rangle &= \cdots
\end{aligned}$$

The FESR's cut the OPE series

FESR at vacuum

Vacuum current correlators

- Axial – Axial correlator
- Pseudoscalar – Axial correlator
- Pseudoscalar – Pseudoscalar correlator

	QCD	Had
$A_\mu(x)$	$\bar{d}(x)\gamma_\mu\gamma_5 u(x)$	$-\sqrt{2}f_\pi\partial_\mu\pi^-(x)$
$\partial\cdot A(x)$	$(m_u+m_d)\bar{d}(x)i\gamma_5 u(x)$	$\sqrt{2}m_\pi^2 f_\pi\pi^-(x)$

Relatd with spin 0 (pions) and spin 1 (a_1)

Axial – Axial correlator

$$\begin{aligned}\Pi^A_{\mu\nu}(q^2) &= i\int d^4x\, e^{iqx}\,\langle 0|T[A_\mu(x)A^\dagger_\nu(0)]|0\rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_0(q^2) + g_{\mu\nu}\,\Pi_d(q^2)\end{aligned}$$

Pseudoscalar – Axial correlator

$$\begin{aligned}\Pi_{5\nu}(q^2) &= i\int d^4x\, e^{iqx}\,\langle 0|T\,[i\partial\cdot A(x)\,A^\dagger_\nu(0)]|0\rangle \\ &= q_\nu\,\Pi_5(q^2)\end{aligned}$$

Pseudoscalar – Pseudoscalar correlator

$$\psi_5(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\,[\partial\cdot A(x)\,\partial\cdot A^\dagger_\nu(0)]|0\rangle$$

Axial – Axial correlator

$$\begin{aligned}\Pi^A_{\mu\nu}(q^2) &= i\int d^4x\, e^{iqx}\,\langle 0|T[A_\mu(x)A^\dagger_\nu(0)]|0\rangle \\ &= (q_\mu q_\nu - q^2 g_{\mu\nu})\,\Pi_0(q^2) + g_{\mu\nu}\,\Pi_d(q^2)\end{aligned}$$

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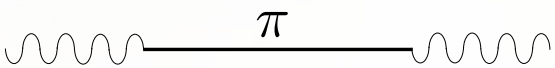
Pseudoscalar – Pseudoscalar correlator

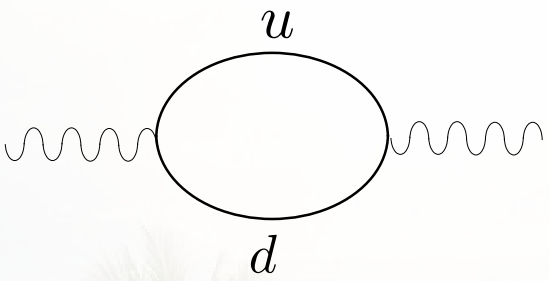
$$\psi_5(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\,[\partial\cdot A(x)\,\partial\cdot A^\dagger_\nu(0)]|0\rangle$$

$\Pi_d,\ \Pi_5$ and Ψ_5 are related through Ward identities

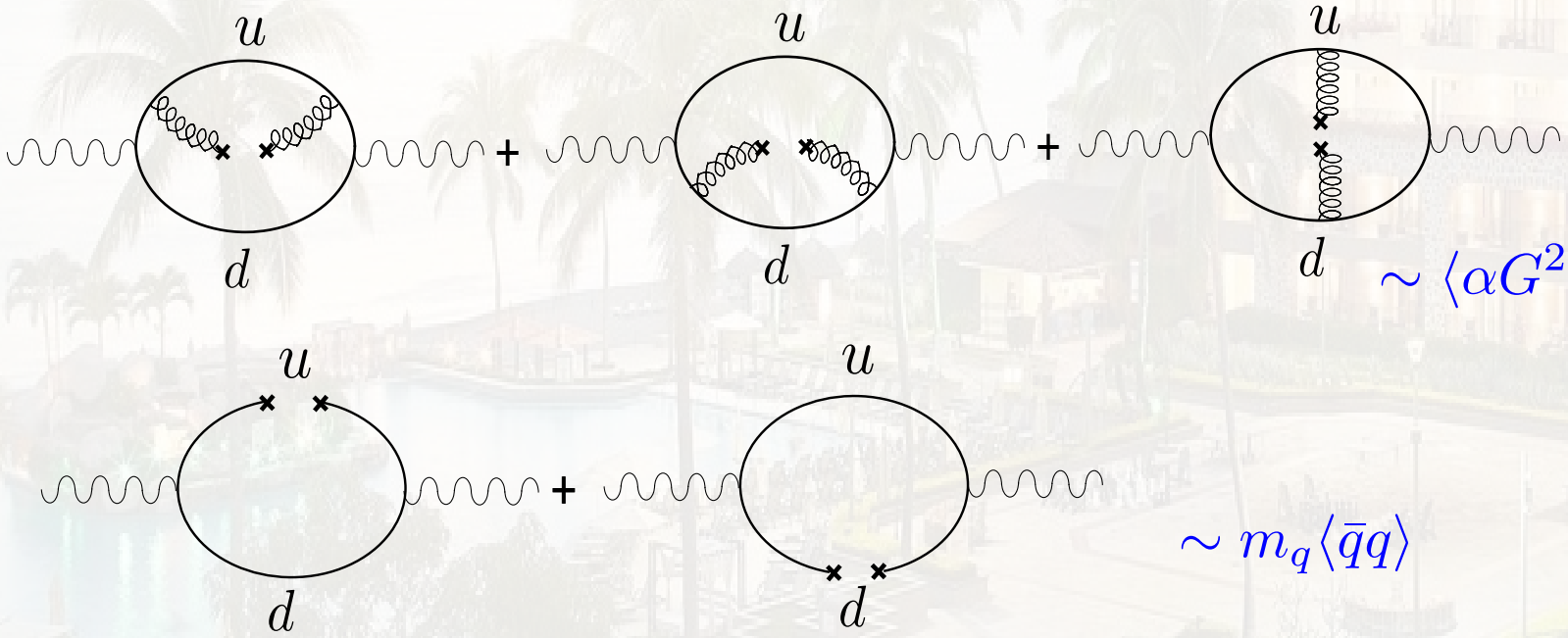
Feynman diagrams $\langle T J(x) J'(y) \rangle$

Had 

(pions only)

pQCD 

+ radiative corrections

OPE O_4 

FESR and OPE up to $\langle O_4 \rangle$

$$\int ds\, \Pi_0 \rightarrow \qquad 2\, f_\pi^2 = \frac{s_0}{4\,\pi^2}$$

$$\int ds\, s\, \Pi_0 \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = \frac{s_0^2}{8\,\pi^2} - 2 m_q\, \langle \bar{q} q \rangle - \frac{1}{12\pi}\, \langle \alpha_s\, G^2 \rangle$$

$$\int ds\, \Pi_5 \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = -\, 4 m_q \langle \bar{q} q \rangle + \frac{3}{8\pi^2} {m_q}^2 s_0$$

$$\int ds\, \Psi_5 \rightarrow \qquad 2 f_\pi^2\, m_\pi^4 = \frac{3 m_q^2\, s_0^2}{4\,\pi^2} - 4 m_q^3\, \langle \bar{q} q \rangle + \frac{m_q^2}{2\pi}\, \langle \alpha_s\, G^2 \rangle$$

$m_q = (m_u + m_d)/2$

$\langle \bar{q} q \rangle = \langle \bar{u} u + \bar{d} d \rangle / 2$

neglecting corrections $\sim {m_q}^2 / s_0$

FESR and OPE up to $\langle O_4 \rangle$

$$\int ds\, \Pi_0 \rightarrow \qquad 2\, f_\pi^2 = \frac{s_0}{4\,\pi^2}$$

$$\int ds\, s\, \Pi_0 \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = \frac{s_0^2}{8\,\pi^2} - 2 m_q \langle \bar{q} q \rangle - \frac{1}{12\pi} \langle \alpha_s\, G^2 \rangle$$

$$\int ds\, \Pi_5 \rightarrow \qquad 2\, f_\pi^2\, m_\pi^2 = -\,4 m_q \langle \bar{q} q \rangle + \frac{3}{8\pi^2} {m_q}^2 s_0 \qquad \text{(G-MOR)}$$

$$\int ds\, \Psi_5 \rightarrow \qquad 2 f_\pi^2\, m_\pi^4 = \frac{3 m_q^2\, s_0^2}{4\,\pi^2} - 4 m_q^3 \langle \bar{q} q \rangle + \frac{m_q^2}{2\pi} \langle \alpha_s\, G^2 \rangle$$

$m_q = (m_u + m_d)/2$

$\langle \bar{q} q \rangle = \langle \bar{u} u + \bar{d} d \rangle / 2$

neglecting corrections $\sim {m_q}^2 / s_0$

Set of parameters

$$m_\pi,\ f_\pi,\ m_u,\ m_d,\ s_0,\ \langle\bar{u}u\rangle,\ \langle\bar{d}d\rangle,\ \langle\alpha_s G^2\rangle,$$

considering $$\frac{m_u - m_d}{m_u + m_d}\frac{\langle\bar{u}u - \bar{d}d\rangle}{\langle\bar{u}u + \bar{d}d\rangle} \ll 1$$

$$m_\pi,\ f_\pi,\ (m_u + m_d),\ s_0,\ \langle\bar{u}u + \bar{d}d\rangle,\ \langle\alpha_s G^2\rangle,$$

Inputs: $m_\pi = 139.6\ \text{MeV}, \qquad f_\pi = 92.3\ \text{MeV}$

$$s_0 = 0.67\ \text{GeV} \qquad m_q = 9.75\ \text{MeV}$$

$$\langle\alpha_s G^2\rangle = 0.21\ \text{GeV}^4 \qquad \langle\bar{q}q\rangle = (-202\ \text{MeV})^3$$

FESR at finite magnetic field

Minimal coupling $\partial_\mu \to D_\mu = \partial_\mu - ie\mathcal{A}$

Hadronic sector

$$A_\mu(x) = -\sqrt{2}(f_\pi^{\parallel} D_\mu^{\parallel} + f_\pi^{\perp} D_\mu^{\perp})\pi(x)$$

$$D^\mu A_\mu(x) = -\sqrt{2} f_\pi^{\parallel} m_\pi^2 \pi(x)$$

$$\frac{f_\pi^{\perp}}{f_\pi^{\parallel}} = v_\perp^2$$

Ward identities are preserved

$$D^\mu \Pi_{\mu\nu} = \Pi_{5\nu} + \text{condensate}$$

$$D^\nu \Pi_{5\nu} = \Psi_5 + \text{condensate}$$

Propagators

$$S_q(x,y) = e^{ie_q\phi(x,y)} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \sum_n S_q^{(n)}(k)$$

quarks

$$D_\pi(x,y) = e^{ie_\pi\phi(x,y)} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \sum_n D_\pi^{(n)}(k)$$

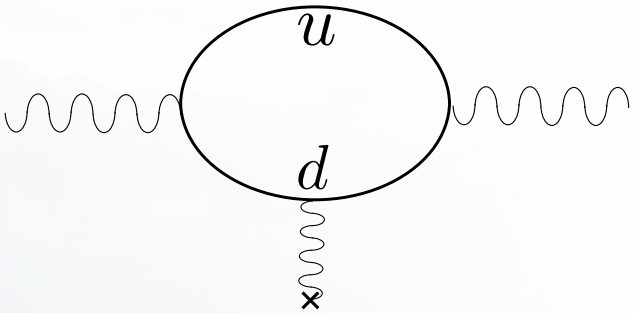
pions

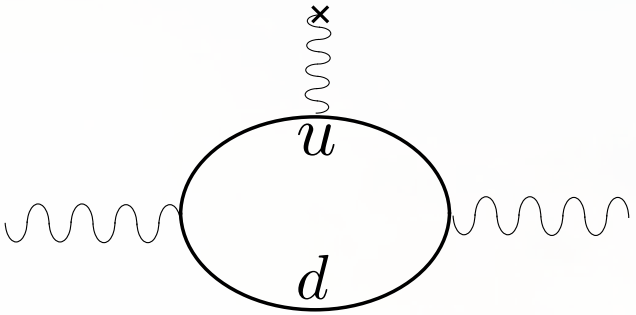
$$\phi(x,y) = -\frac{1}{2} F_{\mu\nu} x^\mu y^\nu$$

Schwinger phase in the symmetric gauge
(Schwinger – Fock gauge)

$$S_q^{(n)},\ D_\pi^{(n)} \sim \left(\frac{B}{k^2 - m^2} \right)^n$$

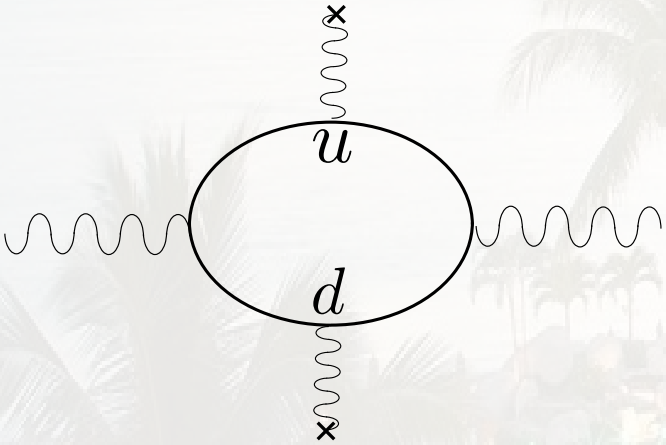
New diagrams

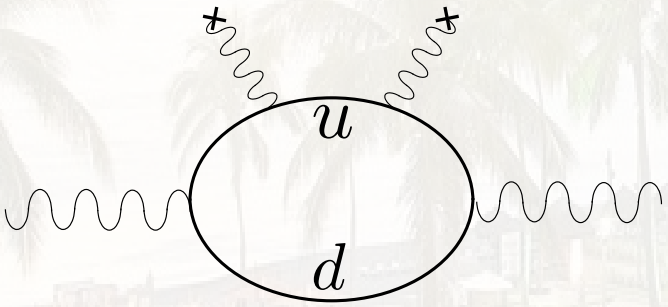


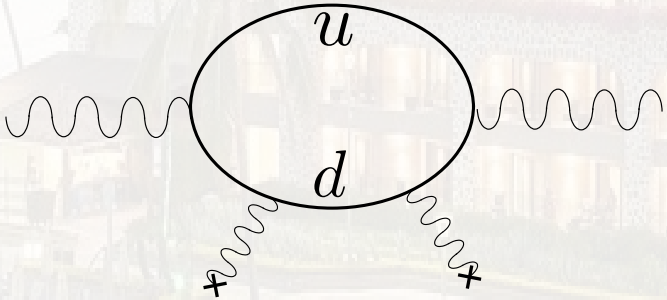


$\sim i\epsilon^{\perp}_{\mu\nu}$

$\sim i\epsilon^{\perp}_{\mu\nu}q^{\nu}$







The FESR's cut the series in *B* too!

Tensor and vector structures

External magnetic field → $F_{\mu\nu} = B\epsilon_{0\mu\nu3} \equiv B\boldsymbol{\epsilon^{\perp}_{\mu\nu}}$

Tensorial structures → combination of $q_\mu, \quad g_{\mu\nu}, \quad i\epsilon^{\perp}_{\mu\nu}$

$$\Pi^{A}_{\mu\nu}(q) = g^{\parallel}_{\mu\nu}\,\Pi^{\parallel}_{1}(q^2_{\parallel}, q^2_{\perp}) + g^{\perp}_{\mu\nu}\,\Pi^{\perp}_{1}(q^2_{\parallel}, q^2_{\perp}) + i\epsilon^{\perp}_{\mu\nu}\,\tilde{\Pi}_{1}(q^2_{\parallel}, q^2_{\perp})$$

$$+q^{\parallel}_{\mu}\,q^{\parallel}_{\nu}\,\Pi^{\parallel}_{0}(q^2_{\parallel}, q^2_{\perp}) + \text{ combinations of } \quad q^{\parallel}_{\mu}, \quad q^{\perp}_{\mu}, \quad i\epsilon^{\perp}_{\mu\nu}q^{\nu}$$

$$\Pi_{5\mu}(q) = q^{\parallel}_{\mu}\Pi^{\parallel}_{5}(q^2_{\parallel}, q^2_{\perp}) + q^{\perp}_{\mu}\Pi^{\perp}_{5}(q^2_{\parallel}, q^2_{\perp}) + i\epsilon^{\perp}_{\mu\nu}q^{\nu}\tilde{\Pi}_{5}(q^2_{\parallel}, q^2_{\perp})$$

We work in the frame $q_\perp = 0$

New parameters

$$f_\pi \begin{cases} f_\pi^{\parallel} \\ f_\pi^{\perp} \end{cases}$$

$$m_q\langle\bar{q}q\rangle = \langle\bar{q}i\not{D}q\rangle \begin{cases} \langle\bar{q}i\not{D}_{\parallel}q\rangle \\ \langle\bar{q}i\not{D}_{\perp}q\rangle \end{cases}$$

$$m_q\langle\bar{q}\sigma_{12}q\rangle = \langle\bar{q}\sigma_{12}i\not{D}q\rangle \begin{cases} \langle\bar{q}\sigma_{12}i\not{D}_{\parallel}q\rangle \\ \langle\bar{q}\sigma_{12}i\not{D}_{\perp}q\rangle \end{cases}$$

$$\langle\alpha_s G^2\rangle \begin{cases} 2\langle\alpha_s(G^a_{03})^2\rangle \\ 2\langle\alpha_s(G^a_{12})^2\rangle \\ 2\langle\alpha_s[(G^a_{01})^2+(G^a_{02})^2+(G^a_{31})^2+(G^a_{32})^2]\rangle \end{cases}$$

New parameters

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Results

First work (PRD 92 016006)

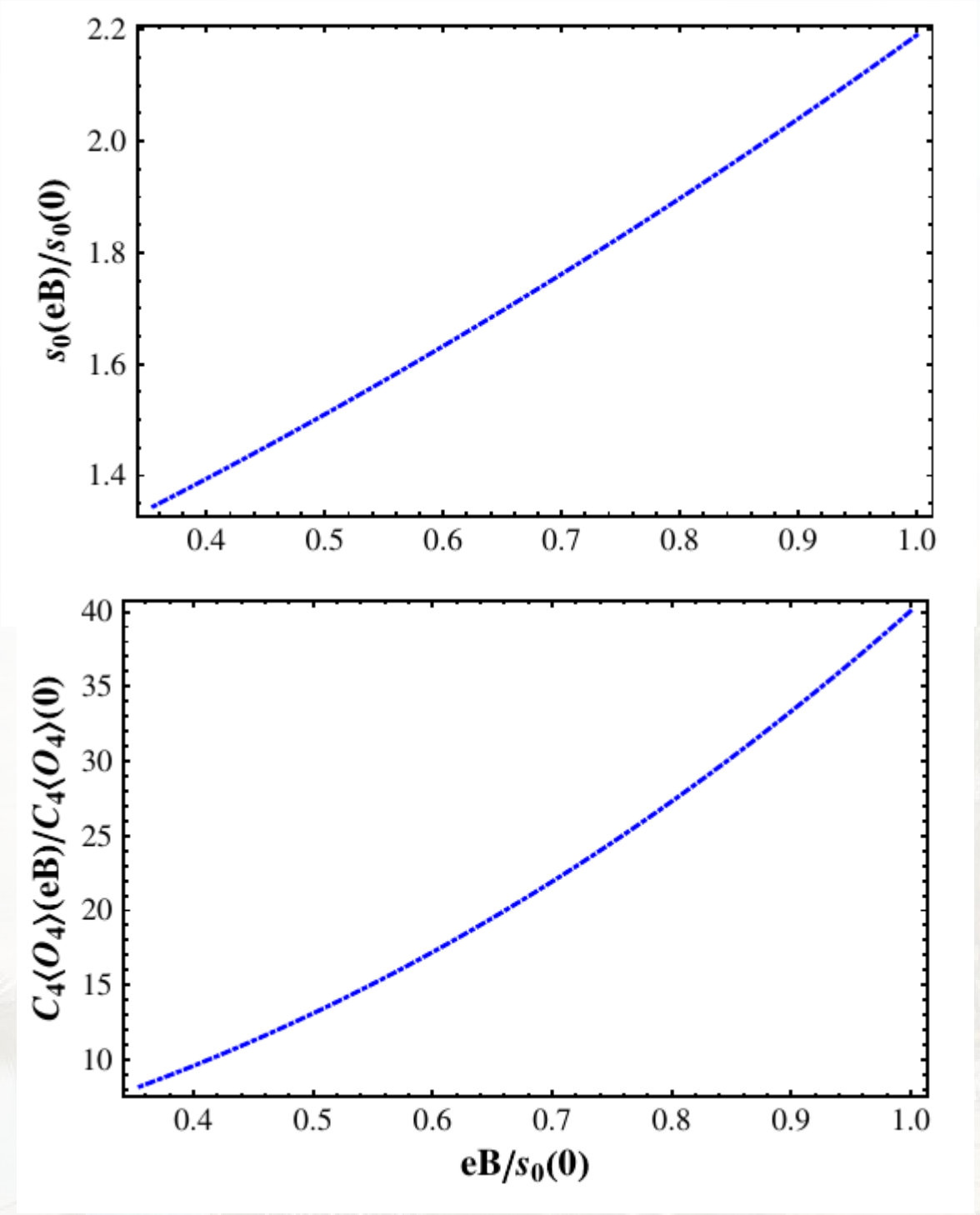
$$\Pi = \frac{q^\mu q^\nu}{q^4}\Pi^A_{\mu\nu}$$

Sum rules $\int ds\Pi, \quad \int dss\Pi,$ GMOR

Chiral limit: $m_\pi \to 0, \quad m_q \to 0, \quad \frac{m_\pi^2}{m_q} \to \text{constant}$

Input: Lattice results (PRD 86 071502)

LLL and NLLL in the hadronic sector


2.2
2.0
1.8
1.6
1.4
$s_0(eB)/s_0(0)$
0.4
0.5
0.6
0.7
0.8
0.9
1.0
40
35
30
25
20
15
10
$C_4\langle O_4\rangle(eB)/C_4\langle O_4\rangle(0)$
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$eB/s_0(0)$

FESR's at finite magnetic field (PRD **98** 034015)

$$2f_\pi^2 = \frac{s_0}{4\pi^2}$$

$$2f_\pi^2 m_\pi^2 = \frac{1}{8\pi^2}\left\{ s_0^2 - \frac{2}{9}(eB)^2\left[10\ln(s_0/{m_q}^2) - 27\right]\right\} + 2m_q\langle\bar{q}q\rangle - \frac{1}{12\pi}\langle\alpha_s G^2\rangle$$

$$2f_\pi^2 m_\pi^2 = -4m_q\langle\bar{q}q\rangle + \frac{3}{2\pi^2}m_q^2 s_0$$

$$2f_\pi^2 m_\pi^4 = \frac{3m_q^2}{4\pi^2}\left\{ s_0^2 - \frac{20}{27}(eB)^2\left[\ln(s_0/{m_q}^2) - 1\right]\right\} - 4m_q^3\langle\bar{q}q\rangle + \frac{m_q^2}{2\pi}\langle\alpha_s G^2\rangle$$

$m_u \approx m_d$ neglecting corrections $\sim {m_q}^2/s_0$

Inputs

1. Chiral condensate from LQCD or NJL

$$\langle \bar{u}u + \bar{d}d \rangle (B)$$

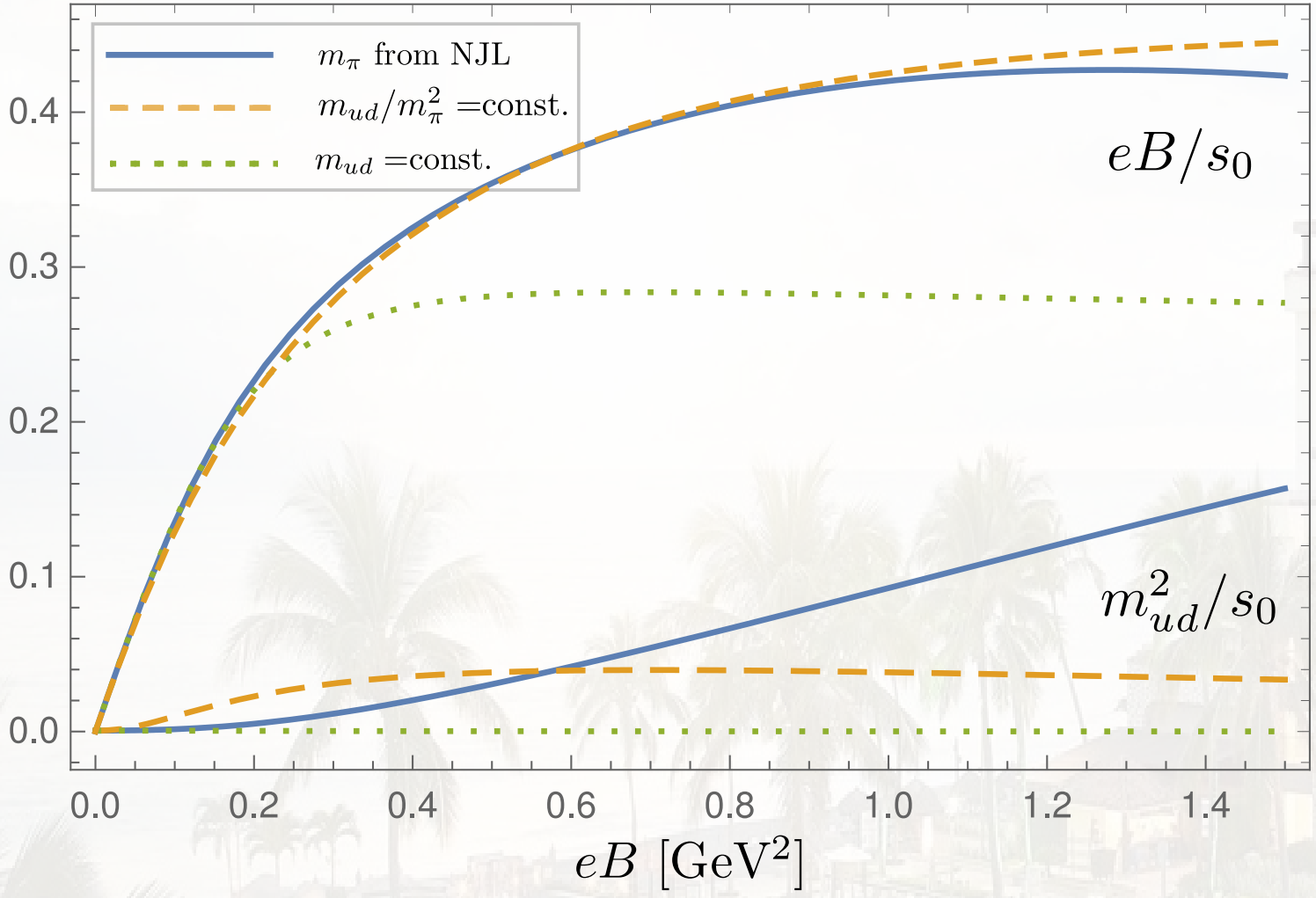
2. A. Pion mass from NJL

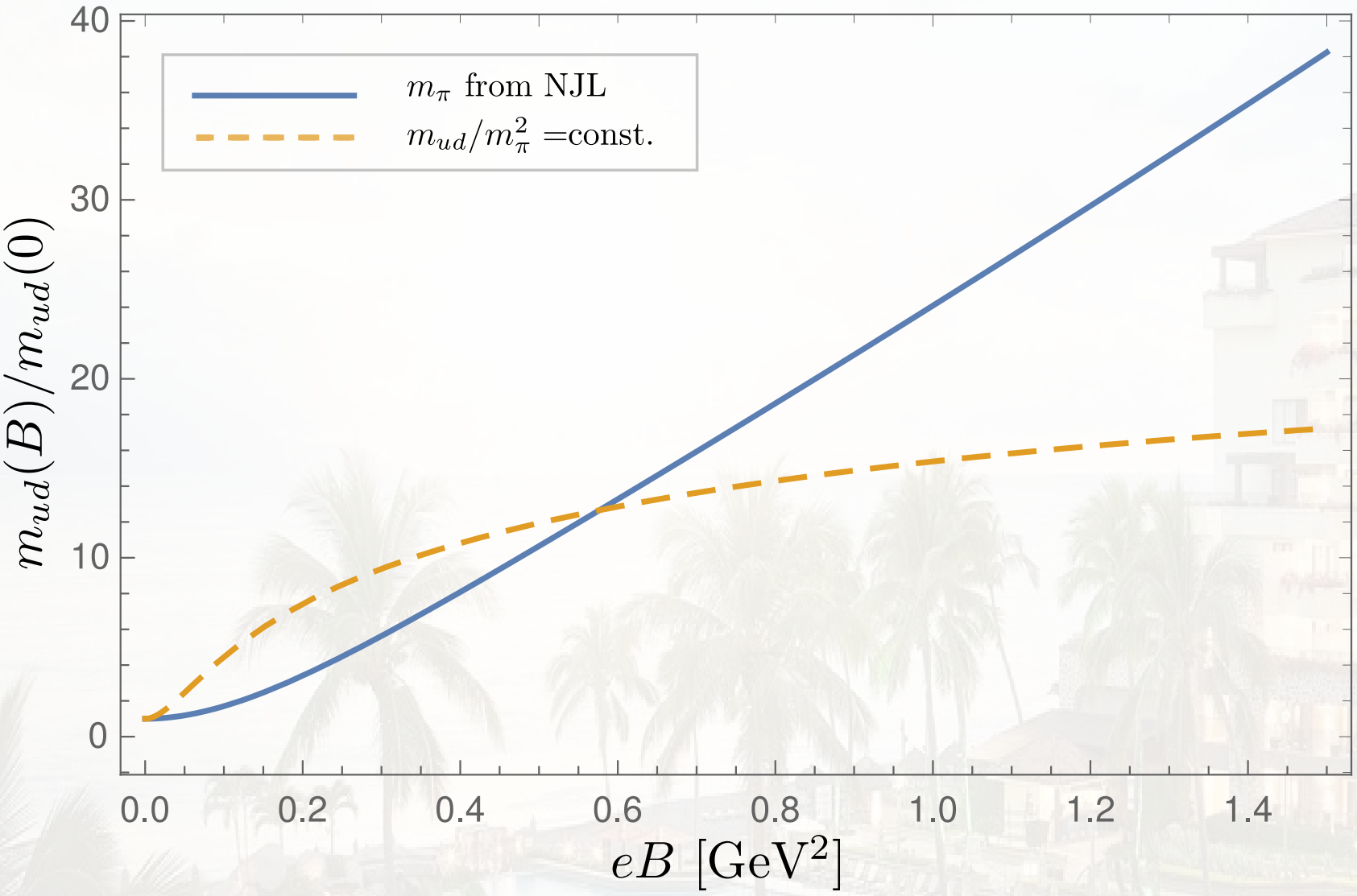
   B. Condition $m_q(B)/m_\pi^2(B) = \text{const.}$

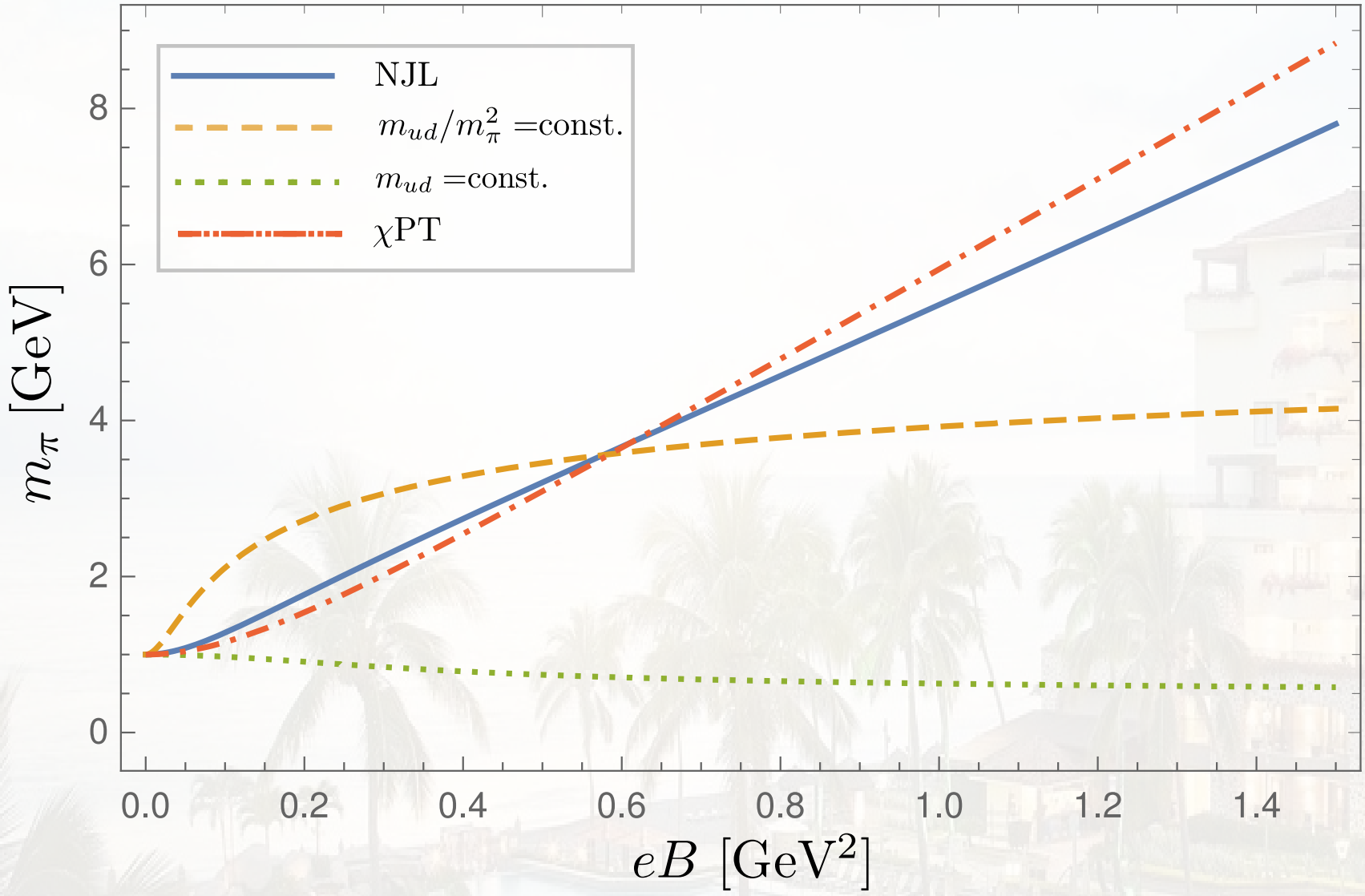
   C. Condition $m_q = \text{const}$

LQCD: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor,
S. D. Katz, and A. Schafer,
Phys. Rev. D 86, 071502 (2012)

NJL: M. Coppola, D. Gomez Dumm, and N. N. Scoccola,
Phys. Lett. B 782, 155 (2018)

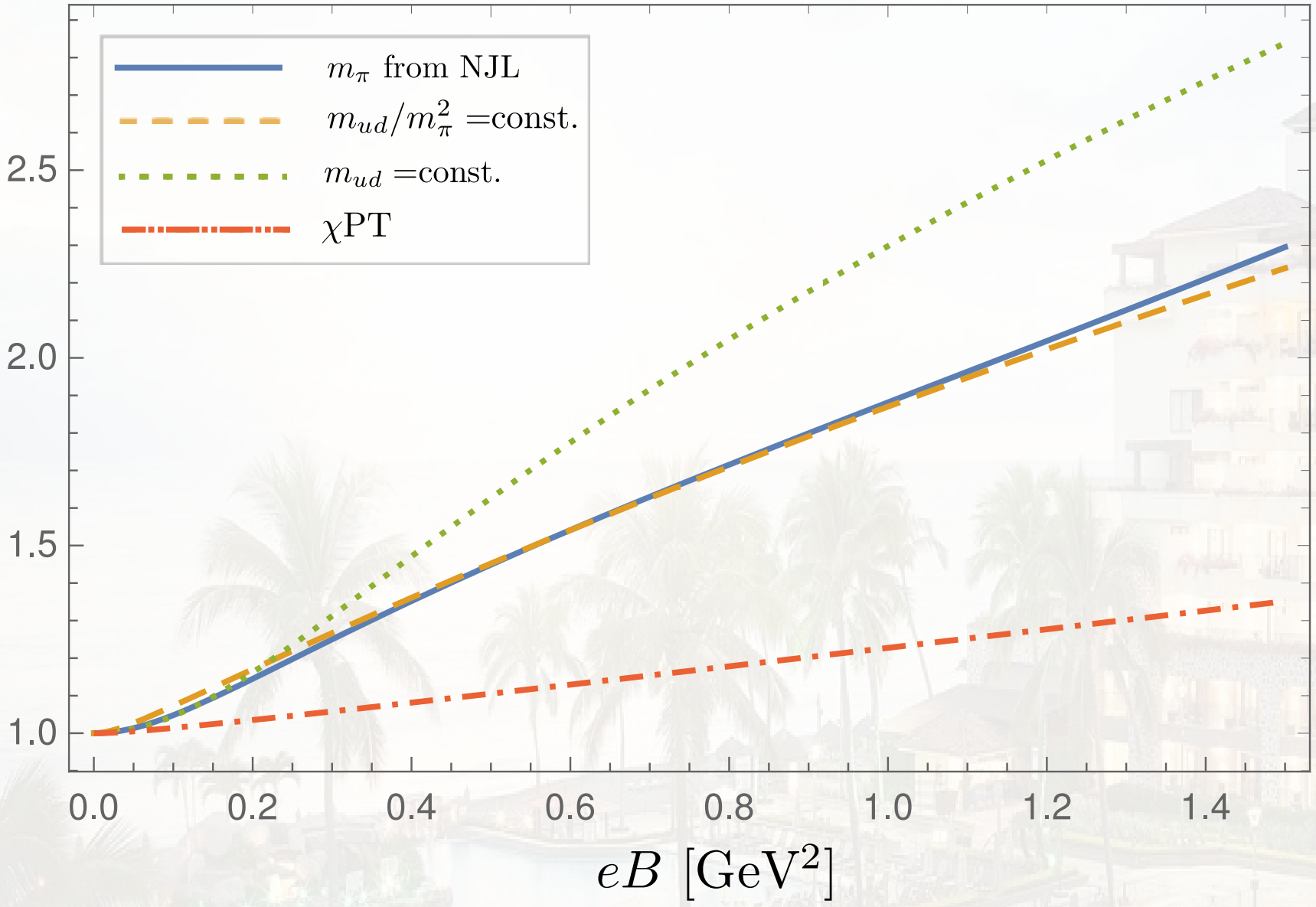

m_π from NJL
m_{ud}/m_π^2 =const.
m_{ud} =const.
eB/s_0
m_{ud}^2/s_0
0.0
0.1
0.2
0.3
0.4
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
eB [GeV^2]

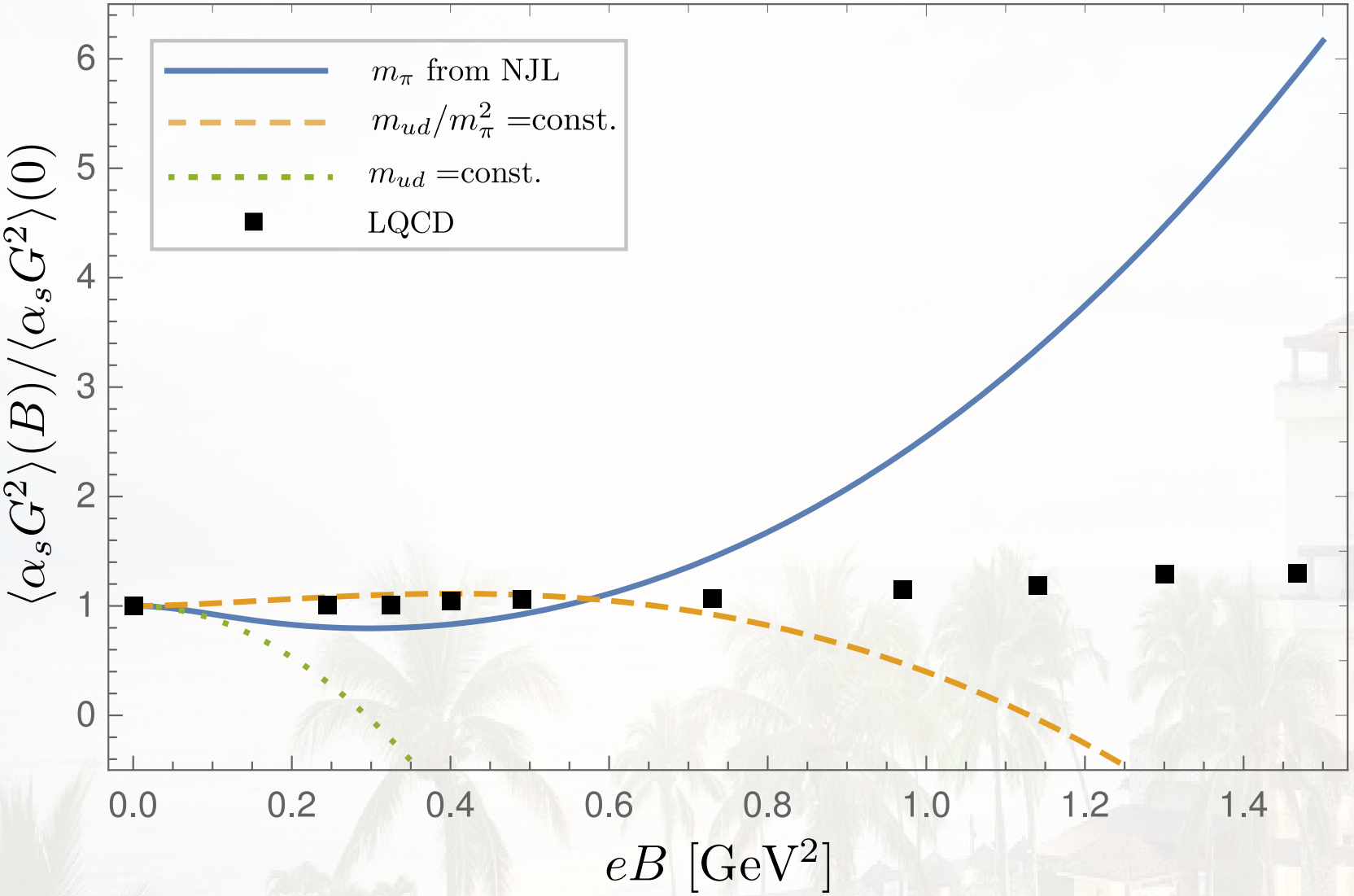

m_π from NJL
m_{ud}/m_π^2 =const.
$m_{ud}(B)/m_{ud}(0)$
0
10
20
30
40
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
eB [GeV^2]



χPT J. O. Andersen, J. High Energy Phys. 10 (2012) 005

$$\frac{f_\pi(B)}{f_\pi(0)} = \sqrt{\frac{s_0(B)}{s_0(0)}}$$





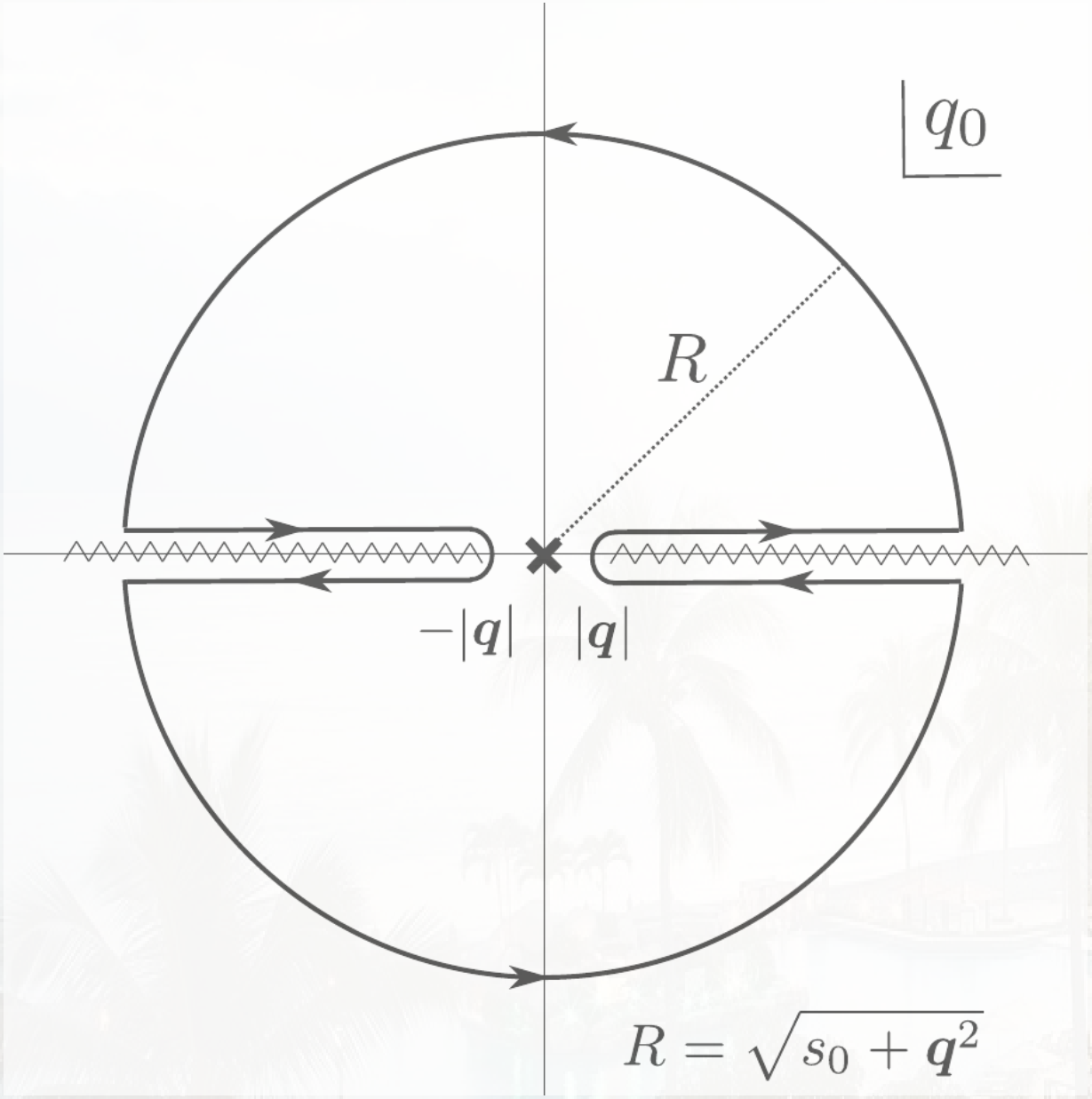
LQCD

M. D'Elia, E. Meggiolaro, M. Mesiti, and F. Negro, Phys. Rev. D 93, 054017 (2016).

Decreasing <GG>

N. O. Agasian and I. A. Shushpanov, Phys. Lett. B 472, 143(2000).

About Finite temperature and chemical potential



Chemical potential

- FESR's cut the series
- New condensates easily isolated
- Dominant scale s_0

Finite temperature

- FESR's do not cut the series
- More inputs to test approximations

Conclusions

- Magnetic evolution of QCD and Hadronic parameters
- Quark mass must change with magnetic field
- All the parameters grow (except gluon condensate)
- Valid for large values of *B*
- Relation $m_q/m_\pi^2 = \mathrm{const.}$ seems to be valid at finite *B*
- Pion decay constant our most robust prediction
- Gluon condensate needs a deep treatment

Outlook

- Finite temperature (soon)
- Finite baryon density (soon)
- $\langle \bar{q} \sigma_{12} q \rangle$
- Baryon parameters (g_A soon)
- New condensates
- Different quark masses (vector channel)

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THANKS!