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LHC ELECTROWEAK PRECISION SUBGROUP

INTERNAL NOTE – DRAFT

Benchmark Comparison of Analytic p_T Resummation

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ABSTRACT: We specify the proposed and tentatively agreed setup and first steps following the discussions in March, and also incorporating further feedback from April 2 at Durham workshop.

1 Short Summary

It was agreed to proceed in a logical way, which will allow to document successive steps of the benchmarking, from pure resummation to full resummation plus fixed-order matching (this latter step is not discussed further at the moment).

It was also agreed to keep things as simple as possible for the first steps and to monitor progress every 3-4 weeks, with a first follow-up meeting end of April-early May.

Prompted by very valid questions raised about the goal of this exercise, it was tentatively agreed that:

- Since this has never really been done before at this level of detail, there would be real added value in publishing the results of these comparisons (as compared to a Yellow Report, which does not mean one cannot include a suitable version of such a publication in a Yellow Report). This would be jointly authored by all interested participating resummation groups. The precise scope can be defined later.
- It is clear that this benchmarking of resummation is also of great interest to experimentalists working on the W mass measurement, particularly of course for the approach relying on the very precise Z boson measurements and their extrapolation to the W boson.
- The study should not take 5 years as the 1606 arXiv report, but rather be seen on the timescale of about six months to one year.

2 General Setup

2.1 Process Definition

We consider Drell-Yan, $pp \to Z/\gamma^* \to \ell^+\ell^-$ (and later on $pp \to W \to \ell\nu$). We consider the triple-differential cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}q_T} \qquad \text{with} \qquad Q \equiv \sqrt{m_{\ell\ell}^2}\,, \quad Y \equiv Y_{\ell\ell}\,, \quad q_T \equiv |\vec{p}_{T\,\ell\ell}|\,,$$

i.e., Q, Y, q_T are the total dilepton invariant mass, rapidity, and transverse momentum magnitude. The leptonic decay into massless leptons is included as part of the process, but at the moment treated as fully inclusively, i.e., without any fiducial cuts on the leptons.

We consider Z/γ^* and W production at leading order in the electroweak interactions, without photon-induced or other nonresonant contributions.

2.2 Parameter Inputs

We use the following measured input values

$$\begin{aligned} m_Z^{\rm os} &= 91.1876\,\mathrm{GeV}\,, & \Gamma_Z^{\rm os} &= 2.4952\,\mathrm{GeV}\,, \\ m_W^{\rm os} &= 80.385\,\mathrm{GeV}\,, & \Gamma_W^{\rm os} &= 2.085\,\mathrm{GeV}\,, \\ \alpha_s(m_Z^{\rm os}) &= 0.118\,, & G_F &= 1.1663787\times 10^{-5}\,\mathrm{GeV}^{-2}\,. \end{aligned} \tag{2.1}$$

For the CKM matrix elements we use the PDG2018 values,

$$\begin{aligned} |V_{ud}| &= 0.97446\,, & |V_{us}| &= 0.22452\,, & |V_{ub}| &= 0.00365\,, \\ |V_{cd}| &= 0.22438\,, & |V_{cs}| &= 0.97359\,, & |V_{cb}| &= 0.04214\,, \\ |V_{td}| &= 0.00896\,, & |V_{ts}| &= 0.04133\,, & |V_{tb}| &= 0.999105\,. \end{aligned} \tag{2.2}$$

For the PDFs we use the `NNPDF31_nnlo_as_0118_luxqed` set (LHAPDF ID 325100).

2.3 LO EW Scheme

For the LO EW scheme, we work in the fixed-width pole scheme, with the V propagator given by

$$\frac{1}{q^2 - (m_V^{\rm pole})^2 + \mathrm{i}\, m_V^{\rm pole}\Gamma_V^{\rm pole}}\,, \tag{2.3}$$

and the pole masses and widths given by

$$m_Z^{\rm pole} = \frac{m_Z^{\rm os}}{\sqrt{1+(\Gamma_Z^{\rm os}/m_Z^{\rm os})^2}} = 91.15348061918276\,\mathrm{GeV}\,, \tag{2.4}$$

$$\Gamma_Z^{\rm pole} = \frac{\Gamma_Z^{\rm os}}{\sqrt{1+(\Gamma_Z^{\rm os}/m_Z^{\rm os})^2}} = 2.4942663787728243\,\mathrm{GeV}\,, \tag{2.5}$$

$$m_W^{\rm pole} = \frac{m_W^{\rm os}}{\sqrt{1+(\Gamma_W^{\rm os}/m_W^{\rm os})^2}} = 80.35797360987756\,\mathrm{GeV}\,, \tag{2.6}$$

$$\Gamma_W^{\rm pole} = \frac{\Gamma_W^{\rm os}}{\sqrt{1+(\Gamma_W^{\rm os}/m_W^{\rm os})^2}} = 2.0842989982782196\,\mathrm{GeV}\,. \tag{2.7}$$

For the electroweak couplings we combine the pole scheme with the G_μ scheme, which means $\{m_Z^{\rm pole}, m_W^{\rm pole}, G_F\}$ serve as the independent inputs. The resulting weak mixing angle and electromagnetic coupling are

$$\sin^2\theta_w = 1 - \left(\frac{m_W^{\rm pole}}{m_Z^{\rm pole}}\right)^2 = 0.22283820939806087\,, \tag{2.8}$$

$$\alpha_{\rm em} = \frac{\sqrt{2}G_F}{\pi}\, m_W^{\rm pole}\sin^2\theta_w = 1/132.35723363577097 = 0.007555310522369057\,. \tag{2.9}$$

To avoid any numerical noise from roundoff errors, the numerical values should either be calculated explicitly from the inputs in eq. (2.1) using these relations or else the exact numerical values as given above should be used.

2.4 LO Reference Cross Section

Leading order reference cross sections for 2 generations are as follows:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y}(m_Z^{\mathrm{pole}}, 0) = 41.936446\,\mathrm{pb/GeV}, \tag{2.10}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y}(m_Z^{\mathrm{pole}}, 2.4) = 43.895720\,\mathrm{pb/GeV}, \tag{2.11}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q}(m_Z^{\mathrm{pole}}) = 329.861412\,\mathrm{pb}, \tag{2.12}$$

$$\int_{66}^{116} dQ \frac{\mathrm{d}\sigma}{\mathrm{d}Q} = 1278.04222(14)\,\mathrm{pb} \tag{2.13}$$

Leading order reference cross sections for 5 generations are as follows:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y}(m_Z^{\mathrm{pole}}, 0) = 66.231965\,\mathrm{pb/GeV}, \tag{2.14}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y}(m_Z^{\mathrm{pole}}, 2.4) = 53.392154\,\mathrm{pb/GeV}, \tag{2.15}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q}(m_Z^{\mathrm{pole}}) = 434.071353\,\mathrm{pb}, \tag{2.16}$$

$$\int_{66}^{116} dQ \frac{\mathrm{d}\sigma}{\mathrm{d}Q} = 1679.57115(15)\,\mathrm{pb} \tag{2.17}$$

3 Pure Resummation Benchmark

3.1 Resummation Setup

Order	Boundary cond. (FO singular)	Anomalous dimensions γ_i (noncusp)	Γ_{cusp}, β	FO matching (nonsingular)
LL	1	-	1-loop	-
NLL	1	1-loop	2-loop	-
NLL$'$ (+NLO$_0$)	α_s	1-loop	2-loop	α_s
NNLL (+NLO$_0$)	α_s	2-loop	3-loop	α_s
NNLL$'$ (+NNLO$_0$)	α_s^2	2-loop	3-loop	α_s^2
N^{3}LL (+NNLO$_0$)	α_s^2	3-loop	4-loop	α_s^2
N^{3}LL$'$ (+N^{3}LO$_0$)	α_s^3	3-loop	4-loop	α_s^3
N^{4}LL (+N^{3}LO$_0$)	α_s^3	4-loop	5-loop	α_s^3

The resummation orders are defined in the above table. The (+N^{n}LO$_0$) in the order refers to whether or not the matching to the full fixed $\mathcal{O}(\alpha_s^n)$ corrections in the last column is included.

One important goal is to investigate and quantify various sources of possible systematic differences between different formalisms. Examples are formally subleading logarithmic corrections that are treated intrinsically differently in different resummation approaches, different methods to turn off the resummation at large q_T and/or for matching to the fixed-order results, different treatments of flavor thresholds, different methods for estimating higher-order uncertainties, and so on.

To start with, we consider 3 levels of resummation:

1. Canonical resummation, i.e., resum as close as possible canonical logarithms of $\ln(q_T/Q)$ or $\ln(Qb_T/b_0)$ in b_T-space native to each formalism.
    - All scales that are naturally $\sim Q$ should be fixed to Q, e.g. fixed-order factorization and renormalization scales $\mu_F = \mu_R = Q$, hard resummation scales $\mu_H = Q$ (e.g. in SCET) or $Q_{\rm res} = Q$ (e.g. in CSS-based formalisms).
    - Low scales should be set to their natural values, e.g., b_0/b_T in b_T-space or the relevant k_T in Radish, etc.
    - No further replacement or modification of logarithms.
2. Nominal resummation with all resummation choices as native to each prediction.
    - Include turning off resummation at large q_T, e.g. $Q_{\rm res} = Q/2$, profile scales, modified logarithms $\ln(b_T) \to \ln(1 + b_T)$, etc.
3. Level 2 above plus matching to full fixed-order, corresponding to the final resummed predictions.
    - This level will only be considered later on in step 3 and beyond, so more detailed specifications will come later ...

Some additional specifications:

- PDF evolution: As provided by LHAPDF, i.e., take $f_i(\mu)$ at whatever scale μ it natively enters each prediction from LHAPDF.
- α_s evolution: Start from input $\alpha_s(91.1876\,\text{GeV}) = 0.118$, use native evolution implemented by each prediction as relevant at a given resummation order.
- Use strictly $n_f = 5$ massless flavors everywhere at all scales, including α_s running. (The only exception is the PDF, for which LHAPDF is treated as a black box.)
- Nonperturbative cutoff: Use implementation native to each prediction of a minimal nonperturbative cutoff to avoid the Landau pole (e.g. some form of b^* prescription or equivalent). But do not include additional nonperturbative form factors or similar. If possible/well-defined, use a cutoff of $1\,\text{GeV}$, e.g. for the b^* prescription this amounts to taking $b_0/b_{\rm max} = 1\,\text{GeV}$. This value allows us to later include a variation between e.g. $0.5 - 1.5\,\text{GeV}$ if needed.
- The numerical accuracy of the final predictions (at least for levels 1 and 2) should be at least 10^{-4} relative accuracy.

These are meant to start simple and remove any unnecessary or obvious differences. For now, these should be applied at both level 1 and level 2. For level 2 (and beyond), we can consider relaxing/varying some of these in a future step in order to study their impact (e.g. the n_f and PDF treatment).

3.2 Step 1

We first address pure resummation without any fixed-order matching (levels 1 and 2 above). In the first step, we also only consider central-value predictions.

1. Process: Z/γ^* production at $\sqrt{s} = 13$ TeV.
   - (a) Z/γ^* coupling only to first generation of quarks (u, $\bar{u}$, d, $\bar{d}$).
   - (b) Z/γ^* coupling to all five generation of quarks.
   - → If it it appears useful, we can also consider finer splits in subsequent steps.
2. Hard phase space: Evaluate cross section at fixed $Q = m_Z$. (Alternatively one can instead use a very narrow integration window if this minimises the work for certain predictions.)
   - (a) At fixed points $Y = 0$ and $Y = 2.4$
   - (b) Fully integrated over Y
   - → More points can be added in parallel for step 2.
3. Resummation: Include the following orders at level 1 and if feasible level 2:
   - (a) NLL (and optionally LL)
   - (b) one of NLL′ or NNLL
   - (c) if possible/available one of NNLL′ or N^{3}LL
   - → More orders and level-2 predictions can be added in parallel for step 2.
4. q_T binning: Evaluate the spectrum at fixed points in q_T for the q_T spectrum and if applicable at fixed points in b_0/b_T for the b_T-space spectrum. This avoids smearing out features or introducing possible binning artifacts, especially at low q_T, which will eventually be important for W/Z ratio. If not possible, either use a very narrow integration window or (least desirable) plain integrated bins.
   - (a) Linear: q_T or $b_0/b_T = \{1, 2, 3, ..., 39, 40, 45, ..., 95, 100\}$.
   - (b) Logarithmic: $\log_{10} q_T$ or $\log_{10}(b_0/b_T) = \{-1, -0.9, ..., 1.9, 2\}$.
     Note: Implement this by directly changing variables in the differential cross sections, i.e., as linearly spaced points for $\mathrm{d}\sigma/\mathrm{d}(\log_{10} q_T)$, as opposed to logarithmically spaced points for $\mathrm{d}\sigma/\mathrm{d}q_T$.
   - → These will be adjusted later for higher Q values.

3.3 Step 2

- Add missing orders as well as missing results for level 2.
- Add points in $Y = 0, 1.2, 2.4, 3.6$.
- Add points in $Q = 66, 116, 300, 1000\,\mathrm{GeV}$.

In addition, we should discuss and prioritize

- Adding results for $\sqrt{s} = 8$ TeV (for all the above phase-space points).

- Adding results for W.
- Adding uncertainties.

3.4 Step 3 and beyond

Will include matching to FO (level 3). Details will be specified later.